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Tracking Control of Multiple Nonlinear Systems via Information Interchange

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Abstract— **This paper considers tracking control of multiple nonlinear systems with a desired trajectory which is not available to each system. With the aid of information interchange between systems, distributed robust/adaptive control laws are proposed such that the state of each system asymptotically converges to the desired trajectory. Simulation results show the effectiveness of the proposed control laws.**

I. INTRODUCTION

Seeking consensus has been an extensive research area in recent years due to its wide applications in computer science, management science, social science, controls, robotics, etc. For multiple systems, the consensus problem is generally defined as designing distributed control laws such that all systems converge to one common state vector that could be either stationary or time-varying.

In literature, there are many research papers on designing consensus algorithms such that the states of multiple systems converge to a constant vector. In [1], alignment of multiple discrete-time agents was investigated by Vicsek et al.. A distributed control law was proposed for each agent based on the average of its own heading and the headings of its neighbors. The simulation results in [1] demonstrated that the proposed control law can make all agents move in the same direction even if the nearest neighbors change with time. In [2], the authors provided a convergence proof for Vicsek's decentralized control model with the aid of graph theory and matrix theory. In [3], consensus problems for networks consisting of multiple continuous-time systems were considered. Consensus algorithms were proposed for fixed and switching communication cases. In [4], the results obtained in [2, 3] were extended and improved conditions were presented for time varying communication graph. In [5], the stability of multiple agents with nonlinear models in discrete time and time-dependent communication links was considered. Necessary and/or sufficient conditions for the convergence of the state of each individual agent to a consensus vector were proposed with the aid of graph theory and convexity techniques. In [6], the results for discrete-time systems reported in [5] were extended to continuous time systems.

In addition to the consensus problem where the states of multiple systems converge to a static state, there are also many papers that proposed consensus algorithms where multiple systems reach agreement on a common time-varying state. In [7, 8], tracking control of multiple first-order linear systems with an active leader was discussed. Distributed dynamic controllers were proposed with the aid of distributed estimators. The proposed control laws can make the tracking error between the state of a system and the state of an active leader as small as possible by choosing a large control parameter. In [9, 10], distributed tracking via a variable structure approach was considered for multiple first-order and second-order systems. Distributed discontinuous controllers were proposed such that the state of each system converges to a desired trajectory within finite time under the condition that the desired trajectory is available to a portion of the group of systems. In [11], adaptive distributed control laws were proposed for multiagent nonlinear systems with uncertainty. However, the derivatives of the neighbors' states are required. In [12], a passivity framework was proposed to steer the differences between output variables of a group of members to a prescribed compact set. The proposed framework can be applied to solve consensus problems of multiple systems. In [13], adaptive motion coordination of multiple systems was studied. Distributed adaptive control laws were proposed such that a reference velocity is tracked by each system with the aid of communications between systems. Flocking of multiple systems may be considered as a consensus problem. In [14], flocking of multiple second-order agents was solved with the aid of potential functions under the assumption that the desired trajectory is available to each agent. In [15], flocking of multiple systems was discussed for fixed and switching communication cases such that the velocity of each agent reaches an agreement. In [16, 17], flocking algorithms of multiple second-order linear systems were proposed under the assumptions that the information of a virtual leader is available to a portion of systems with appropriate assumptions on the virtual leader. For surveys on consensus problems, readers may refer to [18, 19].

In this paper, we consider the consensus problem of multiple high-order nonlinear systems with a reference system whose state is available to a portion of the systems. With the aid of information interchange between neighbors, several types of distributed control laws are proposed. The contributions of this paper are are two fold: 1) the tracking control problem of multiple systems with limited information of a desired trajectory is solved with the aid of neighbors' state information; and 2) systematic controller design methods are proposed with the aid of Lyapunov techniques. The proposed approach can be extended to deal with the consensus problem of multiple nonlinear systems with uncertainty.

The remaining sections are organized as follows. In Section II, the control problem discussed in this paper is defined. In Section III, distributed control laws are proposed. In

Section IV, simulation results are presented. The last section concludes this paper.

II. Problem Statement

Consider a group of m identical nonlinear systems. The j-th system has the following form

$$\dot{x}_{ij} = f_i(\bar{x}_{ij}) + x_{i+1,j}, \text{ for } i = 1, \ldots, n-1 \quad (1)$$
$$\dot{x}_{nj} = f_n(\bar{x}_{nj}) + u_j \quad (2)$$

where $\bar{x}_{ij} = [x_{1j}, \ldots, x_{ij}]^\top$. The function f_i is smooth and is assumed to be known. $u_j \in \Re$ is the control input of system j.

