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Disturbance and Delay Robustness of Gradient Feedback Systems Based on Static Noncooperative Games with Application to PEV Charging

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***Abstract*— This paper considers interconnected dynamical systems derived from static noncooperative games. Designing such a gradient-type feedback aims at robustly driving state variables to the vicinities of Nash equilibria in the presence of uncertainty and variation. A recently-developed small-gain framework for integral input-to-state stable systems is employed to investigate global robustness of the gradient-type dynamical system with respect to disturbances and time-delays. Stability and robustness criteria are presented, and Lyapunov-Krasovskii functionals are constructed. The developed theory is applied to the plug-in electric vehicles charging problem of allocating the charging activities to the overnight electric demand valley for reducing the impact on the electric grid. A decentralized charging scheme with guaranteed robustness is proposed. Its usefulness is illustrated by numerical simulations.**

I. INTRODUCTION

Preparing for the expanding population of electric vehicles is one of the most important issues in establishing a smart grid. A plug-in electric vehicle (PEV) and a plug-in hybrid vehicle (PHV) are equipped with rechargeable batteries that can be restored to full charge by connecting a plug to an external electric power source (a normal electric wall socket or a quick charging station). PEVs and PHVs[3] can actively participate in establishing a smart grid as energy loads and storages as well [8]. The impact of energy consumption by PEVs on the electrical grid depends on the hour and pattern of charging. As a dramatical increase of PEVs in population is anticipated, the electric utility companies are interested in the effective use of off-peak hours of the power demand to avoid possible destructive impacts and to reduce CO_2 emissions. Battery charging activities during daytime need to be somehow restricted, and most of the PEV load needs to be allocated to nighttime. An ideal is to prevent any new peak load exceeding the natural peak [11], [7].

To fill the overnight demand valley of the electrical grid, an off-line recursive algorithm has been proposed for scheduling single cycle PEV charging of large population very recently [9]. The scheduling problem is formulated into a finite horizon noncooperative dynamic game in which PEV owners (players) make decisions independently. Such a strategy would be more acceptable by the users than decisions made by authorities. The proposed method is decentralized so that each charging agent only uses its own local information and the one-dimensional average value of all charging agents. The decentralization renders the local computational complexity independent of the PEV population size. In order to minimize the total charging cost of each agent, the method in [9] resorts to prediction of non-PEV demand over the entire charing interval of all agents and assumes that charging activities of all agents are synchronized for single charging cycles. If the non-PEV demand curve prediction is inaccurate, the minimization is of no use. In fact, the explicit use of the prediction lacks robustness to demand variation. Taking account of asynchronous charging activities and recurrent charging cycles, this paper proposes a simple formulation of the PEV charging problem and shows that such simple scheduling is satisfactory for the purpose of the valley-filling. This paper replaces the dynamic noncooperative game by a "static" noncooperative game and proposes dynamic "feedback" implementation in the form of a gradient algorithm which updates local charging rates "online" taking account of changes of the background demand. This paper proves that the "closed-loop" of the proposed charging algorithm is globally stable and robust with respect to the demand variation and the communication delay.

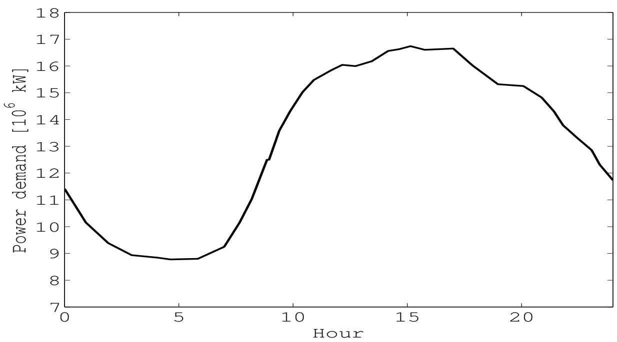

Fig. 1. Load curve for Kyushu region in Japan on Aug. 3, 2001.

