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A Disturbance Rejection-Flatness Based Linear Output Feedback Control Approach for Tracking Tasks on a Chua's Circuit.

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Abstract— **A linear output feedback controller is developed for trajectory tracking problems defined on a modified version of Chua's circuit. The circuit modification considers the introduction of a suitable external control input channel guided by a) the induction of the flatness property on a measurable output signal of the circuit and b) the physical viability of the control input. A linear active disturbance rejection control, based on a high gain linear disturbance observer, is implemented on a laboratory prototype. We show that the state-dependent disturbance can be approximately, but arbitrarily closely, estimated through a linear, high-gain, observer, called a Generalized Proportional Integral (GPI) observer, which contains a linear combination of a sufficient number of extra iterated integrals of the output estimation error. Experimental results are presented in the output reference trajectory tracking of a signal generated by an unrelated chaotic system of the Lorenz type. Laboratory experiments illustrate the proposed linear methodology for effectively controlling chaos.**

I. Introduction

The problem of controlling chaotic systems has been approached from many perspectives. Efforts have been presented outside the realm of feedback control (feedforward or open loop schemes). In the seminal paper of Ott, Grebogi and Yorke, [16], the control problem is solved using results from perturbation theory. Those methodologies are suitable for fast systems but they exhibit a lack of robustness. Regarding feedback control methods, considerable attention has been devoted to the control of chaotic systems using a variety of nonlinear control strategies. Passivity based-control has been proposed where chaotic dynamical system stabilization can be analyzed in terms of energy properties (see [5], [18], [19]). Other techniques include: active control [14], adaptive control [26], backstepping design [17], flatness-based control [15] and sliding mode control [28] among others.

The induction of the differential flatness property on the controlled version of Chua's circuit allows for the trivialization of the controller design task, reducing the feedback control problem to that of a linear controllable time-invariant system. The introduction of the flatness concept is due to Fliess and his colleagues in [3]. The reader is also referred to the books [20], and [13], for background on flatness. For an approximate, yet effective, disturbance observer design, the input-to-flat output relation can be simplified to a linear perturbed equation. The perturbation input lumps, both, external disturbance inputs and state dependent nonlinear terms, into a single, uniformly absolutely bounded, disturbance function. This viewpoint transforms the original control task into one of robustly controlling, via a linear output feedback controller, an externally perturbed chain of integrators (see[21] and [22] for robotic applications and [23] for a typical power electronics problem). The additive disturbance input can be effectively, though approximately, estimated via a linear, high gain, observer. This information is used in approximately canceling, at the controller stage, its effects on the trajectory tracking quality. The effects of the unknown disturbance input on the output reconstruction error dynamics, at the observer stage, may be attenuated via a suitable linear combination of iterated integral injections of the output estimation error. This is precisely the *dual* procedure to that characterizing disturbance input attenuation in Generalized Proportional Integral (GPI) Control (see [4]) and, hence, the observers we advocate may be properly called GPI observers. Both in GPI control and GPI observer design, the need for appropriate linear combinations of iterated integral errors is completely equivalent to the hypothesis of a, self-updating, internal model of the perturbation input as a time polynomial approximation.

GPI observer-based, output feedback control of nonlinear uncertain systems is very much related to methodologies known as: *Disturbance Accommodation Control* (DAC), and *Active Disturbance Rejection Control* (ADRC). These approaches deal with the problem of canceling, from the controller's actions, endogenous and exogenous unknown, additive, disturbance inputs affecting the system. Perturbation effects are made available via suitable linear, or nonlinear, estimation efforts. The reader is invited to read the works of Prof. C.D. Johnson (see [10], [11]), those of the late Prof. Jingqing Han [8] and the contributions made by Z. Gao and his colleagues (see [6], [7], [24], [25]). GPI observers are also intimately related to a radically new viewpoint in nonlinear state estimation, based on differential algebra, developed by Fliess *et al.* [2]). GPI observers for chaotic system linear state reconstruction is presented in Cortés *et al.* [1].

