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Economic Model Predictive Control Using Lyapunov Techniques: Handling Asynchronous, Delayed Measurements and Distributed Implementation

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Abstract— **This work focuses on economic model predictive control of nonlinear systems. First, an economic model predictive control algorithm that efficiently handles asynchronous and delayed measurements is presented and its application to a chemical process example is demonstrated. This algorithm uses suitable Lyapunov-based constraints to ensure closed-loop stability for a well-defined set of initial conditions. Second, a distributed economic model predictive control architecture for nonlinear systems is presented. In this architecture, the distributed controllers communicate in a sequential fashion, optimize their inputs through maximizing a plant-wide (global) economic objective function and guarantee practical stability of the closed-loop system.**

I. Introduction

Economic model predictive control (EMPC) refers to a class of model predictive control formulations in which the cost functional expresses directly economic optimization considerations of the plant under consideration, rather than penalizing (as it is usually the case, see, for example, [1], [2]) the deviations of the plant states and of the manipulated inputs from desired steady-state values. As a result, EMPC may lead to the computation of time-varying optimal operating policies for the plant in contrast to MPC with traditional cost functionals which typically leads to stabilization of the plant at the desired steady state.

While there have been several calls, particularly within process control, for the integration of model predictive control (MPC) and economic optimization of processes (e.g., [3]) as early as two decades ago, the subject of EMPC has received relatively little attention. Recently, in [4], general ideas of a combined steady-state optimization and linear MPC scheme were reported. In [5], MPC schemes using an economics-based cost function were proposed and the stability properties were established using a suitable Lyapunov function. The MPC schemes in [5] adopt a terminal constraint which requires that the closed-loop system state settles to a steady-state at the end of each optimal input trajectory calculation (i.e., end of the prediction horizon). In a recent paper [6], the approach in [5] was extended to deal with cyclic process operation. Even though a rigorous stability analysis is included in [5], [6], it is difficult, in general, to characterize, a priori, the set of initial conditions starting from where feasibility and closed-loop stability of the proposed MPC schemes are guaranteed. In a recent work [7], we presented an EMPC scheme for nonlinear systems that utilizes suitable Lyapunov-based stability constraints. The proposed EMPC is designed via Lyapunov-based techniques and has two different operation modes. The first operation mode corresponds to the periods in which the cost function should be optimized (e.g., normal production periods); and in this operation mode, the MPC maintains the closed-loop system state within a pre-defined stability region and optimizes the cost function to its maximum extent. The second operation mode corresponds to operation in which the system is driven by the MPC to an appropriate steady-state within the closed-loop system stability region.

In this work, we extend the results in [7] into two directions. First, we present an EMPC algorithm for nonlinear systems that efficiently handles asynchronous and delayed measurements using suitable Lyapunov-based constraints to ensure stability for a well-defined set of initial conditions, and demonstrate its application to a chemical process example. Second, we present a distributed EMPC (DEMPC) architecture for nonlinear systems. In this architecture, the distributed controllers communicate in a sequential fashion, optimize their inputs through maximizing a plant-wide (global) economic objective function and guarantee practical stability of the closed-loop system.

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II. Preliminaries

A. Notation

The operator $|\cdot|$ is used to denote Euclidean norm of a vector, and a continuous function $\alpha : [0, a) \rightarrow [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. The symbol Ω_r is used to denote the set $\Omega_r := \{x \in R^{n_x} : V(x) \leq r\}$ where V is a scalar function, and the operator '/' denotes set subtraction, that is, $A/B := \{x \in R^{n_x} : x \in A, x \notin B\}$. The symbol $diag(v)$ denotes a matrix whose diagonal elements are the elements of vector v and all the other elements are zeros.

B. Class of nonlinear systems

We consider a class of nonlinear systems which is composed of m subsystems where each of the subsystems can be described by the following state-space model:

$$\dot{x}_i(t) = f_i(x, u_i, w_i) \quad (1)$$

where $i = 1, \ldots, m$, $x_i(t) \in R^{n_{x_i}}$ denotes the vector of state variables of subsystem i, $u_i(t) \in R^{n_{u_i}}$ and $w_i(t) \in R^{n_{w_i}}$ denote the set of control (manipulated) inputs and disturbances associated with subsystem i, respectively. The variable $x \in R^{n_x}$ denotes the state of the whole system which is composed of the states of the m subsystems, that is $x = [x_1^T \cdots x_i^T \cdots x_m^T]^T$. The dynamics of x can be described in a compact form as follows:

$$\dot{x}(t) = f(x(t), u_1(t), \ldots, u_m(t), w(t)) \quad (2)$$

where $w = [w_1^T \cdots w_i^T \cdots w_m^T]^T$ is assumed to be bounded, that is, $w(t) \in W$ with $W := \{w \in R^{n_w} : |w| \leq \theta, \theta > 0\}$. The m sets of inputs are restricted to be in m nonempty convex sets $U_i \subseteq R^{n_{u_i}}$, $i = 1, \ldots, m$, which are defined as $U_i := \{u_i \in R^{n_{u_i}} : |u_i| \leq u_i^{\max}\}$ where $u_i^{\max}$, $i = 1, \ldots, m$, are the magnitudes of the input constraints. We assume that f is a locally Lipschitz vector function and that the origin is an equilibrium point of the unforced nominal system (i.e., system of Eq. 2 with $u_i(t) = 0$, $i = 1, \ldots, m$, $w(t) = 0$ for all t) which implies that $f(0, \ldots, 0) = 0$.

C. Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $h(x) = [h_1(x) \ \cdots \ h_m(x)]^T$ which renders the origin of the nominal closed-loop system asymptotically stable with $u_i = h_i(x)$, $i = 1, \ldots, m$, while satisfying the input constraints for all the states x inside a given stability region. Using converse Lyapunov theorems [8], [9], this assumption implies that there exist class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ and a continuously differentiable Lyapunov function $V(x)$ for the nominal closed-loop system which is continuous and bounded in $O \subseteq R^{n_x}$ where O is an open neighborhood of the origin, that satisfy the following inequalities:

$$\begin{aligned} &\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \\ &\frac{\partial V(x)}{\partial x} f(x, h_1(x), \ldots, h_m(x), 0) \leq -\alpha_3(|x|) \\ &\left| \frac{\partial V(x)}{\partial x} \right| \leq \alpha_4(|x|),\ h_i(x) \in U_i,\ i = 1, \ldots, m \end{aligned} \quad (3)$$

for all $x \in O$. We denote the region $\Omega_\rho \subseteq O$ (Ω_ρ is a level set of $V(x)$) as the stability region of the closed-loop system under the Lyapunov-based controller $h(x)$. Note that explicit stabilizing control laws that provide explicitly defined regions of attraction for the closed-loop system have been developed using Lyapunov techniques for specific classes of nonlinear systems, particularly input-affine nonlinear systems; the reader may refer to [10], [9], [11] for results in this area including results on the design of bounded Lyapunov-based controllers by taking explicitly into account constraints for broad classes of nonlinear systems.

By continuity, the local Lipschitz property assumed for the vector field f and taking into account that the manipulated inputs u_i, $i = 1, \ldots, m$ are bounded, there exists a positive constant M such that:

$$|f(x, u_1, \ldots, u_m, w)| \leq M \quad (4)$$

for all $x \in \Omega_\rho$ and $u_i \in U_i$, $i = 1, \ldots, m$. By the continuous differentiable property of the Lyapunov function $V(x)$ and the Lipschitz property assumed for the vector field f, there exist positive constants L_x, L_w, L'_x and L'_w such that:

$$\begin{aligned} &|f(x, u_1, \ldots, u_m, w) - f(x', u_1, \ldots, u_m, 0)| \\ &\qquad \leq L_x |x - x'| + L_w |w| \\ &\left| \frac{\partial V(x)}{\partial x} f(x, u_1, \ldots, u_m, w) - \frac{\partial V(x')}{\partial x} f(x', u_1, \ldots, u_m, 0) \right| \\ &\qquad \leq L'_x |x - x'| + L'_w |w| \end{aligned} \quad (5)$$

for all $x, x' \in \Omega_\rho$, $u_i \in U_i$, $i = 1, \ldots, m$ and $w \in W$.

III. Lyapunov-based economic MPC with asynchronous and delayed measurements

In this section, we consider the design of Lyapunov-based EMPC (LEMPC) for nonlinear systems subject to asynchronous and delayed measurements. We assume that the state of the system of Eq. 2, $x(t)$, is available at asynchronous time instants $\{t_{a \geq 0}\}$ which is a random increasing sequence of time and the interval between two consecutive time instants is not fixed. We also assume that there are delays involved in the measurements. In order to model delays in measurements, an auxiliary variable d_a is introduced to indicate the delay corresponding to the measurement received at time t_a, that is, at time t_a, the measurement $x(t_a - d_a)$ is received. In order to study the stability properties in a deterministic framework, we assume that there exists an upper bound T_m on the interval between two successive measurements (i.e., $\max_a \{t_{a+1} - t_a\} \leq T_m$) and an upper bound D on the delays (i.e., $d_a \leq D$). These assumptions are reasonable from a process control perspective. Because the delays are time-varying, it is possible that at a time instant t_a, the controllers may receive a measurement $x(t_a - d_a)$ which does not provide new information (i.e., $t_a - d_a < t_{a-1} - d_{a-1}$) and the maximum amount of time the system might operate in open-loop following t_a is $D + T_m - d_a$. This upper bound will be used in the formulation of LEMPC for systems subject to asynchronous and delayed measurements. The reader may refer to [12] for more discussion on the modeling of asynchronous and delayed measurements.

A. LEMPC implementation strategy

At each asynchronous sampling time, when a delayed measurement that contains new information is received, we propose to take advantage of the nominal system model and the manipulated inputs that have been applied to estimate the current system state from the delayed measurement. Based on the estimate of the current system state, an MPC optimization problem is solved in order to decide the optimal future input trajectory that will be applied until the next measurement containing new information is received. We introduce an LEMPC design which maximizes a cost function accounting for specific economic considerations. This LEMPC has two operation modes.

Specifically, we assume that from the initial time t_0 up to a specific time t', the LEMPC operates in the first operation mode to maximize the economic cost function while maintaining the closed-loop system state in the stability region Ω_ρ. In this operation mode, in order to account for the asynchronous and delayed measurement as well as the disturbances, we consider another region $\Omega_{\hat{\rho}}$ with $\hat{\rho} < \rho$. When a delayed measurement containing new information is received at a sampling time, the current system state is estimated. If the estimated current state is in the region $\Omega_{\hat{\rho}}$, the LEMPC maximizes the cost function within the region $\Omega_{\hat{\rho}}$; if the estimated current state is in the region $\Omega_\rho/\Omega_{\hat{\rho}}$, the LEMPC first drives the system state to the region $\Omega_{\hat{\rho}}$ and then maximizes the cost function within $\Omega_{\hat{\rho}}$. The relation between ρ and $\hat{\rho}$ will be characterized in Eq. 13 in Theorem 1.

After time t', the LEMPC operates in the second operation mode and calculates the inputs in a way that the state of the closed-loop system is driven to a neighborhood of the desired steady-state (i.e., $x = 0$) while taking into account asynchronous and delayed measurements.

The implementation strategy of the proposed LEMPC for systems subject to asynchronous and delayed measurements can be summarized as follows:

1. If a measurement $x(t_a - d_a)$ containing new information is received at t_a, the controller estimates the current system state, $\check{x}(t_a)$. Else, go to Step 5.
2. If $t_a < t'$, go to Step 3. Else, go to Step 4.
3. If $\check{x}(t_a) \in \Omega_{\hat{\rho}}$, go to Step 3.1. Else, go to Step 3.2.
   - 3.1. The controller maximizes the economic cost function within $\Omega_{\hat{\rho}}$. Go to Step 5.
   - 3.2. The controller drives the system state to the region $\Omega_{\hat{\rho}}$ and then maximizes the economic cost function within $\Omega_{\hat{\rho}}$. Go to Step 5.
4. The controller drives the system state to a small neighborhood of the origin.
5. Go to Step 1 ($a \longleftarrow a + 1$).