In order to achieve the abovementioned goal, this paper tackles a general problem of asymptotic globally stable attainment of Nash equilibria in static noncooperative games with N players by means of gradient algorithms. The gradient control originates from a power control of wireless communication networks [2], [1]. To investigate global robustness of such gradient systems with respect to disturbance and time delay, this paper employs the framework of integral input-to-state stability (iISS), which was first attempted in [6]. Compared with the related study [6], this paper deals with a more general class of gradient systems, and derives less conservative criteria for stability and robustness. The proposed framework and criteria are applied to the problem of PEV charging. Simulation studies are also presented.

II. GLOBAL ROBUSTNESS OF GRADIENT SCHEME

This section considers the first-order gradient algorithm which dynamically computes and updates Nash equilibria of

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[3]PHVs are not distinguished from PEVs from now on.

static noncooperative games. The purpose is to investigate its robustness with respect to disturbances and time-delays.

Let N be the number of players. Player's actions (a strategy profile) are denoted by the vector $x = [x_1, x_2, ..., x_N]^T \in \mathbb{R}_+^N := [0, \infty)^N$. Each $x_i \in \mathbb{R}_+ := [0, \infty)$ describes the i-th player's action (a strategy). Although the action space $\mathbb{R}_+^N$ can be replaced by $\mathbb{R}^N$ without essential changes in this paper, this paper takes the orthant $\mathbb{R}_+^N$ since strategies are often unilateral in practical circumstances. Consider the noncooperative game defined by the minimization of the following cost functions:

$$J_i(x) = P_i(x) - U_i(x), \quad i = 1, 2, ..., N. \tag{1}$$

Each cost function J_i is assumed to be twice continuously differentiable. Let x_{-i} denote the strategy profile of all players except for the i-th player. The cost J_i depends on the strategy profile chosen, i.e., on the strategy chosen by the the i-th player as well as the strategies chosen by all the other players x_{-i}. A strategy profile $x^* \in \mathbb{R}_+^N$ is said to be a Nash equilibrium (NE) if no unilateral deviation in strategy by any single player is profitable for that player. If we select P_i and U_i such that $\partial P_i / \partial x_i \geq 0$ and $\partial U_i / \partial x_i \geq 0$, the first term P_i in (1) may be called a price function for the strategy chosen by the i-th player, while the second term U_i in (1) may be referred to as a utility function which is a measure of relative happiness of the i-th player. We assume that the cost functions J_i yield at least one NE $x = x^* \in \mathbb{R}_+^N$. If the NE $x = x^*$ is unique, it can be considered as the most desirable selfish strategy profile in the sense of the cost functions (1). To steer x to x^*, the dynamics governed by

$$\dot{x}_i(t) = \left(-\lambda_i \frac{\partial J_i(x(t))}{\partial x_i} \right)^+_{x_i}, \ t \in \mathbb{R}_+, \ x(0) \in \mathbb{R}_+^N \tag{2}$$

would be a natural choice [1], [2]. If $x = x^*$ of the system (2) is globally asymptotically stable, it is the unique NE of the game defined by (1). The parameter $\lambda_i > 0$ is the step-size determining the speed of descent. The projection

$$(f_i(t, x))^+_{x_i} = \begin{cases} 0 & \text{if } x_i = 0 \text{ and } f_i(t, x) < 0 \\ f_i(t, x) & \text{otherwise} \end{cases}$$

guarantees the positivity of x_i's in (2). In other words, the set $\mathbb{R}_+^N$ is positively invariant for the solutions of (2).