This article is organized as follows: Section II presents an introduction to linear control of nonlinear differentially flat systems using a GPI observer-based control. The description of the controlled Chua's circuit is given in Section III where the main problem is formulated. The experimental implementation of our proposed approach, in a laboratory prototype, and the corresponding trajectory tracking results are presented in Section IV. Finally, Section V presents some

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conclusions and suggestions for further work in this area.

II. Linear Generalized Proportional Integral Observer based control of nonlinear flat systems

Consider the following problem: It is desired to estimate the phase variables of the scalar, n-th order, nonlinear system

$$y^{(n)} = \phi(t, y, \dot{y}, ..., y^{(n-1)}) \tag{1}$$

for which it is assumed that a solution, $y(t)$, exists, uniformly in t, for every given set of initial conditions: $y_0, \dot{y}_0, ..., y_0^{(n-1)}$, specified at time $t = 0$. Assume, however, that the function $\phi(\cdot)$ is completely unknown. Any solution, $y(t)$, of the above differential equation trivially satisfies: $y^{(n)}(t) = \phi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))$. We assume based on this fact that, as a time function, the n-th derivative of the solution $y(t)$, given by, $y^{(n)}(t)$, admits, m, further time derivatives which are all uniformly absolutely bounded. In other words, there exists a constant K such that, [1]

$$\sup_t |\phi^{(m)}(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))| \leq K \tag{2}$$

Setting $y_1 = y$, $y_2 = \dot{y}$,...,$y_n = y^{(n-1)}$, a state space model for such an uncertain system is given by,

$$\begin{aligned} \dot{y}_j &= y_{j+1}, \quad j = 1, ..., n-1 \\ \dot{y}_n &= \phi(t, y_1, y_2, ..., y_n) \end{aligned} \tag{3}$$

We propose the following observer for the phase variables, $\{y_1, y_2, ..., y_n\}$, associated with y, characterized by the states $\hat{y}_1, ..., \hat{y}_n$, and complemented by m output estimation error iterated integral injections, characterized by the variable, z_1. We have

$$\begin{aligned} \dot{\hat{y}}_j &= \hat{y}_{j+1} + \lambda_{n+m-j}(y_1 - \hat{y}_1), \quad j = 1, ..., n-1 \\ \dot{\hat{y}}_n &= z_1 + \lambda_m(y_1 - \hat{y}_1) \\ \dot{z}_i &= z_{i+1} + \lambda_{m-i}(y_1 - \hat{y}_1), \quad i = 1, ..., m-1 \\ \dot{z}_m &= \lambda_0(y_1 - \hat{y}_1) \end{aligned} \tag{4}$$

Let the estimation error, e_y, be defined as $e_y = e_1 = y_1 - \hat{y}_1 = y - \hat{y}_1$ with $e_2 = y_2 - \hat{y}_2$, etc.,

$$\begin{aligned} \dot{e}_j &= e_{j+1} - \lambda_{n+m-j}e_1, \quad j = 1, ..., n-1 \\ \dot{e}_n &= \phi(t, y_1(t), y_2(t), ..., y_n(t)) - z_1 - \lambda_m e_1 \\ \dot{z}_i &= z_{i+1} + \lambda_{m-i}e_1, \quad i = 1, ..., m-1 \\ \dot{z}_m &= \lambda_0 e_1 \end{aligned} \tag{5}$$

It is not difficult to see that the estimation error, $e_y = e_1$, satisfies, after elimination of all variables z, the following $n+m$-th order perturbed linear differential equation,

$$\begin{aligned} e_y^{(n+m)} + \lambda_{n+m-1}e_y^{(n+m-1)} + \cdots + \lambda_1\dot{e}_y + \lambda_0 e_y \\ = \phi^{(m)}(t, y_1(t), y_2(t), ..., y_n(t)) \end{aligned} \tag{6}$$

[1]This assumption cannot be verified a priori when $\phi(\cdot)$ is completely unknown. However, in cases where the nonlinearity is known except for some of its parameters, as it is the case of the Chua's system, its validity can be assessed with some work.

