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$\mathscr{H}_\infty$ Control of Discrete-Time Switched Itô Stochastic Systems Via Dynamic Output Feedback

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Abstract— **This paper is concerned with the $\mathscr{H}_\infty$ dynamic output feedback (DOF) control problem for discrete-time switched Itô stochastic hybrid systems. By applying the average dwell time method and the piecewise Lyapunov function technique, a sufficient condition is first proposed, which guarantees the closed-loop switched stochastic system to be mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance. Then the solvability condition for the $\mathscr{H}_\infty$ DOF control is also established, by which the DOF controller can be found by solving a set of linear matrix inequalities (LMIs).**

I. INTRODUCTION

Switched systems have attracted much attention in the past decade due to the fact that they have extensive applications, such as power systems, transmission and stepper motors, constrained robotics, and automated high ways. Typically, a switched system consists of a number of subsystems, and a switching law, which defines a specific subsystem being activated during a certain interval of time. The switching rule in such systems is usually considered to be arbitrary, and if the switching signals are governed by stochastic processes, the corresponding system is termed as jump systems (e.g., Markovian jump systems [7]). The motivation to study such systems is mainly in twofold. Firstly, from the practical application point of view, switching among different system structures is an essential feature of many real-world systems. Secondly, from the control point of view, multi-controller switching provides an effective mechanism to cope with complex systems and/or systems with large uncertainties. Switched systems have been studied in a large number of papers, such as, stability and stabilization [1], [2], [3], [4], [8]; $\mathscr{H}_\infty$ control problem [10]; and model reduction [11].

Recently, there is enormous growth of interest in using the dwell time approach to deal with the switched systems [3], [8], [10], [11]. To mention a few, Hespanha and Morse [3] investigated the stability of switched systems with average dwell time; Sun *et al*. [8] used the average dwell time approach to study the exponential stability and $\mathscr{L}_2$-gain for delay switched systems; Wu *et al*. [10] considered the stability, stabilization and $\mathscr{H}_\infty$ control of switched stochastic systems; Wu and Zheng [11] applied this approach to investigate the weighted $\mathscr{H}_\infty$ model reduction for linear switched systems with time-varying delay.

On the other hand, stochastic systems play an important role in many branches of science and engineering applications, thus have been received much attention during the past decades. Many results reported on stochastic systems can be found in the literature, see, e.g., [5], [6], [9], [12] and references therein. Recently, there are some results reported on the stochastic systems with Markovian switching; see, for example, Niu *et al.* [5] investigated the sliding mode control for Itô stochastic systems with Markovian switching; Wang *et al.* [9] investigate the stabilization of bilinear uncertain time-delay stochastic systems with Markovian jumping parameters; Xu and Chen [12] study the robust $\mathscr{H}_\infty$ control problem for uncertain discrete-time stochastic bilinear systems with Markovian switching. The above-mentioned results are all based on the Markovian switching. When the switching signal is arbitrary, the results should be very different. This motivates us to study some interesting topics on stochastic systems whose parameters operate by an arbitrary switching signal, that is, the switched stochastic systems. These research should be interesting and challenging since they integrate the switched hybrid systems into that of the stochastic systems, and thus theoretically and practically significant.

In this paper, we shall investigate the mean-square exponential stability and the $\mathscr{H}_\infty$ dynamic output feedback (DOF) control problem control problems for discrete-time switched stochastic systems. The average dwell time approach combined with the piecewise Lyapunov function technique is applied to derive the main results. The advantage of using this approach to the switched system is mainly in twofold. Firstly, this approach uses a mode-dependent Lyapunov function, which avoids some conservativeness caused by using a common Lyapunov function for all the subsystems. At this point, the present approach is superior than the quadratic approach (a common Lyapunov function approach) for the switched systems. The other advantage of using the present approach is that the obtained result is not just an asymptotic stability condition, but an exponential one. Specifically, the problems to be studied can be formulated as:

1. *Stability Analysis.* Propose a condition guaranteeing the mean-square exponential stability of the discrete-time switched stochastic system.
2. *$\mathscr{H}_\infty$ DOF Control.* Design a DOF controller such that the closed-loop system is mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance.

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II. System Description and Preliminaries

Consider a discrete-time nonlinear switched stochastic system with time-delays, which can be described by the following dynamical equation:

$$\begin{aligned} x(k+1) &= A(\alpha_k)x(k) + A_d(\alpha_k)x(k-d(k)) \\ &\quad +A_\tau(\alpha_k)f\left(x(k-\tau)\right) + B_u(\alpha_k)u(k) + B_\upsilon(\alpha_k)\upsilon(k) \\ &\quad + \left[C(\alpha_k)x(k) + C_d(\alpha_k)x(k-d(k)) + D_\upsilon(\alpha_k)\upsilon(k)\right]\omega(k), \\ z(k) &= L(\alpha_k)x(k), \\ x(\theta) &= \phi(\theta), \quad -\max\{\tau, d_2\} < \theta \leq 0, \end{aligned} \tag{1}$$

for $k = 1, 2, \ldots$, where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ represents the control input; $\upsilon(k) \in \mathbb{R}^p$ is the noise signal that belongs to $\ell_2[0, +\infty)$; $z(k) \in \mathbb{R}^q$ is the controlled output; $\omega(k)$ is a zero-mean real scalar process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_k)_{k\in\mathbb{N}}$ of σ-algebras $\mathcal{F}_k \subset \mathcal{F}$ generated by $(\omega(k))_{k\in\mathbb{N}}$. The stochastic process $\{\omega(k)\}$ is independent, which is assumed to satisfy

$$\mathbf{E}\{\omega(k)\} = 0, \quad \mathbf{E}\{\omega^2(k)\} = \delta, \quad k = 0, 1, \ldots$$