The communications between the m systems are assumed to be bi-directional and are described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V}$ is a set of the indices of the systems and $\mathcal{E}$ is a set of edges that describe the communications between the systems. If the state of system i is available to system j, system i is said to be a neighbor of system j. The index numbers of all neighbors of system j form a neighbor set and is denoted by $\mathcal{N}_j$. However, it does not see itself in the list of its neighbors, i.e., $i \notin \mathcal{N}_i$. A graph is called connected if for any two nodes there is a set of edges that connect the two nodes.

Given a reference system $x_{*0} = [x_{10}, \ldots, x_{n0}]^\top$ that satisfies

$$\dot{x}_{i0} = f_i(\bar{x}_{i0}) + x_{i+1,0}, \text{ for } i = 1, \ldots, n-1 \quad (3)$$
$$\dot{x}_{n0} = f_n(\bar{x}_{n0}) + u_0 \quad (4)$$

where $\bar{x}_{i0} = [x_{10}, \ldots, x_{i0}]^\top$ and u_0 are known time-varying functions. We assume that the state x_{*0} is only available to a portion of the m systems. The control problem considered in this paper is defined as follows.

Control Problem: For system j, design a control law based on its own state information, its neighbor's state information (i.e., x_{*i} for $i \in \mathcal{N}_j$), and x_{*0} if it is available such that

$$\lim_{t\to\infty}(x_{*j} - x_{*0}) = 0, \quad 1 \le j \le m. \quad (5)$$

Remark 1: In the control problem, the desired trajectory is available to only a subset of systems. Tracking controller design for a single nonlinear system cannot solve our control problem.

In order to solve the control problem, the following assumption is made on the reference system (3)-(4).

Assumption 1: The variable x_{*0} and its derivative $\dot{x}_{*0}$ are bounded.

III. Control Law Design

The m systems in (1)-(2) and the desired system in (3)-(4) are in strict-feedback forms [20]. With the aid of the backstepping techniques in [20], we define a new variable $z_{*j} = [z_{1j}, \ldots, z_{nj}]^\top$ with

$$z_{ij} = x_{ij} - \alpha_{ij} \quad (6)$$

for $i = 1, \ldots, n$ and $j = 0, 1, 2, \ldots, m$, where

$$\alpha_{1j} = 0 \quad (7)$$
$$\alpha_{2j}(\bar{x}_{1j}) = -k_1 z_{1j} - f_1(\bar{x}_{1j}) \quad (8)$$
$$\alpha_{i+1,j}(\bar{x}_{ij}) = -k_i z_{ij} - f_i(\bar{x}_{ij}) - z_{i-1,j} + \dot{\alpha}_{ij}(\bar{x}_{i-1,j}), \ 3 \le i \le n-1 \quad (9)$$

and $k_i > 0$. Then, we have

$$\dot{z}_{1j} = -k_1 z_{1j} + z_{2j} \quad (10)$$
$$\dot{z}_{ij} = -k_i z_{ij} - z_{i-1,j} + z_{i+1,j}, \quad 2 \le i \le n-1 \quad (11)$$
$$\dot{z}_{nj} = f_n(\bar{x}_{nj}) + u_j - \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \quad (12)$$

for $j = 0, 1, \ldots, m$.

For the $(m+1)$ systems in (10)-(12), we have the following result.

Lemma 1: For the $(m+1)$ systems in (10)-(12), if

$$\lim_{t\to\infty}(z_{nj} - z_{n0}) = 0, \quad \forall 1 \le j \le m \quad (13)$$

then (5) holds.