Clearly, if $\phi^{(m)}(\cdot)$ is uniformly absolutely bounded, then choosing the gain coefficients, λ_j, $j = 0, 1, ..., n+m-1$, so that the characteristic polynomial in the complex variable s,

$$p_o(s) = s^{n+m} + \lambda_{n+m-1}s^{n+m-1} + \cdots + \lambda_1 s + \lambda_0 \tag{7}$$

exhibits all its roots sufficiently far from the imaginary axis, in the left half of the complex plane, then the trajectories for e_y and for its time derivatives globally asymptotically converge, in an exponentially dominated manner, towards a small as desired vicinity of the origin of the estimation error phase space, $\{e_y, \dot{e}_y, ..., e_y^{(n+m-1)}\}$, where they remain ultimately bounded. The further away the roots are located in the left half of the complex plane the smaller the vicinity of ultimate boundedness around the origin of the estimation error phase space. To prove this result we proceed as follows: Let $x = (e_1, ..., e_{n+m})^T$ denote the phase variables of (6). The perturbed linear system (6) is of the form: $\dot{x} = Ax + b\phi^{(m)}(t)$, with A being a Hurwitz matrix written in companion form and b is a vector of zeroes except for the last component being equal to 1. The matrix $Q = A + A^T$ is a symmetric negative definite matrix with the largest (real negative) eigenvalue denoted by: $\sigma_{max}(Q) < 0$. The Lyapunov function candidate, $V(x) = \frac{1}{2}\|x\|^2$, exhibits a strictly negative time derivative everywhere outside the sphere: $\|x\|^2 \leq K^2/(\sigma_{max}(Q))^2$. Hence, all trajectories starting outside this sphere converge towards its interior, and all those trajectories starting inside this sphere will never abandon it. The more negative the real parts of all the eigenvalues of A, the larger $(\sigma_{max}(Q))^2$ and smaller the radius of the ultimate bounding sphere in the x space.

From (5) it follows that

$$z_1 = \phi(t, y_1(t), y_2(t), ..., y_n(t)) - \lambda_m e_1 - \dot{e}_n \tag{8}$$

Hence, as e_1 and e_n evolve towards the small bounding sphere in the estimation error phase space, the trajectory of z_1 tracks arbitrarily close the unknown function, $\phi(t, y_1(t), y_2(t), ..., y_n(t))$. Clearly, z_i, converges towards a vicinity of $\phi^{(i-1)}(t)$, $i = 1, ..., m$. From the definition of the estimation errors for y, and its time derivatives, it follows that $\hat{y}_j$, $j = 1, ..., n$, reconstruct, in an arbitrarily close fashion, the time derivatives of y.

Let u be a scalar input and let, $\psi(t, y, u)$, be a perfectly known smooth scalar function with, $\partial\psi/\partial u$, uniformly bounded away from zero for all y; so that $\psi(t, y, \theta(t, y, v)) = v$ with $u = \theta(t, y, v)$ whenever $\psi(t, y, u) = v$. The above result extends immediately to nonlinear flat systems, with flat output y, of the form: $y^{(n)} = \phi(t, y, \dot{y}, ..., y^{(n-1)}) + \psi(t, y, u)$, by considering the observer:

$$\begin{aligned} \dot{\hat{y}}_j &= \hat{y}_{j+1} + \lambda_{n+m-j}(y_1 - \hat{y}_1), \quad j = 1, 2, ..., n-1 \\ \dot{\hat{y}}_n &= \psi(t, y, u) + z_1 + \lambda_m(y_1 - \hat{y}_1) \\ \dot{z}_i &= z_{i+1} + \lambda_{m-i}(y_1 - \hat{y}_1), \quad i = 1, 2, ..., m-1 \\ \dot{z}_m &= \lambda_0(y_1 - \hat{y}_1) \end{aligned} \tag{9}$$

