2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Event-Triggered Output Feedback Control for Networked Control Systems using Passivity: Triggering Condition and Limitations

Han Yu and Panos J. Antsaklis

Abstract— **Most of the work on event-triggered control in the literature is based on static state-feedback controllers. In this paper, a dynamic output feedback based event-triggered control scheme is introduced for stabilization of Input Feed-forward Output Feedback Passive (IF-OFP) networked control systems(NCSs), which is a more general case compared with the passive and the OFP systems investigated in our previous work [20]. The triggering condition is derived based on the passivity theorem which allows us to characterize a large class of output feedback stabilizing controllers. We show that under the triggering condition derived in this paper, the control system is finite-gain $\mathcal{L}_2$ stable in the presence of bounded external disturbances. We provide some analysis on the inter-event time which reveals the difficulties of obtaining a common lower bound on the inter-event time when the full-state information of the plant is not used. Some challenges of applying event-triggered control to real time NCSs are also discussed.**

I. Introduction

The majority of feedback control laws nowadays are implemented on digital platforms since microprocessors offer many advantages of running real-time operating systems. In such an implementation, the control task consists of sampling the outputs of the plant, computing and implementing new control signals. Traditionally, the control task is executed periodically, since this allows the closed-loop system to be analyzed and the controller to be designed using the well-developed theory on sampled-data systems. However, the control strategy obtained based on this approach is conservative in the sense that resource usage(i.e., sampling rate, CPU time) is more frequent than necessary since stability is guaranteed under sufficiently fast periodic execution of the control action. To overcome this drawback of periodic paradigm, several researchers suggested the idea of event-triggered control. Although in the literature, the terminology refers to the triggering mechanism as event-based-sampling[12], to event-driven sampling[13], Lebesgue sampling[5], deadband control[14], level-crossing sampling[15], state-triggered sampling[6] and self-triggered sampling[9] with slightly different meanings, in all cases, the control signals are kept constant until violation of a condition on certain signals of the plant triggers re-computation of the control signals. The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired level of performance makes event-triggered control very appealing in networked control systems(NCSs).

Most of the results on event-triggered control are obtained under the assumption that the feedback control law provides input-to-state stability(ISS) in the sense of [16] with respect to the measurement errors, see [6]-[10]. The ISS framework provides insight into the triggering condition by exploring the relation between stabilization and the current full-state information. However, in many control applications, the full state information is not available, so it is important to study stability and performance of event-triggered control systems with output feedback based control strategies.

In our previous work[20], a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive(OFP) NCSs. The triggering condition is derived based on the output feedback passivity indices of the plant. Since in many control applications, dynamic controllers are preferred to static controllers due to the considerations of performance, fault tolerance, robustness, etc, it is important to our previous results to more general systems with dynamic output feedback controllers. In this paper, we propose an event-triggered control scheme for Input Feed-forward Output Feedback Passive(IF-OFP) systems, which is a more general framework compared with our previous results. The triggering condition is derived based on passivity theorems (see for example the textbooks [2], [17] and [3]), which allow us to find a large class of output feedback stabilizing controllers(static or dynamic). We show that with the triggering condition derived in this paper, the control system is finite-gain $\mathcal{L}_2$ stable in the presence of bounded external disturbances. We also provide some analysis on the inter-event time which reveals the difficulties of obtaining a common lower bound on the inter-event time when we cannot use the full-state information for feedback control action. Some challenges of applying output feedback based event-triggered control to real time NCSs are discussed. Another related work can be found in [18].

The rest of this paper is organized as follows: we introduce some background on passive and dissipative systems in section II; the problem is stated in section III; our main results are provided in section IV; concluding remarks are made in section V.

II. Background Material

Consider the following dynamic system, which could be linear or nonlinear:

$$H_p : \begin{cases} \dot{x}_p = f_p(x_p, u_p) \\ y_p = h_p(x_p, u_p) \end{cases} \tag{1}$$

where $x_p \in X_p \subset \mathbb{R}^n$, $u_p \in U_p \subset \mathbb{R}^m$ and $y_p \in Y_p \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X_p, U_p and Y_p are the state, input and output spaces, respectively.

Han Yu and Panos J. Antsaklis are both with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA hyu@nd.edu, antsaklis.1@nd.edu

The representation $\phi_p(t, t_0, x_{p0}, u_p)$ is used to denote the state at time t reached from the initial state x_{p0} at time t_0.

Definition 1(Supply Rate)[1]: The supply rate $\omega_p(t) = \omega_p(u_p(t), y_p(t))$ is a real valued function defined on $U_p \times Y_p$, such that for any $u_p(t) \in U_p$ and $x_{p0} \in X_p$ and $y_p(t) = h_p(\phi_p(t, t_0, x_{p0}, u_p), u_p)$, $\omega_p(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega_p(\tau)| d\tau < \infty. \tag{2}$$

Definition 2(Dissipative System)[1]: System H_p with supply rate $\omega_p(t)$ is said to be **dissipative** if there exists a nonnegative real function $V_p(x) : X_p \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_{p0} \in X_p$ and $u_p \in U_p$,

$$V_p(x_{p1}) - V_p(x_{p0}) \leq \int_{t_0}^{t_1} \omega_p(\tau) d\tau, \tag{3}$$

where $x_{p1} = \phi_p(t_1, t_0, x_{p0}, u_p)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 3(Passive System)[1]: System H_p is said to be **passive** if there exists a storage function $V_p(x_p)$ such that

$$V_p(x_{p1}) - V(x_{p0}) \leq \int_{t_0}^{t_1} u_p^T(\tau) y_p(\tau) d\tau, \tag{4}$$

if $V_p(x_p)$ is $\mathcal{C}^1$, then we have

$$\dot{V}_p(x_p) \leq u_p^T(t) y_p(t), \ \forall t \geq 0. \tag{5}$$

One can see that passive system is a special class of dissipative system with supply rate $\omega_p(t) = u_p^T(t) y_p(t)$.