B. LEMPC formulation

When a measurement containing new information is received at t_a, the MPC is evaluated to obtain the future input trajectories based on the received system state value $x(t_a - d_a)$. Specifically, the optimization problem of the proposed LEMPC for systems subject to asynchronous and delayed measurements at t_a is as follows:

$$\max_{u_1,\ldots,u_m \in S(\Delta)} \int_{t_a}^{t_a+N\Delta} L(\check{x}(\tau), u_1(\tau), \ldots, u_m(\tau))d\tau \quad \text{(6a)}$$

$$\text{s.t. } \dot{\check{x}}(t) = f(\check{x}(t), u_1(t), \ldots, u_m(t), 0) \quad \text{(6b)}$$

$$u_i(t) = u_i^*(t),\ i = 1, \ldots, m,\ t \in [t_a - d_a, t_a) \quad \text{(6c)}$$

$$u_i(t) \in U_i,\ i = 1, \ldots, m,\ t \in [t_a, t_a + N\Delta) \quad \text{(6d)}$$

$$\check{x}(t_a - d_a) = x(t_a - d_a) \quad \text{(6e)}$$

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h_1(\hat{x}(t_a + l\Delta)), \ldots, h_m(\hat{x}(t_a + l\Delta)), 0),$$
$$\forall t \in [t_a + l\Delta, t_a + (l + 1)\Delta),\ l = 0, \ldots, N - 1 \quad \text{(6f)}$$

$$\hat{x}(t_a) = \check{x}(t_a) \quad \text{(6g)}$$

$$V(\check{x}(t)) \leq \hat{\rho},\ \forall t \in [t_a, t_a + N\Delta),$$
$$\text{if } t_a \leq t' \text{ and } V(\check{x}(t_a)) \leq \hat{\rho} \quad \text{(6h)}$$

$$V(\check{x}(t)) \leq V(\hat{x}(t)),\ \forall t \in [t_a, t_a + N_{Da}\Delta),$$
$$\text{if } t_a > t' \text{ or } \hat{\rho} < V(\check{x}(t_a)) \leq \rho \quad \text{(6i)}$$

where $\check{x}$ is the predicted trajectory of the system with control inputs calculated by this LEMPC, $u_i^*(t)$ with $i = 1, \ldots, m$ denotes the actual inputs that have been applied to the system, $x(t_a - d_a)$ is the received delayed measurement, $\hat{x}$ is the predicted trajectory of the system with the control inputs determined by $h(x)$ implemented in a sample-and-hold fashion, and N_{Da} is the smallest integer that satisfies $T_m + D - d_a \leq N_{Da}\Delta$. The optimal solution to this optimization problem is denoted by $u_i^{a,*}(t|t_a)$, $i = 1, \ldots, m$, which is defined for $t \in [t_a, t_a + N\Delta)$.

There are two types of calculations in the optimization problem of Eq. 6. The first type of calculation is to estimate the current state $\check{x}(t_a)$ based on the delayed measurement $x(t_a - d_a)$ and input values have been applied to the system from $t_a - d_a$ to t_a (constraints of Eqs. 6b, 6c and 6e). The second type of calculation is to evaluate the input trajectory of u_i ($i = 1, \ldots, m$) based on $\check{x}(t_a)$ while satisfying the input constraint of Eq. 6d and the stability constraints of Eqs. 6h-6i. Note that the length of N_{Da} depends on the current delay d_a, and thus, it may have different values at different time instants and has to be updated before solving the optimization problem of Eq. 6.

The manipulated inputs of the LEMPC of Eq. 6 for systems subject to asynchronous and delayed measurements are defined as follows:

$$u_j(t) = u_j^{a,*}(t|t_a),\ \forall t \in [t_a, t_{a+i}) \quad (7)$$

for all t_a such that $t_a - d_a > \max_{l<a} t_l - d_l$ and for a given t_a, the variable i denotes the smaller integer that satisfies $t_{a+i} - d_{a+i} > t_a - d_a$ and $j = 1, \ldots, m$.

C. Stability analysis

In this subsection, we present the stability properties of the proposed LEMPC of Eq. 6 in the presence of asynchronous and delayed measurements. In order to proceed, we need the following propositions.

Proposition 1 (c.f. [13]): Consider the systems:

$$\begin{aligned} \dot{x}_a(t) &= f(x_a(t), u_1(t), \ldots, u_m(t), w(t)) \\ \dot{x}_b(t) &= f(x_a(t), u_1(t), \ldots, u_m(t), 0) \end{aligned} \quad (8)$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$. There exists a $\mathcal{K}$ function $f_W(\cdot)$ such that:

$$|x_a(t) - x_b(t)| \leq f_W(t - t_0), \quad (9)$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and all $w(t) \in W$ with $f_W(\tau) = L_w\theta(e^{L_x\tau} - 1)/L_x$.

Proposition 1 provides an upper bound on the deviation of the state trajectory obtained using the nominal model, from the actual system state trajectory when the same control input trajectories are applied. Proposition 2 below bounds the difference between the magnitudes of the Lyapunov function of two different states in Ω_ρ.

Proposition 2 (c.f. [13]): Consider the Lyapunov function $V(\cdot)$ of the system of Eq. 2. There exists a quadratic function $f_V(\cdot)$ such that:

$$V(x) \leq V(\hat{x}) + f_V(|x - \hat{x}|) \tag{10}$$

for all $x, \hat{x} \in \Omega_\rho$ with $f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + M_v s^2$ where M_v is a positive constant.

Proposition 3 below ensures that if the nominal system controlled by $h(x)$ implemented in a sample-and-hold fashion and with open-loop state estimation starts in Ω_ρ, then it is ultimately bounded in $\Omega_{\rho_{\min}}$.

