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Gaussian mixture PHD filter for multiple maneuvering extended targets tracking

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Abstract— **This paper addresses the problem of tracking multiple maneuvering extended targets in the framework of random finite set theory. An elliptical model is adopted for exploiting sensor measurements of target extent, and the target dynamics is described by a jump Markov linear system which combines the shape parameters into the kinematic state vector. As each extended target gives rise to unlabeled multiple measurements per time step, all the received measurements are partitioned into a number of subsets so that the measurements in each subset are expected to stem from the same source. In addition, the best-fitting Gaussian approximation approach is employed to circumvent the difficulty of multiple model mixing in the Gaussian mixture probability hypothesis density (GM-PHD) filter. A numerical example is provided to compare the performance of the proposed filter with that of the GM-PHD filter without measurement partition.**

I. INTRODUCTION

Extended target tracking has received much attention in recent years due to the development of high resolution sensors [1]–[6]. Unlike the conventional target tracking problem, in which the target is modeled as a point and at most a single measurement is received per time step, the extended target tracking involves describing a target as a set of points source and each of which may be the origin of a sensor measurement. Another feature of the extended target tracking is that the target extent information can be incorporated within a tracking algorithm to improve the tracking performance, especially for tracking closely spaced targets [3]. Many strategies have been proposed for extracting the target extent information such as the stick model [4], the spatial distribution model [5], and the elliptical model [6]. It should be pointed out that most of the existing literature focus on single extended target tracking. The problem of tracking time-varying number of extended targets in the cluttered environment remains a challenge.

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The random finite set (RFS) approach to multi-target tracking has been developed by Mahler [7]–[9], in which the multi-target state and the multi-target measurement are represented as random finite sets. The main advantage of the RFS formulation is that the difficulty caused by data association is avoided. Moreover, it is natural to investigate the problem of target appear and disappear at any time. Based on the finite set statistics (FISST) theory, a rigorous Bayesian framework has been proposed for multi-target tracking. However, the optimal multi-target Bayes filter is generally intractable due to the combinatorial nature of the multi-target densities and multiple set integrals. The probability hypothesis density (PHD) filter, which aims to recursively propagate the first-order moment or the intensity function associated with the multi-target posterior density, provides a computational tractable alternative. Recently, two implementations of the PHD filter including the sequential Monte Carlo PHD (SMC-PHD) [10]–[13] and the Gaussian mixture PHD (GM-PHD) [14]–[16] are developed. An added advantage of the GM-PHD filter is that it allows the state estimates to be extracted from the posterior intensity in a much more efficient and reliable manner than the SMC-PHD filter [14]. For tracking maneuvering targets, the switching multiple model method has been shown to be highly effective [17]. Multiple model PHD filters have been proposed for tracking multiple maneuvering targets [18], [19]. Nevertheless, the problem of multiple model mixing in the GM-PHD filter has not been resolved effectively. To circumvent this difficulty, the best-fitting Gaussian (BFG) approximation approach has been utilized in our previous work [20] and it has been shown that the BFG-based GM-PHD filter provides better tracking performance with less computational cost.

The problem of tracking multiple extended targets has been studied in the RFS framework [21], and the extended PHD recursion has been developed. As each extended target gives rise to unlabeled multiple measurements per time step, the main difficulty arising is how to partition the measurement set into a number of subsets such that the measurements in the subsets stem from the same source. In [22], the extended GM-PHD filter has been carried out by using a heuristic measurement partition scheme. However, the target extent information is not incorporated and the proposed filter is restricted to deal with the linear measurement equations. In addition, the single model approach is used and thus it is not preferred to handle maneuvering targets.

In this paper, we propose a new GM-PHD filter for tracking multiple maneuvering extended targets. For the sake of simplicity, the investigation is confined to the practically important case of elliptical targets and the target kinematic state vector is augmented by additional state variables characterizing their extensions such as the shape parameters. The tracking problem is then to infer the structure of targets as well as their kinematical properties involved. By describing the target dynamics as a coordinated turn model with Markovian switching parameter, the BFG approximation approach is employed as mentioned in the previous discussion. A three-element measurement vector is formed including the range and the bearing to the target centroid, and the down-range extent along the sensor-target line-of-sight (LOS). A Poisson model is used to generate multiple measurements for each extended target. As pointed out in [3] that the extended Kalman filter (EKF) is prone to divergence due to the high nonlinearity of measurements, the unscented transform [23] technique is used to overcome this problem in this paper. The measurement partition scheme is proposed based on the Mahalanobis distance between measurements which is similar to that in [22]. A simulation example is provided to illustrate the effectiveness of the proposed filter and to compare the performance with that of GM-PHD filter.