Definition 4(IF-OFP systems)[2]: System H_p is said to be **Input Feed-forward Output Feedback Passive**(IF-OFP) if it is dissipative with respect to the supply rate

$$\omega_p(u_p, y_p) = u_p^T y_p - \rho_p y_p^T y_p - \nu_p u_p^T u_p, \ \forall t \geq 0, \tag{6}$$

for some $\rho_p, \nu_p \in \mathbb{R}$.

For the rest of this paper, we will denote an IF-OFP system with m inputs and m outputs by *IF-OFP*$(\nu, \rho)^m$ and we will call (ν, ρ) *passivity indices* of the system. Note that in general, an IF-OFP system may not be passive unless both indices ν and ρ are nonnegative.

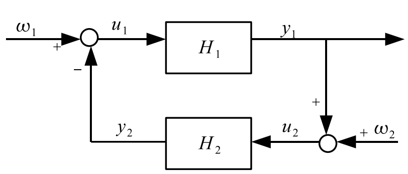

Fig. 1: Feedback Interconnection of Two IF-OFP Systems

Theorem 1 (Passivity Theorem)[17]: Consider the feedback interconnection as shown in Fig.1, and suppose each feedback component is with proper input and output dimensions and satisfies the inequality

$$\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i, \ \text{for } i = 1, 2, \tag{7}$$

for some storage function $V_i(x_i)$. Then, the closed-loop map from $\omega = [\omega_1^T, \omega_2^T]^T$ to $y = [y_1^T, y_2^T]^T$ is finite gain $\mathcal{L}_2$ stable if

$$0 < \rho_1 + \nu_2 < \infty, \quad 0 < \rho_2 + \nu_1 < \infty. \tag{8}$$

III. Problem Statement

We consider the control system given in (1). We assume H_p is IF-OFP$(\nu_p, \rho_p)^m$ with storage function V_p. Based on Theorem 1, we know that if we design an IF-OFP$(\nu_c, \rho_c)^m$ controller with storage function V_c such that $0 < \rho_c + \nu_p < \infty$, $0 < \rho_p + \nu_c < \infty$, then the closed-loop system is finite gain $\mathcal{L}_2$ stable.

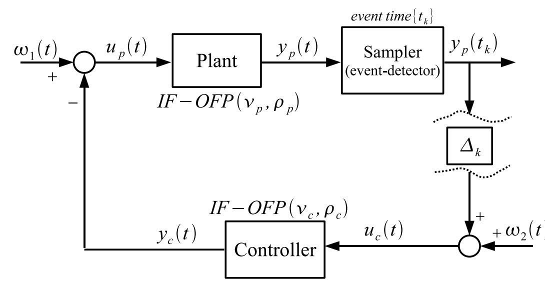

Fig. 2: Event-Triggered Control

In real time, the implementation of the feedback control law is typically done by sampling the plant output $y_p(t)$ at time instants $t_0, \ t_1, \ \ldots,$ computing the control action $y_c(t)$ and updating the input to the plant at time instants $t_0 + \Delta_0, \ t_1 + \Delta_1, \ \ldots,$ where $\Delta_k \geq 0$, for $k = 0, 1, 2, \ldots$ represents the network induced delay from the sampler to the network controller(here, we assume the delay from the controller to the actuator is negligible). This means a sequence of measurements $y_p(t_k)_s$ corresponds to a sequence of controller's input updates $u_c(t_k + \Delta_k)_s$. In event-triggered NCSs, an event-detector (an embedded hardware in the sampler) is used to monitor the output of the plant with sufficiently fast sampling rate, an updated output measurement is sent to the network controller only when the size of the output novelty error $\tilde{e}_p(t) = y_p(t) - y_p(t_k)$ (for $t \in [t_k, t_{k+1})$) exceeds a certain threshold(triggering condition), where $y_p(t_k)$ denotes the last transmitted output information of the plant. See the event-triggered networked control scheme as shown in Fig. 2, where $\omega_1(t)$ and $\omega_2(t)$ denote the external disturbances at the plant's side and at the controller's side respectively.

We summarize the problems we try to solve in this paper as follows. If the plant is IF-OFP$(\nu_p, \rho_p)^m$, what should be the output feedback stabilizing controller and the triggering condition? Is the passivity condition shown in Theorem 1 still sufficient to guarantee $\mathcal{L}_2$ stability of the control system? What is the interaction between the triggering condition, the achievable $\mathcal{L}_2$ gain and the characterization of the output feedback stabilizing controllers? Moreover, what is the inter-event time implicitly determined by the triggering condition?

