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Robust self-tuning PID-like control with a filter for a class of discrete time systems

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Abstract—**Most real processes are high order and hence, an approximate model is usually used in practice. Considering the mismatching of model-process order, in this paper, a robust self-tuning PID-like controller is proposed by combining a pole assignment self-tuning PID controller with a filter. To design the PID-like controller, a reduced order model is introduced, whose linear parameters are identified by a normalized project algorithm with deadzone. The gains of the PID-like controller are obtained by pole assignment, which together with other parameters are tuned on-line according to the certainty equivalent principle. By resorting to time varying operation, the bounded input bounded output (BIBO) stability conditions and convergence conditions of the closed-loop system are presented.**

I. INTRODUCTION

A proportional-integral-derivative (PID) controller has the advantages of structure simpleness and implementation easiness. Since 1920, PID control has received widespread use in industry processes. Today, more than 90% of various practical control systems employ PID control, even though lots of new control techniques have been proposed. The widely usage of PID control attracts more and more control engineers and theorists to design PID controllers in reverse.

As we know, the key of designing a PID controller is the determination of its proportional, integral and derivative gains. Conventional PID controllers with fix gains might be difficult to maintain the desired control performance and stability during operation. Therefore, a great deal of attention has been focused on adaptive or self-tuning of PID controller gains. For example, in [1], the gains of the PID controller are tuned via pole assignment. In [2, 3], the generalized minimum variance control is utilized to tune the PID controller gains. There are also other adaptive or self-tuning schemes which can be found in literature [4-6]. However, the above adaptive or self-tuning PID control approaches are designed according to reduced order models, which might degrade or even become unstable when the underlying process dynamics are high order in nature.

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Most stability results in adaptive or self-tuning control systems are based on the assumption that the model used in the control structure is an accurate representation of the process[7-9]. However, the orders of most real processes are high and difficult to be accurately obtained. Hence, an approximate reduced order model is usually used in practice. Due to this model-process order mismatch, straightforward application of the stable algorithms found in literature may lead to instability problems when unmodeled dynamics are present. In [10], for a discrete-time linear system with unmodeled dynamics, based on a reduced order linear model, a stable adaptive control scheme is proposed using a normalized parameter estimation scheme.

Inspired by the above idea, in this paper, for a class of discrete time system, by combining the pole assignment self-tuning PID controller with a filter, a robust self-tuning PID-like controller is proposed. In consideration of the mismatching of model-process order, a reduced order model for the design of the controller is introduced. The linear parameters of the reduced order model are identified by a normalized project algorithm with deadzone. The high part is viewed as the unmodeled dynamics. The gains of the PID-like controller are obtained by pole assignment, which together with other parameters are tuned on-line according to the certainty equivalent principle. Bounded-input-bounded-output (BIBO) stability conditions and convergence conditions of the closed-loop system are presented by resorting to time varying operation.

The rest of the paper is organized as follows. The system under consideration and the control objective are represented in Section II. In Section III, the PID-like controller for the known system is provided. Section IV develops the proposed robust self-tuning PID-like controller. Section V provides the conditions of BIBO stability and convergence of the closed-loop system. Finally some conclusions are drawn in Section VI.

II. PROBLEM DESCRIPTIONS

Let a single-input single-output discrete time dynamical system be described by the following NARMA model:

$$y(t+1) = f[y(t),\cdots,y(t-n_a+1),u(t),\cdots,u(t-n_b)] \quad (2.1)$$

where $u(t)$ and $y(t)$ are the system input and output, respectively; $f[\cdot]$ is a smooth nonlinear function; $n_a \geq 2$, $n_b \geq 1$ are the system orders.