where $\delta > 0$ is a known scalar. In addition, $\phi(\theta), \ -\max\{\tau, d_2\} < \theta \leq 0$ are the initial conditions; $\{(A(\alpha_k), A_d(\alpha_k), A_\tau(\alpha_k), B_u(\alpha_k), B_\upsilon(\alpha_k), C(\alpha_k), C_d(\alpha_k), D_\upsilon(\alpha_k), L(\alpha_k)) : \alpha_k \in \mathcal{N}\}$ is a family of matrices parameterized by an index set $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\alpha_k : \mathbb{Z}^+ \to \mathcal{N}$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $\mathcal{N}$. At an arbitrary discrete time k, the value of α_k, denoted by α for simplicity, might depend on k or $x(k)$, or both, or may be generated by any other hybrid scheme. We assume that the sequence of subsystems in switching signal α_k is unknown *a priori*, but its instantaneous value is available in real time. For the switching time sequence $k_0 < k_1 < k_2 < \cdots$ of switching signal α, the holding time between $[k_l, k_{l+1}]$ is called the dwell time of the currently engaged subsystem, where $l \in \mathcal{N}$. The delay $d(k)$ satisfying $1 \leq d_1 \leq d(k) \leq d_2$, where d_1 and d_2 are constant positive scalars representing the minimum and maximum delays, respectively. In addition, $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is nonlinear function, which satisfies the following assumption.

Assumption 1: For the nonlinear function $f(\cdot)$, there exist matrices Π_1 and Π_2 such that

$$\left(f(x) - \Pi_1 x\right)^T \left(f(x) - \Pi_2 x\right) \leq 0, \quad x \in \mathbb{R}^n. \tag{2}$$

As is well known, the state feedback controller design requires that the current system state is fully accessible. While in practical applications, it is usually either not accessible or hard to access. In such cases, one option is to assume availability of a measured output signal given by

$$\begin{aligned} y(k) &= E(\alpha_k)x(k) + E_d(\alpha_k)x(k-d(k)) \\ &\quad +E_\tau(\alpha_k)f\left(x(k-\tau)\right) + F_\upsilon(\alpha_k)\upsilon(k) + [G(\alpha_k)x(k) \\ &\quad +G_d(\alpha_k)x(k-d(k)) + H_\upsilon(\alpha_k)\upsilon(k)]\,\omega(k), \end{aligned} \tag{3}$$

where $y(k) \in \mathbb{R}^r$ is the measured output; $E(\alpha_k)$, $E_d(\alpha_k)$, $E_\tau(\alpha_k)$, $F_\upsilon(\alpha_k)$, $G(\alpha_k)$, $G_d(\alpha_k)$ and $H_\upsilon(\alpha_k)$ are real constant matrices.

Here, we are interested in designing a DOF controller of general structure described by

$$\begin{aligned} \hat{x}(k+1) &= A_c(\alpha_k)\hat{x}(k) + B_c(\alpha_k)y(k), \\ u(k) &= C_c(\alpha_k)\hat{x}(k), \end{aligned} \tag{4}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the controller state; $A_c(\alpha_k)$, $B_c(\alpha_k)$ and $C_c(\alpha_k)$ are matrices to be determined.

Augmenting the model of (1) to include the state of controller (4), we obtain the closed-loop system as

$$\begin{aligned} \xi(k+1) &= \tilde{A}(\alpha_k)\xi(k) + \tilde{A}_d(\alpha_k)M\xi(k-d(k)) \\ &\quad +\tilde{A}_\tau(\alpha_k)M\tilde{f}\left(x(k-\tau)\right) + \tilde{B}_\upsilon(\alpha_k)\upsilon(k) \\ &\quad + \Big[\tilde{C}(\alpha_k)M\xi(k) + \tilde{C}_d(\alpha_k)M\xi(k-d(k)) \\ &\quad +\tilde{D}_\upsilon(\alpha_k)\upsilon(k)\Big]\,\omega(k), \\ z(k) &= \tilde{L}(\alpha_k)\xi(k), \\ x(\theta) &= \phi(\theta), \quad -\max\{\tau, d_2\} < \theta \leq 0, \end{aligned} \tag{5}$$

where $\xi(k) \triangleq \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T$, $\tilde{f}\left(x(k-\tau)\right) \triangleq \begin{bmatrix} f^T\left(x(k-\tau)\right) & f^T\left(\hat{x}(k-\tau)\right) \end{bmatrix}^T$ and

$$\begin{aligned} \tilde{A}(\alpha_k) &\triangleq \begin{bmatrix} A(\alpha_k) & B_u(\alpha_k)C_c(\alpha_k) \\ B_c(\alpha_k)E(\alpha_k) & A_c(\alpha_k) \end{bmatrix}, \\ \tilde{A}_d(\alpha_k) &\triangleq \begin{bmatrix} A_d(\alpha_k) \\ B_c(\alpha_k)E_d(\alpha_k) \end{bmatrix}, \tilde{D}_\upsilon(\alpha_k) \triangleq \begin{bmatrix} D_\upsilon(\alpha_k) \\ B_c(\alpha_k)H_\upsilon(\alpha_k) \end{bmatrix}, \\ \tilde{A}_\tau(\alpha_k) &\triangleq \begin{bmatrix} A_\tau(\alpha_k) \\ B_c(\alpha_k)E_\tau(\alpha_k) \end{bmatrix}, \tilde{B}_\upsilon(\alpha_k) \triangleq \begin{bmatrix} B_\upsilon(\alpha_k) \\ B_c(\alpha_k)F_\upsilon(\alpha_k) \end{bmatrix}, \\ \tilde{C}(\alpha_k) &\triangleq \begin{bmatrix} C(\alpha_k) \\ B_c(\alpha_k)G(\alpha_k) \end{bmatrix}, \ \tilde{C}_d(\alpha_k) \triangleq \begin{bmatrix} C_d(\alpha_k) \\ B_c(\alpha_k)G_d(\alpha_k) \end{bmatrix}, \\ \tilde{L}(\alpha_k) &\triangleq \begin{bmatrix} L(\alpha_k) & 0 \end{bmatrix}, \ M \triangleq \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned} \tag{6}$$

Remark 1: For each possible value α_k, we will denote it by i, that is, $\alpha_k = i$, $i \in \mathcal{N}$. Corresponding to the switching signal α_k, we have the switching sequence $\{(i_0, k_0), (i_1, k_1), \ldots, (i_l, k_l), \ldots, \mid i_l \in \mathcal{N}, \ l = 0, 1, \ldots\}$ with $k_0 = 0$, which means that the i_lth subsystem is activated when $k \in [k_l, k_{l+1})$.