Let $\partial J_i / \partial x_i$ be decomposed into

$$\frac{\partial J_i(x)}{\partial x_i} = A_i(x_i) + B_i(x), \tag{3}$$

where $A_i : \mathbb{R}_+ \to \mathbb{R}_+$ and $B_i : \mathbb{R}_+^N \to \mathbb{R}_+$ are continuous functions whose existence is guaranteed by the smoothness of $J_i's$. The part of $A_i(x_i)$ is determined only by the information of the i-th player, while $B_i(x)$ needs information of the other players. Taking account of exchange and aggregation of information and disturbances arising from information processing and communication, time delays and noises are incorporated into (2) as follows:

$$\dot{x}_i(t) = \left(-\lambda_i \left[A_i(x_i(t)) + B_i(x(t - T_i)) + d_i(t) \right] \right)^+_{x_i}, \quad i = 1, 2, ..., N, \tag{4}$$

where the constants $T_i \in \mathbb{R}_+$, $i = 1, 2, ..., N$, represent time-delays, and the signals $d_i(t) \in \mathbb{R}$, $i = 1, 2, ..., N$, are noises which are measurable, locally essentially bounded functions of $t \in \mathbb{R}_+$. Let the disturbance vector be $d = [d_1, d_2, ..., d_N]^T$. The deviation from the equilibrium is defined by $\tilde{x} = x - x^*$. The initial time is $t = 0$, and the initial functions $x_i(\tau) \in \mathbb{R}_+^N$ defined for $\tau \in [-\max_i T_i, 0]$ are supposed to be continuous.

In (4), the functions $B_i(x)$ define static interaction with which N dynamical subsystems of x_i's are coupled each other. To investigate global stability and robustness of such an interconnected system, we can make use of the basic idea of the asymmetric small-gain technique for time-delay systems with static components [4]. In fact, we can obtain the following simple result which not only extends a similar result in [6] to the general system (4), but also introduces the new flexibility of the additional parameters η_i.

Theorem 1: Suppose that

$$\frac{dA_i(x_i)}{dx_i} > 0, \ \forall x_i \in \mathbb{R}_+, \ i = 1, 2, ..., N. \tag{5}$$

If there exists $c > 1$ and $\zeta_i > 0$, $i = 1, 2, ..., N$, such that

$$\sum_{i=1}^{N} \zeta_i \left(A_i(x_i) - A_i(x_i^*) \right)^2 \geq c \sum_{i=1}^{N} \zeta_i \left(B_i(x) - B_i(x^*) \right)^2 \quad , \forall x \in \mathbb{R}_+^N \tag{6}$$

holds, then the system (4) is iISS with respect to input d and state $\tilde{x}$ for arbitrary time-delays T_i. In addition, it is ISS with respect to input d and state $\tilde{x}$ if $\lim_{x_i \to \infty} A_i(x_i) = \infty$ holds for $i = 1, 2, ..., N$ additionally.

The integral input-to-state stability (iISS) of the system (4) implies that $x = x^*$ is globally asymptotically stable for $d(t) \equiv 0$, which ensures that $x = x^*$ is the unique NE. The input-to-state stability (ISS) ensures that the magnitude of the state $\tilde{x}(t)$ remains bounded as long as the magnitude of the disturbance $d(t)$ is bounded. The iISS ensures the boundedness of the magnitude of $\tilde{x}(t)$ if the energy of d is finite. These properties are defined globally on $(x(t), d(t)) \in \mathbb{R}_+^N \times \mathbb{R}^N$. The iISS is strictly weaker than the ISS [12], [13], [10]. Although the condition (6) refers to the NE $x = x^*$, the computation of the NE is not required in may cases where the functions A_i and B_i share similar shapes. A simplest case is that both A_i and B_i are affine functions.