IV. Main Results

In this section, we will assume that the network induced delay from the sampler to the controller is negligible. The delay problem will be addressed in our companion paper [21], where network induced delays with bounded "jitters" from the sampler to the controller and from the controller to the actuator are considered.

Theorem 2. Consider the control system as shown in Fig.2, where the plant is IF-OFP$(\nu_p, \rho_p)^m$ with a $\mathcal{C}^1$ storage function V_p, the controller is IF-OFP$(\nu_c, \rho_c)^m$ with a $\mathcal{C}^1$ storage function V_c, and $0 < \nu_c + \rho_p < \infty$, $0 < \nu_p + \rho_c < \infty$. Assume that the network induced delay $\Delta_k \equiv 0$, $\forall k$. If the event time t_k is explicitly determined by the following triggering condition

$$\|\tilde{e}_p(t)\|_2 = \frac{\delta}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2,\ \forall t \geq 0, \tag{9}$$

$$\text{where}\ \ \zeta = \left[\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right]^{\frac{1}{2}}, \tag{10}$$

with $\delta \in (0, 1]$ and $0 < \alpha,\ \beta < 1$, then the control system is finite gain $\mathcal{L}_2$ stable from $\omega(t) = [\omega_1^T(t),\ \omega_2^T(t)]^T$ to $y(t) = [y_p^T(t),\ y_c^T(t)]^T$.

Proof: Since the plant is IF-OFP$(\nu_p, \rho_p)^m$ and the controller is IF-OFP$(\nu_c, \rho_c)^m$, we have

$$\begin{aligned} \dot{V}_p(t) &\leq u_p^T(t)y_p(t) - \nu_p u_p^T(t)u_p(t) - \rho_p y_p^T(t)y_p(t), \\ \dot{V}_c(t) &\leq u_c^T(t)y_c(t) - \nu_c u_c^T(t)u_c(t) - \rho_c y_c^T(t)y_c(t). \end{aligned} \tag{11}$$

Consider a storage function for the system given by $V = V_c + V_p$, then for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} \dot{V} &\leq \big[\omega_1(t) - y_c(t)\big]^T y_p(t) - \rho_p y_p^T(t)y_p(t) \\ &\quad - \nu_p\big[\omega_1(t) - y_c(t)\big]^T\big[\omega_1(t) - y_c(t)\big] \\ &\quad + \big[\omega_2(t) + y_p(t_k)\big]^T y_c(t) - \rho_c y_c^T(t)y_c(t) \\ &\quad - \nu_c\big[\omega_2(t) + y_p(t_k)\big]^T\big[\omega_2(t) + y_p(t_k)\big] \\ &= \omega_1^T(t)y_p(t) - \nu_p\omega_1^T(t)\omega_1(t) + 2\nu_p\omega_1^T(t)y_c(t) \\ &\quad + \omega_2^T(t)y_c(t) - \nu_c\omega_2^T(t)\omega_2(t) - 2\nu_c\omega_2^T(t)\big[y_p(t) - \tilde{e}_p(t)\big] \\ &\quad - y_c^T(t)y_p(t) - (\nu_p+\rho_c)y_c^T(t)y_c(t) - \rho_p y_p^T(t)y_p(t) \\ &\quad + [y_p(t) - \tilde{e}_p(t)]^T y_c(t) - \nu_c y_p^T(t_k)y_p(t_k). \end{aligned} \tag{12}$$

Since $2\nu_c\omega_2^T(t)\tilde{e}_p(t) \leq |\nu_c|\omega_2^T(t)\omega_2(t) + |\nu_c|\tilde{e}_p^T(t)\tilde{e}_p(t)$ and $y_p^T(t_k)y_p(t_k) = y_p^T(t)y_p(t) + \tilde{e}_p^T(t)\tilde{e}_p(t) - 2\tilde{e}_p^T(t)y_p(t)$, let

$$A = \begin{bmatrix} 1 & 2\nu_p \\ -2\nu_c & 1 \end{bmatrix},\quad B = \begin{bmatrix} \nu_p & 0 \\ 0 & \nu_c - |\nu_c| \end{bmatrix}, \tag{13}$$

we can get

$$\begin{aligned} \dot{V} &\leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - (\nu_p+\rho_c)y_c^T(t)y_c(t) \\ &\quad - \tilde{e}_p^T(t)y_c(t) + |\nu_c|\tilde{e}_p^T(t)\tilde{e}_p(t) - \rho_p y_p^T(t)y_p(t) \\ &\quad - \nu_c y_p^T(t)y_p(t) + 2\nu_c\tilde{e}_p^T(t)y_p(t) - \nu_c\tilde{e}_p^T(t)\tilde{e}_p(t), \end{aligned} \tag{14}$$

if we choose $0 < \alpha, \beta < 1$ and let

$$C = \begin{bmatrix} (1-\beta)(\rho_p+\nu_c) & 0 \\ 0 & (1-\alpha)(\nu_p+\rho_c) \end{bmatrix}, \tag{15}$$

then we can further get

$$\begin{aligned} \dot{V} &\leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t) \\ &\quad - \Big\|\sqrt{\alpha(\nu_p+\rho_c)}y_c(t) + \frac{1}{2\sqrt{\alpha(\nu_p+\rho_c)}}\tilde{e}_p(t)\Big\|_2^2 \\ &\quad - \beta(\rho_p+\nu_c)\|y_p(t)\|_2^2 + 2\nu_c\tilde{e}_p^T(t)y_p(t) \\ &\quad + \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\tilde{e}_p(t)\|_2^2, \end{aligned} \tag{16}$$

thus

$$\begin{aligned} \dot{V} &\leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t) \\ &\quad + \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\tilde{e}_p(t)\|_2^2 \\ &\quad - \beta(\rho_p+\nu_c)\|y_p(t)\|_2^2 + 2\nu_c\tilde{e}_p^T(t)y_p(t). \end{aligned} \tag{17}$$

