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Adaptive Backstepping Control for a Miniature Autonomous Helicopter

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Abstract— **An adaptive backstepping control algorithm is presented for trajectory tracking of a miniature autonomous helicopter with inertial parameter uncertainties. The control algorithm is designed based on a simplified helicopter model in cascaded form with the backstepping technology. The inertial parameter uncertainties are compensated online with parameter adaptive update laws. The closed-loop stability analysis for the un-simplified complete helicopter model under this control algorithm is provided. Simulation results demonstrate the performances of the proposed approach.**

I. INTRODUCTION

Helicopter has many advantages over ordinary fixed-wing vehicles (for instances, hovering, vertical taking-off & landing, and low-velocity flight); consequently, controller design for autonomous helicopters became one of the foci in some recent studies. However, nonlinearities, uncertainties and couplings in the helicopter model lead to some difficulties in the controller design, especially for model-based design approaches.

Generally, controllers for autonomous helicopters can be classified into three categories– 1) controllers for hovering, 2) controllers for path-following, and 3) controllers for trajectory tracking. The task of hovering control are often solved with linear controller and extensive utility of the aerodynamic derivatives [7]. Comparatively, path-following and trajectory tracking control tasks are usually completed by nonlinear approaches, such as approximate linearization [11], backstepping technology [3]–[6] and so on.

Backstepping control is a Lyapunov-based approach, the advantages of which includes the accommodation of nonlinearities and the avoidance of wasteful cancelations [8]. So far, backstepping methodology has been employed by many researchers to trajectory tracking of autonomous helicopters. C. Lee's backstepping design [3] for the autonomous helicopter realizes the asymptotical tracking of a simplified helicopter model, but the controller performance on the complete model is not discussed. E. Frazzoli introduces a backstepping approach combined with Riemannian geometry[4] and proves bounded tracking of the helicopter; however, the obtained controller is expressed with some fairly complicated symbols thus difficult to implement by engineers. H.R. Pota utilized backstepping approach to control the helicopter velocity [5], but the controller does not guarantee the stability (or boundedness) of the attitude. Although the backstepping controller proposed by I.A. Raptis [6] is proved to assure both tracking of the simplified cascaded system and boundedness of the attitude, the stability condition for the complete model with coupled terms remains theoretically untreated. Besides, parameter uncertainties in the helicopter model are considered by none of the above researches.

In this paper, an adaptive backstepping control algorithm is presented to achieve the trajectory tracking of a miniature autonomous helicopter with constant inertial parameter uncertainties. In this approach, rotation matrix is considered to describe the attitude kinematics of the helicopter [6][11], so that its simplified dynamical model appears cascaded. Based on the simplified model, detailed design procedures of the adaptive backstepping control algorithm are provided, and projection algorithms [10] are introduced to the adaptive laws for adjusting parameters such that the estimated parameters are locally bounded. Closed-loop stability analysis for the complete helicopter model with coupled terms shows that the tracking error is bounded under the proposed control algorithm.

The rest of this paper is arranged as following. In section II, the mathematical model of the helicopter is derived, and the objective of the controller design is stated. In section III, a detailed designing procedure of the adaptive backstepping controller is proposed, and stability analysis is also presented. Simulation results are then displayed in section IV. Finally, conclusion is given in section V, with some future works being suggested.

II. PROBLEM STATEMENT

A. Mathematical Modeling for Miniature Helicopter

Two reference frames are adopted for mathematical modeling:

a) The earth reference frame(ERF): This frame is fixed to the earth, with the origin locating at a fix point on the ground. The x axis points to the north and the z axis points upright. The y axis can be confirmed by the right-hand rule for the dextrorotational helicopter or the left-hand rule for the levorotational helicopter.

b) The fuselage reference frame(FRF): This frame is fixed to the helicopter fuselage. The origin locates at c.g.(center of gravity) of the helicopter fuselage, with the x_b axis pointing to the head of the helicopter. The z_b axis is perpendicular to the x_b axis and points upright. The y_b axis can be confirmed by right-hand rule for the dextrorotational helicopter or left-hand rule for the levorotational one.

The mathematical model of the miniature unmanned helicopter could be derived by Newton–Euler equations [6][11]:

$$\dot{P} = V \tag{1}$$

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$$m\dot{V} = -mg_3 + R_t(\gamma)F \tag{2}$$

$$\dot{R}_t(\gamma) = R_t(\gamma)S(\omega) \tag{3}$$

$$J\dot{\omega} = -S(\omega)J\omega + Q \tag{4}$$

where $P \triangleq [x,y,z]^T$ and $V \triangleq [u,v,w]^T$ are position and velocity of c.g. of the helicopter in ERF, respectively; m denotes the mass; $g_3 \triangleq [0,0,g]^T$ and g is the gravitational acceleration; $\gamma \triangleq [\phi,\theta,\psi]^T$ stands for the attitude in ERF; the rotational matrix from FRF to ERF is given by

$$R_t = [R_{ij}] \triangleq \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & c\phi c\psi s\theta + s\phi s\psi \\ c\theta s\psi & s\psi s\theta s\phi + c\phi c\psi & c\phi s\psi s\theta - s\phi c\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