It is possible to decompose (6) into N independent conditions if B_i's have certain properties. For instance, if

$$B_i(x) = \sum_{j=1}^{N} b_{i,j}(x_j), \quad i = 1, 2, ..., N \tag{7}$$

holds, the property (6) with $\zeta_1 = ... = \zeta_N = 1$ is implied by

$$\left(A_i(x_i) - A_i(x_i^*) \right)^2 \geq cN \sum_{j=1}^{N} \left(b_{j,i}(x_i) - b_{j,i}(x_i^*) \right)^2, \quad \forall x_i \in \mathbb{R}_+, \ i = 1, 2, ..., N. \tag{8}$$

If $b_{i,i} = 0$ holds, the condition (8) can be replaced by

$$\left(A_i(x_i) - A_i(x_i^*) \right)^2 \geq c(N-1) \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(b_{j,i}(x_i) - b_{j,i}(x_i^*) \right)^2. \tag{9}$$

The following theorem allows us to make use of nonidentical ζ_i's without computing them.

Theorem 2: Suppose that (5) holds. If there exist $z_{i,j} > 0$, $i, j = 1, 2, ..., N$, such that

$$(B_i(x) - B_i(x^*))^2 \le \sum_{j=1}^{N} z_{i,j} \left(A_j(x_j) - A_j(x_j^*)\right)^2, \quad \forall x \in \mathbb{R}_+^N, \ i = 1, 2, ..., N \tag{10}$$

$$\rho(Z) < 1, \quad Z := \begin{bmatrix} z_{11} & \cdots & z_{1,N} \\ \vdots & \ddots & \vdots \\ z_{N,1} & \cdots & z_{N,N} \end{bmatrix} \tag{11}$$

are satisfied, the system (4) is iISS with respect to input d and state $\tilde{x}$ for arbitrary time-delays T_i. In addition, it is ISS with respect to input d and state $\tilde{x}$ if $\lim_{x_i \to \infty} A_i(x_i) = \infty$ holds for $i = 1, 2, ..., N$ additionally.

In the above theorem, the spectral radius is denoted by $\rho(\cdot)$. The advantage of using nonidentical ζ_i's is that a large nonlinear gain of a subsystem is allowed to be compensated by a small nonlinear gain of another subsystem. In contrast, the stability criteria (8) and (9) uniformly require gains of all the subsystems to to be small.

III. Time-Varying Time Delays

This section allows time delays to be time-varying as

$$\dot{x}_i(t) = \left(-\lambda_i \left[A_i(x_i(t)) + B_i(x(t - T_i(t)))\right]\right)_{x_i}^{+}, \quad i = 1, 2, ..., N. \tag{12}$$

The functions $T_i(t) \in [0, \bar{T}]$, $i = 1, 2, ..., N$, represent time-varying time-delays which are bounded. The constant $\bar{T} \in \mathbb{R}_+$ denotes the maximum involved delay. We assume that the functions $T_i(t)$ are differentiable and satisfy

$$\frac{dT_i(t)}{dt} \le U_i < 1, \ i = 1, 2, ..., N, \ \forall t \in \mathbb{R}_+ \tag{13}$$

for some $U_i \ge 0$, $i = 1, 2, ..., M$.

Theorem 3: Suppose that (5) holds. If there exists $c > 1$ and $\zeta_i > 0$, $i = 1, 2, ..., N$, such that

$$\sum_{i=1}^{N} \zeta_i (1 - U_i) \left(A_i(x_i) - A_i(x_i^*)\right)^2 \ge c \sum_{i=1}^{N} \zeta_i \left(B_i(x) - B_i(x^*)\right)^2, \ \forall x \in \mathbb{R}_+^N \tag{14}$$

is satisfied, the equilibrium $x = x^*$ of the system (12) is globally asymptotically stable.

IV. PEV Charging for Valley-filling in the Grid

A. A Decentralized Scheme

This section proposes and investigates a charging control to reduce burden of PEV charging on the grid by assigning the aggregated PEV load to off peak hours of general demand. Since the population of PEVs should be very large, we need to approach the charging problem in a decentralized manner. Centralized approaches may be also too dictatorial to be accepted by PEV owners. A simple strategy would be that each PEV owner decreases the charging rate if the electricity retail price is high. This section illustrates that such a simple strategy can achieve the valley-filling maintaining global stability of the overall system. In this paper, we do not separate charging stations (or domestic electric wall sockets) from individual PEVs. A PEV is disconnected if its battery is charged completely, and another PEV may be connected to the same charging station. A station is connected to the electric grid, i.e., active when a PEV is charging. Let $x_i(t) \ge 0$, $i = 1, 2, ..., N$, denote the local charging rates, where N is the number of active charging stations. Define the cost function of the i-th charging station as