Given a smooth output reference trajectory, $y^*(t)$, a GPI observer-based controller, including an active disturbance rejection term z_1, is readily given by,

$$u = \theta(t, y_1, v), \tag{10}$$
$$v = -z_1 + [y^*(t)]^{(n)} - \sum_{i=0}^{n-1} \kappa_i(\hat{y}_{i+1} - [y^*(t)]^{(i)})$$

where the set of parameters κ_i, $i = 0, ..., n-1$ is specified so that the associated polynomial, $p_c(s)$, in the complex variables s:

$$p_c(s) = s^n + \kappa_{n-1}s^{n-1} + \cdots + \kappa_1 s + \kappa_0 \tag{11}$$

is a Hurwitz polynomial with all roots located sufficiently far from the imaginary axis in the complex plane.

The closed loop tracking error system, with $e = y - y^*(t)$, evolves according to the n-th order perturbed linearly dominated dynamics

$$e^{(n)} + \sum_{i=0}^{n-1} \kappa_i e^{(i)} = \phi(t, y_1(t), y_2(t), ..., y_n(t)) - z_1 + \sum_{i=0}^{n-1} \kappa_i e_{i+1} \tag{12}$$

where we have used, as before, the observer estimation error vector components: $e_y = e_1 = y_1 - \hat{y}_1 = y - \hat{y}_1$, $e_2 = y_2 - \hat{y}_2$, etc., Given the asymptotic, exponentially dominated, convergence of z_1 to an arbitrarily small neighborhood of $\phi(t, y_1(t), y_2(t), ..., y_n(t))$ and the convergence of the observer estimation errors e_i, $i = 1, ..., n$ to a small vicinity of the origin of the estimation error phase space, the linear dynamics (12) is perturbed by a right hand side whose terms are signals uniformly absolutely bounded by a sufficiently small neighborhood of the origin. The previous Lyapunov stability result similarly applies and the tracking error, e, converges to a small as desired vicinity of zero.

As a final remark, it should be pointed out that low pass filtering is required for the implementation of the GPI observer-based algorithm, along with appropriate "clutching" of the observer output signals (see [21] and [22] for details). Real life noises do not preclude the application of high gain observers, as it can be inferred from the experimental results here presented.

III. LINEAR CONTROL OF CHUA'S CIRCUIT

Consider the modified version of Chua's circuit, depicted in figure 1, including an external current source, of value u, acting as a control input variable to the circuit,

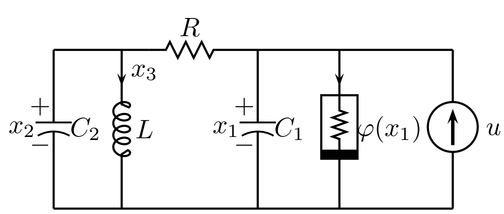

Fig. 1. Controlled Chua's circuit

The mathematical model of the circuit is given by:

$$C_1\dot{x}_1 = \frac{1}{R}(x_2 - x_1) - \varphi(x_1) + u$$
$$C_2\dot{x}_2 = \frac{1}{R}(x_1 - x_2) - x_3$$
$$L\dot{x}_3 = x_2, \quad y = x_3 \tag{13}$$

with the nonlinear function, $\varphi(x_1)$, given by:

$$\varphi(x_1) = m_0 x_1 + \frac{(m_1 - m_0)}{2}(|x_1 + B_p| - |x_1 - B_p|)$$

where x_1, x_2, represent the voltages on the capacitors, C_1, C_2. The variable x_3 represents the current through the inductor, L, while $\varphi(x_1)$ is the current through the nonlinear resistor (known as Chua's diode), m_0, m_1, B_p are fixed, but possibly unknown, constants.