Definition 1: For switching signal and any $k_i > k_j > k_0$, let $N_{\alpha_k}(k_j, k_i)$ be the switching numbers of α_k over the interval $[k_j, k_i]$. If for any given $N_0 > 0$ and $T_a > 0$, we have $N_{\alpha_k}(k_j, k_i) \leq N_0 + (k_i - k_j)/T_a$, then T_a and N_0 are called average dwell time and the chatter bound, respectively.

Definition 2: The equilibrium $x^* = 0$ of the discrete-time switched time-delay system in (1) with $u(k) = 0$ and $\upsilon(k) = 0$ is said to be mean-square exponentially stable under α_k if the solution $x(k)$ satisfies

$$\mathbf{E}\left\{\|x(k)\|\right\} \leq \eta\rho^{(k-k_0)}\left\|x(k_0)\right\|_{C^1}, \quad \forall k \geq k_0,$$

for constants $\eta \geq 1$ and $0 < \rho < 1$, and

$$\|x(k_0)\|_{C^1} \triangleq \underbrace{\left\{\|x(k+\theta)\|, \|\varsigma(k+\theta)\|, \|f(\varsigma(k+\theta))\|\right\}}_{\sup_{-\max\{\tau,d_2\}<\theta\leq 0}},$$

where $\varsigma(\theta) \triangleq x(\theta+1) - x(\theta)$.

Definition 3: For $0 < \beta < 1$ and $\gamma > 0$, the system in (1) with $u(t) = 0$ is said to be mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance γ under α_k, if it is mean-square exponentially stable with $\upsilon(t) = 0$, and under zero initial condition, that is, $x(\theta)=\phi(\theta)=0,\ -\max\{\tau,d_2\}<\theta\leq 0$, it holds for all nonzero $v(t)\in\ell_2[0,\infty)$ that

$$\mathbf{E}\left\{\sum_{s=k_0}^{\infty}\beta^s z^T(s)z(s)\right\}<\gamma^2\sum_{s=k_0}^{\infty}\omega^T(s)\omega(s). \tag{7}$$

III. Main Results

A. Performance Analysis

In this section, we will investigate weighted $\mathscr{H}_\infty$ performance for the closed-loop system in (5).

Theorem 1: For given constants $\beta>0$ and $\gamma>0$, supposed that there exist matrices $P(i)>0$, $Q(i)>0$ and $R(i)>0$ such that matrix inequality (8) (shown at the top of the next page) holds for $i\in\mathcal{N}$, Then the closed-loop system in (5) is mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance level γ for any switching signal with average dwell time satisfying $T_a>T_a^*=\text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$, where $\mu\geq 1$ satisfies

$$P(i)\leq\mu P(j),\quad Q(i)\leq\mu Q(j),\quad R(i)\leq\mu R(j). \tag{9}$$

In (8), $\Phi_{11}(i)\triangleq-\beta P(i)+\beta(d_2-d_1+1)Q(i)-\digamma_1$ with $\digamma_1\triangleq\text{diag}(H_1,H_1)$, $\digamma_2\triangleq\text{diag}(H_2,H_2)$, $H_1\triangleq\frac{\Pi_1^T\Pi_2+\Pi_2^T\Pi_1}{2}$ and $H_2\triangleq\frac{\Pi_1^T+\Pi_2^T}{2}$.

Proof. Choose a Lyapunov function of the form:

$$\begin{cases} V(\xi_k,\alpha_k)\triangleq\sum_{i=1}^{4}V_i(\xi_k,\alpha_k),\\ V_1(\xi_k,\alpha_k)\triangleq\xi^T(k)P(\alpha_k)\xi(k),\\ V_2(\xi_k,\alpha_k)\triangleq\sum_{l=k-d(k)}^{k-1}\beta^{k-l}\xi^T(l)Q(\alpha_k)\xi(l),\\ V_3(\xi_k,\alpha_k)\triangleq\sum_{s=-d_2+1}^{-d_1}\sum_{l=k+s}^{k-1}\beta^{k-l}\xi^T(l)Q(\alpha_k)\xi(l),\\ V_4(\xi_k,\alpha_k)\triangleq\sum_{l=k-\tau}^{k-1}\beta^{k-l}f^T(\xi(l))R(\alpha_k)f(\xi(l)), \end{cases} \tag{10}$$

where $P(\alpha_k)>0$, $Q(\alpha_k)>0$ and $R(\alpha_k)>0$ are real matrices to be determined.