Proposition 3 (c.f. [13]): Consider the nominal sampled trajectory $\hat{x}(t)$ of the system of Eq. 2 in closed-loop for a controller $h(x)$, which satisfies the condition of Eq. 3, obtained by solving recursively:

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h_1(\hat{x}(t_k)), \ldots, h_m(\hat{x}(t_k)), 0) \tag{11}$$

where $t \in [t_k, t_{k+1})$ with $t_k = t_0 + k\Delta,\ k = 0, 1, \ldots$. Let $\Delta, \epsilon_s > 0$ and $\rho > \rho_s > 0$ satisfy:

$$-\alpha_3\left(\alpha_2^{-1}(\rho_s)\right) + L'_x M\Delta \leq -\epsilon_s/\Delta. \tag{12}$$

Then, if $\hat{x}(t_0) \in \Omega_\rho$ and $\rho_{\min} < \rho$ where $\rho_{\min} = \max\{V(x(t+\Delta)) : V(x(t)) \leq \rho_s\}$, the following inequality holds: $V(\hat{x}(t)) \leq V(\hat{x}(t_k)),\ \forall t \in [t_k, t_{k+1})$ and $V(\hat{x}(t_k)) \leq \max\{V(\hat{x}(t_0)) - k\epsilon_s, \rho_{\min}\}$.

Theorem 1 below provides sufficient conditions under which the LEMPC of Eq. 6 guarantees that the closed-loop system state is always bounded in Ω_ρ and is ultimately bounded in a small region containing the origin.

Theorem 1: Consider the system of Eq. 2 in closed-loop under the LEMPC design of Eq. 6 based on a controller $h(x)$ that satisfies the condition of Eq. 3. Let $\epsilon_s > 0$, $\Delta > 0$, $\rho > \hat{\rho} > 0$ and $\rho > \rho_s > 0$ satisfy the condition of Eq. 12 and satisfy:

$$\hat{\rho} \leq \rho - f_V(f_W(N_D\Delta)) \tag{13}$$

and

$$-N_R\epsilon_s + f_V(f_W(N_D\Delta)) + f_V(f_W(D)) < 0 \tag{14}$$

where f_W and f_V are defined in Propositions 1 and 2 respectively, N_D is the smallest integer satisfying $N_D\Delta \geq T_m + D$ and N_R is the smallest integer satisfying $N_R\Delta \geq T_m$. If $N \geq N_R$, $\hat{\rho} \geq \rho_s$, $x(t_0) \in \Omega_\rho$, $d_0 = 0$, then the closed-loop state $x(t)$ of the system of Eq. 1 is always bounded in Ω_ρ and is ultimately bounded in $\Omega_{\rho_a} \subset \Omega_\rho$ where $\rho_a = \rho_{\min} + f_V(f_W(N_D\Delta)) + f_V(f_W(D))$ with $\rho_{\min}$ defined in Proposition 3.

The detailed proof of Theorem 1 is provided in [14] and it is omitted here due to space limitations and our decision to include an application example in this manuscript.

D. Application to a chemical process example

Consider a well-mixed, non-isothermal continuous stirred tank reactor (CSTR) where an irreversible second-order exothermic reaction $A \rightarrow B$ takes place [15]. A is the reactant and B is the product. The feed to the reactor consists of pure A at flow rate F, temperature T_0 and molar concentration C_{A0}. Due to the non-isothermal nature of the reactor, a jacket is used to remove/provide heat to the reactor. The dynamic equations describing the behavior of the system, obtained through material and energy balances under standard modeling assumptions, are given below:

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - k_0 e^{\frac{-E}{RT}} C_A^2 \tag{15a}$$

$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{-\Delta H}{\sigma C_p} k_0 e^{\frac{-E}{RT}} C_A^2 + \frac{Q}{\sigma C_p V} \tag{15b}$$

where C_A denotes the concentration of the reactant A, T denotes the temperature of the reactor, Q denotes the rate of heat input/removal, V represents the volume of the reactor, $\Delta H, k_0$, and E denote the enthalpy, pre-exponential constant and activation energy of the reaction, respectively and C_p and σ denote the heat capacity and the density of the fluid in the reactor, respectively. The process model of Eq. 15 is numerically simulated using an explicit Euler integration method with integration step $h_c = 10^{-4}\ hr$. For a detailed description of this chemical process including the values of the process parameters, please refer to [14]. The process model has one unstable and two stable steady-states. The control objective is to regulate the process in a region around the unstable steady-state (C_{As}, T_s) to maximize the production rate of B. There are two manipulated inputs. One of the inputs is the concentration of A in the inlet to the reactor, C_{A0}, and the other manipulated input is the external heat input/removal, Q. The steady-state input values associated with the steady-state are denoted by C_{A0s} and Q_s, respectively. The process model of Eq. 15 belongs to the following class of nonlinear systems:

$$\dot{x}(t) = f(x(t)) + g_1(x(t))u_1(t) + g_2(x(t))u_2(t) + w(t)$$

where $x^T = [C_A - C_{As}\ T - T_s]$ is the state, $u_1 = C_{A0} - C_{A0s}$ and $u_2 = Q - Q_s$ are the inputs, $f = [f_1\ \ f_2]^T$ and $g_i = [g_{i1}\ g_{i2}]^T$ $(i = 1, 2)$ are vector functions. The inputs are subject to constraints as follows: $|u_1| \leq 3.5\ kmol/m^3$ and $|u_2| \leq 5 \times 10^5\ KJ/hr$. $w = [w_1\ w_2]^T$ is the bounded disturbance vector (Gaussian white noise with variances $\sigma_1 = 0.5\ kmol/m^3$ and $\sigma_2 = 10\ K$) with $|w_1| \leq 0.5\ kmol/m^3$ and $|w_2| \leq 10\ K$. The economic measure that we consider in this example is as follows [15]:

$$L(x, u_1, u_2) = \frac{1}{t_f}\int_0^{t_f} k_0 e^{-\frac{E}{RT(\tau)}} C_A^2(\tau) d\tau \tag{16}$$

where $t_f = 1\ hr$ is the final time of the simulation. This economic objective function corresponds to maximizing the average production rate over process operation for $t_f = 1\ hr$. We also consider that there is limitation on the amount of material which can be used over the period t_f. Specifically, the control input trajectory of u_1 should satisfy the following constraint:

$$\frac{1}{t_f}\int_0^{t_f} u_1(\tau)d\tau = 1\ kmol/m^3. \quad (17)$$

This constraint means that the average amount of u_1 during one period is fixed. For the sake of simplicity, we will refer to Eq. 17 as the integral constraint. In the simulations, we consider a quadratic Lyapunov function $V(x) = x^T P x$ with $P = \text{diag}([1\ 0.01])$. The LEMPC horizon is $N = 10$.