The rest of this paper is organized as follows. The problem of tracking multiple maneuvering extended targets is formulated in Section II. In Section III, the BFG approximation approach and the measurement partition scheme are presented to develop the extended GM-PHD filter based on the unscented transform technique. In Section IV, a numerical example is provided to compare the performance of the proposed filter with that of the standard GM-PHD filter. Conclusion and future work are given in Section V.

II. Problem Formulation

A. Elliptical model for target extent

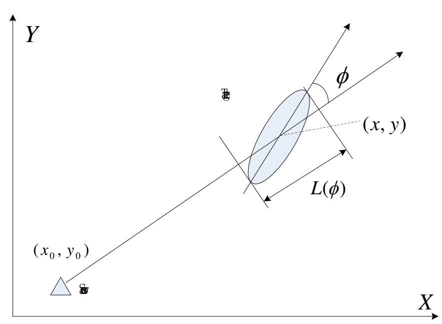

Fig. 1: Elliptical model for target extent.

As in [3] we propose to model the shape of the target as an ellipse. It is assumed that all the targets are moving on a plane and that their major axes are parallel to their velocity vectors, respectively. Then, as shown in Fig.1, the down-range extent for an ellipsoidal target can be described by

$$L(\phi) = l\sqrt{\cos^2\phi + r^2\sin^2\phi} \tag{1}$$

where ϕ is the angle between the major axis of the ellipse and the sensor-target LOS. l is the length of the major axis and r is the ratio of the lengths of the minor and major axes.

B. Extended target dynamics and measurement models

In this paper, our aim is to estimate the target state and the target shape parameters simultaneously. For this purpose, we define the state vector as $\mathbf{x}_k = (x_k\ \dot{x}_k\ y_k\ \dot{y}_k\ l_k\ r_k)^T$, where $(x_k\ y_k)$ and $(\dot{x}_k\ \dot{y}_k)$ represent the target position and velocity components, respectively. Since the dynamics of a maneuvering target is usually modeled by multiple switching regimes, also known as jump Markov systems, we consider the following coordinated turn model

$$\mathbf{x}_{k+1} = \begin{bmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & -\frac{1-\cos(\omega T)}{\omega} & 0 & 0 \\ 0 & \cos(\omega T) & 0 & -\sin(\omega T) & 0 & 0 \\ 0 & \frac{1-\cos(\omega T)}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} & 0 & 0 \\ 0 & \sin(\omega T) & 0 & \cos(\omega T) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \frac{T^2}{2} & 0 & 0 & 0 \\ T & 0 & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & T \end{bmatrix} \mathbf{w}_k \tag{2}$$

where ω denotes the turn rate and T is the sampling time period. $\mathbf{w}_k$ is assumed to be zero-mean white Gaussian noise with covariance Q_k.

We assume that at any time the target motion obeys one of M dynamic behavior models, which can be described by the above coordinated turn model with different turn rate. For example, the model (2) represents a left turn for $\omega > 0$ and a right turn for $\omega < 0$. Specially, the model (2) becomes the nearly constant velocity model when $\omega = 0$. The switching between models is governed by a first-order Markov chain θ_k with known transition probability matrix Π, whose elements are $\pi_{ij} \triangleq \mathbb{P}\{\theta_k = j|\theta_{k-1} = i\}$.

A high resolution sensor is used to provide measurements of range and bearing to the target centroid, as well as the down-range extent $L(\phi)$, i.e.,

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k \tag{3}$$

where h is the measurement function, and $\mathbf{v}_k$ is zero-mean white Gaussian noise with covariance R_k.