where $c(\cdot)$ and $s(\cdot)$ are the shorts for $\cos(\cdot)$ and $\sin(\cdot)$, respectively; $\omega \triangleq [p,q,r]^T$ represents the angular velocity in FRF; $S(\cdot)$ denotes the skew-symmetric matrix such that $S(\omega)J\omega = \omega \times J\omega$; the inertial matrix is given by

$$J \triangleq \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}$$

Resultant forces and torques exerted on fuselage in FRF are given by

$$F \triangleq \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T_m \sin\varepsilon \\ -T_m \sin\eta - T_t \\ T_m \cos\eta \cos\varepsilon \end{bmatrix} \tag{5}$$

and

$$Q \triangleq \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} T_m h_m \sin\eta + T_t h_t + Q_m \sin\varepsilon \\ T_m l_m + T_m h_m \sin\varepsilon + Q_t - Q_m \sin\eta \\ -T_m l_m \sin\eta + T_t l_t - Q_m \cos\varepsilon \cos\eta \end{bmatrix} \tag{6}$$

where T_m, Q_m, T_t and Q_t represent the thrusts and the counteractive torques generated by the main rotor and the tail rotor, respectively; h_m, h_t, l_m, l_t are the vertical and horizonal distances between c.g. of the helicopter and centers of the rotors, respectively; ε and η are the longitudinal and lateral flapping angles, respectively. Since the flapping dynamics of the main rotor is extremely fast compared with the fuselage dynamics, the flapping dynamics is negligible in this research. The relationship between the thrusts and the collective pitch is given by [2]

$$T_i = t_{ci}\rho s_i A_i \Omega_i^2 R_i^2 \tag{7}$$

$$t_{ci} = \frac{1}{4}\left[-\frac{a_i}{4}\sqrt{\frac{s_i}{2}} + \sqrt{\frac{a_i^2 s_i}{32} + \frac{2}{3}a_i\theta_i}\right]^2 \tag{8}$$

and the relationship between the thrust and the torque is given by:

$$Q_i = q_{ci}\rho s_i A_i \Omega_i^2 R_i^3 \tag{9}$$

$$q_{ci} = \frac{\delta_d}{8} + 1.13 t_{ci}^{\frac{3}{2}}\sqrt{\frac{s_i}{2}} \tag{10}$$

where subscripts $i = m$ and t represent the main rotor and the tail rotor accordingly; ρ, s_i, a_i, A_i, Ω_i and R_i denote density of the local air, solidity of the rotor disc, slope of the lift curve, area of the rotor disc and radius of the rotor disc, respectively; δ_d is the drag coefficient of the rotor which often has a typical value of 0.012 [2].

From above model we know that the motion of the helicopter is controlled by θ_m, θ_t, ε, and η.

B. Objective of the Trajectory Tracking for Miniature Helicopter

In this research, it is assumed that m and J are unknown constant parameters with known bounds M_1 and M_2, i.e. $\|m\| \leqslant M_1$ and $\|\rho\| \triangleq \left\|[I_{xx}, I_{yy}, I_{zz}, I_{xz}]^T\right\| \leqslant M_2$, where $\|\cdot\|$ denotes the Euclidean norm for vectors and the induced Euclidean norm for matrice. Our objective is to design a trajectory tracking control algorithm such that the controlled autonomous miniature helicopter can track any feasible command trajectory $P_r = [x_r, y_r, z_r]^T$ and yaw angle ψ_r with limited errors.

In following research, the non-vanishing coupling terms demolishing the cascaded structure of the helicopter model are treated as bounded disturbances; thus the best expectation is bounded tracking. Under the adaptive backstepping controller designed in the following sections, it is proved that the tracking error of the miniature autonomous helicopter becomes bounded.

III. ADAPTIVE BACKSTEPPING CONTROL ALGORITHM DESIGN

A. Model simplification

Because the helicopter model (1)–(4) is strongly coupled, it should be simplified to facilitate controller design. Since the cyclic flapping angles and the tail rotor thrust are fairly small according to the physical properties of the helicopter [1][9][11], it is reasonable to take

$$F_x \approx 0,\ F_y \approx 0,\ F_z \approx T_m$$

in (5) for simplifying the model, and it follows that

$$R_t(\gamma)F = R_3 T_m \tag{11}$$

where R_3 denotes the third column of $R_t(\gamma)$ and $\|R_3\| = 1$. Substituting (11) into (2) enables the helicopter model to appear cascaded, which facilitates the backstepping control design. The neglected terms

$$\Delta_1 \triangleq \begin{bmatrix} T_m \sin\varepsilon \\ -T_m \sin\eta - T_t \\ T_m(\cos\varepsilon\cos\eta - 1) \end{bmatrix} \tag{12}$$

will be considered later in stability analysis.

