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A control problem of PM synchronous motor by internal model design

Zhaowu Ping and Jie Huang

***Abstract*— In this paper, we study a speed tracking and load torque disturbance rejection problem of PM synchronous motor by internal model design. We first formulate the problem into a global robust output regulation problem of a special class of multivariable systems. Then we further convert the output regulation problem into a global stabilization problem of an augmented system composed of the original plant and an internal model. As the augmented system does not take any known special form, we have to develop a specific tool to deal with the stabilization problem. In particular, a generalized changing supply function technique applicable to non-ISS (input-to-state stable) systems is developed. This technique, in conjunction with a particular nonlinear internal model, leads to an effective solution to the problem.**

I. INTRODUCTION

Permanent magnet (PM) synchronous motor can be modeled as follows [15]:

$$\begin{aligned}
\frac{d\theta_r}{dt} &= \omega_r \\
\frac{d\omega_r}{dt} &= \frac{3p\Phi_v}{2J}i_q + \frac{3p}{2J}(L_d - L_q)i_d i_q - \frac{B}{J}\omega_r - \frac{1}{J}T_L \\
\frac{di_d}{dt} &= -\frac{R_s}{L_d}i_d + \frac{pL_q}{L_d}i_q\omega_r + \frac{1}{L_d}u_d \\
\frac{di_q}{dt} &= -\frac{R_s}{L_q}i_q - \frac{pL_d}{L_q}i_d\omega_r - \frac{p\Phi_v}{L_q}\omega_r + \frac{1}{L_q}u_q
\end{aligned} \quad (1)$$

where θ_r is rotor position, ω_r is speed, i_d and i_q are dq frame stator currents, T_L is load torque, u_d and u_q are dq frame stator voltages, L_d and L_q are dq axes inductances, Φ_v is rotor flux, R_s is stator resistance, J is inertia, B is viscous friction coefficient and p is the number of pole pairs. When dq axes inductances are equal, i.e. $L_d = L_q$, PM synchronous motor is also called surface-mounted PM synchronous motor. A basic control problem for PM synchronous motors is to design a feedback control law such that the solution of the closed-loop system is globally bounded, and the speed ω_r asymptotically tracks a desired reference input, and the d axis current i_d is asymptotically regulated to zero. This problem can also be called speed tracking control and load torque disturbance rejection problem.

For the special case where $L_d = L_q$, the problem has been extensively studied since 2000 [1], [8], [9], [11], [14], [16]. In particular, Ping and Huang formulated the above problem into a global robust output regulation problem of a class of multivariable systems [11]. The output regulation approach allows the reference input of the motor speed, and the unknown torque to be generated by a class of autonomous linear system called exosystem and tolerates uncertainties of all the motor parameters. In contrast, for the general case where $L_d \neq L_q$, the problem has received relative little attention. To our knowledge, Zhu et al. considered the problem based on the feedback linearization approach and an extended observer [15]. However, their approach needs to know the exact knowledge of the motor parameters and the control law relies on the reference input as well as its first and second derivatives. Guo et al. studied the above problem via feedback dissipative Hamiltonian realization approach [3]. Their approach allows the load torque and stator resistance to be unknown, but they need to assume that the reference input to be constant and the viscous friction coefficient B to be zero.

In this paper, we will also formulate the speed tracking control and load torque disturbance rejection problem for the general case where $L_d \neq L_q$ into a global robust output regulation problem. It turns out that, by means of a class of nonlinear internal models, the problem can be converted into a global robust stabilization problem of a so-called augmented system composed of the original plant and an internal model. The augmented system is a two-input, two-output nonlinear system subject to both static and dynamic uncertainties. As will be seen in Sections II and III, the presence of the coupling term $\frac{3p}{2J}(L_d - L_q)i_d i_q$ in (1) significantly complicates the above problem. In particular, the augmented system does not possess any known form, and is thus not amenable to the approach in [11]. As a result, we will develop a generalized changing supply function technique and gain assignment method to overcome the difficulty. This technique will lead to an effective solution to the motor control problem. Compared with [15], we allow the motor parameters R_s, J, B to be unknown with known bounds. Compared with [3], we don't need to assume the reference input to be constant and allows the viscous friction coefficient B to be nonzero and uncertain.

The rest of the paper is organized as follows. In Section II, we first formulate the speed tracking and load torque disturbance rejection problem as a robust output regulation problem, and then convert the global robust output regulation problem of the given plant into a global stabilization problem of an augmented system. In Section III we establish some technical lemmas for tackling the stabilization problem of the augmented system. In Section IV we apply the results in Section III to obtain the solvability conditions for the PM synchronous motor control problem, and evaluate the effectiveness of the control law by computer simulations. In Section V, we conclude the paper with some remarks.

Zhaowu Ping and Jie Huang are with Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong, China. This work has been supported in part by the Research Grants Council of the Hong Kong Special Administration Region under grant No. 412609.

II. Problem formulation and Preliminaries

To formulate the speed tracking and load torque disturbance rejection problem of (1) as a robust output regulation problem, define the following exosystem

$$\dot{v} = A_1 v = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v \quad (2)$$

where $v = \text{col}(v_1, v_2, v_3)$. Then (2) can generate any combination of a sine function with arbitrary amplitudes and initial phase and an arbitrary constant. In particular, with initial value given by $v(0) = \text{col}(A\sin\phi, A\cos\phi, \frac{1}{J}T_L)$, the solution of (2) is such that

$$v_1(t) = y_d(t), v_3(t) = \frac{1}{J}T_L. \quad (3)$$

Let $x_{1,1} = \omega_r, x_{1,2} = i_q, x_{2,1} = i_d, u_1 = u_q, u_2 = u_d, a_{11} = \frac{B}{J}, a_{12} = \frac{3p}{2J}(L_d - L_q), b_{11} = \frac{3p\Phi_v}{2J}, a_{13} = \frac{R_s}{L_q}, a_{14} = \frac{p\Phi_v}{L_q}, a_{15} = \frac{pL_d}{L_q}, b_{12} = \frac{1}{L_q}, a_{21} = \frac{R_s}{L_d}, a_{22} = \frac{pL_q}{L_d}, b_{21} = \frac{1}{L_d}$.