To be specific, the measurement function is given by

$$h(\mathbf{x}_k) = \begin{bmatrix} \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2} \\ \arctan[(x_k - x_0)/(y_k - y_0)] \\ L(\phi(\mathbf{x}_k)) \end{bmatrix} \tag{4}$$

where $(x_0\ y_0)$ is the location of the sensor, and

$$L(\phi(\mathbf{x}_k)) = \frac{l_k}{u_k v_k}\sqrt{(\dot{x}_k x'_k + \dot{y}_k y'_k)^2 + r_k^2(\dot{y}_k x'_k + \dot{x}_k y'_k)^2} \tag{5}$$

with

$$x'_k = x_k - x_0, \quad y'_k = y_k - y_0,$$
$$u_k = \sqrt{x'^2_k + y'^2_k}, \quad v_k = \sqrt{\dot{x}^2_k + \dot{y}^2_k}. \tag{6}$$

C. PHD recursion for multiple extended targets tracking

Consider a multi-target tracking scenario, the aim involves the joint estimation of an unknown and time-varying number of targets as well as their individual states from a sequence of noise-corrupted measurements with uncertain origins. Since the number of measurements may vary as not all targets generate measurements and the existence of clutter, it is natural to represent the multi-target state and multi-target measurement as two random finite sets (RFSs) [7]

$$X_k \triangleq \{\mathbf{x}_{k,1}, \cdots, \mathbf{x}_{k,n_k}\} \subset \mathcal{X} \tag{7}$$

$$Z_k \triangleq \{\mathbf{z}_{k,1}, \cdots, \mathbf{z}_{k,m_k}\} \subset \mathcal{Z} \tag{8}$$

where $\mathbf{x}_{k,1}, \cdots, \mathbf{x}_{k,n_k} \in \mathcal{X}$ are the target states, $\mathbf{z}_{k,1}, \cdots, \mathbf{z}_{k,m_k} \in \mathcal{Z}$ are the received measurements. $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Z} \subset \mathbb{R}^p$ denote the state and observation space, respectively. n_k and m_k denote the number of targets and the number of received measurements at time k, respectively. As each extended target generates more than one measurement, the measurements $\mathbf{z}_{k,i}$ and $\mathbf{z}_{k,j}$ might stem from the same target for $i \neq j$. We assume that the number of measurements generated by each extended target per time step is a Poisson distributed with rate $\epsilon(\mathbf{x})$.

By utilizing the FISST theory, the first-order moment of an RFS X on $\mathcal{X}$ is a non-negative function $\nu(\mathbf{x})$ which is also called as the PHD or the intensity function. For the multiple extended targets tracking, let $\nu_{k-1}(\mathbf{x})$ and $\nu_{k|k-1}(\mathbf{x})$ be the respective intensities for multi-target update and prediction recursion at time $k-1$. The prediction equation is given by

$$\nu_{k|k-1}(\mathbf{x}) = \int [p_S(\xi) f(\mathbf{x}|\xi) + \beta_{k|k-1}(\mathbf{x}|\xi)]\nu_{k-1}(\xi) d\xi + \gamma_k(\mathbf{x}) \tag{9}$$

where $f(\mathbf{x}|\xi)$ is the single target transition density, $p_S(\xi)$ is the probability of target survival, $\beta_{k|k-1}(\mathbf{x}|\xi)$ denotes the intensity of the spawned target RFS, and $\gamma_k(\mathbf{x})$ denotes the intensity of the spontaneously birth target RFS. The posterior intensity $\nu_k(\mathbf{x})$ is then updated as [21]

$$\nu_k(\mathbf{x}) = L_{Z_k}(\mathbf{x})\nu_{k|k-1}(\mathbf{x}) \tag{10}$$

where the measurement pseudo-likelihood function is

$$L_{Z_k}(\mathbf{x}) = 1 - (1 - e^{-\epsilon(\mathbf{x})})p_D(\mathbf{x}) + e^{-\epsilon(\mathbf{x})} p_D(\mathbf{x}) \sum_{p \angle Z_k} w_p \sum_{W \in p} \frac{\epsilon(\mathbf{x})^{|W|}}{d_W} \prod_{\mathbf{z} \in W} \frac{\varphi_{\mathbf{z}}(\mathbf{x})}{\lambda_k c_k(\mathbf{z})} \tag{11}$$