We assume that the state measurements of the process are available asynchronously at time instants $\{t_{a\geq 0}\}$ with an upper bound $T_m = 6\Delta$ on the maximum interval between two successive asynchronous state measurements, where Δ is the controller and sensor sampling time and is chosen to be $\Delta = 0.01\ hr = 36\ sec$. To model the time sequence $\{t_{a\geq 0}\}$, we use an upper bounded Poisson process. The Poisson process is defined by the number of events per unit time W. The interval between two successive concentration sampling times (events of the Poisson process) is given by $\Delta_a = \min\{-ln\chi/W, T_m\}$, where χ is a random variable with uniform probability distribution between 0 and 1. This generation ensures that $\max_a\{t_{a+1} - t_a\} \leq T_m$. In this example, W is chosen to be $W = 25$. A gaussian random process is used to generate the associated delay sequence $\{d_{a\geq 0}\}$ with $d_a \leq D$ while $D = 3\Delta$. To ensure that the integral constraint is satisfied through the period t_f, at every sampling time in which the LEMPC obtains the optimal control input trajectory, it utilizes the previously computed inputs u_1 to constrain the first step value of the control input trajectory u_1 at the current sampling time. Based on the cost function formulation, for maximization purposes, it is expected that C_A and T should be increased which results in the fact that at the beginning of the closed-loop simulation u_1 should rise to its maximum value and after a while it will go down to its lowest value to satisfy the integral constraint. We assume that the decrease of the Lyapunov function starts from the beginning of the simulation (i.e., $t' = 0$) for part of the system state (i.e., temperature). To maximize the production rate, we pick a temperature set-point near the boundary of the stability region ($T = 430\ K$), considering the constraints on the control input Q. Due to the fact that the first differential equation (C_A) in Eq 15 is input-to-state-stable (ISS) with respect to T, and the contractive constraint of Eq. 18h (see Eq. 18) ensures that the temperature converges to the set-point, the stability of the closed-loop system is guaranteed in the operating range of interest. To this end, we define $V_T(t_k) = (T(t_k) - 430)^2$. The LEMPC formulation for the chemical process example in question subject to asynchronous and delayed state measurements has the following form:

$$\max_{u_1,u_2\in S(\Delta)} \frac{1}{N\Delta}\int_{t_a}^{t_a+N\Delta} \left[k_0 e^{-\frac{E}{RT(\tau)}} C_A^2(\tau)\right] d\tau \quad (18a)$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t)) + \sum_{i=1}^{2} g_i(\tilde{x}(t))u_i^*(t),$$

$$\forall t \in [t_a - d_a, t_a) \quad (18b)$$

$$\dot{\check{x}}(t) = f(\check{x}(t)) + \sum_{i=1}^{2} g_i(\check{x}(t))u_i(t),$$

$$\forall t \in [t_a, t_a + N\Delta) \quad (18c)$$

$$u_1(t) \in g_\zeta, \forall t \in [t_a, t_a + N\Delta) \quad (18d)$$

$$\check{x}(t_a - d_a) = x(t_a - d_a) \quad (18e)$$

$$\check{x}(t) \in \Omega_{\hat{\rho}} \quad (18f)$$

$$u_i(t) \in U_i, i = 1, 2 \quad (18g)$$

$$V_T(t_a + (l+1)\Delta) \leq \beta V_T(t_a + l\Delta),$$

$$l = 0, \ldots, N_{Da} - 1 \quad (18h)$$

where $x(t_a)$ is the measurement of the process state at sampling time t_a and $\beta = 1/1.1 = 0.909$ and the constraint of Eq. 18d implies that the first N_{Da} steps value of u_1 should be chosen to satisfy the integral constraint where the explicit expression of g_ζ can be computed based on Eq. 17 and the magnitude constraint on u_1. The constraint of Eq. 18h forces the Lyapunov function, based on the temperature, to decrease for N_{Da} sampling times.

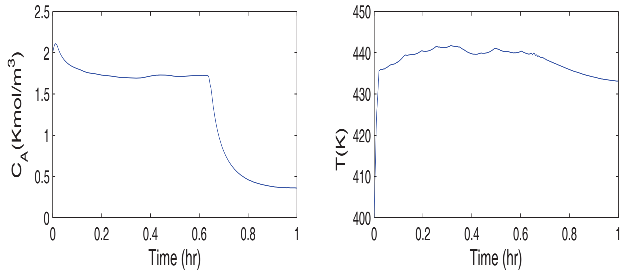

Fig. 1. State trajectories of the process under the LEMPC design of Eq. 18 for initial condition $(C_A(0), T(0)) = (2\ kmol/m^3, 400\ K)$ subject to asynchronous and delayed measurements and bounded disturbances.

The simulations were carried out using Java programming language in a Pentium 3.20 GHz computer. The optimization problems were solved using the open source interior point optimizer Ipopt [16]. Figures 1 and 2 show the state and manipulated input profiles, respectively, starting from the initial condition $(2\ kmol/m^3, 400\ K)$ under bounded process disturbances and subject to delayed and asynchronous measurement samplings. From these figures, we can see that u_1 goes up to its allowable maximum value to increase the reactant concentration as much as possible early on (due to the second-order dependence of the reaction rate on reactant concentration) and the temperature rises as fast as possible when the temperature initial condition is below $430\ K$ to maximize the reaction rate to maintain the maximum possible reaction rate. From these figures, we can also see that the practical stability of the closed-loop system is ensured in the presence of asynchronous and delayed measurements. This is because in the design of the LEMPC of Eq. 18, asynchronous and delayed measurements are taken explicitly into account.

Also, we have carried out a set of simulations to confirm that the application of the LEMPC design with the integral constraint on u_1 improves the economic objective function

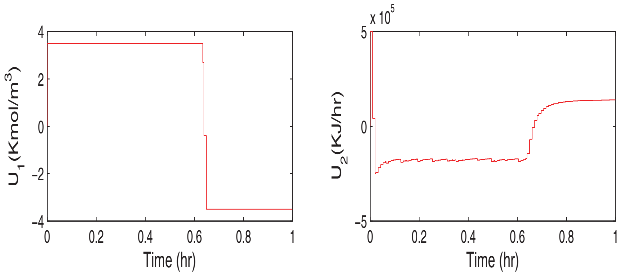

Fig. 2. Manipulated input trajectories under the LEMPC design of Eq. 18 for initial condition $(C_A(0), T(0)) = (2\ kmol/m^3, 400\ K)$ subject to asynchronous and delayed measurements and bounded disturbances.