Then system (1) can be put in the following form.

$$\begin{aligned} \dot{x}_{1,1} &= -a_{11}x_{1,1} - v_3 + b_{11}x_{1,2} + a_{12}x_{2,1}x_{1,2} \\ \dot{x}_{1,2} &= -a_{13}x_{1,2} - a_{14}x_{1,1} + b_{12}u_1 - a_{15}x_{1,1}x_{2,1} \\ \dot{x}_{2,1} &= -a_{21}x_{2,1} + a_{22}x_{1,1}x_{1,2} + b_{21}u_2 \\ e &= \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} x_{1,1} - v_1 \\ x_{2,1} \end{bmatrix}. \end{aligned} \quad (4)$$

Denote the nominal values of the motor parameters by $\bar{R}_s, \bar{J}, \bar{B}$ and let $\text{col}(R_s, J, B) = \text{col}(\bar{R}_s, \bar{J}, \bar{B}) + w$ where $w \in R^3$ is the deviation of the motor parameters from their nominal values. As a result, the system (4) and the exosystem (3) can be put in the following compact form:

$$\begin{aligned} \dot{x} &= f(x, u, v, w) \\ \dot{v} &= A_1 v \\ e &= h(x, u, v, w) \end{aligned} \quad (5)$$

where

$$x = \begin{bmatrix} x_{1,1} \\ x_{1,2} \\ x_{2,1} \end{bmatrix}, h(x, u, v, w) = \begin{bmatrix} x_{1,1} - v_1 \\ x_{2,1} \end{bmatrix},$$

$$f(x, u, v, w) = \begin{bmatrix} -a_{11}x_{1,1} - v_3 + b_{11}x_{1,2} + a_{12}x_{2,1}x_{1,2} \\ -a_{13}x_{1,2} - a_{14}x_{1,1} + b_{12}u_1 - a_{15}x_{1,1}x_{2,1} \\ -a_{21}x_{2,1} + a_{22}x_{1,1}x_{1,2} + b_{21}u_2 \end{bmatrix}.$$

Let V be some known compact subset of R^3 containing the origin, and W a known compact subset of R^3 such that $w \in W$ implies $R_2 \geq \bar{R}_s + w_1 \geq R_1$,$J_2 \geq \bar{J} + w_2 \geq J_1$, and $B_2 \geq \bar{B} + w_3 \geq B_1$ for some known positive numbers $R_i, J_i, B_i, i = 1, 2$. Then the global robust output regulation problem of (5) means the design of a state feedback control law such that, for any initial condition of the closed-loop system, any $w \in W$, and any $v(0) \in V$, the solution of the closed-loop system is globally bounded, and the tracking error e approaches the origin asymptotically. Clearly, the solvability of the global robust output regulation problem of (5) implies the solution of the speed tracking and load torque disturbance rejection problem of the motor (1).

Various versions of the global robust output regulation problem have been extensively studied since the 1990s [2], [5], [7], [12]. In particular, [5] established a framework for converting a global robust output regulation problem of a plant into a global stabilization problem of an augmented system. In what follows, we will derive the augmented system for (5) based on the framework of [5].

First note that, associated with (5) are the following partial differential equations:

$$\begin{aligned} \frac{\partial \mathbf{x}(v, w)}{\partial v} A_1 v &= f(\mathbf{x}(v, w), \mathbf{u}(v, w), v, w) \\ 0 &= h(\mathbf{x}(v, w), \mathbf{u}(v, w), v, w) \end{aligned} \quad (6)$$

where $\mathbf{x} : R^3 \times R^3 \mapsto R^3$ and $\mathbf{u} : R^3 \times R^3 \mapsto R^2$ are two smooth functions vanishing at the origin. (6) is known as regulator equations [6].

As $b_{11}, b_{12}, b_{21} > 0$, it can be readily verified that the solution of the regulator equations (6) exists globally and take the following polynomial form in v.

$$\begin{aligned} \mathbf{x}_{1,1}(v, w) &= v_1, \mathbf{x}_{2,1}(v, w) = 0 \\ \mathbf{x}_{1,2}(v, w) &= b_{11}^{-1}a_{11}v_1 + b_{11}^{-1}\omega v_2 + b_{11}^{-1}v_3 \\ \mathbf{u}_1(v, w) &= b_{12}^{-1}(-b_{11}^{-1}\omega^2 + b_{11}^{-1}a_{11}a_{13} + a_{14})v_1 \\ &\quad + b_{12}^{-1}(b_{11}^{-1}a_{11}\omega + b_{11}^{-1}a_{13}\omega)v_2 \\ &\quad + b_{12}^{-1}b_{11}^{-1}a_{13}v_3 \\ \mathbf{u}_2(v, w) &= -b_{11}^{-1}a_{22}b_{21}^{-1}v_1(a_{11}v_1 + \omega v_2 + v_3). \end{aligned}$$

It can be verified that the solution of the regulator equations (6) satisfies the following equations:

$$\begin{aligned} &\mathbf{x}_{1,2}^{(3)}(v, w) + \omega^2 \dot{\mathbf{x}}_{1,2}(v, w) = 0, \\ &\mathbf{u}_1^{(3)}(v, w) + \omega^2 \dot{\mathbf{u}}_1(v, w) = 0, \\ &\mathbf{u}_2^{(5)}(v, w) + 5\omega^2 \mathbf{u}_2^{(3)}(v, w) + 4\omega^4 \dot{\mathbf{u}}_2(v, w) = 0. \end{aligned}$$