We can obtain

$$\begin{aligned} \dot{V} &\leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t) \\ &\quad + \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\tilde{e}_p(t)\|_2^2 \\ &\quad + 2\nu_c\tilde{e}_p^T(t)y_p(t) + \frac{\nu_c^2}{\frac{1}{4\alpha\left(\nu_p+\rho_c\right)} + |\nu_c| - \nu_c}\|y_p(t)\|_2^2 \\ &\quad - \left[\beta(\rho_p+\nu_c) + \frac{\nu_c^2}{\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c}\right]\|y_p(t)\|_2^2, \end{aligned} \tag{18}$$

and one can verify that if

$$\|\tilde{e}_p(t)\|_2 \leq \frac{1}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2,\ \forall t \geq 0, \tag{19}$$

we have

$$\dot{V} \leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t),\ \forall t \geq 0. \tag{20}$$

Let $c = \min\{(1-\alpha)(\nu_p+\rho_c),\ (1-\beta)(\rho_p+\nu_c)\}$, $a = \|A\|_2$, and $b = \|B\|_2$, we can get

$$\begin{aligned} \dot{V} &\leq -c\|y(t)\|_2^2 + a\|\omega(t)\|_2\|y(t)\|_2 + b\|\omega(t)\|_2^2 \\ &= -\frac{1}{2c}\Big(a\|\omega(t)\|_2 - c\|y(t)\|_2\Big)^2 + \frac{a^2}{2c}\|\omega(t)\|_2^2 \\ &\quad - \frac{c}{2}\|y(t)\|_2^2 + b\|\omega(t)\|_2^2 \leq \frac{k^2}{2c}\|\omega(t)\|_2^2 - \frac{c}{2}\|y(t)\|_2^2, \end{aligned} \tag{21}$$

where $k^2 = a^2 + 2bc$. Integrating (21) over $[0, \tau]$ and using $V(x) \geq 0$, then taking the square root, we arrive at

$$\|y_\tau\|_{\mathcal{L}_2} \leq \frac{k}{c}\|\omega_\tau\|_{\mathcal{L}_2} + \sqrt{\frac{2V(0)}{c}}, \tag{22}$$

where y_τ and ω_τ denote the truncated signals of $y(t)$ and $\omega(t)$. Note that the triggering condition (9) ensures that (19) is satisfied, which completes the proof. ■

Remark 1. Since $\|\tilde{e}_p(t)\|_2 = \|y_p(t) - y_p(t_k)\|_2$, we have $\|\tilde{e}_p(t)\|_2 \geq \|y_p(t_k)\|_2 - \|y_p(t)\|_2$, thus $\|y_p(t)\|_2 \geq \|y_p(t_k)\|_2 - \|\tilde{e}_p(t)\|_2$, for $t \in [t_k, t_{k+1})$. Based on this, if we define

$$\sigma_o = \frac{1}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right],$$

then one can verify that a sufficient condition for (19) to be satisfied is given by

$$\|\tilde{e}_p(t)\|_2 \leq \frac{\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2, \quad \text{for } t \in [t_k, t_{k+1}), \forall k. \quad (23)$$

So an alternative triggering condition to (9) is given by

$$\|\tilde{e}_p(t)\|_2 = \frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2, \quad \text{for } t \in [t_k, t_{k+1}),\ \forall k, \quad (24)$$

with $\delta \in (0, 1]$. This is a tighter triggering condition which will be used later for the analysis of the inter-event time.

Remark 2. If the output $y(t)$ is a class-$\mathcal{K}$ function of the whole state $x(t) = [x_p^T(t), x_c^T(t)]^T$ (i.e., $y(t) = \alpha(x(t))$, where $\alpha(\cdot) \in$ class-$\mathcal{K}$), in view of (21), one can further get the input-to-state stability (ISS) results (where the input is the external disturbance $\omega(t)$) if the storage function $V(x)$ is positive definite.

Remark 3. In view of (9) and (22), one can see that both the triggering condition and the achievable $\mathcal{L}_2$ gain are related to the passivity indices of the plant and the controller. In general, for larger values of $\nu_c + \rho_p$ and $\nu_p + \rho_c$, we can obtain a larger triggering threshold σ_o and a smaller $\mathcal{L}_2$ gain of the system, which implies a better performance of the control system.

The triggering condition (9) in Theorem 2 explicitly determines when an updated output information of the plant should be sent to the controller for control action update to ensure the $\mathcal{L}_2$ stability of the system in the absence of network induced delays. Another problem that needs to be addressed is how often the data transmission occurs based on the triggering condition? This problem is not easy in general, especially when the dynamics of the plant are highly nonlinear and only output information can be used to generate the control actions. Moreover, in the presence of external disturbances, the "zeno" inter-event time maybe unavoidable. The following proposition provides a way to estimate the lower bound of the inter-event time when we restrict the output of the plant to be a memoryless function belonging to a bounded sector of the state. One should be aware that while our analysis is similar to [6], there are other ways in the literature to estimate the inter-event time based on different assumptions, see [8], [9], [10]. Thus, it is possible to derive a less conservative result by taking different approaches and different assumptions. Based on the assumptions shown in the following proposition, the impact of the disturbances on the inter-event time can be shown explicitly.