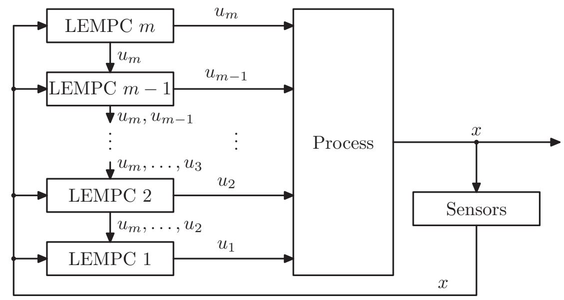

Fig. 3. Distributed LEMPC architecture.

compared to the case that the system operates at a steady-state satisfying the integral constraint. It should be mentioned that this comparison is performed under the case that there is no process disturbance and under synchronous state feedback sampling. This steady-state is computed by assuming that the reactant material amount is equally distributed in the interval $[0, t_f]$. To carry out this comparison, we have computed the total cost of each scenario based on the index as follows:

$$J = \frac{1}{t_M} \sum_{i=0}^{M} \left[k_0 e^{-\frac{E}{RT(t_i)}} C_A^2(t_i) \right]$$

where $t_0 = 0\ hr$, $t_M = 1\ hr$ and $M = 100$. To be consistent in this comparison, we set u_1 to a constant value over the simulation time, $t_f = 1\ hr$, such that it satisfies the integral constraint while letting u_2 be computed by the controller. By comparing the cost function values, we find that in the proposed LEMPC design via time-varying operation (starting from $(C_A, T) = (2\ kmol/m^3, 400\ K)$), the cost function achieves a higher value (19299.47) compared to the case of steady-state operation (17722.07) (i.e., equal in time distribution of the reactant). Also, by starting from $(C_A, T) = (2\ kmol/m^3, 440\ K)$, the cost function achieves a higher value (19459.67) compared to the case of steady-state operation (17852.85).

IV. Distributed LEMPC

As the number of manipulated inputs increases as it is the case in the context of control of large-scale chemical plants, the evaluation time of a centralized MPC may increase significantly. This may impede the ability of centralized MPC to carry out real-time calculations within the limits imposed by process dynamics and operating conditions. Moreover, a centralized control system for large-scale systems may be difficult to organize and maintain and is vulnerable to potential process faults. To overcome these issues, in this work, we propose to utilize a sequential distributed EMPC architecture as shown in Fig. 3. In this architecture, each set of the m sets of control inputs is calculated using an LEMPC. The distributed controllers are connected using one-directional communication network, evaluated in sequence. We will refer to the controller computing u_i associated with subsystem i as LEMPC i. In this section, we propose two different implementation strategies for the sequential distributed EMPC architecture and we assume that the state x of the system is sampled synchronously and the time instants at which we have state measurements are indicated by the time sequence $\{t_{k\geq 0}\}$ with $t_k = t_0 + k\Delta,\ k = 0, 1, \ldots$ where t_0 is the initial time and Δ is the sampling time.

A. Implementation strategy I

In this implementation strategy for the distributed EMPC architecture, all the distributed controllers are evaluated in sequence and once at each sampling time. Specifically, at a sampling time, t_k, when a measurement is received, the distributed controllers evaluate their future input trajectories in sequence starting from LEMPC m to LEMPC 1. Once a controller finishes evaluating its own future input trajectory, it sends its own future input trajectory and the future input trajectories it received to the next controller (i.e., LEMPC j sends input trajectories of u_i, $i = m, \ldots, j$, to LEMPC $j-1$).

This implementation strategy implies that LEMPC j, $j = m, \ldots, 2$, does not have any information about the values that u_i, $i = j-1, \ldots, 1$ will take when the optimization problem of LEMPC j is solved. In order to make a decision, LEMPC j, $j = m, \ldots, 2$ must assume trajectories for u_i, $i = j-1, \ldots, 1$, along the prediction horizon. To this end, the Lyapunov-based controller $h(x)$ is used. In order for the distributed EMPC to inherit the stability properties of the controller $h(x)$, each control input u_i, $i = 1, \ldots, m$ must satisfy a constraint that guarantees a given minimum contribution to the decrease rate of the Lyapunov function $V(x)$. Specifically, the proposed design of the LEMPC j, $j = 1, \ldots, m$, is based on the following optimization problem:

$$\max_{u_j \in S(\Delta)} \int_{t_k}^{t_{k+N}} L(\tilde{x}^j(\tau), u_1(\tau), \ldots, u_m(\tau)) d\tau \quad (19a)$$

$$\text{s.t. } \dot{\tilde{x}}^j(t) = f(\tilde{x}^j(t), u_1(t), \ldots, u_m(t), 0) \quad (19b)$$

$$u_i(t) = h_i(\tilde{x}^j(t_{k+l})),\ i = 1, \ldots, j-1, \quad \forall t \in [t_{k+l}, t_{k+l+1}),\ l = 0, ..., N-1 \quad (19c)$$

$$u_i(t) = u_i^*(t|t_k), i = j+1, \ldots, m \quad (19d)$$

$$u_j(t) \in U_j,\ i = 1, \ldots, m \quad (19e)$$

$$\tilde{x}^j(t_k) = x(t_k) \quad (19f)$$

$$V(\tilde{x}^j(t)) \leq \tilde{\rho},\ \forall t \in [t_k, t_{k+N}), \quad \text{if } t_k \leq t' \text{ and } V(x(t_k)) \leq \tilde{\rho} \quad (19g)$$

$$\frac{\partial V(x(t_k))}{\partial x} f(x(t_k), u_1^n(t_k), \ldots, u_{j-1}^n(t_k), u_j(t_k), \ldots, u_m(t_k)) \leq \frac{\partial V(x(t_k))}{\partial x} f(x(t_k), u_1^n(t_k), \ldots, u_j^n(t_k),$$