$$J_i(x(t)) = F_i(x_i(t)) + P\left(\frac{D(t) + Na(t)}{C}\right) + h_i \left(w_i x_i(t) - a(t)\right)^2, \quad i = 1, 2, ..., N. \tag{15}$$

The function

$$a(t) = \frac{\sum_{j=1}^{N} x_j(t)}{N}$$

is the average of local charging rates. The electricity retail price is specified by the continuously differentiable function $P : \mathbb{R}_+ \to \mathbb{R}$. It is assumed that the retail price depends only on the current demand level [9]. The electricity demand is the sum of the total PEV charging and the background demand unrelated to the PEV charging. The grid generation capacity is denoted by the constant $C > 0$. Consider a charging strategy in which the i-th station individually minimizes their own cost J_i. The continuously differentiable function $F_i : \mathbb{R}_+ \to \mathbb{R}$ penalizes low charging rate to describe the primary benefit of charging for each vehicle. The term $h_i(x_i(t) - a(t))^2$ imposes homogeneity on individual charging rates, where the non-negative constant h_i is its weighting parameter. The positive constants w_i specify the ratios between charging rates of the stations in achieving the homogeneity. Then for the cost function (15), the gradient-type system defined by (2) in Section II becomes

$$\dot{x}_i(t) = \left(-\lambda_i \left[F_i'(x_i(t)) + \frac{1}{C} P'\left(\frac{D(t) + Na(t)}{C}\right) + 2h_i \left(1 - \frac{1}{N}\right)(w_i x_i(t) - a(t))\right]\right)_{x_i}^{+}, \quad i = 1, 2, ..., N, \tag{16}$$

where $\lambda_i > 0$. The proposed strategy (16) is decentralized since each charging station only needs its own charging rate, the non-PEV demand and the average of all charging rates broadcasted by the grid. The aggregation of PEV and non-PEV demand and broadcasting it to all of the charging stations may not be done instantaneously. Taking this point into account, we use (4) which replaces (16) by

$$\dot{x}_i(t) = \left(-\lambda_i \left[F_i'(x_i(t)) + \frac{1}{C} P'\left(\frac{D(t) + Na(t - T_i)}{C}\right) + 2h_i \left(1 - \frac{1}{N}\right)(w_i x_i(t) - a(t - T_i))\right]\right)_{x_i}^{+} \tag{17}$$

for $i = 1, 2, ..., N$, where $T_i \in \mathbb{R}_+$, $i = 1, 2, ..., N$, denote time-delays. Notice that $D(t - \delta_i)$ makes no difference to dynamics since D is an external signal. The time shift merely results in a delayed response to the retail price by δ_i.

Assumption 1: Each $P_i(x)$ and $F_i(x_i)$ are twice continuously differentiable and satisfies

$$\frac{dP(s)}{ds} \geq 0, \quad \frac{d^2P(s)}{ds^2} > 0, \quad \forall s \in \mathbb{R}_+ \tag{18}$$

$$\frac{dF_i(s)}{ds} < 0, \quad \frac{d^2F_i(s)}{ds^2} > 0, \quad \forall s \in \mathbb{R}_+. \tag{19}$$