The counteractive torque of the tail rotor Q_t contributes a tiny part of M, and is also negligible; so the torques in (6) can be simplified by

$$Q = Q_A\tau + Q_B \tag{13}$$

where

$$Q_A = \begin{bmatrix} h_t & Q_m & T_m h_m \\ 0 & T_m h_m & -Q_m \\ l_t & 0 & -T_m l_m \end{bmatrix},\ Q_B = \begin{bmatrix} 0 \\ T_m l_m \\ -Q_m \end{bmatrix}$$

and $\tau \triangleq [T_t, \varepsilon, \eta]^T$. Invertibility of Q_A can be proved by

$$|Q_A| = -(h_t h_m l_m + l_t h_m^2) T_m^2 - l_t Q_m^2 \neq 0.$$

The neglected terms

$$\Delta_2 \triangleq \begin{bmatrix} Q_m(\sin\varepsilon - \varepsilon) + T_m h_m(\sin\eta - \eta) \\ Q_t - Q_m(\sin\eta - \eta) + T_m h_m(\sin\varepsilon - \varepsilon) \\ Q_m(1 - \cos\varepsilon\cos\eta) + T_m l_m(\eta - \sin\eta) \end{bmatrix} \quad (14)$$

will also be considered in the stability analysis.

B. Recursive backstepping design

Step 1: For the position kinematics (1), the velocity V can be viewed as the input. Obviously, the position tracking error $P_e \triangleq P - P_r$ can be stabilized by choosing

$$V = V_c \triangleq -K_{1p} P_e - K_{1i} \int_0^t P_e \mathrm{d}t + \dot{P}_r \quad (15)$$

where K_{1p} and K_{1i} are constant positive definite matrices.

Set the Lyapunov candidate

$$L_1 = \frac{1}{2} P_e^T P_e + \frac{1}{2} \int_0^t P_e^T \mathrm{d}t K_{1i} \int_0^t P_e \mathrm{d}t$$

Its derivative can be obtained as

$$\dot{L}_1 = -P_e^T K_{1p} P_e \leqslant 0$$

Step 2: To backstep, define the velocity tracking error $V_e \triangleq V - V_c$ and the mass estimation error $\tilde{m} \triangleq \hat{m} - m$. Selecting the Lyapunov candidate

$$L_2 = L_1 + \frac{m}{2} V_e^T V_e + \frac{1}{2} \int_0^t V_e^T \mathrm{d}t K_{2i} \int_0^t V_e \mathrm{d}t + \frac{1}{2\gamma_1} \tilde{m}^2 \quad (16)$$

we have

$$\begin{aligned} \dot{L}_2 =& -P_e^T K_{1p} P_e + P_e^T V_e + m V_e^T \dot{V}_e + V_e^T K_{2i} \int_0^t V_e \mathrm{d}t + \frac{1}{\gamma_1} \tilde{m}\dot{\hat{m}} \\ =& -P_e^T K_{1p} P_e + P_e^T V_e + m V_e^T (-g_3 + \frac{1}{m} R_3 T_m - \dot{V}_c) \\ & + V_e^T K_{2i} \int_0^t V_e \mathrm{d}t + \frac{1}{\gamma_1} \tilde{m}\dot{\hat{m}} \end{aligned}$$

where $R_3 T_m$ is given by (11). Design

$$R_3 T_m = \mu_c \triangleq R_{3c} T_m \triangleq \hat{m} X - K_{2p} V_e - K_{2i} \int_0^t V_e \mathrm{d}t - P_e \quad (17)$$

where K_{2p} and K_{2i} are constant positive definite matrices, and $X \triangleq g_3 + \dot{V}_c$, the derivative of Lyapunov candidate (16) can be obtained by

$$\begin{aligned} \dot{L}_2 =& -P_e^T K_{1p} P_e + P_e^T V_e - V_e^T K_{2p} V_e + \tilde{m} V_e^T X - V_e^T P_e + \frac{1}{\gamma_1} \tilde{m}\dot{\hat{m}} \\ =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e + \tilde{m} X^T V_e + \frac{1}{\gamma_1} \tilde{m}\dot{\hat{m}} \end{aligned}$$

If choose $\|\hat{m}(0)\| < M_1$ and design its adaptive update law by following projection algorithm:

$$\dot{\hat{m}} = \begin{cases} -\gamma_1 X^T V_e, & \text{if } \|\hat{m}\| < M_1 \\ & \text{or } \|\hat{m}\| = M_1,\ \hat{m}^T X^T V_e > 0 \\ 0, & \text{if } \|\hat{m}\| = M_1,\ \hat{m}^T X^T V_e \leqslant 0 \end{cases} \quad (18)$$

it follows that

$$\dot{L}_2 = -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e + \varpi_1 \frac{\hat{m} X^T V_e}{\hat{m}^2} \tilde{m}\hat{m}$$

where

$$\varpi_1 = \begin{cases} 0, & \text{if } \|\hat{m}\| < M_1 \\ & \text{or } \|\hat{m}\| = M_1 \text{ and } \hat{m} X^T V_e > 0 \\ 1, & \text{if } \|\hat{m}\| = M_1 \text{ and } \hat{m} X^T V_e \leqslant 0 \end{cases} \quad (19)$$

If $\varpi_1 = 0$, $\dot{L}_2 \leqslant 0$; if $\varpi_1 = 1$, we know $\|\hat{m}\| = M_1$ and $\hat{m} X^T V_e \leqslant 0$, so

$$\tilde{m}\hat{m} = \frac{1}{2}(\|\hat{m}\|^2 + \|\hat{m} - m\|^2 - \|m\|^2) \geqslant 0$$

and $\varpi_1 \frac{\hat{m} X^T V_e}{\hat{m}^2} \tilde{m}\hat{m} \leqslant 0$, which also indicates $\dot{L}_2 \leqslant 0$.

