2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

On the presence of equilibrium points in PI control systems with send-on-delta sampling

Manuel Beschi, Antonio Visioli, Sebastián Dormido and José Sánchez

***Abstract*— This paper deals with the presence of equilibrium points and limit-cycles in a PI control system with send-on-delta sampling. In particular, sufficient conditions on the controller parameters for the existence of equilibrium points are given. These conditions can be usefully exploited for the tuning of the event-based PI controller parameters, thus making the overall design easier. Simulation results are provided as illustrative examples.**

I. INTRODUCTION

It is well known that in some processes a small steady-state control error of the process output around the set-point does not constitute a hard design constraint but, however, the reduction of the information exchanged between the agents that take part in the control loop (sensors, controllers, actuators) is one of the tightest requirements. Indeed, the reduction of the information flow is a relevant issue especially when there are constraints on the communication rate, for instance when data are exchanged in a distributed control system by wired or wireless networks [9], [7], [10]. In these situations, cutting down the traffic load is a key point because the more traffic, the higher possibility of lost data and stochastic time delays. This prevents the occurrence of large latencies and delay jitter and the CPU utilization is also reduced. In particular, a framework where the reduction of the exchanged traffic is an essential issue is in wireless networks and specially using battery-powered or limited computational power devices [21], [1]. Therefore, the higher information flow reduction, the higher decrease of computing operations and transmissions, and thus the longest lifetime of batteries.

In this context, one of the most convenient strategies is the use of event-based sampling and control approaches. Indeed, during last years event-based sampling and control techniques have been addressed by a large number of researchers (see, for example, [3], [19], [6]) also in the context of Proportional-Integral-Derivative (PID) controllers [2], [11], [5], [13]. Between the different event-based sampling strategies, one of the most common is the so called send-on-delta sampling (also known as deadband sampling) where feedback control actions are computed when the process output is outside a certain detection band located around the set-point value [16], [12]. Once the process is inside the detection band, new control actions are not produced until the process leaves the region as a consequence of disturbances or of a change of the set-point value. The controller employed is usually a PID controller with variable sampling period [15], [17], [18].

Actually, (time-based) PID controllers are the most employed controllers in industry owing to their advantageous cost-benefit ratio. In fact, they are capable to provide a satisfactory performance for many processes and the settings of their parameters are relatively easy also because of the large number of tuning rules that are available [8]. However, in event-based control the events occur asynchronously and therefore the tuning of the PID controller parameters is in general more challenging, as the timing of the events influences the system performance and limit cycles may arise [14] (note that the presence of limit cycles is a typical problem in general event-based control systems [4]). Further, in addition to the PID gains, the threshold values employed in the control algorithm (see Section 2) have also to be tuned. Indeed, the tuning of a PID controller with deadband sampling has not been explicitly addressed in the literature until now, at least to the authors' knowledge.

In this paper, sufficient conditions on the controller parameters for the existence of equilibrium points are given. On the other hand, starting from them, conditions for the existence of a limit cycle can be easily derived. The cases of event-based P, I and a PI controllers are considered (the derivative action is not considered because its implementation is very critical with a variable, and possibly long, sampling period). It is believed that these conditions can be usefully employed for the tuning of the overall controller. The paper is organized as follows. In Section 2 the problem is formulated. Sufficient conditions for the presence of equilibrium points (or, from another point of view, the conditions for the presence of a limit cycle) are given, with illustrative examples, in Section 3, 4 and 5 for the P, I, and PI case respectively. Conclusions are explained in Section 6.

II. PROBLEM FORMULATION

We consider the control scheme shown in Figure 1 where the process is described by the following general transfer function

$$G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{a_n s^n + \cdots + a_1 s + a_0} e^{-Ls}. \tag{1}$$

where L is the dead time, $b_0 \neq 0$ and $m < n$. We assume that all the poles belong to the open left half plane with the exception of a possible pole at the origin, namely, we can have a self-regulating or a non self-regulating (integral) process.

The control action $u(t)$ is generated by an event-based PI controller where the proportional and integral actions are enabled once the process output is outside a predefined band around the set-point value r. In particular, a send-on-delta sampling strategy is applied. The proportional action

$u^P(t) = K_p e(t)$ (where K_p is the proportional gain) is recalculated every time that the aboslute error is greater than Δ_p and the absolute value of the difference between the current error $e(t) = r(t) - y(t)$ and the error in the last crossing $e(t_l^P)$ is greater than Δ_p, that is:

$$|e(t) - e(t_l^P)| \geq \Delta_p. \tag{2}$$

The integral action $u^I(t) = K_i IE(t)$ (where K_i is the integral gain) is calculated every time that

$$|IE(t) - IE(t_l^I)| \geq \Delta_i \tag{3}$$

where the integral of the error $IE(t)$ is defined as

$$IE(t) = \int e(t) f(e(t)) dt, \qquad f(e) = \begin{cases} 1 & \text{if } |e| > \varepsilon \\ 0 & \text{if } |e| \leq \varepsilon \end{cases} \tag{4}$$

where ε is the desired maximum error (deadband).