Let

$$\begin{aligned} \theta_1(v, w) &= \begin{bmatrix} \mathbf{x}_{1,2}(v, w) & \dot{\mathbf{x}}_{1,2}(v, w) & \mathbf{x}_{1,2}^{(2)}(v, w) \end{bmatrix}^T \\ \theta_2(v, w) &= \begin{bmatrix} \mathbf{u}_1(v, w) & \dot{\mathbf{u}}_1(v, w) & \mathbf{u}_1^{(2)}(v, w) \end{bmatrix}^T \\ \theta_3(v, w) &= \begin{bmatrix} \mathbf{u}_2(v, w) & \dot{\mathbf{u}}_2(v, w) & \mathbf{u}_2^{(2)}(v, w) \\ \mathbf{u}_2^{(3)}(v, w) & \mathbf{u}_2^{(4)}(v, w) \end{bmatrix}^T. \end{aligned}$$

Let $g(x, u) = \text{col}(x_{1,2}, u_1, u_2)$ with its i-th component being denoted by $g_i(x, u)$. Then it can be verified that

$$\begin{aligned} \dot{\theta}_i(v, w) &= \Phi_i \theta_i(v, w) \\ g_i(x(v, w), u(v, w)) &= \Psi_i \theta_i(v, w), \ i = 1, 2, 3 \end{aligned} \quad (7)$$

where

$$\Phi_1 = \left[\begin{array}{c|c} 0_{2\times1} & I_{2\times2} \\ \hline 0 & -\omega^2 \ \ 0 \end{array}\right]_{3\times3}, \Psi_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \left[\begin{array}{c|c} 0_{2\times1} & I_{2\times2} \\ \hline 0 & -\omega^2 \ \ 0 \end{array}\right]_{3\times3}, \Psi_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \left[\begin{array}{c|cccc} 0_{4\times 1} & & I_{4\times 4} & & \\ \hline 0 & -4\omega^4 & 0 & -5\omega^2 & 0 \end{array}\right]_{5\times 5},$$
$$\Psi_3 = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array}\right].$$

System (7) is called a steady-state generator of (5) with output $g(x,u)$ [5].

For $i = 1, 2, 3$, let $M_i \in R^{\sigma_i \times \sigma_i}$ and $N_i \in R^{\sigma_i \times 1}$ be a pair of controllable matrices with M_i Hurwitz, and define the following dynamic compensator

$$\begin{aligned} \dot{\eta}_1 &= M_1\eta_1 + N_1 x_{1,2} + N_1 b_{11}^{-1} a_{12} x_{1,2} x_{2,1} \\ \dot{\eta}_2 &= M_2\eta_2 + N_2 u_1 \\ \dot{\eta}_3 &= M_3\eta_3 + N_3 u_2. \end{aligned} \qquad (8)$$

It can be verified that (8) is an internal model of (4) with output $x_{1,2}, u_1, u_2$ as described in [5].

Let T_i be the unique nonsingular matrix satisfying the Sylvester equation

$$T_i\Phi_i - M_i T_i = N_i \Psi_i, i = 1, 2, 3. \qquad (9)$$

The existence of T_i is guaranteed since (M_i, N_i) is controllable, (Φ_i, Ψ_i) is observable, and the spectra of Φ_i and M_i are disjoint [10].

Performing on the system composed of (4) and (8) the following coordinate and input transformation

$$\begin{aligned} z_{1,1} &= \eta_1 - T_1\theta_1(v,w) - N_1 b_{11}^{-1}\bar{x}_{1,1} \\ z_{1,2} &= \eta_2 - T_2\theta_2(v,w) - N_2 b_{12}^{-1}\bar{x}_{1,2} \\ z_{2,1} &= \eta_3 - T_3\theta_3(v,w) - N_3 b_{21}^{-1}\bar{x}_{2,1} \\ \bar{x}_{1,1} &= x_{1,1} - v_1, \bar{x}_{2,1} = x_{2,1} \\ \bar{x}_{1,2} &= x_{1,2} - \Psi_1 T_1^{-1}\eta_1 \\ \bar{u}_1 &= u_1 - \Psi_2 T_2^{-1}\eta_2 \\ \bar{u}_2 &= u_2 - \Psi_3 T_3^{-1}\eta_3 \end{aligned} \qquad (10)$$

yields the following system

$$\begin{aligned} \dot{z}_{1,1} &= M_1 z_{1,1} + d_1\bar{x}_{1,1} \\ \dot{\bar{x}}_{1,1} &= \bar{f}_{1,1}(z_{1,1}, \bar{x}_{1,1}, \bar{x}_{2,1}, \mu) + (b_{1,1}(\mu) + a(\mu)\bar{x}_{2,1})\bar{x}_{1,2} \\ \dot{z}_{1,2} &= M_2 z_{1,2} + Q_{1,2}(z_{1,1}, \bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) \\ \dot{\bar{x}}_{1,2} &= \bar{f}_{1,2}(z_{1,1}, z_{1,2}, \bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) + b_{1,2}(\mu)\bar{u}_1 \\ \dot{z}_{2,1} &= M_3 z_{2,1} + Q_{2,1}(\bar{X}_2, \bar{x}_{2,1}, \mu) \\ \dot{\bar{x}}_{2,1} &= \bar{f}_{2,1}(\bar{X}_2, z_{2,1}, \bar{x}_{2,1}, \mu) + b_{2,1}(\mu)\bar{u}_2 \end{aligned} \qquad (11)$$

where $\mu = \text{col}(v,w), \bar{X}_1 = \text{col}(z_{1,1}, \bar{x}_{1,1}), \bar{X}_2 = \text{col}(\bar{X}_1, z_{1,2}, \bar{x}_{1,2}), b_{1,1}(\mu) = b_{11}, b_{1,2}(\mu) = b_{12}, b_{2,1}(\mu) = b_{21}, a(\mu) = a_{12}$ and