Without loss of generality, the origin $(u,y)=(0,0)$ is assumed to be an equilibrium point of the above system (2.1). Expanding (2.1) around the origin $(u,y)=(0,0)$ by using Taylor series, we can obtain the following equivalent model:

$$\begin{aligned} y(t+1) &= -a_1 y(t) - \cdots - a_{n_a} y(t-n_a+1) \\ &\quad + b_0 u(t) + \cdots + b_{n_b} u(t-n_b) \\ &\quad + v[y(t),\cdots,y(t-n_a+1),u(t),\cdots,u(t-n_b)] \end{aligned} \tag{2.2}$$

where $a_i = (-1)\cdot \partial f[\cdot]/\partial y(t-i+1)\big|_{\substack{u=0\\y=0}}$, $i=1,\cdots,n_a$; $b_j = \partial f[\cdot]/\partial u(t-j)\big|_{\substack{u=0\\y=0}}$, $j=0,\cdots,n_b$; $v[\cdot]$ is the high order nonlinear term. To simplify the description, we define

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a} \tag{2.3}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b} \tag{2.4}$$

$$x(t) = \{y(t),\cdots,y(t-n_a+1),u(t-1),\cdots,u(t-n_b)\} \tag{2.5}$$

Then (2.2) can be rewritten as

$$A(z^{-1})y(t+1) = B(z^{-1})u(t) + v[x(t),u(t)] \tag{2.6}$$

In the following, the system described by (2.6) will be taken into account directly and the results obtained are global. Since (2.6) is equivalent to (2.1) in a neighborhood of the origin, the obtained results are also local with respects to (2.1). Although (2.6) looks like a linear system, it is actually the real description of the nonlinear system (2.1) around the origin.

To design the required PID-like controller, the following equivalent reduced order model is introduced:

$$\overline{A}(z^{-1})y(t+1) = \overline{B}(z^{-1})u(t) + \overline{v}[x(t),u(t)] \tag{2.7}$$

where $\overline{A}(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}$, $\overline{B}(z^{-1}) = b_0 + b_1 z^{-1}$ are respectively the reduced order parts of $A(z^{-1}), B(z^{-1})$; $\overline{v}[\cdot] = -\overline{\overline{A}}(z^{-1})y(t+1) + \overline{\overline{B}}(z^{-1})u(t) + v[x(t),u(t)]$ with $\overline{\overline{A}}(z^{-1}) = A(z^{-1}) - \overline{A}(z^{-1})$, $\overline{\overline{B}}(z^{-1}) = B(z^{-1}) - \overline{B}(z^{-1})$ represents the unmodeled dynamics.

Assumption 1.

(1) The system orders n_a, n_b or their upper bounds are known and the system parameters a_i, b_j, $i=1,\cdots,n_a$, $j=0,\cdots,n_b$ lie in a compact region;

(2) The root of the polynomial $\overline{B}(z^{-1})$ lies in the unit circle such that $\overline{B}(z^{-1})$ is stable;

(3) There exist unknown constants α_0, α_1 such that the unmodeled dynamics $\overline{v}[x(t),u(t)]$ satisfies

$$\max_{0\le\tau\le t} \overline{v}[x(\tau),u(\tau)] \le \alpha_0 \| X(t-1) \| + \alpha_1 \tag{2.8}$$

where $0<\alpha_0<\overline{\alpha}$ with $\overline{\alpha}$ being defined in the sequel, $0<\alpha_1<\infty$, $X(t) = \left[y(t),\cdots,y(t-n_a+1),u(t),\cdots,u(t-n_b)\right]^{\mathrm{T}}$.

The control objective is, for the unknown system (2.6) or (2,7), to determine a self-tuning PID-like controller so that all the signals in the closed-loop system remain bounded, while the output $y(t)$ tracks reference input $w(t)$, and the influence of the unmodeled dynamics is suppressed to the lowest possible level.

III. PID-LIKE CONTROLLER

To effectively control the system (2.7), by combining the conventional PID controller with a filter, the PID-like controller is proposed, whose structure is shown in the following figure.

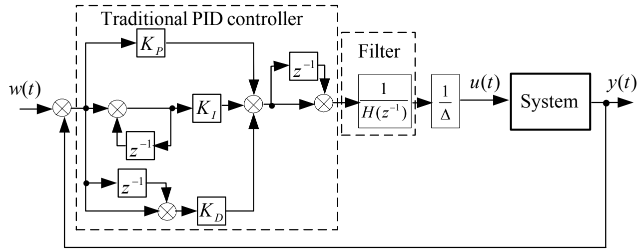

Figure 1. Structure of the PID-like controller

From Figure 1, the PID-like controller can be formulated as

$$\begin{aligned} H(z^{-1})\Delta u(t) &= K_P \Delta\varepsilon(t) + K_I \varepsilon(t) \\ &\quad + K_D[\Delta\varepsilon(t) - \Delta\varepsilon(t-1)] \end{aligned} \tag{3.1}$$

where $\Delta = 1 - z^{-1}$ with z^{-1} being the unit back shift operator is defined as the differential term; $\varepsilon(t) = w(t) - y(t)$ with $w(t)$ being the reference signal is defined as the system tracking error; K_P, K_I, K_D are respectively the proportional, integral and derivative gains; $H(z^{-1}) = 1 + hz^{-1}$ is viewed as a filter.