Step 3: Command trajectories of this step is acquired by

$$T_m = \|\mu_c\|,\ R_{3c} = \frac{\mu_c}{T_m} \quad (20)$$

And the attitude kinematics can be described by

$$\dot{R}_3 = \dot{R}_t e_3 = R_t S(\omega) e_3 = -R_t S(e_3) \omega \quad (21)$$

where $e_3 \triangleq [0,0,1]^T$. Since $R_3 = [R_{13}, R_{23}, R_{33}]^T$ and $\|R_3\| = 1$, R_{33} depends entirely on R_{13} and R_{23}. Extracting the first two lines of (21) yields

$$\dot{\bar{R}}_3 = \begin{bmatrix} \dot{R}_{13} \\ \dot{R}_{23} \end{bmatrix} = \begin{bmatrix} -R_{12} & R_{11} \\ -R_{22} & R_{21} \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \triangleq \hat{R} \omega_2$$

where the invertibility of $\hat{R}$ is obvious.

Define $\bar{R}_{3e} \triangleq \bar{R}_3 - \bar{R}_{3c}$, where $\bar{R}_{3c}$ represents the vector composed by the first two elements of R_{3c}, and choose the Lyapunov candidate

$$L_3 = L_2 + \frac{1}{2} \bar{R}_{3e}^T \bar{R}_{3e} + \frac{1}{2} \int_0^t \bar{R}_{3e}^T \mathrm{d}t K_{3i} \int_0^t \bar{R}_{3e} \mathrm{d}t$$

we get

$$\begin{aligned} \dot{L}_3 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e + T_m V_e^T R_{3e} \\ & + \bar{R}_{3e}^T \dot{\bar{R}}_{3e} + \bar{R}_{3e}^T K_{3i} \int_0^t \bar{R}_{3e} \mathrm{d}t \\ =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e + T_m V_e^T R_{3e} \\ & + \bar{R}_{3e}^T (\dot{\bar{R}}_3 - \dot{\bar{R}}_{3c}) + \bar{R}_{3e}^T K_{3i} \int_0^t \bar{R}_{3e} \mathrm{d}t \\ =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e + T_m V_e^T R_{3e} \\ & + \bar{R}_{3e}^T (\hat{R}\omega_2 - \dot{\bar{R}}_{3c}) + \bar{R}_{3e}^T K_{3i} \int_0^t \bar{R}_{3e} \mathrm{d}t \end{aligned}$$

Assigning the angular velocity command

$$\omega_2 = \omega_{2c} \triangleq \hat{R}^{-1}(-K_{3p} \bar{R}_{3e} - K_{3i} \int_0^t \bar{R}_{3e} \mathrm{d}t + \dot{\bar{R}}_{3c} - T_m \hat{R} \delta) \quad (22)$$

where $\delta = [\delta_1, \delta_2]^T$, $\delta_1 = u_e - \frac{R_{13} + R_{13c}}{R_{33} + R_{33c}} w_e$ and $\delta_2 = v_e - \frac{R_{23} + R_{23c}}{R_{33} + R_{33c}} w_e$; K_{3p} and K_{3i} are constant positive definite matrices; we have

$$\dot{L}_3 = -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} \leqslant 0$$

Step 4: Before backstepping for the attitude dynamics, the controller for the yaw angle ψ has to be designed. An augmentation approach for generating ψ_r is introduced by using the command trajectory x_r, y_r as follows:

$$\dot{\psi}_r = \frac{\dot{x}_r\ddot{y}_r - \ddot{x}_r\dot{y}_r}{\dot{x}_r^2 + \dot{y}_r^2},\ \psi_r = \int_0^t \dot{\psi}_r \mathrm{d}t \tag{23}$$

where $\psi_r(0) = \mathrm{atan2}(\dot{y}_r(0), \dot{x}_r(0))$.

Consider the yaw angle kinematics [6]:

$$\dot{\psi} = \frac{s\phi}{c\theta}q + \frac{c\phi}{c\theta}r$$

where r is regarded as the pseudo input. Define $\psi_e \triangleq \psi - \psi_r$ and choose the Lyapunov candidate

$$L_4 = L_3 + \frac{1}{2}\psi_e^2 + \frac{k_{\psi i}}{2}\left(\int_0^t \psi_e \mathrm{d}t\right)^2$$

it follows that

$$\begin{aligned}
\dot{L}_4 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} \\
&+ \psi_e \dot{\psi}_e + k_{\psi i}\psi_e \int_0^t \psi_e \mathrm{d}t \\
=& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} \\
&+ \psi_e(\dot{\psi} - \dot{\psi}_r) + k_{\psi i}\psi_e \int_0^t \psi_e \mathrm{d}t \\
=& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} \\
&+ \psi_e(\frac{s\phi}{c\theta}q + \frac{c\phi}{c\theta}r - \dot{\psi}_r) + k_{\psi i}\psi_e \int_0^t \psi_e \mathrm{d}t
\end{aligned}$$