It is worth stressing that, from a practical point of view, the considered event-based control approach is implemented by sampling the process variable and by evaluating the event-based conditions as fast as possible. This is called the *compound approach* or the *fast sampling approach* where the asynchronous events are presynchronized by using a fast periodic sampling [7].

Thus, by denoting as T the sampling period of the sensor, the following overall control algorithm can be outlined.

Sensor unit

1) if $|r - y| > \Delta_p$ then set $e_p(t) = r - y$;
   else set $e_p(t) = 0$;
2) if $|e_p(t) - e_p(t_l^P)| > \Delta_p$ then generate a P event by sending $e_p(t)$ to the control unit and set $e_p(t_l^P) = e_p(t)$;
3) if $|r - y| > \varepsilon$ then set $e_i(t) = r - y$;
   else set $e_i(t) = 0$;
4) set $IE = IE + T e_i(t)$;
5) if $|IE(t) - IE(t_l^I)| > \Delta_i$ then generate an I event by sending $IE(t)$ to the control unit and set $IE(t_l^I) = IE(t)$.

Control unit

1) if a P event is received, then set $u^P(t) = K_p e_p(t)$;
2) if a I event is received, then set $u^I(t) = K_i IE(t)$;
3) set $u(t) = u^P(t) + u^I(t)$.

Note that if $K_i = 0$ then a proportional controller results while if $K_p = 0$ then an integral controller results. It appears that, with respect to a standard time-based PI controller, the proposed algorithm has more parameters to tune. Indeed, in addition to the proportional and integral gains the threshold values Δ_p, Δ_i, and ε have to be suitably selected.

The aim of the following sections is to define conditions on the controller parameters which allow the system to have at least an equilibrium point for every value of the set-point r and of the constant disturbance amplitude D. In fact, equilibrium points which depend on a limited set of set-point values r and disturbance amplitudes D have not practical relevance, because in general their values are not known *a priori*. Note that no considerations are done with respect to the region of the attraction of the equilibrium points. Thus, the presence of an equilibrium point does not imply that it will be attained by the control system.

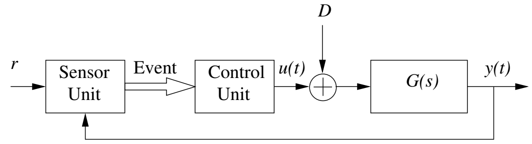

Fig. 1. Scheme of the event-based control system.

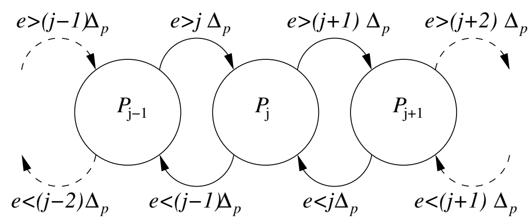

Fig. 2. P event unit automaton.

III. EVENT-BASED PROPORTIONAL CONTROLLER

In a proportional event-based control strategy, the behavior of the sensor unit can be described as an automaton. In fact, it is possible to define a state P_j, where $e(t) \in [(j-1)\Delta_p, (j+1)\Delta_p]$; when the error reaches the threshold $(j+1)\Delta_p$ the automaton jumps to the state P_{j+1} and the sensor unit sends to the controller unit the value $(j+1)\Delta_p$; as soon as the error is less than $(j-1)\Delta_p$ the automaton jumps to the state P_{j-1} and the sensor sends the value $(j-1)\Delta_p$. Figure 2 shows the automaton representation. In a generic state P_j the control system is an open-loop system with a constant control variable u_j^P, therefore it is possible to define a steady-state output $y_{ss,j} = G(0)\left(u_j^P + D\right)$. This output is an equilibrium point if $e_{ss,j} \in [(j-1)\Delta_p, (j+1)\Delta_p]$. In fact, as Figure 3 shows, if the latter condition is not satisfied, the automaton would jump in another state.

It is important to notice that $e_{ss,j} \in [(j-1)\Delta_p, (j+1)\Delta_p]$ is a necessary condition only of the existence of the equilibrium point, because it does not give information about the region of attraction of the equilibrium point. In fact, the system can reach the equilibrium point with a finite number of transitions, or can admit a limit cycle which involves two or more states, as a consequence of a periodic sequence of events that are generated because of the characteristics of the transient response of the system. This aspect is exemplified in Figure 4, where the equilibrium point exists but the system does not attain it. The necessary condition can be used to find sufficient conditions of the values of the controller parameters for which there is at least one possible equilibrium point, indifferently by the values of D. By taking into account that the control action, with an event-based proportional controller, can assume the following values

$$u_{ss,j}^P = K_p e_p(t) = K_p j \Delta_p, \ j \in \mathbb{Z} \tag{5}$$

the following propositions can be stated.