To select the parameters of the PID-like controller (3.1), let us substituting (3.1) into the system (2.7), then we have

$$\begin{aligned} &[H(z^{-1})\Delta\overline{A}(z^{-1}) + z^{-1}\overline{B}(z^{-1})G(z^{-1})]y(t+1) \\ &= \overline{B}(z^{-1})G(z^{-1})w(t) + H(z^{-1})\Delta\overline{v}[x(t),u(t)] \end{aligned} \tag{3.2}$$

where $G(z^{-1}) = g_0 + g_1 z^{-1} + g_2 z^{-2}$, $g_0 = K_P + K_I + K_D$, $g_1 = -K_P - 2K_D$, $g_2 = K_D$. Therefore, the gains of the PID-like controller can be calculated respectively by

$$K_P = -g_1 - 2g_2,\ K_I = g_0 + g_1 + g_2,\ K_D = g_2.$$

The characteristic polynomial $T(z^{-1})$ of the closed-loop system is

$$T(z^{-1}) = H(z^{-1})\Delta\overline{A}(z^{-1}) + \overline{B}(z^{-1})G(z^{-1})z^{-1} \tag{3.3}$$

To guarantee the stability of the closed-loop system, the zero should be assigned in the unit circle, i.e., the polynomials $G(z^{-1})$ and $H(z^{-1})$ should be chosen such that the following formula is satisfied:

$$\begin{aligned} \det\{T(z^{-1})\} &= \det\{H(z^{-1})\Delta\overline{A}(z^{-1}) \\ &\quad + z^{-1}\overline{B}(z^{-1})G(z^{-1})\} \ne 0, \quad |z|>1 \end{aligned} \tag{3.4}$$

IV. ROBUST SELF-TUNING PID-LIKE CONTROLLER

Since the purpose of this paper is to design a self-tuning PID-like controller for the unknown system (2.7), some parameter identification algorithm for the reduced order linear part of the system is needed.

To identify the reduced order linear parameters of the system (2.7), we first define $X_r(t) = [y(t), y(t-1), u(t), u(t-1)]^{\mathrm{T}}$, $\theta_r = [-a_1, -a_2, b_0, b_1]^{\mathrm{T}}$, $X_m(t) = [y(t-2),\cdots,$

$y(t-n_a+1), u(t-2), \cdots, u(t-n_b)]^{\mathrm{T}}$, $\theta_m = [-a_3, \cdots, -a_{n_a}$, $b_2, \cdots, b_{n_b}]^{\mathrm{T}}$, then, the system (2.7) can be rewritten as

$$y(t+1) = \theta_r^{\mathrm{T}} X_r(t) + \theta_m^{\mathrm{T}} X_m(t) + v[x(t), u(t)] \tag{4.1}$$

Since $\overline{v}[x(t), u(t)] = \theta_m^{\mathrm{T}} X_m(t) + v[x(t), u(t)]$ could be unbounded, to identify the reduced order linear parameters using the project algorithm with deadzone, it should be normalized. Now we define the normalization factor as

$$N(t) = \max\{\max_{1 \le i \le n_X} | X_i(t-1) |, \ c\} \tag{4.2}$$

where $X_i(t)$ is the ith element of $X(t) = [X_r(t)^{\mathrm{T}}, X_m(t)^{\mathrm{T}}]^{\mathrm{T}}$; $n_X = n_a + n_b + 1$ is the element number of $X(t)$; c is a given positive constant. Then the identification model of the reduced order linear parameters is obtained as

$$y_N(t+1) = \theta_r^{\mathrm{T}} X_{r,N}(t) + \overline{v}_N[x(t), u(t)] \tag{4.3}$$

where $y_N(t+1) = y(t+1)/N(t)$, $X_{r,N}(t) = X_r(t)/N(t)$, $\overline{v}_N[x(t), u(t)] = \overline{v}[x(t), u(t)]/N(t)$.