If r is designed by

$$r = r_c \triangleq \frac{-s\phi}{c\phi}q - \frac{c\theta}{c\phi}\left(k_{\psi p}\psi_e + k_{\psi i}\int_0^t \psi_e \mathrm{d}t - \dot{\psi}_r\right) \tag{24}$$

where $k_{\psi p}$ and $k_{\psi i}$ are constant positive numbers, then

$$\dot{L}_4 = -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \leqslant 0$$

Step 5: Define $\omega_c \triangleq [\omega_{2c}^T, r_c]^T$, $\omega_e \triangleq \omega - \omega_c$, $\tilde{J} \triangleq \hat{J} - J$, $\rho \triangleq [I_{xx}, I_{yy}, I_{zz}, I_{xz},]^T$ and $\tilde{\rho} \triangleq \hat{\rho} - \rho$. Select the Lyapunov candidate

$$L_5 = L_4 + \frac{1}{2}\omega_e^T J \omega_e + \frac{1}{2}\int_0^t \omega_e^T \mathrm{d}t K_{4i} \int_0^t \omega_e \mathrm{d}t + \frac{1}{2\gamma_2}\tilde{\rho}^T\tilde{\rho}$$

its derivative can be obtained by

$$\begin{aligned}
\dot{L}_5 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \\
&+ \bar{R}_{3e}^T \hat{R}\omega_{2e} + \psi_e r_e \frac{c\phi}{c\theta} + \omega_e^T J \dot{\omega}_e + \omega_e^T K_{4i}\int_0^t \omega_e \mathrm{d}t + \frac{1}{\gamma_2}\tilde{\rho}^T\dot{\hat{\rho}}
\end{aligned}$$

where $J\dot{\omega}_e = J\dot{\omega} - J\dot{\omega}_c = -S(\omega)J\omega + Q - J\dot{\omega}_c$. If the control input Q is designed by

$$Q = S(\omega)\hat{J}\omega + \hat{J}\dot{\omega}_c - K_{4p}\omega_e - K_{4i}\int_0^t \omega_e \mathrm{d}t - \xi \tag{25}$$

where $\xi = [\bar{R}_{3e}^T\hat{R},\ \psi_e c\phi/c\theta]^T$, then the derivative of Lyapunov candidate L_5 is given by

$$\begin{aligned}
\dot{L}_5 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \\
&- \omega_e^T K_{4p}\omega_e + \omega_e^T S(\omega)\tilde{J}\omega + \omega_e^T \tilde{J}\dot{\omega}_c + \frac{1}{\gamma_2}\tilde{\rho}^T\dot{\hat{\rho}} \\
=& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \\
&- \omega_e^T K_{4p}\omega_e + \tilde{\rho}^T Y^T \omega_e + \frac{1}{\gamma_2}\tilde{\rho}^T\dot{\hat{\rho}}
\end{aligned}$$

where

$$Y \triangleq \begin{bmatrix} \dot{p}_c & -qr & qr & -pq - \dot{r}_c \\ pr & \dot{q}_c & -pr & p^2 - r^2 \\ -pq & pq & \dot{r}_c & qr - \dot{p}_c \end{bmatrix}$$

If select $\|\hat{\rho}(0)\| < M_2$ and design its adaptive updating laws by projection algorithm:

$$\dot{\hat{\rho}} = \begin{cases} -\gamma_2 Y^T \omega_e, & \text{if } \|\hat{\rho}\| < M_2 \\ & \text{or } \|\hat{\rho}\| = M_2,\ \hat{\rho}^T Y^T \omega_e > 0 \\ -\gamma_2 Y^T \omega_e + \kappa, & \text{if } \|\hat{\rho}\| = M_2,\ \hat{\rho}^T Y^T \omega_e \leqslant 0 \end{cases} \tag{26}$$

where $\kappa \triangleq \gamma_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\hat{\rho}$, it then follows that

$$\begin{aligned}
\dot{L}_5 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \\
&- \omega_e^T K_{4p}\omega_e + \varpi_1 \frac{\hat{m}X^T V_e}{\hat{m}^2}\tilde{m}\hat{m} + \varpi_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\tilde{\rho}^T\hat{\rho}
\end{aligned}$$

where ϖ_1 is defined by (19) and ϖ_2 is given by

$$\varpi_2 = \begin{cases} 0, & \text{if } \|\hat{\rho}\| < M_2 \\ & \text{or } \|\hat{\rho}\| = M_2 \text{ and } \hat{\rho}^T Y^T \omega_e > 0 \\ 1, & \text{if } \|\hat{\rho}\| = M_2 \text{ and } \hat{\rho}^T Y^T \omega_e \leqslant 0 \end{cases} \tag{27}$$

In Step 2, $\varpi_1 \frac{\hat{m}X^T V_e}{\hat{m}^2}\tilde{m}\hat{m} \leqslant 0$ has been derived. Similarly, here $\varpi_2 = 0$ results in $\dot{L}_5 \leqslant 0$, and $\varpi_2 = 1$ (i.e. $\|\hat{\rho}\| = M_2$ and $\hat{\rho}^T Y^T \omega_e \leqslant 0$) implies that

$$\tilde{\rho}^T\hat{\rho} = \frac{1}{2}(\|\hat{\rho}\|^2 + \|\hat{\rho} - \rho\|^2 - \|\rho\|^2) \geqslant 0$$

and $\varpi_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\tilde{\rho}^T\hat{\rho} \leqslant 0$, which also means that $\dot{L}_5 \leqslant 0$.