Notice that the controlled system is flat ([3]), with x_3, being the flat output, or the linearizing output. Indeed, all the system variables, including the control input u, may be expressed in terms of $y = x_3$ and a finite number of its time derivatives:

$$x_1 = RLC_2\ddot{y} + L\dot{y} + Ry$$
$$x_2 = L\dot{y}$$
$$x_3 = y \tag{14}$$

and

$$u = RLC_1C_2y^{(3)} + L(C_1 + C_2)\ddot{y} + RC_1\dot{y} + y + m_0(RLC_2\ddot{y} + L\dot{y} + Ry) + \frac{(m_1 - m_0)}{2} \times \left(\left|RLC_2\ddot{y} + L\dot{y} + Ry + B_p\right| - \left|RLC_2\ddot{y} + L\dot{y} + Ry - B_p\right| \right) \tag{15}$$

Under this complete differential parametrization, and for the purposes of building a GPI observer based controller, the resulting input-to-flat output dynamics, which is also free of any zero dynamics, can be obtained from (15) as the following simplified system:

$$y^{(3)} = \frac{1}{RLC_1C_2}u + \xi(t) \tag{16}$$

where $\xi(t)$ lumps the nonlinear state dependent terms in (15) which contains the uncertain parameters of the Chua's diode. Due to the uniformly absolutely bounded responses of the circuit, the input signal, $\xi(t)$, may be considered as an unknown, but uniformly absolutely bounded, input. As seen above, this signal can be on-line approximately estimated by means of a GPI observer. Such an on-line estimation allows the subsequent canceling of $\xi(t)$ from the input-to-flat output dynamics, through the appropriately devised, GPI observer-based, active disturbance rejection control law. The nonlinear output trajectory tracking controller design task is, thus, reduced to that of a linear output feedback controller on a third order chain of integrators.

A. Problem formulation:

Given as a smooth output reference signal, $y^*(t)$, the output, $z(t)$, of the following perfectly known Lorenz system,

$$\begin{aligned}\dot{z}_1 &= \sigma(z_2 - z_1)\\ \dot{z}_2 &= rz_1 - z_2 - z_1 z_3\\ \dot{z}_3 &= z_1 z_2 - bz_3\\ z(t) &= \epsilon z_1(t),\end{aligned} \tag{17}$$

it is desired to force, the Chua's circuit output, $y = x_3$, to accurately track the given trajectory, $y^*(t) = z(t)$, independently of the possible unmodeled external disturbance inputs and regardless of the endogenous disturbance input, $\xi(t)$, affecting the input-to-flat output simplified dynamics (16).

B. A GPI observer-based disturbance rejection control for Chua's circuit

We propose the following GPI observer-based control, using a lumped disturbance internal model requiring four iterated output error integral injections for its approximate cancellation, i.e, $m = 5$

$$\begin{aligned}u =& RLC_1C_2 \Big[[y^*(t)]^{(3)} - \kappa_2(y_3 - \ddot{y}^*(t)) - \kappa_1(y_2 - \dot{y}^*(t))\\ &-\kappa_0(y_1 - y^*(t))] - \hat{\xi}(t)\Big]\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{y}_1 &= y_2 + \lambda_7(y - y_1)\\ \dot{y}_2 &= y_3 + \lambda_6(y - y_1)\\ \dot{y}_3 &= \frac{1}{RLC_1C_2}u + z_1 + \lambda_5(y - y_1)\\ \dot{z}_1 &= z_2 + \lambda_4(y - y_1)\\ \dot{z}_2 &= z_3 + \lambda_3(y - y_1)\\ \dot{z}_3 &= z_4 + \lambda_2(y - y_1)\\ \dot{z}_4 &= z_5 + \lambda_1(y - y_1)\\ \dot{z}_5 &= \lambda_0(y - y_1)\\ \hat{\xi}(t) &= z_1\\ \hat{y}(t) &= y_1, \quad \hat{\dot{y}}(t) = y_2 \quad \hat{\ddot{y}}(t) = y_3\end{aligned}$$