It is easy to know, in the identification model (4.3), $\overline{v}_N[x(t), u(t)]$ is bounded.

Assumption 2. The bound of $\overline{v}_N[x(t), u(t)]$ is Δ_N and is known.

Based on Assumption 2, the following project algorithm with deadzone is adopted:

$$\hat{\theta}_r(t) = \hat{\theta}_r(t-1) + \frac{\eta(t) X_{r,N}(t-1) e_N(t)}{1 + X_{r,N}(t-1)^{\mathrm{T}} X_{r,N}(t-1)} \tag{4.4}$$

$$\eta(t) = \begin{cases} 1 & \text{If } | e_N(t) | > 4\Delta_N \\ 0 & \text{Otherwise} \end{cases} \tag{4.5}$$

$$e_N(t) = y_N(t) - \hat{\theta}_r(t-1)^{\mathrm{T}} X_{r,N}(t-1) \tag{4.6}$$

where $\hat{\theta}_r(t) = \left[-\hat{a}_1(t), -\hat{a}_2(t), \hat{b}_0(t), \hat{b}_1(t)\right]^{\mathrm{T}}$.

The model error is obtained as

$$e(t) = y(t) - \hat{\theta}_r(t-1)^{\mathrm{T}} X_r(t-1) \tag{4.7}$$

Based on the above identification algorithm, the robust self- tuning PID-like controller is obtained as

$$\begin{aligned} \hat{H}(t, z^{-1}) \Delta u(t) = \hat{K}_P(t) \Delta \varepsilon(t) + \hat{K}_I(t) \varepsilon(t) \\ + \hat{K}_D(t)[\Delta \varepsilon(t) - \Delta \varepsilon(t-1)] \end{aligned} \tag{4.8}$$

where $\hat{K}_P(t) = -\hat{g}_1(t) - 2\hat{g}_2(t)$, $\hat{K}_I(t) = \hat{g}_0(t) + \hat{g}_1(t) + \hat{g}_2(t)$, $\hat{K}_D(t) = \hat{g}_2(t)$; $\hat{G}(t, z^{-1}) := \hat{g}_0(t) \ + \hat{g}_1(t) z^{-1} + \hat{g}_2(t) z^{-2}$ and $\hat{H}(t, z^{-1})$ are computed such that

$$\begin{aligned} \det\{T(z^{-1})\} = \det\{\hat{H}(t, z^{-1}) \Delta \hat{\bar{A}}(t, z^{-1}) \\ + z^{-1} \hat{\bar{B}}(t, z^{-1}) \hat{G}(t, z^{-1})\} \neq 0, \quad | z | > 1 \end{aligned} \tag{4.9}$$

where $\hat{\bar{A}}(t, z^{-1}) = 1 + \hat{a}_1(t) z^{-1} + \hat{a}_2(t) z^{-2}$, $\hat{\bar{B}}(t, z^{-1}) = \hat{b}_0(t) + \hat{b}_1(t) z^{-1}$.

V. STABILITY ANALYSIS

Since the proposed self-tuning PID-like algorithm deals with time-varying operation, the following symbols are introduced to simplify the operation before proceeding with stability analysis. For the given time-varying polynomials:

$$L(t, z^{-1}) = l_0(t) + l_1(t) z^{-1} + \cdots + l_{n_1}(t) z^{-n_1}$$

$$M(t, z^{-1}) = m_0(t) + m_1(t) z^{-1} + \cdots + m_{n_2}(t) z^{-n_2}$$

We define

$$L := L(t, z^{-1}), \quad M := M(t, z^{-1})$$

$$L(t, z^{-1}) M(t, z^{-1}) := \sum_i \sum_j l_i(t) m_j(t) z^{-i-j}$$

$$L(t, z^{-1}) \bullet M(t, z^{-1}) := \sum_i \sum_j l_i(t) m_j(t-i) z^{-i-j}.$$

Lemma 1. The identification algorithm (4.3)-(4.6) have the following properties:

(1) $\| \hat{\theta}_r(t) - \theta_r \| \le \| \hat{\theta}_r(0) - \theta_r \|$, where $\hat{\theta}_r(0)$ is the initial vector of $\hat{\theta}_r(t)$;

(2) $\lim_{t \to \infty} \dfrac{\eta(t)(e_N(t)^2 - 16\Delta_N^2)}{4(1 + X_{r,N}(t-1)^{\mathrm{T}} X_{r,N}(t-1))} = 0$;

(3) $\lim_{t \to \infty} \| \hat{\theta}_r(t) - \hat{\theta}_r(t-k) \| = 0$, for any finite positive integer k.

Proof. Define $\tilde{\theta}_r(t) = \hat{\theta}_r(t) - \theta_r$, then from (4.3) and (4.6) we have

$$\begin{aligned} e_N(t) &= y_N(t) - \hat{\theta}_r(t-1)^{\mathrm{T}} X_{r,N}(t-1) \\ &= -\tilde{\theta}_r(t-1)^{\mathrm{T}} X_{r,N}(t-1) + \overline{v}_N[x(t-1), u(t-1)] \end{aligned} \tag{5.1}$$

Consequently, from (4.4) and (5.1), we have,

$$\begin{aligned} \| \tilde{\theta}_r(t) \|^2 &\le \| \tilde{\theta}_r(t-1) \|^2 + \frac{2\eta(t) e_N(t) \tilde{\theta}_r(t-1)^{\mathrm{T}} X_{r,N}(t-1)}{1 + \| X_{r,N}(t-1) \|^2} \\ &\quad + \frac{\eta(t)^2 e_N(t)^2 \| X_{r,N}(t-1) \|^2}{[1 + \| X_{r,N}(t-1) \|^2]^2} \\ &= \| \tilde{\theta}_r(t-1) \|^2 + \frac{\eta(t)^2 e_N(t)^2 \| X_{r,N}(t-1) \|^2}{[1 + \| X_{r,N}(t-1) \|^2]^2} \\ &\quad + \frac{2\eta(t) e_N(t) [\overline{v}_N[x(t-1), u(t-1)] - e_N(t)]}{1 + \| X_{r,N}(t-1) \|^2} \\ &= \| \tilde{\theta}_r(t-1) \|^2 + \frac{2\eta(t) e_N(t) \overline{v}_N[x(t-1), u(t-1)]}{1 + \| X_{r,N}(t-1) \|^2} \\ &\quad - \frac{\eta(t) e_N(t)^2}{1 + \| X_{r,N}(t-1) \|^2} [2 - \frac{\eta(t) \| X_{r,N}(t-1) \|^2}{1 + \| X_{r,N}(t-1) \|^2}] \\ &\le \| \tilde{\theta}_r(t-1) \|^2 + \frac{2\eta(t) e_N(t) \overline{v}_N[x(t-1), u(t-1)]}{1 + \| X_{r,N}(t-1) \|^2} \\ &\quad - \frac{\eta(t) e_N(t)^2}{1 + \| X_{r,N}(t-1) \|^2} \\ &\le \| \tilde{\theta}_r(t-1) \|^2 - \frac{\eta(t)[e_N(t)^2 - 16\overline{v}_N[x(t-1), u(t-1)]^2}{4(1 + \| X_{r,N}(t-1) \|^2)} \\ &\quad - \frac{\eta(t) e_N(t)^2}{2(1 + \| X_{r,N}(t-1) \|^2)} \end{aligned}$$

$$\le \| \tilde{\theta}_r(t-1)\|^2 - \frac{\eta(t)[e_N(t)^2 - 16\Delta_N^2]}{4(1+\| X_{r,N}(t-1)\|^2)} - \frac{\eta(t)e_N(t)^2}{2(1+\| X_{r,N}(t-1)\|^2)} \tag{5.2}$$