Step 6: Since T_m and $Q = [L, M, N]^T$ have been designed in previous steps, θ_m can be obtained from (7):

$$t_{cm} = \frac{T_m}{\rho s_m A_m \Omega_m^2 R_m^2},\ \theta_m = \frac{3}{2}\left[\sqrt{\frac{s_m t_{cm}}{2}} + \frac{4t_{cm}}{a_m}\right] \tag{28}$$

and Q_m is determined by

$$q_{cm} = \frac{\delta_d}{8} + 1.13 t_{cm}^{\frac{3}{2}}\sqrt{\frac{s_m}{2}},\ Q_m = q_{cm}\rho s_m A_m \Omega_m^2 R_m^3$$

Then $\tau = [T_t, \varepsilon, \eta]^T$ can be obtained from (13):

$$\tau = Q_A^{-1}(Q - Q_B) \tag{29}$$

and the collective pitch of the tail rotor is yielded by

$$t_{ct} = \frac{T_t}{\rho s_t A_t \Omega_t^2 R_t^2},\ \theta_t = \frac{3}{2}\left[\sqrt{\frac{s_t t_{ct}}{2}} + \frac{4t_{ct}}{a_t}\right] \tag{30}$$

which ends the adaptive backstepping design process.

C. Stability analysis

Assumption 1: *The small coupling terms are bounded by* $\|\Delta_1\| < l_v\|\zeta\| + \bar{\Delta}_1$ *and* $\|\Delta_2\| < l_\omega\|\zeta\| + \bar{\Delta}_2$*, where* ζ *is defined by*

$$\zeta \triangleq [\|P_e\|, \|V_e\|, \|\bar{R}_{3e}\|, \|\psi_e\|, \|\omega_e\|]^T$$

and l_v*,* l_ω*,* $\bar{\Delta}_1$ *and* $\bar{\Delta}_2$ *are small positive numbers.*

In the above assumption, the non-vanishing terms $\bar{\Delta}_1$ and $\bar{\Delta}_2$ concerns the values of Δ_1 and Δ_2 at the equilibrium points, which are very small according to the physical properties. The vanishing terms $l_v\|\zeta\|$ and $l_\omega\|\zeta\|$ are based on the fact that Δ_1 and Δ_2 are related to the states of the system.

***Proposition** 1: Consider the helicopter system* (1)–(4) *with unknown constant inertial parameters m and J bounded by* $\|m\| \leqslant M_1$ *and* $\|\rho\| \leqslant M_2$*, and suppose Assumption 1 is satisfied. If the controller is designed by* (15), (17), (20), (22)–(25), (28) *and* (30) *with proper design parameters, and the adaptive laws are assigned as* (18) *and* (26) *with initial values* $\|\hat{m}(0)\| \leqslant M_1$ *and* $\|\hat{\rho}(0)\| \leqslant M_2$*, then*

1) *the estimated parameters* $\hat{m}$ *and* $\hat{\rho}$ *satisfy* $\|\hat{m}\| \leqslant M_1$ *and* $\|\hat{\rho}\| \leqslant M_2$*, respectively;*
2) *tracking errors of the closed loop system are bounded.*

Proof:

1) Set the Lyapunov candidate for $\hat{\rho}$ as $L = \frac{1}{2\gamma_2}\hat{\rho}^T\hat{\rho}$.
   a) If $\|\hat{\rho}\| < M_2$, the boundedness is obvious.
   b) If $\|\hat{\rho}\| = M_2$ and $\hat{\rho}^T Y^T \omega_e > 0$, then

$$\dot{L} = -\hat{\rho}^T Y^T \omega_e < 0$$

   which indicates that $\|\hat{\rho}\|$ is decreasing.
   c) If $\|\hat{\rho}\| = M_2$ and $\hat{\rho}^T Y^T \omega_e \leqslant 0$, then

$$\dot{L} = -\hat{\rho}^T Y^T \omega_e + \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\hat{\rho}^T\hat{\rho} = 0$$

   which means that $\|\hat{\rho}\|$ is non-increasing.