Proof: Let $\bar{z}_{ij} = z_{ij} - z_{i0}$ for $1 \le i \le n$ and $1 \le j \le m$, we have

$$\dot{\bar{z}}_{1j} = -k_1 \bar{z}_{1j} + \bar{z}_{2j} \quad (14)$$
$$\dot{\bar{z}}_{ij} = -k_i \bar{z}_{ij} - \bar{z}_{i-1,j} + \bar{z}_{i+1,j}, \quad 2 \le i \le n-1. \quad (15)$$

Let a Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n-1} \bar{z}_{ij}^2$$

and differentiate V along the solution of (14)-(15), we have

$$\dot{V} = -\sum_{i=1}^{n-1} k_i \bar{z}_{ij}^2 + \bar{z}_{n-1,j}\bar{z}_{nj} \le -\bar{k}V + \sqrt{2}|\bar{z}_{nj}|\sqrt{V}$$

where $\bar{k} = 2\min\{k_i, 1 \le i \le n-2\}$. Let $V_1 = \sqrt{V}$, then

$$D^+V_1 \le -\frac{\bar{k}}{2}V_1 + \frac{\sqrt{2}}{2}|\bar{z}_{nj}|$$

where D^+V_1 is the upper Dini derivative of V_1 [21]. Noting that $\bar{z}_{nj}$ converges to zero, it can be shown that V_1 converges to zero by the comparison lemma in [21]. So, V converges to zero, which means that z_{ij} converge to z_{i0} for $1 \le i \le n-1$ and $1 \le j \le m$. Therefore, (5) holds. ∎

The next lemma presents some results related to a Laplacian matrix of a graph with a weight matrix. These results are useful in proving the proposed results later. The proof of this lemma can be found in [7].

Lemma 2: If a graph $\mathcal{G}$ with weight matrix $\mathcal{A} = [a_{ji}]$ ($a_{ji} = a_{ij} \ge \delta > 0$) is connected, then the matrix $(\mathcal{L} + \mathrm{diag}(\xi))$ has m positive real eigenvalues, where ξ is a nonzero nonnegative vector with nonzero element larger than δ.

A. Distributed Control Laws

The next lemma gives distributed control laws for the m systems such that eqn. (13) holds.

Lemma 3: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is strongly connected and the state of the reference system (3)-(4) is at least available to one of the m systems, then the distributed control laws

$$\begin{aligned} u_j &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) - \mu_j b_j(z_{nj}-z_{n0}) \\ &\quad -\rho_j \mathrm{sign}(s_j) - f_n(\bar{x}_{nj}) + \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \end{aligned} \quad (16)$$

for $1 \le j \le m$ make eqn. (13) satisfied, where constants $a_{ji} = a_{ij} \ge \delta > 0$, $b_j \ge \delta > 0$, ρ_j is a sufficient large number, and

$$s_j = \sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) + \mu_j b_j(z_{nj}-z_{n0}) \quad (17)$$

$$\mu_j = \begin{cases} 1, & \text{if } x_{*0} \text{ is available to system } j \\ 0, & \text{if } x_{*0} \text{ is not available to system } j. \end{cases} \quad (18)$$

Proof: With the control law (16), we have

$$\dot{z}_{n*} = -\mathcal{L}z_{n*} - B(z_{n*}-\mathbf{1}z_{n0}) - \Lambda = -(\mathcal{L}+B)z_{n*} + B\mathbf{1}z_{n0} - \Lambda \quad (19)$$

where $z_{n*} = [z_{n1}, z_{n2}\ldots, z_{nm}]^\top$, $B = \mathrm{diag}[\mu_1 b_1, \ldots, \mu_m b_m]$, $\mathbf{1} = [1, 1, \ldots, 1]^\top$, $\Lambda = [\rho_1 \mathrm{sign}(s_1), \ldots, \rho_m \mathrm{sign}(s_m)]^\top$.

Let $\bar{z}_{n*} = z_{n*} - \mathbf{1}z_{n0}$, we have

$$\dot{\bar{z}}_{n*} = -(\mathcal{L}+B)\bar{z}_{n*} - \Lambda - \mathbf{1}\dot{z}_{n0}. \quad (20)$$

Since the communication graph is connected, $(\mathcal{L}+B)$ is a symmetric positive definite matrix by Lemma 2. Let

$$V = \frac{1}{2}\bar{z}_{n*}^\top(\mathcal{L}+B)\bar{z}_{n*}$$

and differentiate V along (20), we have

$$\begin{aligned} \dot{V} &= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \bar{z}_{n*}^\top(\mathcal{L}+B)(\Lambda + \mathbf{1}\dot{z}_{n0}) \\ &\le -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \bar{z}_{n*}^\top(\mathcal{L}+B)\Lambda + |\dot{z}_{n0}|\sum_{i=1}^m |s_i| \\ &= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \sum_{i=1}^m |s_i|(\rho_i - |\dot{z}_{n0}|) \\ &\le -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} \\ &\le -\sigma V \end{aligned} \quad (21)$$

where we apply the fact that ρ_i is sufficient large and $\rho_i > |\dot{z}_{n0}(t)|$ for any time due to Assumption 1, and σ is a positive constant that depends on $(\mathcal{L}+B)$. Therefore, V exponentially converges to zero. Furthermore, $\bar{z}_{n*}$ exponentially converges to zero and z_{n*} exponentially converges to z_{n0}. ■

With the aid of Lemmas 1-3, we have the following results.