We assume that the plant is IF-OFP$(\nu_p, \rho_p)^m$ with dynamics given by

$$H_p : \begin{cases} \dot{x}_p &= f_p(x_p, u_p) \\ y_p &= h_p(x_p) \end{cases}, \quad (25)$$

and the controller is IF-OFP$(\nu_c, \rho_c)^m$ with dynamics given by

$$H_c : \begin{cases} \dot{x}_c &= f_c(x_c, u_c) \\ y_c &= h_c(x_c, u_c) \end{cases}. \quad (26)$$

Note that we assume that there is no feed-through at the output of the plant. This usually corresponds to the case when the relative degree of the plant is greater than zero and $\nu_p \leq 0$, see [2].

Proposition 1. Consider the networked control system shown in Fig.2, where the plant is IF-OFP$(\nu_p, \rho_p)^m$ with a $\mathcal{C}^1$ storage function $V_p(x_p)$ and the controller is IF-OFP$(\nu_c, \rho_c)^m$ with a $\mathcal{C}^1$ storage function $V_c(x_c)$. Assume that the network induced delay from the sampler to the controller $\Delta_k \equiv 0$, $\forall k$. Let the following assumptions be satisfied:

1) $f_p(x_p, u_p) : \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$ is locally Lipschitz continuous in x_p on a compact set $S_x \subset \mathbb{R}^{n_p}$ with Lipschitz constant L_x;
2) $\|f_p(x_p, u_p) - f_p(x_p, 0)\|_2 \leq L_u\|u_p\|_2$ for all $x_p \in S_x$ with some nonnegative constant L_u;
3) $h_p(x_p) : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$ belongs to a sector (K_1, K_2), with $K_1 x_p^T x_p \leq x_p^T h_p(x_p) \leq K_2 x_p^T x_p$, where $K_1 \in \mathbb{R}$, $K_2 \in \mathbb{R}$ and $0 < K_1 K_2 < \infty$;
4) $\left\|\frac{\partial h_p}{\partial x_p}\right\|_2 \leq \gamma_p$, where $0 < \gamma_p < \infty$;
5) $\nu_p + \rho_c > 0$, $\rho_p + \nu_c > 0$, $\rho_c > 0$, $x_c(t_0) = 0$;
6) $\sup_{t \geq 0}\|\omega_1(t)\|_2 \leq d_1$ and $\sup_{t\geq 0}\|\omega_2(t)\|_2 \leq d_2$, where $0 < d_1, d_2 < \infty$.

Then for any initial condition $x_p(0)$ in a compact set $S_0 \subset S_x$, the inter-event time $\{t_{k+1} - t_k\}$ implicitly determined by the triggering condition (24) is lower bounded by $\tau_k = \frac{1}{C_2}\ln\left(1 + \frac{C_3}{C_1}\right) \geq 0$, where

$$C_1 = \frac{(L_x\zeta_p + L_u\Gamma_c)\|y_p(t_k)\|_2 + L_u(d_1 + \Gamma_c d_2)}{L_x\zeta_p},$$

$$C_2 = \gamma_p L_x \zeta_p \quad \text{and} \quad C_3 = \frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2,$$

$\zeta_p = \max\{\frac{1}{|K_1|}, \frac{1}{|K_2|}\}$, $\Gamma_c = \sqrt{\frac{1+2\rho_c|\nu_c|}{\rho_c^2}}$.

Proof: Since $\tilde{e}_p(t) = y_p(t) - y_p(t_k)$ for $t \in [t_k, t_{k+1})$, we can get for $t \in [t_k, t_{k+1})$

$$\begin{aligned} &\frac{d}{dt}\|\tilde{e}_p(t)\|_2 \leq \|\dot{\tilde{e}}_p(t)\|_2 = \|\dot{y}_p(t)\|_2 = \|\dot{h}_p(x_p)\|_2 \\ &= \left\|\frac{\partial h_p}{\partial x_p}f_p(x_p, 0) + \frac{\partial h_p}{\partial x_p}\left[f_p(x_p, u_p) - f_p(x_p, 0)\right]\right\|_2 \\ &\leq \gamma_p L_x\|x_p\|_2 + \gamma_p L_u\|u_p\|_2 \\ &= \gamma_p L_x\|x_p\|_2 + \gamma_p L_u\|\omega_1 - y_c\|_2 \\ &\leq \gamma_p L_x\|x_p\|_2 + \gamma_p L_u d_1 + \gamma_p L_u\|y_c\|_2. \end{aligned} \quad (27)$$

Since $x_c(t_0) = 0$, with $\rho_c > 0$, one can prove that

$$\|y_{c\tau}\|_{\mathcal{L}_2} \leq \sqrt{\frac{1+2\rho_c|\nu_c|}{\rho_c^2}}\|u_{c\tau}\|_{\mathcal{L}_2}, \forall \tau \geq t_0. \quad (28)$$