In (11), $\varphi_{\mathbf{z}}(\mathbf{x})$ is the spatial measurement distribution. $p_D(\mathbf{x})$ is the probability of detection, λ_k is the Poisson rate that determines the number of clutter per scan, $c_k(\mathbf{z})$ is the spatial distribution of the clutter measurements. In particular, the notation $p \angle Z_k$ means that p partitions the measurement set Z_k into cells W. w_p is the weight of a particular partition and is computed by

$$w_p = \frac{\prod_{W \in p} d_W}{\sum_{p' \in Z_k} \prod_{W \in p'} d_W} \tag{12}$$

where

$$d_W = \delta_{|W|,1} + \left[e^{-\epsilon(\mathbf{x})} \epsilon(\mathbf{x})^{|W|} p_D(\mathbf{x}) \prod_{\mathbf{z} \in W} \frac{\varphi_{\mathbf{z}}(\mathbf{x})}{\lambda_k c_k(\mathbf{z})} \right] \times \nu_{k|k-1}(\mathbf{x}) \tag{13}$$

with $\delta_{i,j}$ being the Kronecker delta function and $|W|$ being the number of the measurements in W.

Remark 1: The main difference between the standard PHD recursion and the extended PHD recursion is that every possible partition of the measurement set has been separated into several cells since an extended target gives rise to multiple measurements per time step. In other words, each measurement is used to update the Gaussian components for the standard GM-PHD filter, while each cell of the partition is used to update the Gaussian components for the extended GM-PHD filter, see [22] for more details.

III. EXTENDED GM-PHD FILTER

A. JMLS with BFG approximation

We rewrite the target motion model (2) in a compact form

$$\mathbf{x}_{k+1} = F_k(\theta_{k+1})\mathbf{x}_k + G_k(\theta_{k+1})\mathbf{w}_k(r_{k+1}) \tag{14}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the target state at time k, $F_k(\theta_{k+1})$ and $G_k(\theta_{k+1})$ denote the transition matrices of model θ_{k+1}. θ_{k+1} specifies the target motion model which is in effect during the time interval $[k, k+1)$. $\mathbf{w}_k(\theta_{k+1})$ is the additive zero-mean white Gaussian noise with covariance $Q_k(\theta_{k+1})$. For the convenience of development, the notations $F_k(r)$, $G_k(r)$ and $Q_k(r)$ are shortly denoted by F_k^r, G_k^r and Q_k^r, respectively.

The purpose of the BFG approximation is to express the dynamics of the JMLS (14) with the linear Gaussian system

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + \mathbf{w}_k \tag{15}$$

where $\mathbf{w}_k$ is a zero-mean white Gaussian random vector with covariance matrix Σ_k, i.e., $\mathbf{w}_k \sim \mathcal{N}(0, \Sigma_k)$. Then, we want to replace the JMLS with a single BFG distribution such that the distribution of $\mathbf{x}_k$ has the same mean and covariance under each model. Similar to the calculation in [24], the system matrix Φ_k and the covariance matrix Σ_k of $\mathbf{w}_k$ can be determined recursively

$$p_{k+1,r} = \sum_{i=1}^{M} \pi_{ir} p_{k,i} \tag{16}$$

$$\Phi_k = \sum_{r=1}^{M} p_{k+1,r} F_k^r \tag{17}$$

$$\Theta_{k+1} = \sum_{r=1}^{M} p_{k+1,r} \left[F_k^r (\Theta_k + \varepsilon_k \varepsilon_k^T)[F_k^r]^T + G_k^r Q_k^r [G_k^r]^T \right] - \Phi_k \varepsilon_k \varepsilon_k^T \Phi_k^T \tag{18}$$

$$\Sigma_k = \Theta_{k+1} - \Phi_k \Theta_k \Phi_k^T \tag{19}$$

$$\varepsilon_{k+1} = \Phi_k \varepsilon_k \tag{20}$$

where $p_{k+1,r}$ is the probability of the event that model r is in effect during the sampling period $[k, k+1)$. ε_k and Θ_k represent the mean and the covariance of $\mathbf{x}_k$, respectively.

B. Measurement partition

Let us present an example to illustrate the partition. Assume that three valid measurements have been received at time k and the measurement set is denoted as $Z_k = \{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \mathbf{z}_{k,3}\}$. Then the measurement set can be partitioned in the following ways [21]: (p_1). $W_1 = \{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \mathbf{z}_{k,3}\}$; (p_2). $W_1 = \{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}\}$, $W_2 = \{\mathbf{z}_{k,3}\}$; (p_3). $W_1 = \{\mathbf{z}_{k,1}, \mathbf{z}_{k,3}\}$, $W_2 = \{\mathbf{z}_{k,2}\}$; (p_4). $W_1 = \{\mathbf{z}_{k,2}, \mathbf{z}_{k,3}\}$, $W_2 = \{\mathbf{z}_{k,1}\}$; (p_5). $W_1 = \{\mathbf{z}_{k,1}\}$, $W_2 = \{\mathbf{z}_{k,2}\}$, $W_3 = \{\mathbf{z}_{k,3}\}$. Note that the number of partitions grows very large as the size of measurements increases. It is therefore to find some possible partitions in order to derive a computationally tractable algorithm. To this end, a simple partition principle is adopted which is based on the Mahalanobis distance between measurements [22].