$$\begin{aligned} &\bar{f}_{1,1}(z_{1,1}, \bar{x}_{1,1}, \bar{x}_{2,1}, \mu) \\ &\quad = d_2 z_{1,1} + d_3\bar{x}_{1,1} + d_4\bar{x}_{2,1}z_{1,1} + d_5\bar{x}_{1,1}\bar{x}_{2,1} \\ &\qquad + c_1(v)\bar{x}_{2,1} \\ &Q_{1,2}(z_{1,1}, \bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) \\ &\quad = d_6 z_{1,1} + d_7\bar{x}_{1,1} + c_2(v)\bar{x}_{2,1} + d_8\bar{x}_{1,2} + d_9\bar{x}_{1,1}\bar{x}_{2,1} \\ &\qquad + d_{10}\bar{x}_{2,1}\bar{x}_{1,2} + d_{11}\bar{x}_{2,1}z_{1,1} \\ &\bar{f}_{1,2}(z_{1,1}, z_{1,2}, \bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) \\ &\quad = d_{12}z_{1,2} + d_{13}z_{1,1} + d_{14}\bar{x}_{1,1} + c_3(v)\bar{x}_{2,1} + d_{15}\bar{x}_{1,2} \\ &\qquad + d_{16}\bar{x}_{1,1}\bar{x}_{2,1} - b_3\bar{x}_{2,1}\bar{x}_{1,2} - b_4\bar{x}_{2,1}z_{1,1} \\ &Q_{2,1}(\bar{X}_2, \bar{x}_{2,1}, \mu) \\ &\quad = c_4(v)z_{1,1} + c_5(v)\bar{x}_{1,1} + d_{17}\bar{x}_{2,1} + c_6(v)\bar{x}_{1,2} \\ &\qquad + d_{18}\bar{x}_{1,1}\bar{x}_{1,2} + d_{19}\bar{x}_{1,1}z_{1,1} + d_{20}\bar{x}_{1,1}^2 \\ &\bar{f}_{2,1}(\bar{X}_{2,1}, z_{2,1}, \bar{x}_{2,1}, \mu) \\ &\quad = d_{21}z_{2,1} + c_7(v)z_{1,1} + c_8(v)\bar{x}_{1,1} + d_{22}\bar{x}_{2,1} \\ &\qquad + c_9(v)\bar{x}_{1,2} + d_{23}\bar{x}_{1,1}\bar{x}_{1,2} + d_{24}\bar{x}_{1,1}z_{1,1} + d_{25}\bar{x}_{1,1}^2 \end{aligned}$$

where the expressions of b_3, b_4, $c_1(v)$, $\cdots$, $c_9(v)$, d_1, $\cdots$, d_{25} are omitted for space limit.

Remark 2.1: System (11) is called the augmented system of the plant (4). It can be shown that the origin is an equilibrium of (4) for all μ [5]. Moreover, if a state feedback control law of the form

$$\bar{u}_1 = \alpha_1(\bar{x}_1),\ \bar{u}_2 = \alpha_2(\bar{x}_1, \bar{x}_2) \qquad (12)$$

where $\bar{x}_1 = \text{col}(\bar{x}_{1,1}, \bar{x}_{1,2}), \bar{x}_2 = \bar{x}_{2,1}$, α_1 and α_2 are globally defined smooth functions vanishing at the origin globally stabilizes the augmented system (11), then the following control law

$$\begin{aligned} u_1 &= \alpha_1(\bar{x}_1) + \Psi_2 T_2^{-1}\eta_2 \\ u_2 &= \alpha_2(\bar{x}_1, \bar{x}_2) + \Psi_3 T_3^{-1}\eta_3 \\ \dot{\eta}_1 &= M_1\eta_1 + N_1 g_1(x,u) + N_1 b_{11}^{-1} a_{12} x_{1,2} x_{2,1} \\ \dot{\eta}_i &= M_i\eta_i + N_i g_i(x,u), i = 2, 3 \end{aligned} \qquad (13)$$

solves the global robust output regulation problem of the original plant (4) [5].

Remark 2.2: As pointed out in [5], a system can have multiple internal models. For example, (8) would still be qualified for being an internal model of (4) if the last term in the first equation of (8) were removed. In this case, (8) would be the so-called canonical linear internal model as used in [11]. It is known that the key to the success of solving an output regulation problem is to be able to find a proper internal model that yields an augmented system which is stablizable and whose stabilization problem is tractable. It can be shown that, due to the presence of the term $a_{12}x_{2,1}x_{1,2}$ in (4), a linear canonical internal model as used in [11] would yield an augmented system of the form (11) but with the first equation taking the following form:

$$\dot{z}_{1,1} = (M_1 + \chi\bar{x}_{2,1})z_{1,1} + F(\bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) \qquad (14)$$

where $\chi = -N_1 b_{11}^{-1} a_{12}\Psi_1 T_1^{-1}$, $F(\bar{x}_{1,1}, \bar{x}_{1,2}, \bar{x}_{2,1}, \mu) = (M_1 N_1 b_{11}^{-1} + N_1 b_{11}^{-1} a_{11})\bar{x}_{1,1} - N_1 b_{11}^{-1} a_{12}\bar{x}_{2,1}(\bar{x}_{1,2} + \Psi_1 T_1^{-1} N_1 b_{11}^{-1}\bar{x}_{1,1} + \mathbf{x}_{1,2}(v,w))$. We have no clue if the stabilization problem of such an augmented system is solvable. That is why we have proposed the nonlinear internal model (8) in this paper.

III. Two Technical Lemmas

Since the term $\bar{x}_{2,1}$ appears in all equations of system (11), system (11) is not in any special form such as the normal form or lower triangular form. The stabilization of such a system has never been encountered before. Consequently, in this section, we have to first establish two technical lemmas tackling the stabilization problem of system (11). The first lemma can be viewed as a generalized changing supply function technique, and the second one a generalized gain assignment technique. The two lemmas are generalized version of the results in [13] and [2] in the sense that they apply to a particular type of non-ISS (input-to-state stable) systems.