For $k\in[k_l,k_{l+1})$, we define $\mathbf{E}\{\Delta V_j(\xi_k,\alpha_k)\}\triangleq\mathbf{E}\{V_j(\xi_{k+1},\alpha_k)-V_j(\xi_k,\alpha_k)\}$, $j=1,2,3,4$, thus we have $\mathbf{E}\{\Delta V(\xi_k,\alpha_k)\}=\sum_{i=1}^4\mathbf{E}\{\Delta V_i(\xi_k,\alpha_k)\}$ with

$$\begin{aligned}\mathbf{E}\{\Delta V_1(\xi_k,\alpha_k)\}=&\mathbf{E}\Big\{\Big[\tilde{A}(\alpha_k)\xi(k)+\tilde{A}_d(\alpha_k)M\xi(k-d(k))\\ &+\tilde{A}_\tau(\alpha_k)Mf(\xi(k-\tau))\Big]^T P(\alpha_k)\Big[\tilde{A}(\alpha_k)\xi(k)\\ &+\tilde{A}_d(\alpha_k)M\xi(k-d(k))+\tilde{A}_\tau(\alpha_k)Mf(\xi(k-\tau))\Big]\\ &+\Big[\tilde{C}(\alpha_k)M\xi(k)+\tilde{C}_d(\alpha_k)M\xi(k-d(k))\Big]^T\delta P(\alpha_k)\\ &\times\Big[\tilde{C}(\alpha_k)M\xi(k)+\tilde{C}_d(\alpha_k)M\xi(k-d(k))\Big]\\ &-\xi^T(k)P(\alpha_k)\xi(k)\Big\},\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{E}\{\Delta V_2(\xi_k,\alpha_k)\}\leq&\mathbf{E}\Big\{-(1-\beta)\sum_{l=k-d(k)}^{k-1}\beta^{k-l}\xi^T(l)Q(\alpha_k)\xi(l)\\ &+\sum_{l=k+1-d_2}^{k-d_1}\beta^{k+1-l}\xi^T(l)Q(\alpha_k)\xi(l)+\beta\xi^T(k)Q(\alpha_k)\xi(k)\\ &-\beta^{d_2+1}\xi^T(k-d(k))Q(\alpha_k)\xi(k-d(k))\Big\},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{E}\{\Delta V_3(\xi_k,\alpha_k)\}=&\\ &\mathbf{E}\Big\{-(1-\beta)\sum_{s=-d_2+1}^{-d_1}\sum_{l=k+s}^{k-1}\beta^{k-l}\xi^T(l)Q(\alpha_k)\xi(l)\\ &+\beta(d_2-d_1)\xi^T(k)Q(\alpha_k)\xi(k)\\ &-\sum_{l=k+1-d_2}^{k-d_1}\beta^{k+1-l}\xi^T(l)Q(\alpha_k)\xi(l)\Big\},\end{aligned} \tag{13}$$

$$\begin{aligned}\mathbf{E}\{\Delta V_4(\xi_k,\alpha_k)\}\leq&\\ &\mathbf{E}\Big\{-(1-\beta)\sum_{l=k-\tau}^{k-1}\beta^{k-l}f^T(\xi(l))R(\alpha_k)f(\xi(l))\\ &+\beta f^T(\xi(k))R(\alpha_k)f(\xi(k))\\ &-\beta^{\tau+1}f^T(\xi(k-\tau))R(\alpha_k)f(\xi(k-\tau))\Big\},\end{aligned} \tag{14}$$

Moreover, Assumption 1 gives

$$\mathbf{E}\left\{\begin{bmatrix}\xi^T(k) & f^T(\xi)\end{bmatrix}\begin{bmatrix}\digamma_1 & -\digamma_2\\ \star & I\end{bmatrix}\begin{bmatrix}\xi(k)\\ f(\xi)\end{bmatrix}\right\}\leq 0. \tag{15}$$

where $\digamma_1$ and $\digamma_2$ are defined in Theorem 1.

Consider (11)–(15), we have

$$\begin{aligned}&\mathbf{E}\{\Delta V(\xi_k,\alpha_k)\}+(1-\beta)\mathbf{E}\{V(\xi_k,\alpha_k)\}\\ &\qquad\triangleq\mathbf{E}\left\{\zeta^T(k)\Phi(\alpha_k)\zeta(k)\right\},\end{aligned} \tag{16}$$

where $\zeta(k)\triangleq\left[\xi^T(k)\ \xi^T(k-d(k))\ f^T(\xi)\ f^T(\xi(k-\tau))\right]^T$, and $\Phi(\alpha_k)$ is defined as

$$\begin{aligned}\Phi(\alpha_k)\triangleq&\begin{bmatrix}\Phi_{11}(\alpha_k) & 0 & \digamma_2 & 0\\ \star & -\beta^{d_2+1}Q(\alpha_k) & 0 & 0\\ \star & \star & \beta R(\alpha_k)-I & 0\\ \star & \star & \star & -\beta^{\tau+1}R(\alpha_k)\end{bmatrix}\\ &+\begin{bmatrix}\tilde{A}^T(\alpha_k)\\ M^T\tilde{A}_d^T(\alpha_k)\\ 0\\ M^T\tilde{A}_\tau^T(\alpha_k)\end{bmatrix}P(\alpha_k)\begin{bmatrix}\tilde{A}^T(\alpha_k)\\ M^T\tilde{A}_d^T(\alpha_k)\\ 0\\ M^T\tilde{A}_\tau^T(\alpha_k)\end{bmatrix}^T\\ &+\begin{bmatrix}M^T\tilde{C}^T(\alpha_k)\\ M^T\tilde{C}_d^T(\alpha_k)\\ 0\\ 0\end{bmatrix}\delta P(\alpha_k)\begin{bmatrix}M^T\tilde{C}^T(\alpha_k)\\ M^T\tilde{C}_d^T(\alpha_k)\\ 0\\ 0\end{bmatrix}^T,\end{aligned}$$

Moreover, by Schur complement to (8), it follows that $\Phi(\alpha_k)<0$, then one can easily achieve $\forall k\in[k_l,k_{l+1})$,

$$\mathbf{E}\{\Delta V(\xi_k,\alpha_k)+(1-\beta)V(\xi_k,\alpha_k)\}<0. \tag{17}$$