Given the measurement set Z_k and a sequence of distance threshold $\{d_i\}_{i=1}^{N_d}$ with $d_i < d_{i+1}$. We can get a partition for each d_i in which the cells constitute the measurements that are no more than d_i apart from their closet cell neighbor. For defining the distance between measurements, we consider two measurements $\mathbf{z}_{k,i}$ and $\mathbf{z}_{k,j}$, both measured with covariance matrix R_k, then the Mahalanobis distance is defined as

$$\Delta = (\mathbf{z}_{k,i} - \mathbf{z}_{k,j}) R_k^{-1} (\mathbf{z}_{k,i} - \mathbf{z}_{k,j})^T \tag{21}$$

which is χ^2 distributed with 3 degrees of freedom. Using an inverse χ^2 distribution, a unit-less distance threshold δ_{P_G} can be obtained for a given probability P_G. Then $\mathbf{z}_{k,i}$ and $\mathbf{z}_{k,j}$ can be considered in the same cell if $\Delta < \delta_{P_G}$. Specially, the threshold d_i can be taken as δ_{P_G} for different P_G.

C. Extended GM-PHD filter based on BFG approximation

Based on the above formulation, the state dynamics and measurement of each extended target can be described by

$$f(\mathbf{x}_{k+1}|\mathbf{x}_k) = \mathcal{N}(\mathbf{x}_{k+1}; \Phi_k \mathbf{x}_k, \Sigma_k) \tag{22}$$

$$g(\mathbf{z}_k|\mathbf{x}_k) = \mathcal{N}(\mathbf{z}_k; h(\mathbf{x}_k), R_k) \tag{23}$$

where Φ_k and Σ_k are calculated by the BFG approximation.

We assume that the survival probability p_S and the detection probability p_D are both state independent, and the intensities of the birth and the spawning RFSs are Gaussian mixtures

$$\gamma_k(\mathbf{x}) = \sum_{j=1}^{J_{\gamma,k}} w_{\gamma,k}^j \mathcal{N}(\mathbf{x}; m_{\gamma,k}^j, P_{\gamma,k}^j) \tag{24}$$

$$\beta_{k|k-1}(\mathbf{x}|\xi) = \sum_{l=1}^{J_{\beta,k}} w_{\beta,k}^l \mathcal{N}(\mathbf{x}; F_{\beta,k}^l \xi + d_{\beta,k}^l, Q_{\beta,k}^l) \tag{25}$$

where $J_{\gamma,k}$, $w_{\gamma,k}^j$, $m_{\gamma,k}^j$ and $P_{\gamma,k}^j$ are given parameters that determine the shape of the birth intensity. $J_{\beta,k}$, $w_{\beta,k}^l$, $F_{\beta,k}^l$, $d_{\beta,k}^l$ and $Q_{\beta,k}^l$ are given parameters that determine the shape of the spawning intensity.

Then, the extended PHD recursion (9)-(11) can be implemented as follows

BFG Approximation Step: Given the mode probability $p_{k,i}$, the mean ε_k and the covariance Θ_k, determine the matrices Φ_k and Σ_k by (16)-(20).

Prediction Step: Given that the posterior intensity $\nu_k(\mathbf{x})$ is a Gaussian mixture

$$\nu_k(\mathbf{x}) = \sum_{j=1}^{J_k} w_k^j \mathcal{N}(\mathbf{x}; m_{k|k}^j, P_{k|k}^j) \tag{26}$$

then the predicted intensity is also a Gaussian mixture with the form

$$\nu_{k+1|k}(\mathbf{x}) = \nu_{S,k+1|k}(\mathbf{x}) + \nu_{\beta,k+1|k}(\mathbf{x}) + \gamma_{k+1}(\mathbf{x}) \tag{27}$$