From (4.5), since $\eta(t)=0$ for $|e_N(t)|<4\Delta_N$ and is 1 otherwise, $\{\|\tilde{\theta}_r(t)\|^2\}$ is a nonincreasing sequence. Hence $\|\tilde{\theta}_r(t)\|^2 \le \|\tilde{\theta}_r(t-1)\|^2 \le \cdots \le \|\tilde{\theta}_r(0)\|^2$ and $\hat{\theta}_r(t)$ is bounded. Moreover,

$$\lim_{M\to\infty}\sum_{t=1}^{M}\frac{\eta(t)[e_N(t)^2-16\Delta_N{}^2]}{4(1+\|X_{r,N}(t-1)\|^2)} \le \|\tilde{\theta}_r(0)\| - \|\tilde{\theta}_r(N)\| < \infty \tag{5.3}$$

Therefore (1) and (2) in Lemma 1 can be easily obtained. From (5.2), we also have

$$\lim_{M\to\infty}\sum_{t=1}^{M}\frac{\eta(t)e_N(t)^2}{2(1+\|X_{r,N}(t-1)\|^2)} \le \|\tilde{\theta}_r(0)\| - \|\tilde{\theta}_r(N)\| < \infty \tag{5.4}$$

therefore

$$\lim_{t\to\infty}\frac{\eta(t)e_N(t)^2}{2(1+\|X_{r,N}(t-1)\|^2)} = 0 \tag{5.5}$$

Since for any finite positive integer k,

$$\begin{aligned}
&\|\hat{\theta}_r(t)-\hat{\theta}_r(t-k)\|^2 \\
&= \| \sum_{j=t-k+1}^{t} \tilde{\theta}_r(j)-\tilde{\theta}_r(j-1)\|^2 \\
&\le k\sum_{j=t-k+1}^{t}\|\tilde{\theta}_r(j)-\tilde{\theta}_r(j-1)\|^2 \\
&\le k\sum_{j=t-k+1}^{t}\frac{\eta(t)^2 e_N(t)^2 X_{r,N}(t-1)^{\mathrm{T}}X_{r,N}(t-1)}{(1+\|X_{r,N}(t-1)\|^2)} \\
&\le k\sum_{j=t-k+1}^{t}\frac{\eta(t)e_N(t)^2}{1+\|X_{r,N}(t-1)\|^2}
\end{aligned} \tag{5.6}$$

then from (5.5), Lemma 1 (3) is obtained. □

Lemma 2. For any positive constant δ, there exist time instant T, such that when $t>T$, the identification algorithm (4.3)-(4.7) have the following properties:

(1) $|e_N(t)|<4\Delta_N+\delta$;

(2) $\max\limits_{0\le\tau\le t}|e(\tau)|<(4\Delta_N+\varepsilon)\cdot\max\{\|X(t-d)\|,c\}$.

Proof. Since $\{X_{r,N}(t)\}=\{X_r(t)/N(t)\}$ is a bounded sequence, from (1) in Lemma 1, we can obtain

$$\lim_{t\to\infty}\eta(t)[|e_N(t)|^2-16\Delta_N^2]\to 0 .$$

Consequently (1) in Lemma 2 is obtained. From (4.6) and (4.7), we have $e(t)=N(t)\cdot e_N(t)$, therefore from Lemma 2 (1), Lemma 2 (2) can be obtained. □

Theorem 1. For the system (2.7) with the self-tuning PID-like control algorithm (4.3)-(4.7), if Assumptions 1-2 are satisfied and if there exists a region Π and a constant $\alpha\in\Pi$ such that $\alpha\ge 4\Delta_N+\delta$, then the closed-loop system is BIBO stable, i.e. the sequence $\{\|X(t)\|\}$ is bounded. Moreover, at steady state, the tracking error $\varepsilon(t)$ tends to zero for the step input.