Now, for an arbitrary piecewise constant switching signal α_k, and for any $k>0$, we let $k_0<k_1<\cdots<k_l<\cdots$, $l=1,\ldots$, denote the switching points of α_k over the interval $(0,k)$. As mentioned earlier, the i_lth subsystem is activated when $k\in[k_l,k_{l+1})$. Therefore, for $k\in[k_l,k_{l+1})$, it holds from (17) that

$$\mathbf{E}\{V(\xi_k,\alpha_k)\}<\beta^{k-k_l}\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\}. \tag{18}$$

Using (9) and (10), we have

$$\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\}\leq\mu\mathbf{E}\left\{V(\xi_{k_l},\alpha_{k_{l-1}})\right\}. \tag{19}$$

$$\begin{bmatrix} \Phi_{11}(i) & 0 & F_2 & 0 & 0 & \tilde{A}^T(i)P(i) & \delta M^T\tilde{C}^T(i)P(i) & \tilde{L}^T(i) \\ \star & -\beta^{d_2+1}Q(i) & 0 & 0 & 0 & M^T\tilde{A}_d^T(i)P(i) & \delta M^T\tilde{C}_d^T(i)P(i) & 0 \\ \star & \star & \beta R(i)-I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}R(i) & 0 & M^T\tilde{A}_\tau^T(i)P(i) & 0 & 0 \\ \star & \star & \star & \star & -\gamma^2 I & \tilde{B}_v^T(i)P(i) & \delta\tilde{D}_v^T(i)P(i) & 0 \\ \star & \star & \star & \star & \star & -P(i) & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\delta P(i) & 0 \\ \star & \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \quad (8)$$

Therefore, it follows from (18)–(19) and the relationship $\vartheta = N_\alpha(k_0,k) \le (k-k_0)/T_a$ that

$$\begin{aligned} \mathbf{E}\{V(\xi_k,\alpha_k)\} &\le \beta^{k-k_l}\mu\mathbf{E}\{V(\xi_{k_l},\alpha_{k_{l-1}})\} \\ &\le \cdots \\ &\le \beta^{(k-k_0)}\mu^{\vartheta}\mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\} \\ &\le (\beta\mu^{1/T_a})^{(k-k_0)}\mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\}. \quad (20) \end{aligned}$$

Notice from (10) that there exist two positive constants a and b ($a \le b$) such that

$$\begin{aligned} \mathbf{E}\{V(\xi_k,\alpha_k)\} &\ge a\mathbf{E}\left\{\|\xi(k)\|^2\right\}, \\ \mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\} &\le b\,\|\xi(k_0)\|_{C^1}^2. \quad (21) \end{aligned}$$

Combining (20) and (21) yields

$$\begin{aligned} \mathbf{E}\left\{\|\xi(k)\|^2\right\} &\le \frac{1}{a}\mathbf{E}\{V(\xi_k,\alpha_k)\} \\ &\le \frac{b}{a}(\beta\mu^{1/T_a})^{(k-k_0)}\,\|\xi(k_0)\|_{C^1}^2. \quad (22) \end{aligned}$$

Furthermore, letting $\rho \triangleq \sqrt{\beta\mu^{1/T_a}}$, it follows that

$$\mathbf{E}\{\|\xi(k)\|\} \le \sqrt{\frac{b}{a}}\rho^{(k-k_0)}\,\|\xi(k_0)\|_{C^1}. \quad (23)$$

By Definition 2, we know that if $0 < \rho < 1$, that is, $T_a > T_a^* = \text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$, the closed-loop system in (5) is mean-square exponentially stable, where function $\text{ceil}(h)$ represents rounding real number h to the nearest integer greater than or equal to h.

Now, we will establish the weighted $\mathscr{H}_\infty$ performance defined in (7), to this end, introduce the following index:

$$\begin{aligned} \mathcal{J} \triangleq \mathbf{E}\{&\Delta V(\xi_k,\alpha_k) + (1-\beta)V(\xi_k,\alpha_k) \\ &+ z^T(k)z(k) - \gamma^2 v^T(k)v(k)\}, \quad (24) \end{aligned}$$

For $k \in [k_l, k_{l+1})$, we have

$$\begin{aligned} \mathbf{E}\{&\Delta V(\xi_k,\alpha_k) + (1-\beta)V(\xi_k,\alpha_k) \\ &+ z^T(k)z(k) - \gamma^2 v^T(k)v(k)\} \\ &\le \mathbf{E}\left\{\chi^T(k)\Pi(\alpha_k)\chi(k)\right\}, \quad (25) \end{aligned}$$

where $\chi(k) \triangleq \begin{bmatrix} \zeta^T(k) & \omega^T(k) \end{bmatrix}^T$ and $\Pi(\alpha_k)$ is shown at the top of the next page. By Schur complement, LMI (8) equals to $\Pi(\alpha_k) < 0$, thus $\mathcal{J} < 0$. Let $\Gamma(k) \triangleq z^T(k)z(k) - \gamma^2 v^T(k)v(k)$, then we have

$$\mathbf{E}\{\Delta V(\xi_k,\alpha_k)\} < \mathbf{E}\{-(1-\beta)V(\xi_k,\alpha_k) - \Gamma(k)\}. \quad (26)$$

Therefore, for $k \in [k_l, k_{l+1})$, it holds from (26) that

$$\mathbf{E}\{V(\xi_k,\alpha_k)\} < \beta^{k-k_l}\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\} - \mathbf{E}\left\{\sum_{s=k_l}^{k-1}\beta^{k-1-s}\Gamma(s)\right\}.$$

Using (9) and (10), we have

$$\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\} \le \mu\mathbf{E}\{V(\xi_{k_l},\alpha_{k_{l-1}})\}.$$