Lemma 3.1: Consider the following system

$$\dot{x} = f(x, u, y, \mu(t)) \quad (15)$$

where $x \in R^n, u \in R, y \in R$, $\mu : [0, \infty) \mapsto R^l$ is a bounded piecewise continuous function. Let $y_m = \text{col}(h(x, u), u)$ where $h(x, u) : R^{n+1} \mapsto R^s$ with $0 \leq s \leq n$ is some measurable output of (15). Suppose, for any compact set $\Sigma \subset R^l$, there exists a C^1 function $V_0(x)$ satisfying $\underline{\gamma}_0(\|x\|) \leq V_0(x) \leq \overline{\gamma}_0(\|x\|)$ for some class K_∞ functions $\underline{\gamma}_0(\cdot)$ and $\overline{\gamma}_0(\cdot)$ such that for all $\mu(t) \in \Sigma$ along any trajectory of system (15),

$$\dot{V}_0 \leq -\gamma_0(\|x\|) + \omega_1(y_m)u^2 + \omega_2(y_m, y)y^2 \quad (16)$$

for some class K_∞ function $\gamma_0(\cdot)$ satisfying $\lim_{s \to 0^+} \sup(\frac{\gamma_0^{-1}(s^2)}{s}) < \infty$ and some smooth nonnegative functions $\omega_1(y_m), \omega_2(y_m, y)$. Then given any smooth function $\Delta(x) > 0$, there exist a C^1 function $W_0(x)$ satisfying $\underline{\delta}_0(\|x\|) \leq W_0(x) \leq \overline{\delta}_0(\|x\|)$ for some class K_∞ functions $\underline{\delta}_0(\cdot)$ and $\overline{\delta}_0(\cdot)$ such that for all $\mu \in \Sigma$ along the trajectory of system (15),

$$\dot{W}_0 \leq -\Delta(x)\|x\|^2 + \bar{\omega}_1(y_m)u^2 + \bar{\omega}_2(y_m, y)y^2 \quad (17)$$

for some known smooth functions $\bar{\omega}_1(y_m) \geq 1, \bar{\omega}_2(y_m, y) \geq 1$.

The proof is omitted due to the space limit.

Remark 3.1: The inequality (17) means that system (15) does not have to be input-to-state stable with x as the state and (u, y) as input. For example, the following system

$$\dot{x} = -x + xy. \quad (18)$$

is a special case of system (15). It is not ISS but satisfies inequality (16) with $y_m = x$ since, along any trajectory of system (15), the positive definite proper function $V_0(x) = x^2$ satisfies

$$\begin{aligned} \dot{V}_0 &= -2x^2 + 2x^2 y \\ &= -2x^2 + 2x \cdot xy \\ &\leq -x^2 + x^2 y^2. \end{aligned} \quad (19)$$

Thus, Lemma 3.1 can be viewed as a generalized changing supply function technique. The interest of this lemma lies in the fact that it will lead to a generalized gain assignment result for non-ISS systems as shown in the following result.

Lemma 3.2: Consider the following system

$$\begin{aligned} \dot{\zeta}_1 &= \varphi_1(\zeta_1, x, y, \mu(t)) \\ \dot{\zeta}_2 &= A\zeta_2 + \varphi_2(\zeta_1, x, y, \mu(t)) \\ \dot{x} &= \phi(\zeta_1, \zeta_2, x, y, \mu) + (b(\mu(t)) + a(\mu(t))y)u \end{aligned} \quad (20)$$

where $\zeta_1 \in R^{n_1}, \zeta_2 \in R^{n_2}, x \in R, u \in R, y \in R$, $\mu : [0, \infty) \mapsto R^l$ is a bounded piecewise continuous function, $A \in R^{n_2 \times n_2}$ is a Hurwitz matrix, $\varphi_1(\zeta_1, x, y, \mu)$, $\varphi_2(\zeta_1, x, y, \mu)$ and $\phi(\zeta_1, \zeta_2, x, y, \mu)$ are sufficiently smooth with $\varphi_1(0, 0, 0, \mu) = 0$, $\varphi_2(0, 0, 0, \mu) = 0$ and $\phi(0, 0, 0, 0, \mu) = 0$ for all $\mu \in R^l$. $b(\mu)$ is a function satisfying, for all μ, $b_m \leq b(\mu) \leq b_M$ with b_m, b_M known positive numbers. Let $y_m = \text{col}(h(\zeta_1, x), x)$ where $h(\zeta_1, x) : R^{n_1+1} \mapsto R^s$ with $0 \leq s \leq n_1$ is some measurable output of the plant (20). Suppose, given any compact subset $\Sigma \subset R^l$, there exists a C^1 function $\bar{V}_1(\zeta_1)$ satisfying $\underline{\gamma}_1(\|\zeta_1\|) \leq \bar{V}_1(\zeta_1) \leq \overline{\gamma}_1(\|\zeta_1\|)$ for some class K_∞ functions $\underline{\gamma}_1(\cdot)$ and $\overline{\gamma}_1(\cdot)$ such that, for all $\mu(t) \in \Sigma$, along any trajectory of system $\dot{\zeta}_1 = \varphi_1(\zeta_1, x, y, \mu)$,

$$\dot{\bar{V}}_1 \leq -\gamma_1(\|\zeta_1\|) + \bar{\pi}_1(y_m)x^2 + \bar{\pi}_2(y_m, y)y^2 \quad (21)$$

where $\gamma_1(\cdot)$ is some known class K_∞ functions satisfying $\lim_{s \to 0^+} \sup(\frac{\gamma_1^{-1}(s^2)}{s}) < \infty$ and $\bar{\pi}_1(y_m), \bar{\pi}_2(y_m, y)$ are some known smooth nonnegative functions. Then there exist a smooth function $\rho(y_m) : R^{s+1} \mapsto [0, \infty)$, a controller of the form

$$u = -\rho(y_m)x + \nu \quad (22)$$

with $\nu \in R$, and a C^1 function $U_1(\zeta_1, \zeta_2, x)$ satisfying $\underline{\alpha}_1(\|\zeta_1, \zeta_2, x\|) \leq U_1(\zeta_1, \zeta_2, x) \leq \overline{\alpha}_1(\|\zeta_1, \zeta_2, x\|)$ for some class K_∞ functions $\underline{\alpha}_1(\cdot)$ and $\overline{\alpha}_1(\cdot)$ such that along the trajectory of the closed-loop system composed of (20) and (22),

$$\dot{U}_1 \leq -\|\zeta_1\|^2 - \|\zeta_2\|^2 - x^2 + \nu^2 + \pi(\bar{y}_m, y)y^2 \quad (23)$$

for some known smooth nonnegative function $\pi(\bar{y}_m, y)$ where $\bar{y}_m = \text{col}(y_m, \nu)$.