where $\gamma_{k+1}(x)$ is given by (24), and

$$\nu_{S,k+1|k}(\mathbf{x}) = p_{S,k+1} \sum_{j=1}^{J_k} w_k^j \mathcal{N}(\mathbf{x}; m_{S,k+1|k}^j, P_{S,k+1|k}^j) \tag{28}$$

$$\nu_{\beta,k+1|k}(\mathbf{x}) = \sum_{j=1}^{J_k} \sum_{l=1}^{J_{\beta,k+1}} w_k^j w_{\beta,k+1}^l \mathcal{N}(\mathbf{x}; m_{\beta,k+1|k}^{j,l}, P_{\beta,k+1|k}^{j,l}) \tag{29}$$

$$m_{S,k+1|k}^j = \Phi_k m_{k|k}^j \tag{30}$$

$$P_{S,k+1|k}^j = \Phi_k P_{k|k}^j \Phi_k^T + \Sigma_k \tag{31}$$

$$m_{\beta,k+1|k}^{j,l} = F_{\beta,k+1}^l m_{k|k}^j + d_{\beta,k+1}^l \tag{32}$$

$$P_{\beta,k+1|k}^{j,l} = F_{\beta,k+1}^l P_{k|k}^j [F_{\beta,k+1}^l]^T + Q_{\beta,k+1}^l \tag{33}$$

Measurement partition step: Utilizing the scheme proposed in Section III-B to obtain the possible partitions and cells.

Update Step: Given that the predicted intensity can be represented as the form of

$$\nu_{k+1|k}(\mathbf{x}) = \sum_{i=1}^{J_{k+1|k}} w_{k+1|k}^i \mathcal{N}(\mathbf{x}; m_{k+1|k}^i, P_{k+1|k}^i) \tag{34}$$

then the posterior intensity is

$$\begin{aligned} \nu_{k+1}(\mathbf{x}) = {} & (1 - (1 - e^{\epsilon(\mathbf{x})}) p_D) \nu_{k+1|k}(\mathbf{x}) \\ & + \sum_{p \angle Z_{k+1}} \sum_{W \in p} \nu_{D,k+1}(\mathbf{x}) \end{aligned} \tag{35}$$

where

$$\nu_{D,k+1}(\mathbf{x}) = \sum_{i=1}^{J_{k+1|k}} w_{k+1}^i \mathcal{N}(\mathbf{x}; m_{k+1|k+1}^i, P_{k+1|k+1}^i) \tag{36}$$

$$w_{k+1}^i = w_p \frac{p_D \Gamma^i}{d_W} \Psi_W^i w_{k+1|k}^i \tag{37}$$

$$w_p = \frac{\prod_{W \in p} d_W}{\sum_{p' \angle Z_{k+1}} \prod_{W' \in p'} d_{W'}} \tag{38}$$

$$d_W = \delta_{|W|,1} + \sum_{i=1}^{J_{k+1|k}} \Gamma^i \Psi_W^i w_{k+1|k}^i \tag{39}$$

$$\Gamma^i = e^{-\epsilon(\mathbf{x})} \epsilon(\mathbf{x})^{|W|} \tag{40}$$

$$\Psi_W^i = \prod_{\mathbf{z} \in W} \frac{\varphi_{\mathbf{z}}(m_{k+1|k}^i)}{\lambda_k c_k(\mathbf{z})} \tag{41}$$

The likelihood function $\varphi_{\mathbf{z}}(m^i_{k+1|k})$ is calculated by using the unscented transform technique, i.e.,

$$\varphi_{\mathbf{z}}(m^i_{k+1|k}) = \mathcal{N}(\mathbf{z}; h(m^i_{k+1|k}), S^i_{k+1}) \tag{42}$$

where

$$S^i_{k+1} = R_{k+1} + \sum_{s=0}^{2n} W_s[\zeta^i_{k,s} - h(m^i_{k+1|k})] \times [\zeta^i_{k,s} - h(m^i_{k+1|k})]^T \tag{43}$$

with $\zeta^i_{k,s}$ being the sigma point and W_s being the weight

$$\zeta^i_{k,0} = m^i_{k+1|k} \tag{44}$$

$$\zeta^i_{k,s} = m^i_{k+1|k} + \left(\sqrt{P^i_{k+1|k}}\right)_s \tag{45}$$

$$\zeta^i_{k,s+n} = m^i_{k+1|k} - \left(\sqrt{P^i_{k+1|k}}\right)_s, \quad s = 1, \cdots, n \tag{46}$$