Thus, by the above two inequalities we have

$$\begin{aligned} \mathbf{E}\{V(\xi_k,\alpha_k)\} &< \beta^{k-k_l}\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\} \\ &\quad -\mathbf{E}\left\{\sum_{s=k_l}^{k-1}\beta^{k-1-s}\Gamma(s)\right\}, \\ \mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\} &< \beta^{k_l-k_{l-1}}\mu\mathbf{E}\{V(\xi_{k_{l-1}},\alpha_{k_{l-1}})\} \\ &\quad -\mu\mathbf{E}\left\{\sum_{s=k_{l-1}}^{k_l-1}\beta^{k_l-1-s}\Gamma(s)\right\}, \\ &\vdots \\ \mathbf{E}\{V(\xi_{k_1},\alpha_{k_1})\} &< \beta^{k_1-k_0}\mu\mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\} \\ &\quad -\mu\mathbf{E}\left\{\sum_{s=k_0}^{k_1-1}\beta^{k_1-1-s}\Gamma(s)\right\}. \end{aligned}$$

Therefore, it follows from the above inequalities and the relationship $\vartheta = N_\alpha(k_0,k) \le (k-k_0)/T_a$ that

$$\begin{aligned} \mathbf{E}\{V(\xi_k,\alpha_k)\} &< \beta^{k-k_l}\mathbf{E}\{V(\xi_{k_l},\alpha_{k_l})\} \\ &\quad -\mathbf{E}\left\{\sum_{s=k_l}^{k-1}\beta^{k-1-s}\Gamma(s)\right\} \\ &< \beta^{k-k_0}\mu^{N_\alpha(k_0,k)}\mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\} \\ &\quad -\beta^{k-k_1}\mu^{N_\alpha(k_0,k)}\mathbf{E}\left\{\sum_{s=k_0}^{k_1-1}\beta^{k_1-1-s}\Gamma(s)\right\} \\ &\quad -\beta^{k-k_2}\mu^{N_\alpha(k_1,k)}\mathbf{E}\left\{\sum_{s=k_1}^{k_2-1}\beta^{k_2-1-s}\Gamma(s)\right\} \\ &\quad -\cdots \\ &\quad -\beta^{k-k_{l-1}}\mu^2\mathbf{E}\left\{\sum_{s=k_{l-2}}^{k_{l-1}-1}\beta^{k_{l-1}-1-s}\Gamma(s)\right\} \\ &\quad -\beta^{k-k_l}\mu\mathbf{E}\left\{\sum_{s=k_{l-1}}^{k_l-1}\beta^{k_l-1-s}\Gamma(s)\right\} \\ &\quad -\mathbf{E}\left\{\sum_{s=k_l}^{k-1}\beta^{k-1-s}\Gamma(s)\right\} \\ &= \beta^{k-k_0}\mu^{N_\alpha(k_0,k)}\mathbf{E}\{V(\xi_{k_0},\alpha_{k_0})\} \\ &\quad -\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{N_\alpha(s,k)}\Gamma(s)\right\}. \quad (27) \end{aligned}$$

$$\Pi(\alpha_k) \triangleq \begin{bmatrix} \Phi_{11}(\alpha_k)+\tilde{L}^T(\alpha_k)\tilde{L}(\alpha_k) & 0 & F_2 & 0 & 0 \\ \star & -\beta^{d_2+1}Q(\alpha_k) & 0 & 0 & 0 \\ \star & \star & \beta R(\alpha_k)-I & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}R(\alpha_k) & 0 \\ \star & \star & \star & \star & -\gamma^2 I \end{bmatrix}$$
$$+\begin{bmatrix} \tilde{A}^T(\alpha_k) \\ M^T\tilde{A}_d^T(\alpha_k) \\ 0 \\ M^T\tilde{A}_\tau^T(\alpha_k) \\ \tilde{B}_v^T(\alpha_k) \end{bmatrix} P(\alpha_k) \begin{bmatrix} \tilde{A}^T(\alpha_k) \\ M^T\tilde{A}_d^T(\alpha_k) \\ 0 \\ M^T\tilde{A}_\tau^T(\alpha_k) \\ \tilde{B}_v^T(\alpha_k) \end{bmatrix}^T + \begin{bmatrix} M^T\tilde{C}^T(\alpha_k) \\ M^T\tilde{C}_d^T(\alpha_k) \\ 0 \\ 0 \\ D_v^T(\alpha_k) \end{bmatrix} \delta P(\alpha_k) \begin{bmatrix} M^T\tilde{C}^T(\alpha_k) \\ M^T\tilde{C}_d^T(\alpha_k) \\ 0 \\ 0 \\ D_v^T(\alpha_k) \end{bmatrix}^T,$$

Under zero initial condition, (27) implies

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{N_\alpha(s,k)}z^T(s)z(s)\right\} < \gamma^2\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{N_\alpha(s,k)}\upsilon^T(s)\upsilon(s)\right\}. \quad (28)$$

Multiplying both sides of (28) by $\mu^{-N_\alpha(0,k)}$ yields

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{-N_\alpha(0,s)}z^T(s)z(s)\right\} < \gamma^2\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{-N_\alpha(0,s)}\upsilon^T(s)\upsilon(s)\right\}. (29)$$

Notice that $N_\alpha(0,s) \le s/T_a$ and $T_a > -\frac{\ln\mu}{\ln\beta}$, we have $N_\alpha(0,s) \le -s\frac{\ln\beta}{\ln\mu}$. Thus, (29) implies

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\mu^{s\frac{\ln\beta}{\ln\mu}}z^T(s)z(s)\right\} < \gamma^2\mathbf{E}\left\{\sum_{s=k_0}^{k-1}\beta^{k-1-s}\upsilon^T(s)\upsilon(s)\right\}.$$

which yields that

$$\mathbf{E}\left\{\sum_{s=k_0}^{\infty}\beta^{s}z^T(s)z(s)\right\} < \mathbf{E}\left\{\sum_{s=k_0}^{\infty}\upsilon^T(s)\upsilon(s)\right\}.$$

By Definition 3, we know that the closed-loop system in (5) is mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance γ under α_k. This completes the proof. ■

Remark 2: In Theorem 1, we propose a sufficient condition for the mean-square exponential stability condition for the considered the discrete-time switched stochastic time-delay system (the closed-loop system) in (5). Here, β plays a key role in controlling the low bound of the average dwell time, which can be seen from $T_a > T_a^* = \text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$, specifically, if β is given a smaller value, the low bound of the average dwell time becomes smaller with a fixed μ, which may result in the instability of the system.