Proof. To simplify the description, in the following, we first note $\hat{\bar{A}}(t,z^{-1})=\hat{\bar{A}}$, $\hat{\bar{B}}(t,z^{-1})=\hat{\bar{B}}$, $\hat{H}(t,z^{-1})=\hat{H}$, $\hat{G}(t,z^{-1})=\hat{G}$. From (4.7), we have

$$\begin{aligned} e(t+1) &= y(t+1)-\hat{\theta}_r(t)^{\mathrm{T}}X_r(t) \\ &= \hat{\bar{A}}y(t+1)-\hat{\bar{B}}u(t) \end{aligned} \tag{5.7}$$

The self-tuning PID-like controller (4.8) can be rewritten as

$$\hat{H}\Delta u(t)=\hat{G}w(t)-\hat{G}y(t) \tag{5.8}$$

The polynomials $\hat{H},\hat{G}$ are chosen such that the following polynomial $T(z^{-1})$ is stable:

$$T(z^{-1})=\hat{H}\Delta\hat{\bar{A}}+\hat{\bar{B}}\hat{G}z^{-1} \tag{5.9}$$

Multiplying both sides of (5.7) by $\hat{H}\Delta$, and using (5.8) and (5.9), we have

$$\begin{aligned}
\hat{H}\Delta e(t+1) &= \hat{H}\Delta\bullet\hat{\bar{A}}y(t+1)-\hat{H}\Delta\bullet\hat{\bar{B}}u(t) \\
&= \hat{H}\Delta\hat{\bar{A}}y(t+1)-\hat{H}\Delta\bullet\hat{\bar{B}}u(t)+[\hat{H}\Delta\bullet\hat{\bar{A}}-\hat{H}\Delta\hat{\bar{A}}]y(t+1) \\
&= T(z^{-1})y(t+1)-\hat{\bar{B}}\hat{G}y(t)-\hat{H}\Delta\bullet\hat{\bar{B}}u(t) \\
&\quad +[\hat{H}\Delta\bullet\hat{\bar{A}}-\hat{H}\Delta\hat{\bar{A}}]y(t+1) \\
&= T(z^{-1})y(t+1)-\hat{\bar{B}}\bullet\hat{G}y(t)-\hat{\bar{B}}\bullet\hat{H}\Delta u(t) \\
&\quad +[\hat{\bar{B}}\bullet\hat{G}-\hat{\bar{B}}\hat{G}]y(t)+[\hat{H}\Delta\bullet\hat{\bar{A}}-\hat{H}\Delta\hat{\bar{A}}]y(t+1) \\
&\quad -[\hat{H}\Delta\bullet\hat{\bar{B}}-\hat{\bar{B}}\bullet\hat{H}\Delta]u(t) \\
&= T(z^{-1})y(t+1)-\hat{\bar{B}}\bullet\hat{G}w(t)+[\hat{\bar{B}}\bullet\hat{G}-\hat{\bar{B}}\hat{G}]y(t) \\
&\quad +[\hat{H}\Delta\bullet\hat{\bar{A}}-\hat{H}\Delta\hat{\bar{A}}]y(t+1)-[\hat{H}\Delta\bullet\hat{\bar{B}}-\hat{\bar{B}}\bullet\hat{H}\Delta]u(t)
\end{aligned} \tag{5.10}$$

From Lemma 1 (3), when $t\to\infty$, the elements in the square brackets of (5.10) tend to zero. Then from the boundedness of $\hat{\theta}_r(t)$, $w(t)$, the fact that $T(z^{-1})$ is stable, there exist positive constants c_1, c_2 such that

$$|y(t+1)|\le c_1+c_2\max_{0\le\tau\le t}|e(\tau+1)| \tag{5.11}$$

Combining (5.10) with the system (2.7), when $t\to\infty$, we have

$$\begin{aligned} A(z^{-1})\hat{H}\Delta e(t+1) &= T(z^{-1})\bar{B}(z^{-1})u(t) \\ &\quad +T(z^{-1})\bar{v}[\cdot]-A(z^{-1})\hat{\bar{B}}\bullet\hat{G}w(t) \end{aligned} \tag{5.12}$$

Since $T(z^{-1})\bar{B}(z^{-1})$ is stable, and $\hat{\theta}_r(t)$ and $w(t)$ are bounded, there exist positive constants c_3, c_4 such that

$$|u(t)|\le c_3+c_4\max_{0\le\tau\le t}|e(\tau+1)|+c_4\max_{0\le\tau\le t}|\bar{v}[x(\tau),u(\tau)]| \tag{5.13}$$

Since $X(t)=[y(t),\cdots,u(t),\cdots]^{\mathrm{T}}$, there exist positive constants c_5,c_6 such that

$$\|X(t)\|\le c_5+c_6\max_{0\le\tau\le t}|e(\tau+1)|+c_6\max_{0\le\tau\le t}|\bar{v}[x(\tau),u(\tau)]| \tag{5.14}$$