$$u_{j+1}(t_k), \ldots, u_m(t_k)),$$
$$\text{if } t_k > t' \text{ or } \tilde{\rho} < V(x(t_k)) \leq \rho \quad (19h)$$

where $\tilde{x}^j$ is the predicted trajectory of the nominal system with u_i, $i = j+1, \ldots, m$, the input trajectory computed by the LEMPC controllers of Eq. 19 evaluated before LEMPC j, u_i, $i = 1, \ldots, j-1$, the corresponding elements of $h(x)$ applied in a sample-and-hold fashion, $u_i^*(t|t_k)$ denotes the future input trajectory of u_i obtained by LEMPC i of the form of Eq. 19, and $u_i^n(t_k)$, $i = 1, \ldots, m$, are inputs determined by $h_i(x(t_k))$ (i.e., $u_i^n(t_k) = h_i(x(t_k))$). The optimal solution to the optimization problem of Eq. 19 is denoted $u_j^*(t|t_k)$ which is defined for $t \in [t_k, t_{k+N})$. The relation between $\tilde{\rho}$ and ρ is characterized in Theorem 2 below.

In the optimization problem of Eq. 19, the constraint of Eq. 19g is only active when $x(t_k) \in \Omega_{\tilde{\rho}}$ in the first operation mode and is incorporated to ensure that the predicted state evolution of the closed-loop system is maintained in the region $\Omega_{\tilde{\rho}}$ (thus, the actual state of the closed-loop system is in the stability region Ω_ρ). Due to the fact that all of the controllers receive state feedback $x(t_k)$ at sampling time t_k, all of the distributed controller operate in the same operation mode by verifying whether $V(x(t_k)) \leq \tilde{\rho}$; the constraint of Eq. 19h is only active in the second operation mode or when $\tilde{\rho} < V(x(t_k)) \leq \rho$ in the first operation mode. This constraint guarantees that the contribution of input u_j to the decrease rate of the time derivative of the Lyapunov function $V(x)$ at the initial time (i.e., t_k), if $u_j = u_j^*(t_k|t_k)$ is applied, is bigger than or equal to the value obtained when $u_j = h_j(x(t_k))$ is applied.

The manipulated inputs of the proposed distributed control design from time t_k to t_{k+1} ($k = 0, 1, 2, \ldots$) are applied in a receding horizon scheme as follows:

$$u_i(t) = u_i^*(t|t_k), \; i = 1, \ldots, m, \; \forall t \in [t_k, t_{k+1}). \quad (20)$$

Theorem 2 below provides sufficient conditions under which the LEMPC of Eq. 19 guarantees that the state of the closed-loop system is always bounded in Ω_ρ and is ultimately bounded in a small region containing the origin.

Theorem 2: Consider the system of Eq. 2 in closed-loop under the distributed LEMPC design of Eq. 19 based on a controller $h(x)$ that satisfies the conditions of Eq. 3. Let $\epsilon_w > 0$, $\Delta > 0$, $\rho > \tilde{\rho} > 0$ and $\rho > \rho_s > 0$ satisfy:

$$\tilde{\rho} \leq \rho - f_V(f_W(\Delta)) \quad (21)$$

and

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + L'_x M\Delta + L'_w\theta \leq -\epsilon_w/\Delta. \quad (22)$$

If $x(t_0) \in \Omega_\rho$, $\rho_s \leq \tilde{\rho}$, $\rho_{\min} \leq \rho$ and $N \geq 1$, then the state $x(t)$ of the closed-loop system is always bounded in Ω_ρ and is ultimately bounded in $\Omega_{\rho_{\min}}$ with $\rho_{\min}$ defined in Proposition 3 .

Proof: The proof consists of three parts. We first prove that the optimization problem of Eq. 19 is feasible for all states $x \in \Omega_\rho$. Subsequently, we prove that, in the first operation mode, under the LEMPC design of Eq. 19, the closed-loop state of the system of Eq. 2 is always bounded in Ω_ρ. Finally, we prove that, in the second operation mode, under the LEMPC of Eq. 19, the closed-loop state of the system of Eq. 2 is ultimately bounded in $\rho_{\min}$.

Part 1: When $x(t)$ is maintained in Ω_ρ (which will be proved in Part 2), the feasibility of the DEMPC of Eq. 19 follows because input trajectory $u_j(t)$, $j = 1, \ldots, m$, such that $u_j(t) = h_j(x(t_{k+q}))$, $\forall t \in [t_{k+q}, t_{k+q+1})$ with $q = 0, \ldots, N-1$ is a feasible solution to the optimization problem of Eq. 19 since such trajectory satisfy the input constraint of Eq. 19e and the Lyapunov-based constraints of Eqs. 19g and 19h. This is guaranteed by the closed-loop stability property of the Lyapunov-based controller $h(x)$; the reader may refer to [17] for more detailed discussion on the stability property of the Lyapunov-based controller $h(x)$.

Part 2: We assume that the LEMPC of Eq. 19 operates in the first operation mode. We prove that if $x(t_k) \in \Omega_{\tilde{\rho}}$, then $x(t_{k+1}) \in \Omega_\rho$; and if $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$, then $V(x(t_{k+1})) < V(x(t_k))$ and in finite steps, the state converges to $\Omega_{\tilde{\rho}}$ (i.e., $x(t_{k+j}) \in \Omega_{\tilde{\rho}}$ where j is a finite positive integer).