Remark 3: Note that when $\mu = 1$ in $T_a > T_a^* = \text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$ we have $T_a > T_a^* = 0$, which means that the switching signal α_k can be arbitrary. In this case, (9) turns out to be $P(i) = P(j) = P$, $Q(i) = Q(j) = P$, $R(i) = R(j) = P$, $\forall i,j \in \mathcal{N}$, and the proposed approach becomes quadratic one thus conservative. On the other hand, when $\beta = 1$ in $T_a > T_a^* = \text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$, we have $T_a = \infty$, that is, there is no switching.

B. $\mathscr{H}_\infty$ Dynamic Output Feedback Control

Now, we are in a position to present a solution to the $\mathscr{H}_\infty$ dynamic output feedback control problem based on Theorem 1, and give the following result.

Theorem 2: Consider the discrete-time nonlinear switched stochastic system in (1). For given constants $\beta > 0$ and $\gamma > 0$, suppose there exist matrices $\mathscr{P}(i) > 0$, $\mathscr{X}(i) > 0$, $\mathscr{Z}(i)$, $\mathscr{Q}_1(i) > 0$, $\mathscr{Q}_2(i)$, $\mathscr{Q}_3(i) > 0$, $\mathscr{R}_1(i) > 0$, $\mathscr{R}_2(i)$, $\mathscr{R}_3(i) > 0$, $\mathscr{A}_c(i)$, $\mathscr{B}_c(i)$ and $\mathscr{C}_c(i)$ such that (30) (shown at the top of the next page) holds for $i \in \mathcal{N}$, where

$$\Omega_{11}(i) \triangleq -\beta\mathscr{P}(i) + \beta(d_2 - d_1 + 1)\mathscr{Q}_1(i) - H_1,$$
$$\Omega_{12}(i) \triangleq -\beta I + \beta(d_2 - d_1 + 1)\mathscr{Q}_2(i) - H_1\mathscr{X}(i),$$
$$\Omega_{22}(i) \triangleq -\beta\mathscr{P}(i)+\beta(d_2 - d_1 + 1)\mathscr{Q}_3(i)-2\mathscr{Z}(i)+H_1^{-1},$$
$$\Omega_{33}(i) \triangleq -\beta^{d_2+1}\mathscr{Q}_1(i),$$
$$\Omega_{34}(i) \triangleq -\beta^{d_2+1}\mathscr{Q}_2(i),$$
$$\Omega_{44}(i) \triangleq -\beta^{d_2+1}\mathscr{Q}_3(i),$$
$$\Omega_{25}(i) \triangleq \mathscr{X}(i)H_2,$$
$$\Omega_{26}(i) \triangleq \mathscr{Z}(i)H_2,$$
$$\Omega_{55}(i) \triangleq \beta\mathscr{R}_1(i) - I,$$
$$\Omega_{56}(i) \triangleq \beta\mathscr{R}_2(i),$$
$$\Omega_{66}(i) \triangleq \beta\mathscr{R}_3(i) - I,$$
$$\Omega_{77}(i) \triangleq -\beta^{\tau+1}\mathscr{R}_1(i),$$
$$\Omega_{78}(i) \triangleq -\beta^{\tau+1}\mathscr{R}_2(i),$$
$$\Omega_{88}(i) \triangleq -\beta^{\tau+1}\mathscr{R}_3(i),$$
$$\Omega_{110}(i) \triangleq A^T(i)\mathscr{P}(i) + E^T(i)\mathscr{B}_c^T(i),$$
$$\Omega_{111}(i) \triangleq A(i),$$
$$\Omega_{210}(i) \triangleq \mathscr{A}_c^T(i),$$
$$\Omega_{211}(i) \triangleq \mathscr{X}(i)A^T(i) + \mathscr{C}_c^T(i)B_u^T(i),$$
$$\Omega_{310}(i) \triangleq A_d^T(i)\mathscr{P}(i) + E_d^T(i)\mathscr{B}_c^T(i),$$
$$\Omega_{311}(i) \triangleq A_d^T(i),$$
$$\Omega_{410}(i) \triangleq \mathscr{X}(i)A_d^T(i)\mathscr{P}(i) + \mathscr{X}(i)E_d^T(i)\mathscr{B}_c^T(i),$$

$$
\begin{bmatrix}
\Omega_{11}(i) & \Omega_{12}(i) & 0 & 0 & H_2 & 0 & 0 & 0 & 0 & \Omega_{110}(i) & \Omega_{111}(i) & \Omega_{112}(i) & \Omega_{113}(i) & \Omega_{114}(i) \\
\star & \Omega_{22}(i) & 0 & 0 & \Omega_{25}(i) & \Omega_{26}(i) & 0 & 0 & 0 & \Omega_{210}(i) & \Omega_{211}(i) & \Omega_{212}(i) & \Omega_{213}(i) & \Omega_{214}(i) \\
\star & \star & \Omega_{33}(i) & \Omega_{34}(i) & 0 & 0 & 0 & 0 & 0 & \Omega_{310}(i) & \Omega_{311}(i) & \Omega_{312}(i) & \Omega_{313}(i) & 0 \\
\star & \star & \star & \Omega_{44}(i) & 0 & 0 & 0 & 0 & 0 & \Omega_{410}(i) & \Omega_{411}(i) & \Omega_{412}(i) & \Omega_{413}(i) & 0 \\
\star & \star & \star & \star & \Omega_{55}(i) & \Omega_{56}(i) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \Omega_{66}(i) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \Omega_{77}(i) & \Omega_{78}(i) & 0 & \Omega_{710}(i) & \Omega_{711}(i) & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \Omega_{88}(i) & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & -\gamma^2 I & \Omega_{910}(i) & \Omega_{911}(i) & \Omega_{912}(i) & \Omega_{913}(i) & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -\mathscr{P}(i) & -I & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\mathscr{X}(i) & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\delta\mathscr{P}(i) & -\delta I & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\delta\mathscr{X}(i) & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -I
\end{bmatrix} < 0, \quad (30)
$$