Thus, we can further obtain

$$\begin{aligned} &\frac{d}{dt}\|\tilde{e}_p(t)\|_2 \leq \gamma_p L_x\|x_p(t)\|_2 + \gamma_p L_u d_1 + \gamma_p L_u\Gamma_c\|u_c(t)\|_2 \\ &= \gamma_p L_x\|x_p(t)\|_2 + \gamma_p L_u d_1 + \gamma_p L_u\Gamma_c\|y_p(t_k) + \omega_2(t)\|_2. \end{aligned} \quad (29)$$

Since $h_p(x_p)$ belongs to the sector (K_1, K_2), one can verify that $\|x_p(t)\|_2 \leq \zeta_p \|y_p(t)\|_2$, and we have

$$\begin{aligned}
&\frac{d}{dt}\|\tilde{e}_p(t)\|_2 \leq \gamma_p L_x \zeta_p \|y_p(t)\|_2 + \gamma_p L_u d_1 \\
&+ \gamma_p L_u \Gamma_c \|y_p(t_k)\|_2 + \gamma_p L_u \Gamma_c \|\omega_2(t)\|_2 \\
&= \gamma_p L_x \zeta_p \|\tilde{e}_p(t) + y_p(t_k)\|_2 + \gamma_p L_u d_1 \\
&+ \gamma_p L_u \Gamma_c \|y_p(t_k)\|_2 + \gamma_p L_u \Gamma_c \|\omega_2(t)\|_2 \\
&\leq \gamma_p L_x \zeta_p \|\tilde{e}_p\|_2 + \gamma_p (L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 \\
&+ \gamma_p L_u (d_1 + \Gamma_c d_2),
\end{aligned} \tag{30}$$

so the evolution of $\|\tilde{e}_p(t)\|_2$ during the time interval $[t_k, t_{k+1})$ is bounded by the solution to

$$\begin{aligned}
\frac{d}{dt}\phi(t) = \gamma_p L_x \zeta_p \phi(t) + \gamma_p (L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 \\
+ \gamma_p L_u (d_1 + \Gamma_c d_2),
\end{aligned} \tag{31}$$

with initial condition $\phi(t_k) = 0$. Thus the time for $\|\tilde{e}_p(t)\|_2$ to evolve from 0 to $\frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$ is lower bounded by the solution to $\phi(t_k + \tau_k) = \frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$. Let

$$C_1 = \frac{(L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 + L_u(d_1 + \Gamma_c d_2)}{L_x \zeta_p},$$

$C_2 = \gamma_p L_x \zeta_p$ and $C_3 = \frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$, we can get

$$\tau_k = \frac{1}{C_2}\ln\big(1 + \frac{C_3}{C_1}\big) \geq 0, \tag{32}$$

and the proof is complete. ■

Remark 4. One can see that when $d_1 = d_2 = 0$ (no external disturbance inputs), then we have

$$\tau_k = \frac{1}{\gamma_p L_x \zeta_p}\ln\big(1 + \frac{\frac{\delta\sigma_o}{1+\sigma_o} L_x \zeta_p}{L_x \zeta_p + L_u \Gamma_c}\big) > 0, \tag{33}$$

and in this case we can get a common lower bound of the inter-event time. Moreover, with a larger triggering threshold σ_o, we can get a larger τ_k. Since σ_o is related to the passivity indices of the plant and the controller, the interactions between the triggering condition, the passivity indices and the inter-event time are implicitly revealed here. However, when the external disturbances ω_1, ω_2 cannot be neglected, τ_k could be extremely small when $y_p(t)$ approaches the origin, and we may get "zeno" inter-event time.

For a linear time-invariant (LTI) IF-OFP plant H_p, by assuming that H_p is detectable, we derive another analysis of the inter-event time (H_c could be linear or nonlinear with the same input and output dimension as H_p). In this case, we do not need to assume that y_p belongs to a bounded sector of x_p. Consider a LTI system given by:

$$H_p : \begin{cases} \dot{x}_p = A_p x_p + B_p u_p \\ y_p = C_p x_p, \end{cases} \tag{34}$$

where $A_p \in \mathbb{R}^{n_p \times n_p}$, $B \in \mathbb{R}^{n_p \times m_p}$, and $C \in \mathbb{R}^{m_p \times n_p}$. Recall that H_p is detectable if with zero input, $C_p x_p = 0$ implies $x_p \to 0$. Note that if H_p is detectable, then there always exits a matrix $L \in \mathbb{R}^{n_p \times m_p}$ such that $A_p + L_p C_p$ is Hurwitz. Moreover, we have the following proposition for detectable LTI systems.

Proposition 2 [19]. For a detectable LTI system H_p, there exists some constant $K > 0$ such that the following bound on the norm of the state x_p holds:

$$\begin{aligned}
\|x_p\|_2 \leq K e^{-\widehat{\lambda} t}\|x_p(0)\|_2 + \frac{K\|B_p\|_2}{\widehat{\lambda}}\|u_p|_{[0,t]}\| \\
+ \frac{K\|L\|_2}{\widehat{\lambda}}\|y_p|_{[0,t]}\|,
\end{aligned} \tag{35}$$

where L is the matrix such that $A_p + LC_p$ is Hurwitz, and $-\widehat{\lambda} > Re\lambda$ for every eigenvalue λ of $A_p + LC_p$. $\|(\cdot)|_{[0,t]}\|$ denotes the essential supremum norm of $(\cdot)$ on the time interval $[0, t]$.