When $x(t_k) \in \Omega_{\tilde{\rho}}$, from the constraint of Eq. 19g, we obtain that $\tilde{x}^1(t_{k+1}) \in \Omega_{\tilde{\rho}}$. By Propositions 1 and 2, we obtain the following inequality:

$$V(x(t_{k+1})) \leq V(\tilde{x}^1(t_{k+1})) + f_V(f_W(\Delta)). \quad (23)$$

Note that LEMPC 1 has access to all of the optimal input trajectories of the other distributed controllers evaluated before it. Since $V(\tilde{x}^1(t_{k+1})) \leq \tilde{\rho}$, if the condition of Eq. 21 is satisfied, we can conclude that:

$$x(t_{k+1}) \in \Omega_\rho.$$

When $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$, from the constraint of Eq. 19h and the condition of Eq. 3, we can obtain:

$$\begin{aligned}
&\frac{\partial V(x(t_k))}{\partial x} f(x(t_k), u_1^*(t_k|t_k), \ldots, u_m^*(t_k|t_k), 0)\\
&\leq \frac{\partial V(x(t_k))}{\partial x} f(x(t_k), h_1(x(t_k)), u_2^*(t_k|t_k), \ldots, u_m^*(t_k|t_k), 0)\\
&\leq \cdots\\
&\leq \frac{\partial V(x(t_k))}{\partial x} f(x(t_k), h_1(x(t_k)), \ldots, h_m(x(t_k)), 0)\\
&\leq -\alpha_3(|x(t_k)|).
\end{aligned} \quad (24)$$

The time derivative of the Lyapunov function along the actual system state $x(t)$ for $t \in [t_k, t_{k+1})$ can be written as follows:

$$\dot{V}(x(t)) = \frac{\partial V(x(t))}{\partial x} f(x(t), u_1^*(t_k|t_k), \ldots, u_m^*(t_k|t_k), w(t)) \quad (25)$$

Adding and subtracting $\frac{\partial V(x(t_k))}{\partial x} f(x(t), u_1^*(t_k|t_k), \ldots,$ $u_m^*(t_k|t_k), 0)$ to/from the above equation and accounting for Eq. 24, we have:

$$\begin{aligned}
&\dot{V}(x(t)) \leq -\alpha_3(|x(t_k)|)\\
&+ \frac{\partial V(x(t))}{\partial x} f(x(t), u_1^*(t_k|t_k), \ldots, u_m^*(t_k|t_k), w(t))\\
&- \frac{\partial V(x(t_k))}{\partial x} f(x(t), u_1^*(t_k|t_k), \ldots, u_m^*(t_k|t_k), 0)
\end{aligned} \quad (26)$$

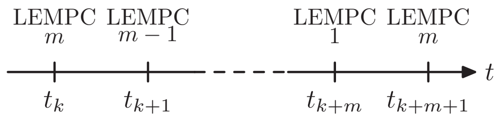

Fig. 4. Distributed controller evaluation sequence.

Due to the fact that the disturbance is bounded (i.e., $|w| \leq \theta$) and the Lipschitz properties of Eq. 5, we can write:

$$\dot{V}(x(t)) \leq -\alpha_3(\alpha_2^{-1}(\rho_s)) + L'_x|x(t) - x(t_k)| + L_w\theta. \quad (27)$$

Taking into account Eq. 4 and the continuity of $x(t)$, the following bound can be written for all $\tau \in [t_k, t_{k+1})$

$$|x(\tau) - x(t_k)| \leq M\Delta. \quad (28)$$

Since $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$, it can be concluded that $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$. Thus, we can write for $t \in [t_k, t_{k+1})$

$$\dot{V}(x(t)) \leq -\alpha_3(\alpha_2^{-1}(\rho_s)) + L'_x M\Delta + L_w\theta. \quad (29)$$

If the condition of Eq. 22 is satisfied, then there exists $\epsilon_w > 0$ such that the following inequality holds for $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$:

$$\dot{V}(x(t)) \leq -\epsilon_w/\Delta, \ \forall t = [t_k, t_{k+1}).$$

Integrating this bound on $t \in [t_k, t_{k+1})$, we obtain that:

$$\begin{aligned} V(x(t_{k+1})) &\leq V(x(t_k)) - \epsilon_w \\ V(x(t)) &\leq V(x(t_k)), \ \forall t \in [t_k, t_{k+1}) \end{aligned} \quad (30)$$

for all $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$. Using Eq. 30 recursively, it is proved that, if $x(t_k) \in \Omega_\rho/\Omega_{\tilde{\rho}}$, the state converges to $\Omega_{\tilde{\rho}}$ in a finite number of sampling times without leaving Ω_ρ.

Part 3: We assume that the DEMPC of Eq. 19 operates in the second operation mode. We prove that if $x(t_k) \in \Omega_\rho$, then $V(x(t_{k+1})) \leq V(x(t_k))$ and the system state is ultimately bounded in an invariant set $\Omega_{\rho_{\min}}$. Following the similar steps as in Part 2, we can derive that the inequality of Eq. 30 hold for all $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$. Using this result recursively, it is proved that, if $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$, the state converges to Ω_{ρ_s} in a finite number of sampling times without leaving Ω_ρ. Once the state converges to $\Omega_{\rho_s} \subseteq \Omega_{\rho_{\min}}$, it remains inside $\Omega_{\rho_{\min}}$ for all times. This statement holds because of the definition of $\rho_{\min}$. This proves that the closed-loop system under the LEMPC of Eq. 19 is ultimately bounded in $\Omega_{\rho_{\min}}$. ■

B. Implementation strategy II

In the implementation strategy introduced in the previous subsection, the evaluation time of the distributed LEMPC at a sampling time is the summation of the evaluation times of all the distributed controllers. For applications in which a small sampling time needs to be used and fast controller evaluation is required, we may distribute the evaluation of the distributed controllers into multiple sampling periods. In this implementation strategy, the distributed controllers are evaluated in sequence but over several sampling times and only one controller is evaluated at each sampling time. Figure 4 shows a possible evaluation sequence of the distributed controllers in this implementation strategy. In Fig. 4, at t_k, LEMPC m is evaluated and it sends the input trajectories of u_m to LEMPC $m-1$; at t_{k+1}, LEMPC $m-1$ is evaluated and it sends u_m and u_{m-1} to LEMPC $m-2$; from time t_{k+2} to t_{k+m}, LEMPC $m-2$ to LEMPC 1 are evaluated in sequence and one complete distributed control system evaluation cycle is carried out. Another controller evaluation cycle starts at t_{k+m+1} with the evaluation of LEMPC m again. In order to guarantee the closed-loop stability of this implementation strategy, the design of the distributed LEMPC of Eq. 19 needs to be modified to account for the multiple sampling time evaluation cycle; details are omitted due to space limitations.

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