$$
\begin{aligned}
\Omega_{411}(i) &\triangleq \mathscr{X}(i)A_d^T(i), \\
\Omega_{710}(i) &\triangleq A_\tau^T(i)\mathscr{P}(i) + E_\tau^T(i)\mathscr{B}_c^T(i), \\
\Omega_{711}(i) &\triangleq A_\tau^T(i), \\
\Omega_{910}(i) &\triangleq B_v^T(i)\mathscr{P}(i) + F_v^T(i)\mathscr{B}_c^T(i), \\
\Omega_{911}(i) &\triangleq B_v^T(i), \\
\Omega_{112}(i) &\triangleq \delta C^T(i)\mathscr{P}(i) + \delta G^T(i)\mathscr{B}_c^T(i), \\
\Omega_{113}(i) &\triangleq \delta C^T(i), \\
\Omega_{212}(i) &\triangleq \delta\mathscr{X}(i)C^T(i)\mathscr{P}(i) + \delta\mathscr{X}(i)G^T(i)\mathscr{B}_c^T(i), \\
\Omega_{213}(i) &\triangleq \delta\mathscr{X}(i)C^T(i), \\
\Omega_{312}(i) &\triangleq \delta C_d^T(i)\mathscr{P}(i) + \delta G_d^T(i)\mathscr{B}_c^T(i), \\
\Omega_{313}(i) &\triangleq \delta C_d^T(i), \\
\Omega_{412}(i) &\triangleq \delta\mathscr{X}(i)C_d^T(i)\mathscr{P}(i) + \delta\mathscr{X}(i)G_d^T(i)\mathscr{B}_c^T(i), \\
\Omega_{413}(i) &\triangleq \delta\mathscr{X}(i)C_d^T(i), \\
\Omega_{912}(i) &\triangleq \delta D_v^T(i)\mathscr{P}(i) + \delta H_v^T(i)\mathscr{B}_c^T(i), \\
\Omega_{913}(i) &\triangleq \delta D_v^T(i), \\
\Omega_{114}(i) &\triangleq L^T(i), \\
\Omega_{214}(i) &\triangleq \mathscr{X}(i)L^T(i).
\end{aligned}
$$

Then the closed-loop $\mathscr{H}_\infty$ dynamic output feedback control system in (5) is mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance level γ for any switching signal with average dwell time satisfying $T_a > T_a^* = \text{ceil}\left(-\frac{\ln\mu}{\ln\beta}\right)$, where $\mu \geq 1$ satisfies that $\forall i, j \in \mathscr{N}$,

$$
\begin{aligned}
\begin{bmatrix} \mathscr{P}(i) & I \\ I & \mathscr{X}(i) \end{bmatrix} &\leq \mu \begin{bmatrix} \mathscr{P}(j) & I \\ I & \mathscr{X}(j) \end{bmatrix}, \\
\begin{bmatrix} \mathscr{Q}_1(i) & \mathscr{Q}_2(i) \\ \star & \mathscr{Q}_3(i) \end{bmatrix} &\leq \mu \begin{bmatrix} \mathscr{R}_1(j) & \mathscr{R}_2(j) \\ \star & \mathscr{R}_3(j) \end{bmatrix}, \\
\begin{bmatrix} \mathscr{R}_1(i) & \mathscr{R}_2(i) \\ \star & \mathscr{R}_3(i) \end{bmatrix} &\leq \mu \begin{bmatrix} \mathscr{R}_1(j) & \mathscr{R}_2(j) \\ \star & \mathscr{R}_3(j) \end{bmatrix}. \quad (31)
\end{aligned}
$$

Moreover, if the above conditions are feasible then a desired weighted $\mathscr{H}_\infty$ dynamic output feedback controller realization is given by

$$
\begin{aligned}
A_c(i) &\triangleq P_2^{-1}(i)\left[\mathscr{A}_c(i) - \mathscr{P}(i)A(i)\mathscr{X}(i) - \mathscr{P}(i)B_u(i)\right. \\
&\quad \left.\times C_c(i)\mathscr{Z}^T(i) - P_2(i)B_c(i)E(i)\mathscr{X}(i)\right]\mathscr{Z}^{-T}(i), \\
B_c(i) &\triangleq P_2^{-1}(i)\mathscr{B}_c(i), \\
C_c(i) &\triangleq \mathscr{C}_c(i)\mathscr{Z}^{-T}(i). \quad (32)
\end{aligned}
$$

The proof of Theorem 2 is omitted due to page limit.

IV. Conclusion

In this paper, the $\mathscr{H}_\infty$ DOF control problem has been investigated for discrete-time switched stochastic systems. A sufficient condition has been proposed, which guarantees the closed-loop switched stochastic system to be mean-square exponentially stable with a weighted $\mathscr{H}_\infty$ performance. Then, the $\mathscr{H}_\infty$ DOF controller has been found by solving a set of LMI. In deriving the main results, the average dwell time approach combined with the piecewise Lyapunov function technique has been applied, which avoids some conservativeness caused by using a common Lyapunov function for all the subsystems.

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