The mean $m^i_{k+1|k+1}$ and covariance $P^i_{k+1|k+1}$ are

$$m^i_{k+1|k+1} = m^i_{k+1|k} + K^i_{k+1}\hat{\mathbf{z}}^i_{k+1} \tag{47}$$

$$P^i_{k+1|k+1} = P^i_{k+1|k} - K^i_{k+1}P^i_{\mathbf{zz}}[K^i_{k+1}]^T \tag{48}$$

$$K^i_{k+1} = P^i_{\mathbf{xz}}[P^i_{\mathbf{zz}}]^{-1} \tag{49}$$

$$P^i_{\mathbf{zz}} = \bar{R}_{k+1} + \sum_{s=0}^{2n} W_s\bar{\mathbf{z}}^i_{k+1}[\bar{\mathbf{z}}^i_{k+1}]^T \tag{50}$$

$$P^i_{\mathbf{xz}} = \sum_{s=0}^{2n} W_s(\zeta^i_{k,s} - m^i_{k+1|k})[\bar{\mathbf{z}}^i_{k+1}]^T \tag{51}$$

where

$$\bar{R}_{k+1} = \begin{bmatrix} R_{k+1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{k+1} \end{bmatrix}, \hat{\mathbf{z}}^i_{k+1} = \begin{bmatrix} \mathbf{z}_1 - h(m^i_{k+1|k}) \\ \vdots \\ \mathbf{z}_{|W|} - h(m^i_{k+1|k}) \end{bmatrix}$$

$$\bar{\mathbf{z}}^i_{k+1} = \begin{bmatrix} h(\zeta^i_{k,s}) - h(m^i_{k+1|k}) \\ \vdots \\ h(\zeta^i_{k,s}) - h(m^i_{k+1|k}) \end{bmatrix}. \tag{52}$$

IV. Simulations

This section provides a numerical example to compare the performance of the proposed filter with that of the standard GM-PHD filter without measurement partition. We consider a two-dimensional scenario with an unknown and time-varying number of extended targets as in [20].

Tracking model: Three models corresponding to different turn rates are used. Model 1 is a nearly constant velocity model and the standard deviation of noise is 5 $\mathrm{m/s^2}$. Model 2 is a coordinated turn model with a clockwise turn rate of $3^\circ/\mathrm{s}$ and the standard deviation of noise is 20 $\mathrm{m/s^2}$. Model 3 is a coordinated turn model with a counterclockwise turn rate of $3^\circ/\mathrm{s}$ and the standard deviation of noise is 20 $\mathrm{m/s^2}$. The switching between three models is governed by a first-order Markov chain with known transition probability matrix

$$\Pi = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \tag{53}$$

The Poisson rate for the number of measurements generated per time step is $\epsilon(\mathbf{x}) = 5$ for each target. The measurement noise $\mathbf{v}_k$ is assumed to be zero-mean white Gaussian with covariance matrix $R = \mathrm{diag}\{100^2, (\pi/180)^2, 5^2\}$. The sensor is located at $(35, -60)$ km, and the average number of clutter returns per unit volume is taken as $\lambda_k = 1.67 \times 10^{-2}$ which corresponding to 10 clutter returns over the surveillance region. It is assumed that targets can appear or disappear in the scene at any time. The spontaneous birth RFS is Poisson with the following intensity

$$\gamma_k(\xi) = 0.1\left[\mathcal{N}(\xi; m^1_\gamma, P_\gamma) + \mathcal{N}(\xi; m^2_\gamma, P_\gamma)\right] \tag{54}$$

where

$$m^1_\gamma = (40, 0, -50, 0, 50, 0.2)^T$$
$$m^2_\gamma = (30, 0, -40, 0, 50, 0.2)^T$$
$$P_\gamma = \mathrm{diag}\{10^6, 10^4, 10^6, 10^4, 5^2, 0.2^2\}$$

The intensity of the Poisson RFS of spawn births is given by

$$\beta_{k|k-1}(x|\xi) = 0.05\mathcal{N}(x; \xi, Q_\beta) \tag{55}$$

where $Q_\beta = \mathrm{diag}\{10^4, 400, 10^4, 20^2, 5^2, 0.2^2\}$.