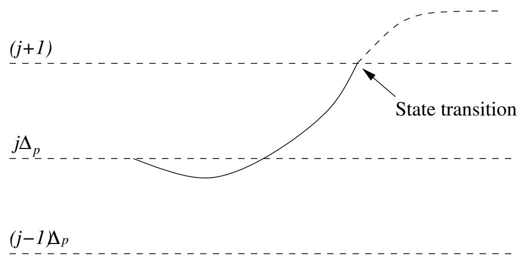

Fig. 3. Case without an equilibrium point.

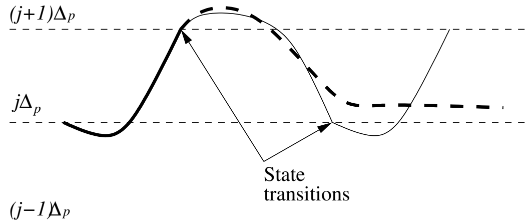

Fig. 4. Case with an equilibrium point which is not attained. Solid thick line: evolution of the error in the state P_j. Dashed thick line: evolution of the error if the first event would not occur. Solid thin line: evolution of the error after the first event.

Proposition 1: If the process (1) has a pole at the origin, there are values of D for which there does not exist an equilibrium point.
Proof. In order for the process output to be constant the steady-state process input should be null. As it is $u^P_{ss,j} + D = 0$ if D is not exactly a multiple of $K_p\Delta_p$ there are no equilibrium points. □

Proposition 2: If the process (1) is asymptotically stable, then a sufficient condition for the presence of an equilibrium point for all the values of r and D is $K_p \le 1/K$ where $K = G(0)$.
Proof. If the process is self-regulating, then the steady-state outputs $y_{ss,j}$ are

$$y_{ss,j} = K(jK_p\Delta_p + D),\ j \in \mathbb{Z}.$$

Thus, $y_{ss,j}$ can be an equilibrium point if:

$$(j-1)\Delta_p \le e_{ss,j} = r - y_{ss,j} \le (j+1)\Delta_p,$$

that is,

$$\begin{array}{l} r - jKK_p\Delta_p - KD \ge (j-1)\Delta_p, \\ r - jKK_p\Delta_p - KD \le (j+1)\Delta_p. \end{array} \tag{6}$$

These conditions can be rewritten as

$$\begin{array}{l} j \le \frac{r-KD}{(1+KK_p)\Delta_p} + \frac{\Delta_p}{(1+KK_p)\Delta_p}, \\ j \ge \frac{r-KD}{(1+KK_p)\Delta_p} - \frac{\Delta_p}{(1+KK_p)\Delta_p}. \end{array} \tag{7}$$

The term $\frac{r-KD}{1+KK_p}$ corresponds to the steady-state error that would be obtained by using a proportional controller with periodic sampling. The quantity $r-KD$ can be expressed as $r-KD = j_f(1+KK_p)\Delta_p+d$ with $j_f = \lfloor\frac{r-KD}{(1+KK_p)\Delta_p}\rfloor \in \mathbb{Z}$ and $d = r - KD - j_f(1+KK_p)\Delta_p \in [0,(1+KK_p)\Delta_p]$, where d is, roughly speaking, the "unquantizable" part of $r-KD$. In this way, the conditions (7) can be rewritten as:

$$j \le j_f + \frac{d+\Delta_p}{(1+KK_p)\Delta_p} \tag{8}$$

$$j \ge j_f + \frac{d-\Delta_p}{(1+KK_p)\Delta_p}. \tag{9}$$

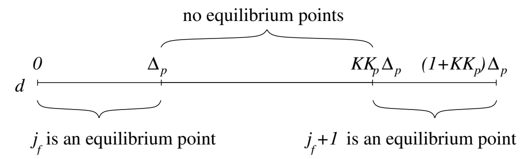

Fig. 5. Relationship between equilibriums and d

It is important to notice that, with the first condition, it is possible to exclude all states P_j with $j > j_f + 1$. In fact, considering a state P_{j_f+h}, with $h \ge 2$ the condition (8) becomes:

$$j_f + h \le j_f + \frac{d+\Delta_p}{(1+KK_p)\Delta_p}$$

and therefore

$$h(1+KK_p)\Delta_p - \Delta_p < (1+KK_p)\Delta_p \le d,$$

which is absurd because $d \in [0,(1+KK_p)\Delta_p]$.
In the same way the state P_j with $j < j_f$ can be excluded by applying the second condition. In fact, considering the state P_{j_f-h}, with $h \ge 1$ the condition (9) becomes:

$$j_f - h \ge j_f + \frac{d-\Delta_p}{(1+KK_p)\Delta_p}$$

or equally:

$$d \le -h(1+KK_p)\Delta_p + \Delta_p,$$

which is absurd because $d \in [0,(1+KK_p)\Delta_p]$.
From the previous considerations, there are only two possible candidates to be equilibrium states, namely P_{j_f} and P_{j_f+1}. Considering the first one, the necessary conditions (8)-(9) can be written, respectively, as:

$$j_f \le j_f + \frac{d+\Delta_p}{(1+KK_p)\Delta_p}, \tag{10}$$

which is always true, and

$$j_f \ge j_f + \frac{d-\Delta_p}{(1+KK_p)\Delta_p}, \tag{11}$$

which is true if $d \le \Delta_p$.
Considering the state P_{j_f+1}, the necessary conditions are:

$$j_f + 1 \le j_f + \frac{d+\Delta_p}{(1+KK_p)\Delta_p} \tag{12}$$

which is verified if $d \ge KK_p\Delta_p$, and

$$j_f + 1 \ge j_f + \frac{d-\Delta_p}{(1+KK_p)\Delta_p} \tag{13}$$

which is always true. As already stressed, we need to find values of K_p and Δ_p which allow the controlled system to have at least one equilibrium point indifferently to the value of d. By looking the previous equations, it is easy to note that: if $d \le \Delta_p$, then P_{j_f} is a equilibrium state; if $d \ge KK_p\Delta_p$, then P_{j_f+1} is an equilibrium state; if $\Delta_p < d < KK_p\Delta_p$, then there is the absence of equilibrium points. To avoid the occurrence of the third condition, it is necessary to chose $KK_p\Delta_p < \Delta_p$, therefore $KK_p \le 1$. The conditions are summarized in Figure 5. □

Remark 1. In this section we consider that the send-on-delta sampled error assumes only values multiple of Δ_p (see for instance (5)). This assumption seems contradictory with the definition (2), however it is clear that applying (2) if the error

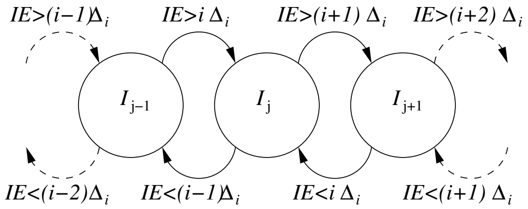

Fig. 6. I event unit automaton.

is a continuous signal, then the send-on-delta sampled error assumes values which are the sum of a multiple of Δ_p and an initial constant term which can be neglected by considering it as a part of the disturbance D. Similar considerations are applied in Sections IV-V.

IV. Event-based integral controller

By following a similar reasoning applied to the P case, the sensor unit of an event-based integral controller has a behavior which can be described as an automaton. In this automaton the state I_i is assumed when $IE = [i-1, i+1]\Delta_i$ with $\Delta_i > 0$; the automaton jumps to the "upper" state if $IE \geq (i+1)\Delta_i$ and to the "downer" state if $IE \leq (i-1)\Delta_i$. When a transition occurs, the sensor sends the new value to the controller unit. Figure 6 shows the automaton representation. Also in this case, when the system remains in a state, the control system is an open-loop system with a constant input u_i^I, therefore it is possible to define a steady-state output $y_{ss,i} = G(0)\left(u_i^I + D\right)$. This is an equilibrium point if $e_{ss,i} \in [-\varepsilon, \varepsilon]$. This necessary condition is easy to explain by noting that if it is false then IE changes its value continuously. As for the P controller, it can be used to find the values of the controller parameters for which there is at least one equilibrium point. By taking into account that the control action of an event-based integral controller can assume the following values:

$$u_{ss,i}^I = K_i IE = K_i i \Delta_i, \; i \in \mathbb{Z}$$

the following propositions can be stated.

Proposition 3: If the process (1) as a pole at the origin, there are values of D for which there does not exists an equilibrium point.

Proof. If the process (1) is non self-regulating, then the steady-state control variable should be null, that is $u_{ss,i}^I + D = 0$. Thus, if D is not exactly a multiple of $K_i\Delta_i$ there are no equilibrium points. □

Proposition 4: If the process (1) is asymptotically stable, then a sufficient condition for the presence of an equilibrium point for all the values of r and D is $K_i \leq (2\varepsilon)/(K\Delta_i)$ where $K = G(0)$.