where the characteristic polynomial associated with the estimation error is given by,

$$p_{ob}(s) = s^8 + \lambda_7 s^7 + \lambda_6 s^6 + \ldots + \lambda_1 s + \lambda_0 \tag{19}$$

and the characteristic polynomial of the predominantly linear closed loop tracking error is

$$p_c(s) = s^3 + \kappa_2 s^2 + \kappa_1 s + \kappa_0 \tag{20}$$

By choosing, $\lambda_0, ..., \lambda_7$, such that $p_{ob}(s)$ exhibits the form, $(s^2+2\zeta_{o1}\omega_{no1}s+\omega_{no1}^2)^2(s^2+2\zeta_{o2}\omega_{no2}s+\omega_{no2}^2)^2$, with ζ_{o1}, ζ_{o2}, ω_{no1}, ω_{no2} strictly positive constants, the reconstruction error $e = y - y_1$ is ultimately confined to a small as desired vicinity of the origin of the reconstruction error phase space, provided the roots of $p_{ob}(s)$ are located sufficiently to the left of the imaginary axis in the complex plane. Analogously, selecting $\kappa_0, \kappa_1, \kappa_2$, based on the desired form of the dominant characteristic polynomial, $(s^2 + 2\zeta_c\omega_c s + \omega_c^2)(s + p_c)$, with $\zeta_c, \omega_c, p_c > 0$, we ensure the tracking error to exponentially converge to a small region around the origin of the tracking error phase space. This vicinity becomes smaller as larger is made, in absolute value, the real part of roots of $p_c(s)$ in the left half of the complex plane.

IV. Experimental results

The previously described linear output feedback control law was devised for an actual Chua's circuit prototype, constructed on the basis of the work of Tôrres and Aguirre, [27], which handles low frequency chaotic oscillations, thus avoiding time re-parameterizations in the model synthesis.

Figure 2 shows the Chua's diode realization, based on operational amplifiers (see [12]), and figure 3 shows the schematic diagram of the inductor equivalent circuit. The following set of circuit parameters was used: $C_1 = 23.5$ $[\mu F]$, $C_2 = 235$ $[\mu F]$, $R = 1550$ $[\Omega]$. The equivalent inductance of the realization was, $L = L_{eq} = 42.3$ [H]. The nonlinear parameters of Chua's diode were set to be: $m_0 = -0.409$ [ms], $m_1 = -0.758$ [ms], $B_p = 1.8$ [V]. The passive components, for the op-amp realization, were chosen as: $R_1 = R_2 = 220$ $[\Omega]$, $R_3 = 2.2$ $[K\Omega]$, $R_4 = R_5 = 22$ $[K\Omega]$, $R_6 = 3.3$ $[K\Omega]$, $R_7 = R_8 = R_9 = 1$ $[K\Omega]$, $R_{10} = 1.8$ $[K\Omega]$ and $C_3 = 23.5$ $[\mu F]$. All operational amplifiers were integrated circuits of the $LF412$ type. The real-time processing was implemented in a MatLab xPC Target environment with a sampling period of 0.1 [ms]. The data acquisition facility and the analog control outputs were provided by a National Instruments PCI-6259 DAQ card. Figure 4 depicts the experimental implementation of the modified Chua's circuit.

Concerning the GPI observer-based control, the design parameters of the characteristic polynomial of the observer were set to be: $\zeta_{o1} = 2$, $\omega_{no1} = 50$, $\zeta_{o2} = 2$, $\omega_{no2} = 70$. The characteristic polynomial of the closed loop control was chosen to be of the form $(s^2 + 2\zeta_c\omega_c s + \omega_c^2)(s + p_c)$, with $\zeta_c = 1$, $\omega_c = 50$, and $p_c = 50$.

We tested the performance of the GPI observer based feedback control in the challenging task of having the controlled Chua's circuit flat output, y, track a particular output, $z(t)$, of an uncontrolled (simulated) Lorenz chaotic system, with $\sigma = 10$, $r = 28$, $b = \frac{8}{3}$ and $\epsilon = 0.75 \times 10^{-4}$.