   In conclusion, $\|\hat{\rho}\| \leqslant M_2$ is guaranteed, if $\|\hat{\rho}(0)\| \leqslant M_2$ is assigned. Boundedness of $\hat{m}$ can be proved similarly.
2) Select L_5 as the candidate Lyapunov function. When the non-vanishing neglected terms Δ_1 and Δ_2 are considered, the derivative of L_5 is given by

$$\begin{aligned}\dot{L}_5 =& -P_e^T K_{1p} P_e - V_e^T K_{2p} V_e - \bar{R}_{3e}^T K_{3p} \bar{R}_{3e} - k_{\psi p}\psi_e^2 \\ & - \omega_e^T K_{4p}\omega_e + \varpi_1 \frac{\hat{m} X^T V_e}{\hat{m}^2}\tilde{m}\hat{m} + \varpi_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\tilde{\rho}^T\hat{\rho} \\ & + V_e^T R_t \Delta_1 + \omega_e^T \Delta_2 \\ \leqslant & -\lambda_{min}(K_{1p})\|P_e\|^2 - \lambda_{min}(K_{2p})\|V_e\|^2 \\ & -\lambda_{min}(K_{3p})\|\bar{R}_{3e}\|^2 - \lambda_{min}(K_{\psi p})\|\psi_e\|^2 \\ & -\lambda_{min}(K_{4p})\|\omega_e\|^2 + \varpi_1 \frac{\hat{m} X^T V_e}{\hat{m}^2}\tilde{m}\hat{m} \\ & + \varpi_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\tilde{\rho}^T\hat{\rho} + \|\Delta_1\|\|R_t\|\|V_e\| + \|\Delta_2\|\|\omega_e\| \\ \leqslant & -(k_\zeta - l_v - l_\omega)\|\zeta\|^2 + (\bar{\Delta}_1 + \bar{\Delta}_2)\|\zeta\| \\ & + \varpi_1 \frac{\hat{m} X^T V_e}{\hat{m}^2}\tilde{m}\hat{m} + \varpi_2 \frac{\hat{\rho}^T Y^T \omega_e}{\|\hat{\rho}\|^2}\tilde{\rho}^T\hat{\rho} \\ \leqslant & -(k_\zeta - l_v - l_\omega)\|\zeta\|^2 + (\bar{\Delta}_1 + \bar{\Delta}_2)\|\zeta\|\end{aligned} \tag{31}$$

   where ϖ_1 and ϖ_2 are given by (19) and (27), $\lambda_{min}(\cdot)$ represents the minimum eigenvalue, and

$$k_\zeta \triangleq \min(\lambda_{min}(K_{ip}), \lambda_{min}(k_{\psi p})), \; i = 1,2,3,4$$

   In (31), we have used $\|R_t\| = 1$ which can be obviously proved by $R_t^T R_t = I_{3\times3}$. If the controller parameters are designed such that $k_\zeta > l_v + l_\omega$, then L_5 decreases when $\|\zeta\| > (\bar{\Delta}_1 + \bar{\Delta}_2)/(k_\zeta - l_v - l_\omega)$, which indicates that the tracking errors are bounded by $\|\zeta\| \leqslant \frac{\bar{\Delta}_1+\bar{\Delta}_2}{k_\zeta - l_v - l_\omega}$.

■

IV. SIMULATION AND DISCUSSION

In following simulation, values concerning the helicopter aerodynamics are obtained from [12]. M_1 and M_2 are assumed to be 15 and 1, respectively. Initial values of the estimated parameters $\hat{m}$ and $\hat{\rho}$, as well as real values of m

TABLE I
INERTIAL PARAMETERS IN SIMULATION

Notations	Values	Notations	Values
$\hat{m}(0)$	10kg	m	8.75kg
$\hat{I}_{xx}(0)$	$0.15\text{kg}\cdot\text{m}^2$	I_{xx}	$0.19\text{kg}\cdot\text{m}^2$
$\hat{I}_{yy}(0)$	$0.3\text{kg}\cdot\text{m}^2$	I_{yy}	$0.34\text{kg}\cdot\text{m}^2$
$\hat{I}_{zz}(0)$	$0.25\text{kg}\cdot\text{m}^2$	I_{zz}	$0.3\text{kg}\cdot\text{m}^2$
$\hat{I}_{xz}(0)$	$0\text{kg}\cdot\text{m}^2$	I_{xz}	$0.05\text{kg}\cdot\text{m}^2$