Theorem 1: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is connected and the state of the reference system (3)-(4) is at least available to one of the m systems, then the distributed control laws (16) make (5) hold.

Proof: With the control laws (16), by Lemma 3 the equations in (13) hold. By Lemma 1, eqn. (5) holds. Therefore, *Problem 2* is solved by the distributed control laws (16). ■

In Theorem 1, ρ_i is sufficient large. By the proof, ρ_j should be chosen such that

$$\rho_j \ge M := \max_{t\in[0,\infty)} |\dot{z}_{n0}(t)|. \quad (22)$$

Since the state of the reference system is not available to each system, the upper bound M is not known to each system. In order to make (22) satisfied, ρ_j must be chosen as large as possible, which means that the magnitude of the control inputs will be large. To reduce the magnitude of the control inputs, ρ_j can be estimated on-line.

Theorem 2: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is connected and the state of the reference system (3)-(4) is at least available to one of the m systems, the distributed control laws

$$\begin{aligned} u_j &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) - \mu_j b_j(z_{nj}-z_{n0}) \\ &\quad -\hat{\rho}_j \mathrm{sign}(s_j) - f_n(\bar{x}_{nj}) + \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \end{aligned} \quad (23)$$

$$\dot{\hat{\rho}}_j = \gamma_j |s_j| \quad (24)$$

for $1 \le j \le m$ make (5) satisfied and $\hat{\rho}_j$ bounded, where constants $a_{ji} = a_{ij} \ge \delta > 0$, $b_j \ge \delta > 0$, $\gamma_j > 0$, s_j and μ_j are defined in (17) and (18), respectively.

Proof: Let $\bar{z}_{n*} = z_{n*} - \mathbf{1}z_{n0}$, we have

$$\dot{\bar{z}}_{n*} = -(\mathcal{L}+B)\bar{z}_{n*} - \hat{\Lambda} - \mathbf{1}\dot{z}_{n0}. \quad (25)$$

where B is defined in Lemma 3, and

$$\hat{\Lambda} = [\hat{\rho}_1 \mathrm{sign}(s_1), \ldots, \hat{\rho}_m \mathrm{sign}(s_m)]^\top. \quad (26)$$

Since the communication graph is connected, $(\mathcal{L}+B)$ is a symmetric positive definite matrix by Lemma 2. Let

$$V = \frac{1}{2}\bar{z}_{n*}^\top(\mathcal{L}+B)\bar{z}_{n*} + \frac{1}{2}\sum_{i=1}^m \gamma_i^{-1}(\hat{\rho}_i - \rho)^2$$

where ρ is a positive constant and $\rho \ge |\dot{z}_{n0}(t)|$ for $t \in [0,\infty)$. Differentiate V along (25), we have

$$\begin{aligned} \dot{V} &= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \bar{z}_{n*}^\top(\mathcal{L}+B)(\hat{\Lambda} + \mathbf{1}\dot{z}_{n0}) \\ &\quad + \sum_{i=1}^m \gamma_i^{-1}(\hat{\rho}_i - \rho)\dot{\hat{\rho}}_i \\ &\le -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \bar{z}_{n*}^\top(\mathcal{L}+B)\hat{\Lambda} + |\dot{z}_{n0}|\sum_{i=1}^m |s_i| \\ &\quad + \sum_{i=1}^m \gamma_i^{-1}(\hat{\rho}_i - \rho)\dot{\hat{\rho}}_i \\ &= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \sum_{i=1}^m |s_i|(\hat{\rho}_i - |\dot{z}_{n0}|) \\ &\quad + \sum_{i=1}^m \gamma_i^{-1}(\hat{\rho}_i - \rho)\dot{\hat{\rho}}_i \end{aligned}$$