From the assumption (2.8), there exists positive constant c_7 such that

$$\|X(t-1)\|\le c_7+c_6\max_{0\le\tau\le t}|e(\tau)|+2c_6\alpha_0\max_{0\le\tau\le t}\|X(\tau-1)\| \tag{5.15}$$

Therefore if we note $\bar{\alpha} = 1/(2c_6)$, then for any $\alpha_0 \in [0, \bar{\alpha})$, there exist positive c_8, c_9 such that

$$\| X(t-1) \| \le c_8 + c_9 \max_{0\le\tau\le t} | e(\tau) | \tag{5.16}$$

If $\{\| X(t) \|\}$ is unbounded, there exists a subsequence $\{t_s\}$, such that $\lim_{t_s\to\infty} \| X(t_s) \| = \infty$. Therefore from (5.16), when $t \to \infty$, we have

$$1 \le \frac{c_8}{\| X(t_s - 1) \|} + \frac{c_9 \max_{0\le\tau\le t_s} | e(\tau) |}{\| X(t_s - 1) \|} \tag{5.17}$$

Define the region $\Pi = (0, 1/c_9]$, then if there exist a constant $\alpha \in \Pi$ such that $\alpha \ge 4\Delta_N + \delta$, (5.17) will contradict to Lemma 1 (2). Therefore $\{\| X(t) \|\}$ is bounded and the closed-loop system is BIBO stable.

From (5.10), we have

$$\begin{aligned}\hat{H}\Delta e(t+1) &= T(z^{-1})\{y(t+1) - w(t+1)\} + [zT(z^{-1}) - \hat{\hat{B}}\hat{G}]w(t)\\ &+ [\hat{\hat{B}}\hat{G} - \hat{\hat{B}}\bullet\hat{G}]w(t) + [\hat{\hat{B}}\bullet\hat{G} - \hat{\hat{B}}\hat{G}]y(t)\\ &+ [\hat{H}\Delta\bullet\hat{\hat{A}} - \hat{H}\Delta\hat{\hat{A}}]y(t+1) - [\hat{H}\Delta\bullet\hat{\hat{B}} - \hat{\hat{B}}\bullet\hat{H}\Delta]u(t)\end{aligned} \tag{5.18}$$

From (5.8), we have

$$\begin{aligned}\hat{H}\Delta e(t+1) &= T(z^{-1})\{y(t+1) - w(t+1)\} + \hat{H}\Delta\hat{\hat{A}}w(t+1)\\ &+ [\hat{\hat{B}}\hat{G} - \hat{\hat{B}}\bullet\hat{G}]w(t) + [\hat{\hat{B}}\bullet\hat{G} - \hat{\hat{B}}\hat{G}]y(t)\\ &+ [\hat{H}\Delta\bullet\hat{\hat{A}} - \hat{H}\Delta\hat{\hat{A}}]y(t+1) - [\hat{H}\Delta\bullet\hat{\hat{B}} - \hat{\hat{B}}\bullet\hat{H}\Delta]u(t)\end{aligned} \tag{5.19}$$

Since when $t \to \infty$, the elements in the square brackets of (5.19) tend to zero, the tracking error $\varepsilon(t)$ satisfies

$$T(z^{-1})\varepsilon(t) = \hat{H}\Delta e(t) + \hat{H}\Delta\hat{\hat{A}}w(t) \tag{5.20}$$

Therefore, at steady state, i.e. when $z = 1$, $\varepsilon(t)$ tends to zero for the step input. □

VI. Conclusion

This paper proposes a robust self-tuning PID-like controller by combining a pole assignment self-tuning PID controller with a filter. Considering the mismatching of model-process order, a reduced order model for the design of the controller is introduced. The linear parameters of the reduced order model are identified by a normalized project algorithm with deadzone. The high part is viewed as the unmodeled dynamics. The gains of the PID-like controller are obtained by pole assignment, which together with other parameters are tuned on-line according to the certainty equivalent principle. BIBO stability condition and convergence condition of the closed-loop system are presented by resorting to time varying operation.

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