The proof is omitted due to the space limit.

Remark 3.2: System (11) is in contrast with system (19) of [11] in that it contains the term: $a(\mu)\bar{x}_{2,1}\bar{x}_{1,2}$ in the $\bar{x}_{1,1}$-subsystem. Therefore, Lemma 3.1 in [11] does not apply to system (11). Thus we need to develop Lemma 3.2 to tackle the stabilization problem of system (11).

Remark 3.3: System (20) contains the following system

$$\begin{aligned} \dot{\zeta} &= A\zeta + \varphi(x, y, \mu(t)) \\ \dot{x} &= \phi(\zeta, x, y, \mu) + (b(\mu(t)) + a(\mu(t))y)u \end{aligned} \quad (24)$$

as a special case. For this case, condition (21) becomes redundant. By Lemma 3.2 with $y_m = x$, there exist a smooth function $\rho(y_m) : R^1 \mapsto [0, \infty)$, a controller of the form (22) with $\nu \in R$, and a C^1 function $U_1(\zeta, x)$ satisfying $\underline{\alpha}_1(\|\zeta, x\|) \leq U_1(\zeta, x) \leq \overline{\alpha}_1(\|\zeta, x\|)$ for some class K_∞ functions $\underline{\alpha}_1(\cdot)$ and $\overline{\alpha}_1(\cdot)$ such that, along any trajectory of the closed-loop system composed of (24) and (22),

$$\dot{U}_1 \leq -\|\zeta\|^2 - x^2 + \nu^2 + \pi(\bar{y}_m, y)y^2 \quad (25)$$

for some known smooth nonnegative function $\pi(\bar{y}_m, y)$ where $\bar{y}_m = \text{col}(y_m, \nu)$.

Remark 3.4: If system (20) does not contain y, then Lemma 3.2 means that the controller (22) with $\nu = 0$ solves the global stabilization problem of system (20).

IV. Solvability of the motor control problem

Let us first introduce the following notations:

$$\begin{aligned}\tilde{x}_{1,1} &= \bar{x}_{1,1}\\ \tilde{x}_{1,2} &= \bar{x}_{1,2}+\rho_{1,1}(\tilde{x}_{1,1})\tilde{x}_{1,1}\\ \bar{u}_1 &= -\rho_{1,2}(\tilde{x}_{1,1},\tilde{x}_{1,2})\tilde{x}_{1,2}\\ \tilde{x}_{2,1} &= \bar{x}_{2,1}\\ \bar{u}_2 &= -\rho_{2,1}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})\tilde{x}_{2,1} \qquad (26)\end{aligned}$$

where $\rho_{1,1},\rho_{1,2},\rho_{2,1}$ are certain smooth nonnegative functions to be specified. Also, let $\tilde{X}_1=\text{col}(z_{1,1},\tilde{x}_{1,1})$, $\tilde{X}_2=\text{col}(\tilde{X}_1,z_{1,2},\tilde{x}_{1,2})$, $\tilde{X}_3=\text{col}(\tilde{X}_2,z_{2,1},\tilde{x}_{2,1})$.

The stabilization problem of system (11) can be done by the following steps.

Step 1: Consider the following system

$$\begin{aligned}\dot{z}_{1,1} &= M_1z_{1,1}+d_1\tilde{x}_{1,1}\\ \dot{\tilde{x}}_{1,1} &= \tilde{f}_{1,1}(z_{1,1},\tilde{x}_{1,1},\bar{x}_{2,1},\mu)+(b_{1,1}(\mu)+a(\mu)\bar{x}_{2,1})\bar{x}_{1,2} \qquad (27)\end{aligned}$$

where $\tilde{f}_{1,1}(z_{1,1},\tilde{x}_{1,1},\bar{x}_{2,1},\mu)=\bar{f}_{1,1}(z_{1,1},\bar{x}_{1,1},\bar{x}_{2,1},\mu)$.

System (27) is in the form of system (24) with $\zeta=z_{1,1},x=\tilde{x}_{1,1},u=\bar{x}_{1,2},y=\bar{x}_{2,1},A=M_1,y_m=\tilde{x}_{1,1}$. By Remark 3.3, with $\nu=\tilde{x}_{1,2},\bar{y}_m=\text{col}(\tilde{x}_{1,1},\tilde{x}_{1,2})$, there exist a smooth nonnegative function $\rho_{1,1}(\tilde{x}_{1,1})$ and a C^1 function $\tilde{U}_1(\tilde{X}_1)$ satisfying $\underline{\alpha}_1(\|\tilde{X}_1\|)\le\tilde{U}_1(\tilde{X}_1)\le\overline{\alpha}_1(\|\tilde{X}_1\|)$ for some class K_∞ functions $\underline{\alpha}_1(\cdot)$ and $\overline{\alpha}_1(\cdot)$ such that, for all $\mu\in\Sigma$, $\tilde{U}_1(\tilde{X}_1)$ satisfies inequality (25).