$$
\begin{aligned}
&= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \sum_{i=1}^m |s_i|(\rho_i - |\dot{z}_{n0}|) \\
&\quad + \sum_{i=1}^m |s_i|(\rho_i - \hat{\rho}_i) + \sum_{i=1}^m \gamma_i^{-1}(\hat{\rho}_i - \rho)\dot{\hat{\rho}}_i \\
&\leq -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} \leq -\beta\bar{z}_{n*}^\top\bar{z}_{n*} \leq 0 \qquad (27)
\end{aligned}
$$

where β (> 0) is the smallest eigenvalue of the matrix $(\mathcal{L}+B)^2$. Therefore, V is bounded. So, $\bar{z}_{n*}$ and $\hat{\rho}_j$ are bounded. By integrating both sides of (27), we can show that $\bar{z}_{n*}^\top\bar{z}_{n*}$ is integrable, i.e., $\bar{z}_{n*}$ is square-integrable. By Barbalat's lemma (Lemma 3.2.5 in [22]), $\bar{z}_{n*}$ converges to zero. Therefore, z_{n*} converges to z_{n0}. By Lemma 1, (5) holds. ■

B. Distributed Approximation Based Control Laws

In this subsection, we propose distributed approximation based adaptive control laws. The control laws can learn the variable $\dot{z}_{n0}$ on-line.

For system j, we choose a vector function $\Phi_j(t) = [\phi_{1j}(t), \phi_{2j}(t), \ldots, \phi_{p_j,j}(t)]^\top$ where p_j is an integer. Given a function $\dot{z}_{n0}(t)$, its best approximation based on the basis vector Φ_j is $\Phi_j^\top\theta_j$, where θ_j is a constant vector. The largest approximation error is ϵ_j, i.e.,

$$
\dot{z}_{n0}(t) = \Phi_j(t)^\top\theta_j + e_j(t) \qquad (28)
$$

and $|e_j(t)| \leq \epsilon_j$. It should be noted that ϵ_j is unknown and may be made small by suitably choosing $\Phi_j(t)$. With the aid of the approximation of $\dot{z}_{n0}$, we have the following results.

Theorem 3: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is connected and the state of the reference system (3)-(4) is at least available to one of the m systems, the distributed adaptive control laws

$$
\begin{aligned}
u_j &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) - \mu_j b_j(z_{nj}-z_{n0}) - f_n(\bar{x}_{nj}) \\
&\quad + \Phi_j(t)^\top\hat{\theta}_j - \rho_j\mathrm{sign}(s_j) + \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \qquad (29) \\
\dot{\hat{\theta}}_j &= -\Gamma_j\Phi_j(t)s_j \qquad (30)
\end{aligned}
$$

for $1 \leq j \leq m$ make (5) satisfied and $\hat{\theta}_j$ bounded, where constants $a_{ji} = a_{ij} \geq \delta > 0$, $b_j \geq \delta > 0$, Γ_j is a positive definite matrix, ρ_j is a sufficient large number, s_j and μ_j are defined in Lemma 3.

Proof: Let $\bar{z}_{n*} = z_{n*} - \mathbf{1}z_{n0}$, then

$$
\dot{\bar{z}}_{n*} = -(\mathcal{L}+B)\bar{z}_{n*} + \Pi - \Lambda - e. \qquad (31)
$$

where B and Λ are defined in the proof of Lemma 3, $e = [e_1, e_2, \ldots, e_n]^\top$, and

$$
\Pi = [\Phi_1^\top(\hat{\theta}_1-\theta_1), \Phi_2^\top(\hat{\theta}_2-\theta_2), \ldots, \Phi_m^\top(\hat{\theta}_m-\theta_m)]^\top.
$$

Let the Lyapunov function

$$
V = \frac{1}{2}\bar{z}_{n*}^\top(\mathcal{L}+B)\bar{z}_{n*} + \frac{1}{2}\sum_{i=1}^m(\hat{\theta}_i-\theta_i)^\top\Gamma_i^{-1}(\hat{\theta}_i-\theta_i).
$$