We let $z = y^*(t)$ become the reference signal for the flat output y of the controlled Chua's circuit and used the observer based GPI control synthesis for the corresponding output reference trajectory tracking problem. We let Chua's circuit evolve without any control action for a period of 30 [s] and then, we switched on the GPI observer based controller. Figure 5 depicts the performance of the controller in the tracking task and it is evident how the tracking error signal, $y - y^*(t)$, remains in a small region around the origin of the error space. On the other hand, figure 6 shows the behavior of the x_1, x_2 state variables of the Chua's circuit. Figure 7 illustrates the results of the disturbance estimation and the acting feedback control input signal.

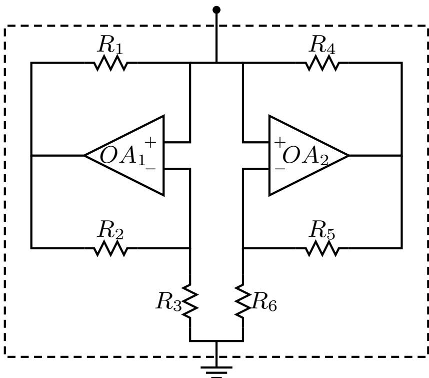

Fig. 2. Operational Amplifier realization of Chua's diode.

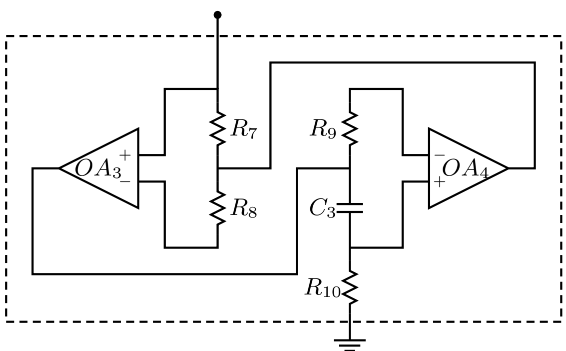

Fig. 3. Equivalent circuit for the inductor in Chua's circuit.

V. Concluding Remarks

In this article, we have proposed a linear active disturbance rejection feedback control approach for the output trajectory tracking task in a controlled version of Chua's circuit with uncertain parameters. The proposed linear controller design methodology is based on the facts that 1) the nonlinear uncontrolled system is easily made to be flat, thanks to the possibilities of choosing a physically meaningful control input channel and 2) one can simplify, for observer design purposes, the highly nonlinear input-to-flat output representation of the circuit to that of a linear perturbed chain of integrations, in which only the order of integration of the system and the control input gain need to be known. We have shown, in an actual laboratory implementation, that the proposed linear output feedback control method efficiently results in a rather accurate closed loop trajectory tracking. This was particularly valid in an output trajectory tracking, on the part of the Chua's circuit, of a chaotic system's output generated by an unrelated Lorenz system.

The proposed linear GPI observer-based control design methodology can be extended to the linear feedback control of some other controlled versions of chaotic, and even

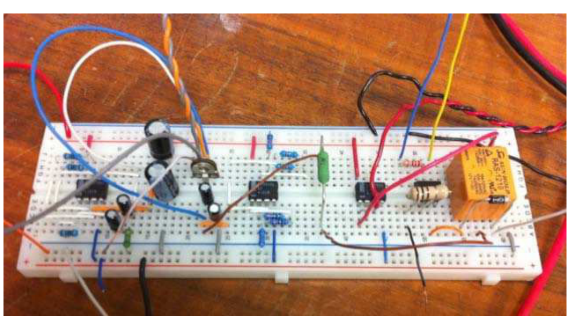

Fig. 4. Modified Chua's circuit prototype.