Assumption 1 guarantees the existence of a unique NE $x^* \in \mathbb{R}^N$ for the minimization of (15) for arbitrary given constant non-PEV demand $D(t) \equiv D^* > 0$. Pick A_i in (3) as

$$A_i(x_i) = F_i'(x_i) + w_i K_i x_i, \tag{20}$$

where $K_i = 2h_i(1 - 1/N) \geq 0$. The properties (19) and $K_i > 0$ guarantee (5). The condition (6) becomes

$$\sum_{i=1}^{N} \zeta_i \left\{F'(x_i) - F'(x_i^*) + w_i K_i (x_i - x_i^*)\right\}^2 \geq c \sum_{i=1}^{N} \zeta_i \left\{ Q(x) - Q(x^*) - \frac{K_i}{N} \sum_{j=1}^{N} (x_j - x_j^*) \right\}^2 \quad \forall x \in \mathbb{R}_+, \tag{21}$$

where

$$Q(x) = \frac{1}{C} P'\left(\frac{D^* + Na(t)}{C}\right).$$

The existence of $c > 1$ satisfying (21) ensures that the delay system (17) and the delay-free system (16) are iISS with respect to the disturbance $D(t) - D^*$.

In the delay-free case where $T_i = 0$, $i = 1, 2, ..., N$, we can take another choice of A_i. Write Q as

$$Q(x) = Q_i(x_i) + R_i(x), \ i = 1, 2, ..., N \tag{22}$$

$$Q_i'(s) \geq 0, \quad \forall s \in \mathbb{R}_+. \tag{23}$$

Define the function $A_i : \mathbb{R}_+ \to \mathbb{R}_+$ by

$$A_i(x_i) = F_i'(x_i) + Q_i(x_i) + w_i K_i x_i. \tag{24}$$

The properties (19), (23) and $K_i > 0$ ensure (5). Then the condition (6) for (24) becomes

$$\sum_{i=1}^{N} \zeta_i \left\{F'(x_i) - F'(x_i^*) + Q_i(x_i) - Q_i(x_i^*) + w_i K_i (x_i - x_i^*)\right\}^2 \geq c \sum_{i=1}^{N} \zeta_i \left\{R_i(x) - R_i(x^*) - \frac{K_i}{N} \sum_{j=1}^{N} (x_j - x_j^*)\right\}^2, \forall x \in \mathbb{R}_+^N. \tag{25}$$

If we pick $h_i = 0$ for all $i = 1, 2, ..., N$, it is not necessary to evaluate (25) on the entire $x \in \mathbb{R}_+^N$. Due to $F'' > 0$ and $P'' > 0$ in Assumption 1, there exists $\hat{x} \in (x_1^*, \infty) \times \cdots \times (x_N^*, \infty)$ such that

$$F_i'(\hat{x}_i) + \frac{1}{C} P'\left(\frac{D^* + \sum_{j=1}^{N} \hat{x}_j}{C}\right) > 0, \quad \begin{array}{l} \forall x_i \in [\hat{x}_i, \infty), \\ x_j \in \mathbb{R}_+, \ j \neq i \end{array}$$

holds for all $i = 1, 2, ..., N$. Therefore, the condition (25) can be replaced by

$$\sum_{i=1}^{N} \zeta_i \left\{F'(x_i) - F'(x_i^*) + Q_i(x_i) - Q_i(x_i^*)\right\}^2 \geq c \sum_{i=1}^{N} \zeta_i \left\{R_i(x) - R_i(x^*)\right\}^2, \quad \forall x \in [0, \hat{x}_1) \times \cdots \times [0, \hat{x}_N) \tag{26}$$

in the case of $h_i = 0$, $i = 1, 2, ..., N$.

For a large population N, exploiting the flexibility of ζ_i's in (21), (25) and (26) would be computationally too heavy. The computation of the spectral radius $\rho(Z)$ to get rid of ζ_i's is also computationally expensive although it is an off-line analysis. For such a large population N, admitting some conservativeness, we can use $\zeta_1 = ... = \zeta_N = 1$ as in (8) and (9). If the charging scheme is completely homogeneous, i.e., the functions F_i and the constants h_i, w_i are chosen identically for all i, all non-zero $z_{i,j}$'s become a single value $z > 0$. The condition $\rho(Z) < 1$ is $z < 1/N$ for (20). If $h_i = 0$ and $R_i(x) = R_i(x_{-i})$ are chosen, the condition $\rho(Z) < 1$ becomes $z < 1/(N-1)$ for (24).