Step 2: Consider the following system

$$\begin{aligned}\dot{\tilde{X}}_1 &= F_1(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)\\ \dot{z}_{1,2} &= M_2z_{1,2}+\tilde{Q}_{1,2}(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)\\ \dot{\tilde{x}}_{1,2} &= \tilde{f}_{1,2}(\tilde{X}_1,z_{1,2},\tilde{x}_{1,2},\bar{x}_{2,1},\mu)+b_{1,2}(\mu)\bar{u}_1 \qquad (28)\end{aligned}$$

where

$$\begin{aligned}&F_1(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)=\\ &\begin{bmatrix} M_1z_{1,1}+d_1\tilde{x}_{1,1}\\ \tilde{f}_{1,1}(z_{1,1},\tilde{x}_{1,1},\bar{x}_{2,1},\mu)+(b_{1,1}(\mu)+a(\mu)\bar{x}_{2,1})\bar{x}_{1,2}\end{bmatrix}\\ &\tilde{Q}_{1,2}(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)=Q_{1,2}(z_{1,1},\bar{x}_{1,1},\bar{x}_{1,2},\bar{x}_{2,1},\mu)\\ &\tilde{f}_{1,2}(\tilde{X}_1,z_{1,2},\tilde{x}_{1,2},\bar{x}_{2,1},\mu)\\ &\quad=\bar{f}_{1,2}(z_{1,1},z_{1,2},\bar{x}_{1,1},\bar{x}_{1,2},\bar{x}_{2,1},\mu)\\ &\qquad+\frac{\partial(\rho_{1,1}(\tilde{x}_{1,1})\tilde{x}_{1,1})}{\partial\tilde{x}_{1,1}}(\tilde{f}_{1,1}(z_{1,1},\tilde{x}_{1,1},\bar{x}_{2,1},\mu)\\ &\qquad+(b_{1,1}(\mu)+a(\mu)\bar{x}_{2,1})\bar{x}_{1,2}).\end{aligned}$$

System (28) is in the form of system (20) with $\zeta_1=\tilde{X}_1,\zeta_2=z_{1,2},x=\tilde{x}_{1,2},u=\bar{u}_1,y=\bar{x}_{2,1},A=M_2,y_m=\text{col}(\tilde{x}_{1,1},\tilde{x}_{1,2})$. Because $\tilde{X}_1$-subsystem satisfies inequality (21), by Lemma 3.2, with $\nu=0,\bar{y}_m=\text{col}(\tilde{x}_{1,1},\tilde{x}_{1,2})$, there exist a smooth nonnegative function $\rho_{1,2}(\tilde{x}_{1,1},\tilde{x}_{1,2})$ and a C^1 function $\tilde{U}_2(\tilde{X}_2)$ satisfying $\underline{\alpha}_2(\|\tilde{X}_2\|)\le\tilde{U}_2(\tilde{X}_2)\le\overline{\alpha}_2(\|\tilde{X}_2\|)$ for some class K_∞ functions $\underline{\alpha}_2(\cdot)$ and $\overline{\alpha}_2(\cdot)$ such that, for all $\mu\in\Sigma$, $\tilde{U}_2(\tilde{X}_2)$ satisfies inequality (23).

Step 3: Consider the following system

$$\begin{aligned}\dot{\tilde{X}}_2 &= F_2(\tilde{X}_2,\tilde{x}_{2,1},\mu)\\ \dot{z}_{2,1} &= M_3z_{2,1}+\tilde{Q}_{2,1}(\tilde{X}_2,\tilde{x}_{2,1},\mu)\\ \dot{\tilde{x}}_{2,1} &= \tilde{f}_{2,1}(\tilde{X}_2,z_{2,1},\tilde{x}_{2,1},\mu)+b_{2,1}(\mu)\bar{u}_2 \qquad (29)\end{aligned}$$

where

$$\begin{aligned}&F_2(\tilde{X}_2,\tilde{x}_{2,1},\mu)\\ &\quad=\begin{bmatrix}F_1(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)\\ M_2z_{1,2}+\tilde{Q}_{1,2}(\tilde{X}_1,\tilde{x}_{1,2},\bar{x}_{2,1},\mu)\\ \tilde{f}_{1,2}(\tilde{X}_1,z_{1,2},\tilde{x}_{1,2},\bar{x}_{2,1},\mu)+b_{1,2}(\mu)\bar{u}_1\end{bmatrix}\\ &\tilde{Q}_{2,1}(\tilde{X}_2,\tilde{x}_{2,1},\mu)=Q_{2,1}(\bar{X}_2,\bar{x}_{2,1},\mu)\\ &\tilde{f}_{2,1}(\tilde{X}_2,z_{2,1},\tilde{x}_{2,1},\mu)=\bar{f}_{2,1}(\bar{X}_2,z_{2,1},\bar{x}_{2,1},\mu).\end{aligned}$$

System (29) is in the form of system (20) with $\zeta_1=\tilde{X}_2,\zeta_2=z_{2,1},x=\tilde{x}_{2,1},u=\bar{u}_2,y\in R^0,y_m=\text{col}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})$. Because $\tilde{X}_2$-subsystem satisfies inequality (21), by Lemma 3.2, with $\nu=0,\bar{y}_m=\text{col}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})$, there exist a smooth function $\rho_{2,1}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})$ and a C^1 function $\tilde{U}_3(\tilde{X}_3)$ satisfying $\underline{\alpha}_3(\|\tilde{X}_3\|)\le\tilde{U}_3(\tilde{X}_3)\le\overline{\alpha}_3(\|\tilde{X}_3\|)$ for some class K_∞ functions $\underline{\alpha}_3(\cdot)$ and $\overline{\alpha}_3(\cdot)$ such that, for all $\mu\in\Sigma$, $\tilde{U}_3(\tilde{X}_3)$ satisfies inequality (23). By Remark 3.4, a control law of the form (26) solves the global stabilization problem of system (11). Finally, by Remark 2.1, we obtain the following main result.