Differentiating V along the solution of the systems in (31) and (30), we have

$$
\begin{aligned}
\dot{V} &= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} + \bar{z}_{n*}^\top(\mathcal{L}+B)(\Pi-\Lambda-e) \\
&\quad + \sum_{i=1}^m(\hat{\theta}_i-\theta_i)^\top\Gamma_i^{-1}\dot{\hat{\theta}}_i \\
&\leq -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \bar{z}_{n*}^\top(\mathcal{L}+B)\Lambda + \sum_{i=1}^m \epsilon_i|s_i| \\
&\quad + \sum_{i=1}^m(\hat{\theta}_i-\theta_i)^\top\Gamma_i^{-1}(\dot{\hat{\theta}}_i + \Gamma_i\Phi_i s_i) \\
&= -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} - \sum_{i=1}^m|s_i|(\rho_i-\epsilon_i) \\
&\leq -\bar{z}_{n*}^\top(\mathcal{L}+B)^2\bar{z}_{n*} \\
&\leq -\sigma\bar{z}_{n*}^\top\bar{z}_{n*} \qquad (32)
\end{aligned}
$$

where σ (> 0) is the smallest eigenvalue of $(\mathcal{L}+B)^2$. Therefore, V is bounded. So, $\bar{z}_{n*}$ and $\hat{\theta}_j$ are bounded. Integrating both sides of (32), we can show that $\bar{z}_{n*}$ is square-integrable. By Barbalat's lemma (Lemma 3.2.5 in [22]), $\bar{z}_{n*}$ converges to zero. Furthermore, z_{n*} converges to z_{n0}. By Lemma 1, (5) holds. ■

With the control laws in theorem 3, ρ_j can be small because ϵ_j can be made small by choosing appropriate Φ_j. However, in many cases it is hard to know how large ρ_j should be. In the next theorem, we estimate ρ_j on-line.

Theorem 4: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is connected and the state of the reference system (3)-(4) is at least available to one of the m systems, the distributed adaptive control laws

$$
\begin{aligned}
u_j &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) - \mu_j b_j(z_{nj}-z_{n0}) \\
&\quad - f_n(\bar{x}_{nj}) + \Phi_j(t)^\top\hat{\theta}_j - \hat{\rho}_j\mathrm{sign}(s_j) + \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \qquad (33) \\
\dot{\hat{\theta}}_j &= -\Gamma_j\Phi_j(t)s_j \qquad (34) \\
\dot{\hat{\rho}}_j &= \gamma_j|s_j| \qquad (35)
\end{aligned}
$$

for $1 \leq j \leq m$ make (5) satisfied and make $\hat{\theta}_j$ and $\hat{\rho}_j$ bounded, where constants $a_{ji} = a_{ij} \geq \delta > 0$, $b_j \geq \delta > 0$, $\gamma_j > 0$, Γ_j is a positive definite matrix, and s_j and μ_j are defined in Lemma 3.

Theorem 4 can be proven by combining the proofs of Theorem 3 and Theorem 2. We omit it here.

If Φ_j is chosen such that $\dot{z}_{n0}$ is in the span space of Φ_j for $1 \leq j \leq m$, i.e., there exists a constant vector θ_j such that

$$
\dot{z}_{n0}(t) = \Phi_j(t)^\top\theta_j. \qquad (36)
$$

Theorem 3 can be written as the following theorem.

Theorem 5: For the m systems in (1)-(2), under Assumption 1, if the communication graph $\mathcal{G}$ is connected and the state of the reference system (3)-(4) is at least available to one of the m systems, the distributed adaptive control laws

$$
\begin{aligned}
u_j &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj}-z_{ni}) - \mu_j b_j(z_{nj}-z_{n0}) \\
&\quad + \Phi_j(t)^\top\hat{\theta}_j - f_n(\bar{x}_{nj}) + \dot{\alpha}_{nj}(\bar{x}_{n-1,j}) \qquad (37)
\end{aligned}
$$

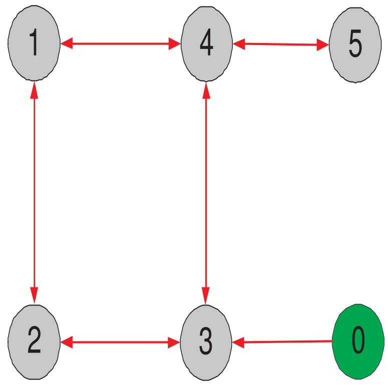

Fig. 1. Communication graph $\mathcal{G}$.

for $1 \le j \le m$ make (5) hold and $\hat{\theta}_j$ bounded, where $\hat{\theta}_j$ is updated in (30), constants $a_{ji} = a_{ij} \ge \delta > 0$, $b_j \ge \delta > 0$, Γ_j is a positive definite matrix, s_j and μ_j are defined in Lemma 3.