Proof. If the process is self-regulating, the steady-state outputs $y_{ss,i}$ are:

$$y_{ss,i} = K(iK_i\Delta_i + D) \text{ with } i \in \mathbb{Z}$$

Thus $y_{ss,i}$ is an equilibrium point if

$$-\varepsilon \leq e_{ss,j} = r - y_{ss,i} \leq \varepsilon$$

or equivalently

$$\begin{array}{c} -iKK_i\Delta_i + r - KD \geq -\varepsilon \\ -iKK_i\Delta_i + r - KD \leq \varepsilon \end{array}$$

These conditions can be rewritten as

$$\begin{array}{c} i \leq \frac{r-KD}{KK_i\Delta_i} + \frac{\varepsilon}{KK_i\Delta_i} \\ i \geq \frac{r-KD}{KK_i\Delta_i} - \frac{\varepsilon}{KK_i\Delta_i} \end{array}$$

Redefining the product $KK_i\Delta_i$ as $\alpha\varepsilon$, the term $r-KD$ can be expressed as $r-KD = i_f\alpha\varepsilon + d$ with $i_f = \lfloor \frac{r-KD}{\alpha\varepsilon} \rfloor \in \mathbb{Z}$ and $d = r - KD - i_f\alpha\varepsilon \in [0, \alpha\varepsilon]$. In this way, the conditions can be rewritten as:

$$\begin{array}{c} i \leq \frac{d}{\alpha\varepsilon} + \frac{1}{\alpha} + i_f \\ i \geq \frac{d}{\alpha\varepsilon} - \frac{1}{\alpha} + i_f \end{array}$$

or:

$$d \geq (\alpha(i - i_f) - 1)\,\varepsilon \tag{14}$$

$$d \leq (\alpha(i - i_f) + 1)\,\varepsilon \tag{15}$$

Condition (14) is certainly true if $(i - i_f) \leq 1/\alpha$, and it is certainly false if $(i - i_f) \geq (1 + 1/\alpha)$. Conversely, condition (15) is certainly true if $(i - i_f) \geq (1 - 1/\alpha)$, and it is certainly false if $(i - i_f) \leq -1/\alpha$. Also in this controller, it is important to find values of the parameters which allow the controlled system to have at least a possible equilibrium point independently from the value of d. The previous conditions are verified together if $1/\alpha \geq (1 - 1/\alpha)$ or, equivalently, if $\alpha \leq 2$, that is $K_i \leq (2\varepsilon)/(K\Delta_i)$. If this condition is not verified an equilibrium point can exist for particular values of d (that is, of D). □

It is worth noting that the number of equilibrium points increases as α decreases. Some illustrative cases are outlined.

- Choosing $\alpha = 2$, if $d < \varepsilon$, then I_{i_f} is an equilibrium state; if $d > \varepsilon$, then I_{i_f+1} is an equilibrium state; if $d = \varepsilon$, then both I_{i_f} and I_{i_f+1} are equilibrium states.
- Choosing $\alpha = 1$, I_{i_f} and I_{i_f+1} are equilibrium states for every values of d.
- Choosing $\alpha = 1/2$, I_{i_f}, I_{i_f+1}, I_{i_f+2} and I_{i_f-1} are equilibrium states for every values of d.

V. Event-based PI controller

If the chosen control strategy is an event-based PI controller, it is possible to define an automaton where a generic state $S_{i,j}$ is the combination between a state P_j on the proportional part and a state I_i on the integral part. Figure 7 shows the automaton representation. In a generic state $S_{i,j}$ the control system is an open-loop system with a constant input $u_{i,j}$, therefore is possible to define a steady-state output $y_{ss,i,j} = G(0)\,(u_{i,j} + D)$. This output corresponds to an equilibrium point if $e_{ss,i,j} \in [(j-1)\Delta_p, (j+1)\Delta_p]$ and $e_{ss,i,j} \in [-\varepsilon, \varepsilon]$.

Note that the value of Δ_p should be greater than ε, because when the error is less than the maximum desirable error, namely $e \in [-\varepsilon, \varepsilon]$, the new value of the control variable must not be computed.

By taking into account that for the event-based PI control strategy the control action can assume the following values

$$u_{ss,i,j} = iK_i\Delta_i + jK_p\Delta_p, \; i, j \in \mathbb{Z}$$

we can state the following propositions.