Theorem 4.1: The state feedback control law of the following form

$$\begin{aligned}u_1 &= -\rho_{1,2}(\tilde{x}_{1,1},\tilde{x}_{1,2})\tilde{x}_{1,2}+\Psi_2T_2^{-1}\eta_2\\ u_2 &= -\rho_{2,1}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})e_2+\Psi_3T_3^{-1}\eta_3\\ \dot{\eta}_1 &= M_1\eta_1+N_1g_1(x,u)+N_1b_{11}^{-1}a_{12}x_{1,2}x_{2,1}\\ \dot{\eta}_i &= M_i\eta_i+N_ig_i(x,u), i=2,3 \qquad (30)\end{aligned}$$

where $\tilde{x}_{1,1}=x_{1,1}-v_1$, $\tilde{x}_{1,2}=x_{1,2}-\Psi_1T_1^{-1}\eta_1+\rho_{1,1}(\tilde{x}_{1,1})e_1$, and $\tilde{x}_{2,1}=x_{2,1}$ solves the robust output regulation problem of system (4).

The control law (30) contains three design functions $\rho_{1,1}$, $\rho_{1,2}$, and $\rho_{2,1}$ and several design parameters M_i,N_i, $i=1,2,3$, and T_1,T_2,T_3. These design functions depends on the nominal values and the boundaries of the uncertainties of the motor parameters, as well as the specific Lyapunov functions used in deriving Theorem 4.1. For the purpose of the computer simulation, various certain parameters of the motor are $p=3,\Phi_v=0.18$V.sec/rad, $L_d=0.022$H, $L_q=0.011$H, the nominal values of various uncertain motor parameters are $\bar{R}_s=1.2\Omega,\bar{B}=0.0001$N.m.sec/rad, $\bar{J}=0.006$Kg.m^2, $\bar{T}_L=0.3$N.m, and the real values of R_s,B,J,T_L are prescribed by $R_s\in[0.5\bar{R}_s,2\bar{R}_s],B\in[0.5\bar{B},2\bar{B}],J\in[0.5\bar{J},2\bar{J}],T_L\in[0.5\bar{T}_L,2\bar{T}_L]$. Also, we assume the frequency of the reference input is $\omega=3$, and the amplitude $A\le2$rad/sec. Under the above setup, various control law design functions and parameters are as follows:

$$\begin{aligned}&\rho_{1,1}(\tilde{x}_{1,1})=1,\\ &\rho_{1,2}(\tilde{x}_{1,1},\tilde{x}_{1,2})=20(1+\tilde{x}_{1,2}^2),\\ &\rho_{2,1}(\tilde{x}_{1,1},\tilde{x}_{1,2},\tilde{x}_{2,1})=20(1+e_2^2+e_2^2\tilde{x}_{1,2}^4+e_1^4e_2^2+e_1^6),\end{aligned}$$

$$M_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -17 & -19 & -3 \end{bmatrix}, N_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -16 & -7 \end{bmatrix}, N_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$M_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -2626 & -4156 & -1857 & -357 & -31 \end{bmatrix},$$
$$N_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T,$$
$$\Psi_1 T_1^{-1} = \begin{bmatrix} 17 & 10 & 3 \end{bmatrix}, \Psi_2 T_2^{-1} = \begin{bmatrix} 10 & 7 & 7 \end{bmatrix},$$
$$\Psi_3 T_3^{-1} = \begin{bmatrix} 2626 & 3832 & 1857 & 312 & 31 \end{bmatrix}.$$

The performance of the control law is evaluated through computer simulation with the uncertain motor parameters taking the following values: $R_s = 2\bar{R}_s, B = 1.5\bar{B}, J = 0.8\bar{J}$. The desired speed is $y_d(t) = 2\sin(3t + \frac{\pi}{2})$rad/sec, and the load $T_L = 0$ for $0 \leq t < 4$s and $T_L = 0.8\bar{T}_L$ for $t \geq 4$s. Initial values for computer simulation are as follows: $\omega_r(0) = 0.1$rad/sec, $i_d(0) = 0.2$A, $i_q(0) = 0.3$A, $v_1(0) = 2, v_2(0) = 0, v_3(0) = 0$, and $\eta_1(0) = \eta_2(0) = \eta_3(0) = 0$.

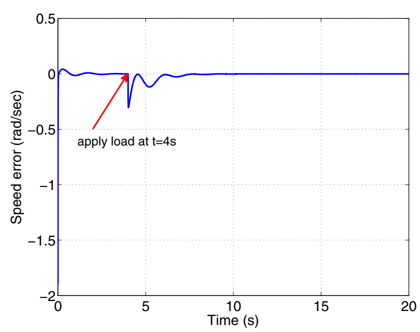

Fig. 1. Speed error with load at $t = 4$s

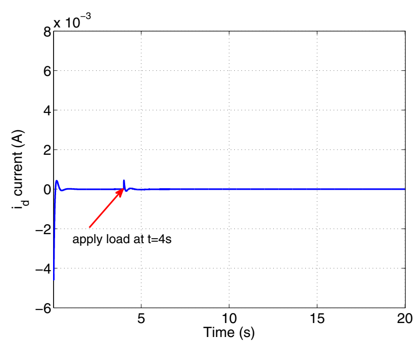

Fig. 2. i_d current with load at $t = 4$s

V. Conclusion

In this paper, we have studied the speed tracking and load torque disturbance rejection problem of PM synchronous motor by internal model design. After formulating the problem into a global robust output regulation problem of a special class of multivariable systems, we have developed a systematic approach that decomposes two-input control problem into two single-input control problems. As the augmented system does not take any known special form, we have developed a generalized changing supply function technique which is applicable to non-ISS (input-to-state stable) systems. This technique, in conjunction with a particular nonlinear internal model has led to an effective solution to the problem. The control performance was verified by simulation results. Compared with existing results about PM synchronous motor control, our approach offers a better robust property and allow the reference speed and load torque disturbance to be a linear combination of finitely many sinusoidal signals of different frequencies and constant signal.

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