In Theorems 3-4, instead of choosing a set of basis functions Φ_j we can use neural networks to estimate $\dot{z}_{n0}$. The update laws for the weights of neural networks can be derived similarly.

In Theorems 1-5, distributed control laws were proposed for the defined control problem. The proposed control laws extended the results in [7, 11, 13, 14, 16, 23, 24] in two aspects: 1). each system is a high-order nonlinear system; and 2). the state of a reference system is available to a portion of the m systems.

In the proposed control laws, $\mathrm{sign}(\cdot)$ function is applied, which may lead to chattering. To avoid chattering, the function $\mathrm{sign}(\omega)$ can be replaced with $\frac{\omega}{\sqrt{\omega^2+e^{-t}}}$ or $\mathrm{sat}(\omega)$. We will not discuss it due to space limitation.

IV. Simulation

To show the effectiveness of the proposed control laws, we consider the consensus problems of five identical systems defined by

$$\dot{x}_{1j} = x_{1j}^2 + x_{2j}, \quad \dot{x}_{2j} = x_{3j}, \quad \dot{x}_{3j} = u_j. \tag{38}$$

By the transformation (6), we have (10)-(12) with $n = 3$. Given a reference system $x_{*0} = [x_{10}, x_{20}, x_{30}]^\top$ generated by

$$\dot{x}_{10} = x_{20} + x_{10}^2, \quad \dot{x}_{20} = x_{30}, \quad \dot{x}_{30} = u_0$$

where $u_0 = -\cos t - 2\cos 2t$ and the initial condition $x_{*0}(0) = [0, 1, 0]^\top$. The communication graph between the five systems is shown in Fig. 1, where the reference system 0 is assumed to be available to system 3. The control laws in Theorem 1 solve the control problem. In the control laws, we choose $k_1 = k_2 = 1$, $b_j = 2$, and $\rho_j = 10$ for $1 \le j \le 5$. Figs. 2-4 show the response of x_{1j}, x_{2j}, x_{3j}, and the reference system, respectively. It can be seen that the control problem is solved even if the reference system is only available to one system.

The distributed adaptive control laws in Theorem 2 also solve the control problem. Figs. 5-7 show the response of x_{1j}, x_{2j}, x_{3j}, and the reference system, respectively. It can be seen that x_{*j} converge to x_{*0}.

If we choose the basis function vector $\Phi_j(t) = [1, \sin t, \cos t, \sin 2t, \cos 2t]^\top$, $\dot{z}_{30}$ is in the span space of Φ_j. The distributed adaptive control laws in Theorem 5 solve *Problem 2*. Figs. 8-10 show the response of x_{1j}, x_{2j}, x_{3j}, and the reference system, respectively. It can be seen that x_{*j} converge to x_{*0}.

V. Conclusion

This paper considered the tracking control problem of multiple nonlinear systems. Distributed robust/adaptive control laws were proposed such that the state of each system converges to the state of a reference system with the aid of local information. Simulation results show the effectiveness of the proposed control laws. In this paper, the communication between systems is assumed to be bidirectional. The future work is to propose distributed control laws for the cases where the communication between systems is unidirectional. Also, in this paper it is assumed that there is no communication delay between systems. This work will be extended to account for communication delays.

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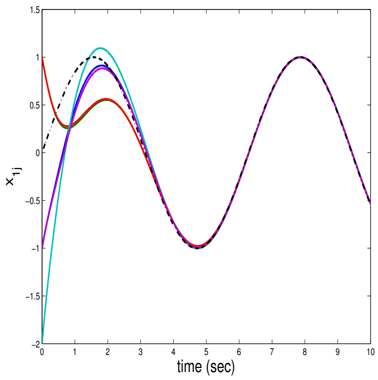

Fig. 2. Response of x_{1j} for $1 \leq j \leq 5$ (solid) and x_{10} (dashdot).

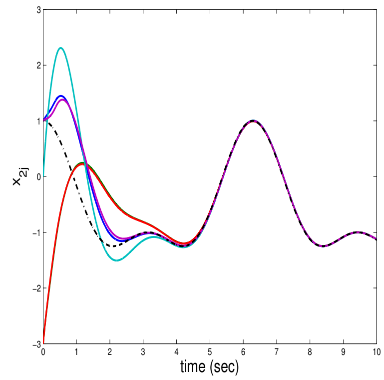

Fig. 3. Response of x_{2j} for $1 \leq j \leq 5$ (solid) and x_{20} (dashdot).