Proposition 3. Consider the networked control system shown in Fig.2, where the plant H_p as given in (34) is detectable and IF-OFP$(\nu_p, \rho_p)^m$ with storage function $V_p(x_p)$, the controller is IF-OFP$(\nu_c, \rho_c)^m$ with storage function $V_c(x_c)$. Assume that the network induced delay from the sampler to the controller $\Delta_k \equiv 0, \forall k$. Let assumption 5)-6) in Proposition 1 be satisfied. Then for any initial condition $x_p(0)$ in a compact set $S_0 \subset S_x$, the inter-event time $\{t_{k+1} - t_k\}$ explicitly determined by the triggering condition (24) is lower bounded by

$$\tau_k = \frac{\frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2}{\widehat{C}_1\|x_p(t_k)\|_2 + \widehat{C}_2\|y_p(t_k)\|_2 + \widehat{C}_3 d_1 + \widehat{C}_4 d_2}$$

where $\widehat{C}_1 = \gamma_p L_x K$, $\widehat{C}_2 = \gamma_p L_x\big[\frac{K L_u \Gamma_c}{\widehat{\lambda}} + \frac{K\|L\|_2}{\widehat{\lambda}}\big(1 + \frac{\delta\sigma_o}{1+\sigma_o}\big)\big] + \gamma_p L_u \Gamma_c$, $\widehat{C}_3 = \frac{\gamma_p L_x K L_u}{\widehat{\lambda}} + \gamma_p L_u$, $\widehat{C}_4 = \frac{\gamma_p L_x K L_u \Gamma_c}{\widehat{\lambda}} + \gamma_p L_u \Gamma_c$, $\gamma_p = \|C_p\|_2$, $L_x = \|A_p\|_2$ and $L_u = \|B_p\|_2$, Γ_c is the same as defined in Proposition 1, K, $\widehat{\lambda}$ and L are the same as defined in Proposition 2.

Proof: One can verify that for $t \in [t_k, t_{k+1})$, with $L_x = \|A_p\|_2$, $L_u = \|B_p\|_2$, and γ_p replaced by $\|C_p\|_2$, we can still arrive at (29). Then, by applying Proposition 2, we can further get for $t \in [t_k, t_{k+1}]$

$$\begin{aligned}
\|x_p\|_2 \leq K e^{-\widehat{\lambda}(t-t_k)}\|x_p(t_k)\|_2 + \frac{K L_u}{\widehat{\lambda}}\|u_p|_{[t_k, t_{k+1})}\| \\
+ \frac{K\|L\|_2}{\widehat{\lambda}}\|y_p|_{[t_k, t_{k+1})}\|,
\end{aligned} \tag{36}$$

one can further obtain

$$K e^{-\widehat{\lambda}(t-t_k)}\|x_p(t_k)\|_2 \leq K\|x_p(t_k)\|_2, \tag{37}$$

$$\frac{K L_u}{\widehat{\lambda}}\|u_p|_{[t_k, t_{k+1})}\| \leq \frac{K L_u}{\widehat{\lambda}} d_1 + \frac{K L_u \Gamma_c}{\widehat{\lambda}}(\|y_p(t_k)\|_2 + d_2), \tag{38}$$

$$\frac{K\|L\|_2}{\widehat{\lambda}}\|y_p|_{[t_k, t_{k+1})}\| \leq \frac{K\|L\|_2}{\widehat{\lambda}}\big(1 + \frac{\delta\sigma_o}{1+\sigma_o}\big)\|y_p(t_k)\|_2 \tag{39}$$

substitute (36)-(39) into (29), we will get

$$\frac{d}{dt}\|\tilde{e}_p(t)\|_2 \leq \widehat{C}_1\|x_p(t_k)\|_2 + \widehat{C}_2\|y_p(t_k)\|_2 + \widehat{C}_3 d_1 + \widehat{C}_4 d_2. \tag{40}$$

So in this case, on can verify that the inter-event time is lower bounded by

$$\tau_k = \frac{\frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2}{\widehat{C}_1\|x_p(t_k)\|_2 + \widehat{C}_2\|y_p(t_k)\|_2 + \widehat{C}_3 d_1 + \widehat{C}_4 d_2}. \quad (41)$$

■

Remark 5. In view of τ_k in Proposition 3, one can see that if $x_p(t) = y_p(t)$(the full-state is available for measure) or if $y_p(t)$ belongs to a bounded sector of $x_p(t)$, then in the absence of external disturbances ω_1 and ω_2, we could get a common lower bound on the inter-event time, which is strictly positive. However, since we have no information on $\|x_p(t_k)\|_2$, it is difficult to get a common lower bound on the inter-event time. In many cases, common lower bound on the inter-event time is only a local property of event-triggered control.