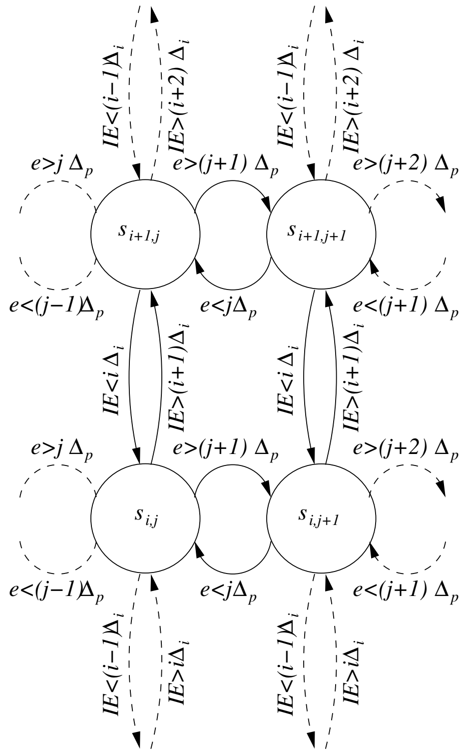

Fig. 7. PI event unit automaton.

Proposition 5: If the process (1) as a pole at the origin, there are values of D for which there does not exists an equilibrium point.
Proof. If the process is non self-regulating, the steady-state process input should be null, that is, $u_{ss,i,j} + D = 0$. Thus, if D is not exactly a multiple of a combination of $K_i\Delta_i$ and $K_p\Delta_p$ there are not equilibrium points and the system certainly has a limit cycle. □

Proposition 6: If the process (1) is asymptotically stable, then a sufficient condition for the presence of an equilibrium point for all the values of r and D is $K_i \leq (2\varepsilon)/(K\Delta_i)$.
Proof. If the process is asymptotically stable, the steady-state outputs $y_{ss,i,j}$ are

$$y_{ss,i,j} = K(iK_i\Delta_i + jK_p\Delta_p + D) \text{ with } i, j \in \mathbb{Z}$$

These outputs are equilibrium points if

$$\begin{aligned} -iKK_i\Delta_i - jKK_p\Delta_p + r - KD &\geq -\varepsilon \\ -iKK_i\Delta_i - jKK_p\Delta_p + r - KD &\leq \varepsilon \\ -iKK_i\Delta_i - jKK_p\Delta_p + r - KD &\geq (j-1)\Delta_p \\ -iKK_i\Delta_i - jKK_p\Delta_p + r - KD &\leq (j+1)\Delta_p \end{aligned} \tag{16}$$

Hence, by considering $\Delta_p > \varepsilon$, there are only three values of j which satisfy all the conditions: -1, 0 and 1. When $j = 0$, the conditions on the steady-state error are equal to the integral case, therefore, recalling that $KK_i\Delta_i = \alpha\varepsilon$, there is at least an equilibrium point if $\alpha \leq 2$.
When $j = -1$ the steady-state errors are

$$e_{ss,i,-1} = -iKK_i\Delta_i + K(K_p\Delta_P - D) + r, \ i \in \mathbb{Z}$$

The term $K(K_p\Delta_p - D) + r$ can be expressed as

$$K(K_p\Delta_P - D) + r = i_{f1}\alpha\varepsilon + d$$

TABLE I
SUFFICIENT CONDITIONS FOR EVENT BASED PI

$\alpha > 2$	Equilibriums points exist only for some values of D, otherwise there is a limit cycle.
$1 < \alpha \leq 2$	There are equilibrium points with $j = 0$.
$0 < \alpha \leq 1$	There are equilibrium points with $j = 0, 1, -1$.

where $i_{f1} = \lfloor \frac{K(K_p\Delta_p - D) + r}{\alpha\varepsilon} \rfloor \in \mathbb{Z}$ and $d = K(K_p\Delta_p - D) + d - i_{f1}\alpha\varepsilon \in [0, \alpha\varepsilon]$. In this way, the conditions (16) can be rewritten as

$$-(i - i_{f1})\alpha\varepsilon + d \geq -min(\varepsilon, 2\Delta_p) = -\varepsilon \tag{17}$$

$$-(i - i_{f1})\alpha\varepsilon + d \leq min(\varepsilon, 0) = 0 \tag{18}$$

Conditions (17) and (18) can be expressed as

$$d \geq (i - i_{f1})\alpha\varepsilon - \varepsilon \tag{19}$$

$$d \leq (i - i_{f1})\alpha\varepsilon \tag{20}$$

Condition (19) is certainly true if $(i - i_{f2}) \leq 1/\alpha$ and is certainly false if $(i - i_{f2}) \geq (1 + 1/\alpha)$, while condition (20) is certainly true if $(i - i_{f2}) \geq 1$ and is certainly false if $(i - i_{f2}) \leq 0$. Previous equations are certainly both verified if $\frac{1}{\alpha} > 1$, or equally $\alpha < 1$.
When $j = 1$ the steady-state outputs are

$$y_{ss,i,-1} = -iKK_i\Delta_i + K(-K_p\Delta_P - D) + r, \ i \in \mathbb{Z}$$

The term $r - K(K_p\Delta_p + D)$ can be expressed as:

$$r - K(K_p\Delta_P + D) = i_{f2}\alpha\varepsilon + d$$

with $i_{f2} = \lfloor \frac{r - K(K_p\Delta_p + D)}{\alpha\varepsilon} \rfloor \in \mathbb{Z}$ and $d = r - K(K_p\Delta_p + D) - i_{f2}\alpha\varepsilon \in [0, \alpha\varepsilon]$. In this way, the four conditions (16) can be rewritten as:

$$-(i - i_{f2})\alpha\varepsilon + d \geq -\varepsilon \tag{21}$$

$$-(i - i_{f2})\alpha\varepsilon + d \leq \varepsilon \tag{22}$$

$$-(i - i_{f2})\alpha\varepsilon + d \geq 0 \tag{23}$$

$$-(i - i_{f2})\alpha\varepsilon + d \leq 2\Delta_p \tag{24}$$

Conditions (21) and (24) are always verified, while conditions (22)-(23) can be expressed as:

$$d \geq (i - i_{f1})\alpha\varepsilon \tag{25}$$

$$d \leq \varepsilon + (i - i_{f1})\alpha\varepsilon \tag{26}$$

Condition (25) is certainly true if $(i - i_f) \leq 0$ and certainly false if $(i - i_{f2}) \geq (1 - 1/\alpha)$ while condition (26) is certainly true if $(i - i_{f2}) \geq (1 - 1/\alpha)$ and certainly false if $(i - i_{f2}) < -1/\alpha$. Both conditions are therefore verified if $(1 - 1/\alpha) > 0$, i.e., $\alpha < 1$. Results are summarized in Table 1. □

Remark 2. Note that in an event-based PI controller, the steady-state sufficient conditions for the existence of at least an equilibrium point concern only the product $KK_i\Delta_i = \alpha\varepsilon$ and there are no conditions on the proportional parameters K_p and Δ_p (provided $\Delta_p > \varepsilon$). Note also that the conditions on the integral part are the same as the event-based integral controller case.

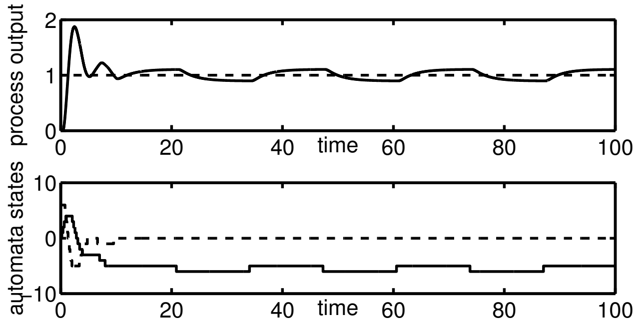

Fig. 8. Simulation with $\alpha = 2.1$. Dashed line: evolution of the automaton P_j. Solid line: evolution of the automaton I_i

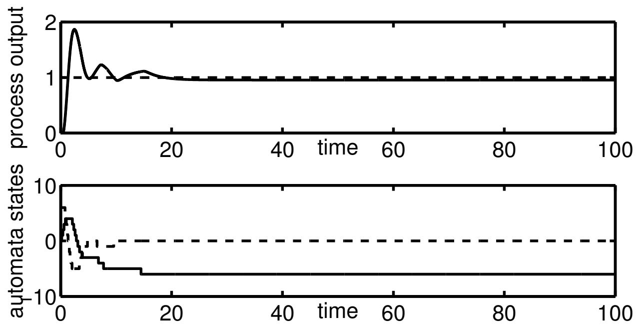

Fig. 9. Simulation with $\alpha = 2.0$. Dashed line: evolution of the automaton P_j. Solid line: evolution of the automaton I_i

VI. Illustrative example

In this section, the unit step response of a second-order-plus-dead-time process (with $K = 1$) controlled by an event-based P controller is analyzed. In particular, two cases are presented, the first where $\alpha = 2.1$ (therefore the sufficient condition is not verified), and the second where $\alpha = 2.0$ (therefore the condition is verified). Both the cases have the same $K_i = 1$. The other parameters are set as $\varepsilon = 0.1$, $\Delta_p = 0.15$, $K_p = 3$ and $D = 2.155$. The considered process is

$$G(s) = \frac{1}{s^2+3s+1} e^{-0.4s} \tag{27}$$

The process output and the evolution of the automaton $S_{i,j}$ are presented for the two cases in Figures 8 and 9.

Remark 3. It is worth stressing that from the above analysis it can be straightforwardly deduced that, from a practical point of view, it is very likely that a limit cycle occurs if $\alpha > 2$ (this does not happen just for specific values of D, see Table 1).