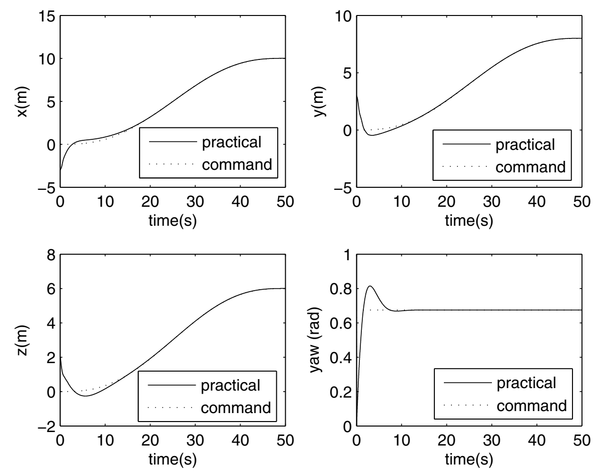

Fig. 1. The trajectory of the controlled autonomous helicopter

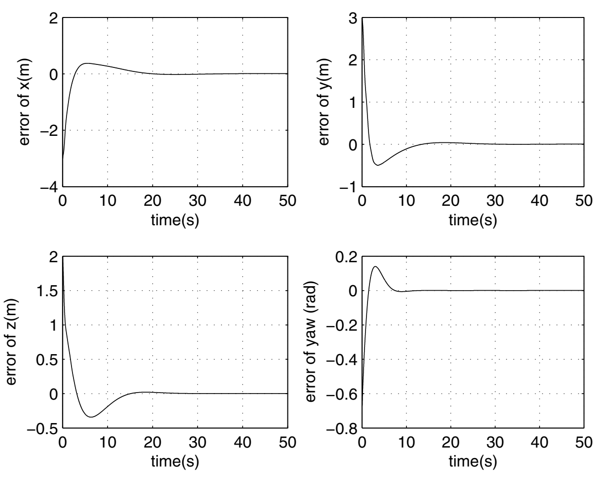

Fig. 2. The tracking errors of the controlled autonomous helicopter

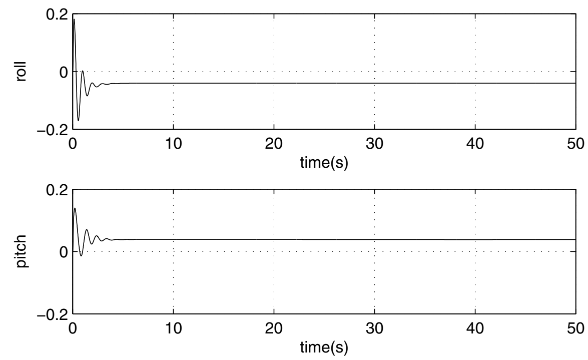

Fig. 3. The roll and pitch angle of the controlled autonomous helicopter

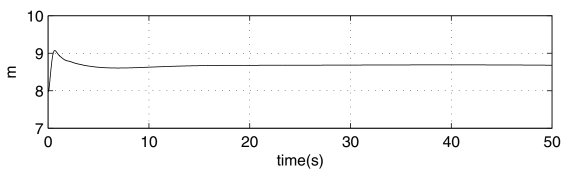

Fig. 4. The estimation for m

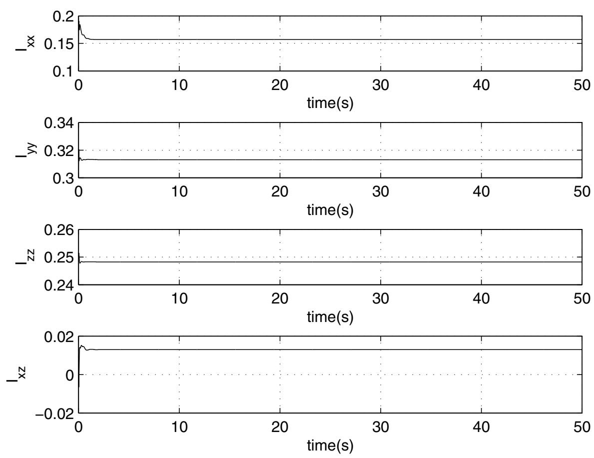

Fig. 5. The estimation for ρ

and ρ, are shown in TABLE I. The command trajectory is given by

$$x_r(t) = 1.92 \cdot 10^{-7} t^5 - 2.4 \cdot 10^{-5} t^4 + 8 \cdot 10^{-4} t^3$$
$$y_r(t) = 1.536 \cdot 10^{-7} t^5 - 1.92 \cdot 10^{-5} t^4 + 6.4 \cdot 10^{-4} t^3$$
$$z_r(t) = 1.152 \cdot 10^{-7} t^5 - 1.44 \cdot 10^{-5} t^4 + 4.8 \cdot 10^{-4} t^3$$

and ψ_r is specified by (23).

Fig. 1–5 display the simulation results for the complete model with error terms (12) and (14) under the control algorithm and the parameter adaptive law provided in Proposition 1. As is illustrated in Fig. 1, the helicopter tracks the command trajectory with some tracking errors. Fig. 2 demonstrates that the tracking errors are bounded, as is expected by Proposition 1. The ultimate bounds of the errors are fairly small, indicating that the side-effects brought by neglecting Δ_1 and Δ_2 are tiny. The roll and pitch angles of the controlled helicopter are maintained in acceptable rages, as are exposed in Fig. 3. The boundedness of estimated parameters are verified by Fig. 4 and Fig. 5.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

An adaptive backstepping approach is proposed in this paper to solve the trajectory tracking problem of an autonomous miniature helicopter with constant inertial parameter uncertainties. The control algorithm is designed through backstepping approach, while the inertial parameter uncertainties are compensated online by adaptive update laws based on projection algorithm. It is proved that the proposed adaptive backstepping control algorithm guarantees the bounded tracking of the miniature autonomous helicopter.

B. Future Works

In this work, derivatives $\dot{R}_{3c}$ and $\dot{\omega}_c$ are obtained from numerical differentiators, because the analytical expressions are rather complicated to implement. Performances of the closed-loop system would improve significantly, if $\dot{R}_{3c}$ and $\dot{\omega}_c$ can be expressed in analytical forms. Moreover, constraints of control inputs are necessary to be included in the research to avoid occasional aggressive attitude. At present, the authors are working at implement the proposed controller on a practical miniature unmanned helicopter.

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