Remark 6. When there is non-trivial network induced delay from the sampler to the controller ($\Delta_k \neq 0$), if the delay is upper bounded by the inter-event time, then stability can still be assumed. Thus, the admissible network induced delay is related to the triggering condition: usually, a larger triggering threshold implies more tolerance to the network induced delay since the inter-event time may be longer. However, as indicated in Remark 4, the obtained admissible network induced delays could be small in the presence of external disturbances since the inter-event time could be very small in that case. In real time NCSs, the network induced delay is usually unknown, it is very likely to have delay larger than the inter-event time. Thus it is not very practical to schedule the control tasks at the sampler side based on the knowledge of the inter-event time. Unfortunately, most existing work on event-triggered control for NCSs neglect this fact and assume that network induced delay is smaller than the inter-event time to ensure the stability of the control system. In [21], we propose a set-up to deal with arbitrary constant network induced delays or delays with bounded "jitters" based on the results shown in the current paper.

V. Conclusion

In this paper, a dynamic output feedback based event-triggered control scheme is introduced for stabilization of IF-OFP NCSs, which extends our previous work in [20] for stabilization of more general dissipative NCSs. The triggering condition is derived based on the passivity theorem which allows us to characterize a large class of output feedback stabilizing controllers. We show that with the triggering condition derived in this paper, the control system is finite-gain $\mathcal{L}_2$ stable in the presence of bounded external disturbances. The interactions between the triggering condition, the achievable $\mathcal{L}_2$ gain of the control system and the inter-event time have been studied in terms of the passivity indices of the plant and the controller. Analysis of the inter-event is provided followed by the discussions on challenges of applying event-triggered output feedback control to NCSs.

VI. Acknowledgments

The partial support of the National Science Foundation under Grants CCF-0819865 and CNS-1035655 is gratefully acknowledged.

References

[1] J. C. Willems, "Dissipative dynamical systems part I: General theory", *Archive for Rational Mechanics and Analysis*, Springer Berlin, Vol.45, No.5, pp.321-351, January, 1972.
[2] R. Sepulchre, M. Jankovic and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, 1997.
[3] W. M. Haddad, VS. Chellaboina, *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*, Princeton University Press, 2008.
[4] K. E. Årzén, "A simple event based PID controller", *Proceedings of 14th IFAC World Congress,* vol.18, pp.423-428, 1999.
[5] K. J. Aström and B.M. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems (I)", *Proceedings of the 41st IEEE Conference on Decision and Control*, pp.2011-2016, 2002.
[6] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transaction on Automatic Control*, vol.52, no.9, pp.1680-1685, September 2007.
[7] M. Mazo and P. Tabuada, "On event-triggered and self-triggered control over sensor/actuator networks," *Proceedings of the 47th Conference on Decision and Control*, pp.435-440, Cancun, December 9-11, 2008.
[8] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked systems with data dropouts and delays," *Hybrid Systems: Computation and Control*, pp.366-380, 2009.
[9] X. Wang and M. D. Lemmon, "Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability", *IEEE Transactions on Automatic Control*, vol.54, no.3 , pp.452-467, March 2009.
[10] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems", *IEEE Transactions on Automatic Control*, vol.55, no.9, pp.2030-2042, September 2010.
[11] J. P. LaSalle, "Some extensions of Liapunov's second method", *IRE Transactions on Circuit Theory*, CT-7, pp.520-527, 1960.
[12] K. J. Aström, "Event Based Control", *Analysis and Design of Nonlinear Control Systems*, Part 3, pp.127-147, 2008.
[13] W. P. M. H. Heemelsa, J. H. Sandeeb, P. P. J. Van Den Boscha, "Analysis of event-driven controllers for linear systems", *International Journal of Control*, Volume 81, Issue 4, pp.571-590, April 2008.
[14] P. G. Otanez, J. R. Moyne, D. M. Tilbury, "Using deadbands to reduce communication in networked control systems", *Proceedings of the American Control Conference*, pp.3015-3020, 2002.
[15] E. Kofman, J. H. Braslavsky, "Level Crossing Sampling in Feedback Stabilization under Data-Rate Constraints", *Proceedings of the 46th IEEE Conference on Decision and Control*, pp.4423-4428, San Diego, CA, December 13-15, 2006.
[16] E. D. Sontag, "Smooth stabilization implies coprime factorization,"*IEEE Transactions on Automatic Control*, vol.34, no.4, pp.435-443, Apr. 1989.
[17] H. Khalil, *Nonlinear systems*, Prentice Hall, 3 edition.
[18] M. C. F. Donkers and W. P. M. H. Heemels, "Output-Based Event-Triggered Control with Guaranteed $\mathcal{L}_\infty$-gain and Improved Event-Triggering", *Proceedings of the 49th IEEE Conference on Decision and Control*, pp.3246-3251, Atlanta, GA, December 15-17, 2010.
[19] M. Krichman, E. D. Sontag, Y. Wang, "Input-Output-to-State Stability", *SIAM J. Control Optim*, vol.39, pp.1874-1928, 1999.
[20] H. Yu and P. J. Antsaklis, "Event-Triggered Real-Time Scheduling For Stabilization of Passive/Output Feedback Passive Systems", *Proceedings of the 2011 American Control Conference*, pp. 1674-1679, San Francisco, CA, June 29-July 1, 2011.
[21] H. Yu and P. J. Antsaklis, "Event-Triggered Output Feedback Control for Networked Control Systems using Passivity: Time-varying Network Induced Delays", *Proceedings of the 50th IEEE Conference on Decision and Control (CDC'11) and ECC'11*, Orlando, Florida, December 12-15, 2011.