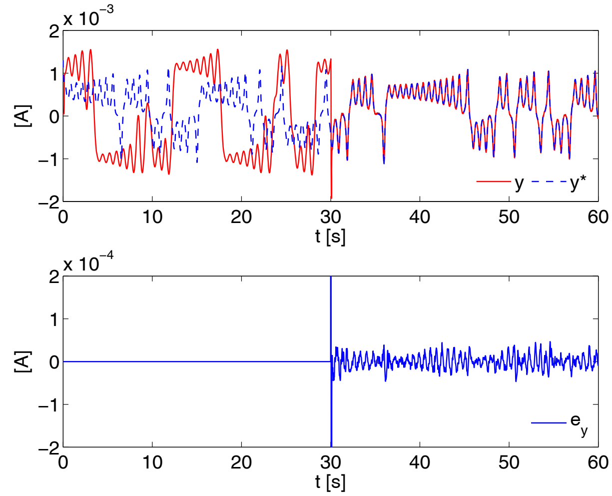

Fig. 5. Performance of the GPI observer based controller, on a Chua's circuit, in the tracking of an output of a Lorenz system.

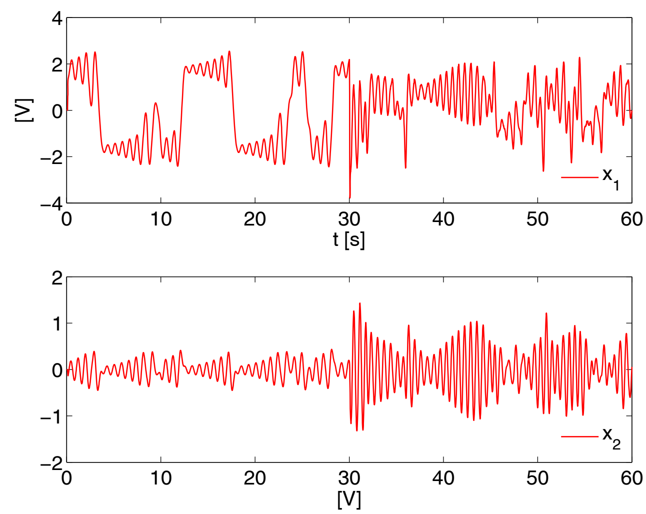

Fig. 6. Behavior of states x_1, x_2 of Chua's circuit in a Lorenz system output trajectory tracking task.

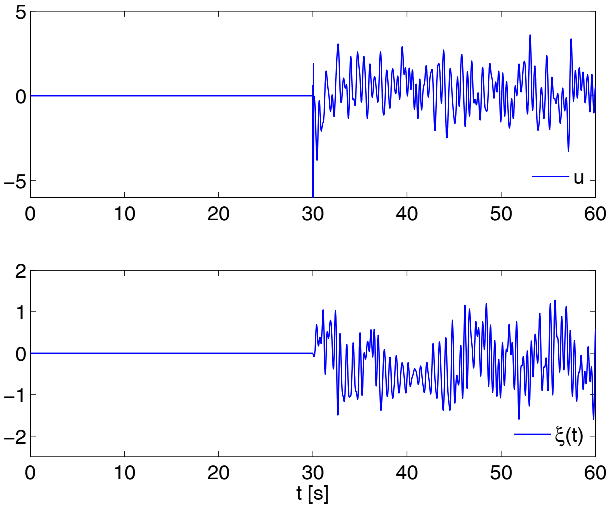

Fig. 7. Control input signal, u, and on-line GPI observer estimation, $\hat{\xi}(t)$, of the lumped disturbance input.

hyper-chaotic, systems. The relative freedom of choosing the control input channel, results in a controlled system which is, generally speaking, "easy" to control when the flatness property is either induced, or invoked on a certain measurable output of the circuit. The use of the proposed controller design method is specially suitable in forced chaotic synchronization tasks, specially when a controlled chaotic system output is made to behave as that of a given, different, uncontrolled chaotic system. This particular possibility may prove to be useful in "controlled chaotic encryption" tasks.

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