VII. Conclusions

In this work conditions on the existence of equilibrium points and limit cycles are investigated. In particular, conditions on the parameters of P, I and PI controllers based on send-on delta sampling are presented. These conditions allow the controlled system to have at least one equilibrium point indifferently from the value of the constant load disturbance. Another important result that has been presented is the presence of limit cycle in processes with a pole at the origin of the complex plane. This fact is caused by the "quantized" nature of the controller, which can not exactly compensate the constant load disturbance. This is relevant because integral (non self-regulating) process are frequently encountered in the process industry and their control has been widely investigated [20].

References

[1] G. Anastasi, M. Conti, M. Di Francesco, and A. Passarella. Energy conservation in wireless sensor networks: A survey. *Ad Hoc Networks*, 7(3):537–568, 2008.

[2] K.E. Årzèn. A simple event-based PID controller. In *Proceedings of 14th World Congress of IFAC*, Bejining, China, 1999.

[3] K.J. Åström. Event based control. *In Astolfi, A. and Marconi, L. (eds.), Analysis and Design of Nonlinear Control Systems: In Honor of Alberto Isidori*, 2008.

[4] A. Cervin and K.J. Johansson. On limit cycles in event-based control systems. In *Proceedings of 46th IEEE International Conference on Decision and Control*, pages 3190–3195, New Orleans, LA, USA, 2007.

[5] S. Durand and N. Marchand. Further results on event-based pid controller. In *Proceedings of 10th European Control Conference (ECC'09)*, Budapest, Hungary, 2009.

[6] W.P.M.H. Heemels, J.H. Sandee, and P.P.J. Van Den Bosch. Analysis of event-driven controllers for linear systems. *International Journal of Control*, 81(4):571–590, 2008.

[7] M. Miskowicz. Send-on-delta: An event-based data reporting strategy. *Sensors*, 6:49–63, 2006.

[8] A. O'Dwyer. *Handbook of PI and PID Tuning Rules*. Imperial College Press, London, UK, 2006.

[9] P. Otanez, J. Moyne, and D. Tilbury. Using deadbands to reduce communication in networked control systems. In *Proceedings of American Control Conference*, Anchorage, USA, 2002.

[10] V. Pawlowskyy, J.L. Guzmán, F. Rodríguez, M. Berenguel, J. Sanchez, and S. Dormido. Event-based control and wireless sensor network for greenhouse diurnal temperature control: A simulated case study. In *Proc. of 13th IEEE International Conference on Emerging Technologies and Factory Automation (EFTA-2008)*, Hamburg, Germany, 2008.

[11] M. Rabi and K.H. Johansson. Event-triggered strategies for industrial control over wireless networks. In *Proceedings of 4th Annual International Conference on Wireless Internet*, Maui, Hawaii, USA, 2008.

[12] J. Sánchez, M.A. Guarnes, S. Dormido, and A. Visioli. Comparative study of event-based control strategies: An experimental approach on a simple tank. In *Proceedings European Control Conference*, Budapest, Hungary, 2009.

[13] J. Sanchez, A. Visioli, and S. Dormido. A two-degree-of-freedom PI controller based on events. *Journal of Process Control (in press)*, 2011.

[14] V. Vasyutynskyy. Limit cycles in PI control loops with absolute deadband sampling. In *Proceedings of 18th IEEE International Conference on Microwave & Telecommunication Technology (CRIMICO'2008)*, Sevastopol, Ukraine, 2008.

[15] V. Vasyutynskyy and K. Kabitzsh. Implementation of PID controller with send-on-delta sampling. In *International Conference Control*, 2006.

[16] V. Vasyutynskyy and K. Kabitzsh. Towards comparison of deadband sampling types. In *IEEE International Symposium on Industrial Electronics (ISIE 2007)*, June 2007.

[17] V. Vasyutynskyy and K. Kabitzsh. First order observers in event-based PID controls. In *IEEE Conference on Emerging Technologies & Factory Automation (ETFA 2009)*, September 2009.

[18] V. Vasyutynskyy and K. Kabitzsh. A comparative study of pid control algorithms adapted to send-on-delta sampling. In *Proceedings of IEEE International Symposium on Industrial Electronics (ISIE)*, Bari, Italy, 2010.

[19] V. Vasyutynskyy and K. Kabitzsh. Event-based control: Overview and generic model. In *Proccedings of 8th IEEE International Workshop on Factory Communication Systems (WFCS)*, May 2010.

[20] A. Visioli and Q.C. Zhong. *Control of Integral Processes with Dead Time*. Springer, London, UK, 2010.

[21] M. Yarvis and M. Zorzi. Special issue on energy efficient design in wireless ad hoc and sensor networks. *Ad Hoc Networks*, 6(8):1183–1184, 2008.