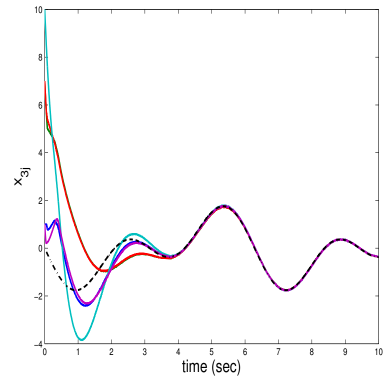

Fig. 4. Response of x_{3j} for $1 \leq j \leq 5$ (solid) and x_{30} (dashdot).

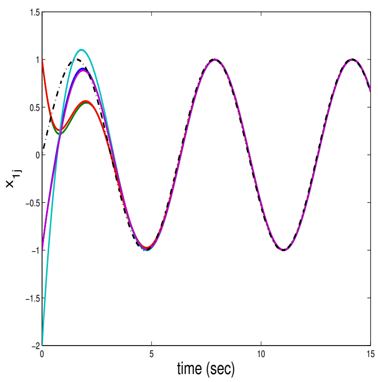

Fig. 5. Response of x_{1j} for $1 \leq j \leq 5$ (solid) and x_{10} (dashdot).

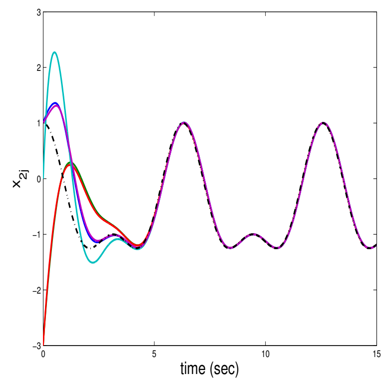

Fig. 6. Response of x_{2j} for $1 \leq j \leq 5$ (solid) and x_{20} (dashdot).

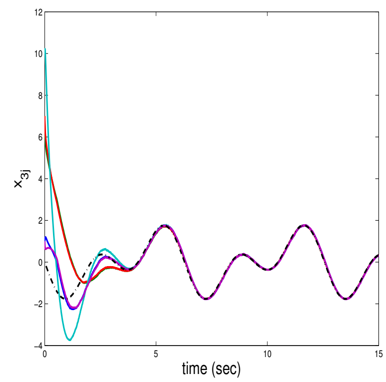

Fig. 7. Response of x_{3j} for $1 \leq j \leq 5$ (solid) and x_{30} (dashdot).

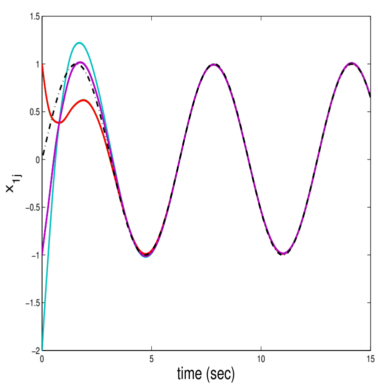

Fig. 8. Response of x_{1j} for $1 \leq j \leq 5$ (solid) and x_{10} (dashdot).

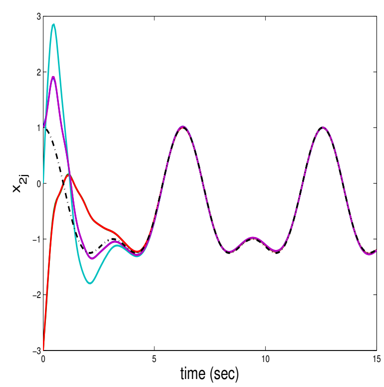

Fig. 9. Response of x_{2j} for $1 \leq j \leq 5$ (solid) and x_{20} (dashdot).

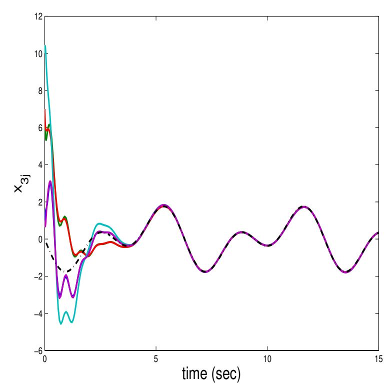

Fig. 10. Response of x_{3j} for $1 \leq j \leq 5$ (solid) and x_{30} (dashdot).

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