Simulation results: In our simulations, the survival and the detection probabilities are set to $p_{S,k} = 0.99$ and $p_{D,k} = 0.99$, respectively. The pruning threshold has been taken as $T_{\mathrm{Th}} = 10^{-7}$, the merging threshold $U_{\mathrm{Th}} = 5$, the weight threshold $w_{\mathrm{Th}} = 0.005$ and the maximum number of Gaussian terms $J_{\max} = 10$ (see [14] for the meanings of these parameters). The criterion known as optimal subpattern assignment (OSPA) metric is used for performance evaluation. The OSPA metric has been considered as a much more natural interpretation for demonstrating localization and cardinality errors in multi-target tracking [25].

The true trajectories of four targets are shown in Fig 2. Target 1 starts at time $k = 1$ with initial position at $(40, -50)$ km and ends at time $k = 100$; Target 2 is spawned from target 1 at time 50 and ends at time 90; Target 3 starts at time $k = 5$ with initial position at $(30, -40)$ km and ends at time $k = 85$; Target 4 is spawned from target 3 at time 25 and ends at time 60. To verify the performance of the proposed filter, 100 Monte Carlo runs are performed with independently generated clutter and measurements for each trial. The position estimates of the proposed filter for one trial shown in Fig. 2 indicate that the filter provides accurate tracking performance. The OSPA distance for $p = 2$ and $c = 200$ is shown in Fig. 3, which suggests that the proposed filter gives more reliable estimates than the standard GM-PHD filter without measurement partition.

To assess the computational requirement of the proposed filter, we compute the averaged CPU time in MATLAB 7.1 on a 2.80 GHz 4 CPU Pentium-based computer operating under Windows XP (Professional). The proposed filter consumed approximately 73.6 s per sample run over 100 time steps, while the standard GM-PHD filter consumed 31.3 s. This is due to the fact that more computations have been done in the measurement partition. Note that both algorithms fit comfortably within the real-time requirements.

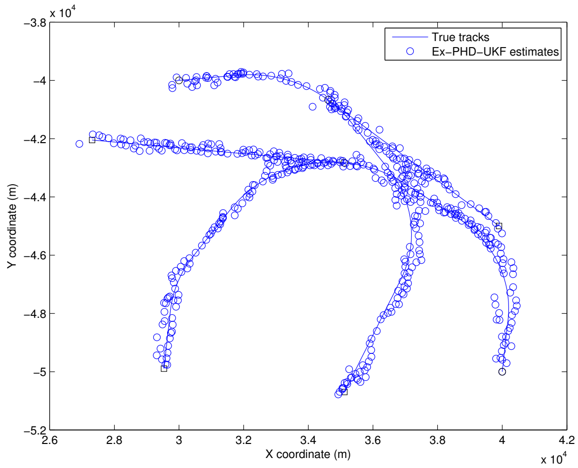

Fig. 2: Position estimates of the Ex-PHD-UKF.

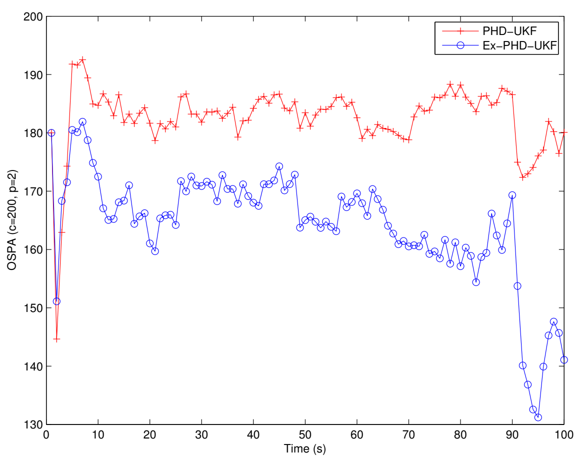

Fig. 3: OSPA distances for the PHD-UKF and the Ex-PHD-UKF.

V. Conclusion

A GM-PHD filter for tracking multiple maneuvering extended targets is developed. Two main features for tracking extended targets are investigated in a unified formulation including unlabeled multiple measurements and target extent information. By employing the BFG approximation approach, the extended GM-PHD filter is implemented based on a heuristic measurement partition scheme and the unscented transform technique. Compared with the standard GM-PHD filter, simulation results show that the proposed filter can achieve better tracking performance with more computational overload. Further study should find a better way to partition the measurement set in a more efficient manner. Extending the proposed approach to accommodate other target extent models is an important problem.

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