Remark 1: A finite-horizon "dynamic" noncooperative game is proposed in [9] which needs prediction of background demand $D(t)$ over the entire charging interval. The strategy profile is computed recursively off-line for each initial condition prior to the actual charging interval. The strategy pays attention to the total cost of individual vehicles for the full charge where all vehicles are synchronized for single charging cycles. On the other hand, the charging scheme proposed in this paper is on-line, i.e., feedback so that it does not require the prediction of the background demand $D(t)$ and can adopt to the actual demand variation. The preference on charing time is indirectly specified by the functions F_i. An aggressive F_i representing quick charging increase the load related to PEV charging in the grid, while a moderate choice of F_i allows us to fill the overnight demand valley for an efficient operation of the grid and reducing individual charing bills. The total cost of individual vehicles for the full charge is not directly minimized in the "static" noncooperative game formulation in this paper.

Remark 2: The stability and robust properties presented in this section are uniform in time, Boundedness properties can be verified even if charging stations are switched between active and inactive states unless the switching interval converge to zero.

B. An Example and Its Simulations

Let

$$P_i(s) = Ls^2, \ F_i(s) = \frac{b_i}{s + g_i}, \ L, b_i, g_i > 0. \tag{27}$$

These choices fulfill Assumption 1. Using

$$Q_i(x_i) = \frac{2L}{C^2} x_i, \quad R_i(x) = \frac{2L}{C^2}\left(D^* + \sum_{j \neq i} x_j\right)$$

we obtain the condition (21) as

$$\sum_{i=1}^{N} \zeta_i \left\{b_i\left(\frac{1}{(x_i^* + g_i)^2} - \frac{1}{(x_i + g_i)^2}\right) + w_i K_i (x_i - x_i^*)\right\}^2 \geq c \sum_{i=1}^{N} \zeta_i \left\{\left(\frac{2L}{C^2} - \frac{K_i}{N}\right) \sum_{j=1}^{N} (x_j - x_j^*)\right\}^2, \forall x \in \mathbb{R}_+. \tag{28}$$

Since $(F'(x_i) - F'(x_i^*))(x_i - x_i^*) > 0$ holds for all $x_i \neq x_i^*$, by virtue of (7), the criterion (8) yields

$$\min_{i=1,2,...,N} h_i(w_i + 1) > \frac{LN^2}{C^2(N-1)} . \tag{29}$$

Although the above condition is more conservative than (28), it is a quick test that is useful for arbitrarily large N. It can be verified that (29) implies $\rho(Z) < 1$ in Theorem 2 and achieves (28) with $\zeta_1 = ... = \zeta_N = 1$. The condition (29) is a natural requirement that the grid generation capacity should be large enough to accommodate a given population of PEVs.

The condition (26) in the case of

$$h_i = 0, \quad i = 1, 2, ..., N \tag{30}$$

is obtained as follows:

$$\sum_{i=1}^{N} \zeta_i \left\{ b_i \left(\frac{1}{(x_i^* + g_i)^2} - \frac{1}{(x_i + g_i)^2} \right) + \frac{2L}{C^2}(x_i - x_i^*) \right\}^2$$
$$\geq c \sum_{i=1}^{N} \zeta_i \left\{ \frac{2L}{C^2} \sum_{j \neq i} (x_j - x_j^*) \right\}^2, \ \forall x \in [0, \hat{x}_i)^N. \tag{31}$$

Making use of $|2(x_i - x_i^*)/(\hat{x}_i + g_i)^3| \leq |1/(x_i^* + g_i)^2 - 1/(x_i + g_i)^2|$ for $x_i \in [0, \hat{x}_i)$ and (7) with $b_{i,i} = 0$, applying (9) to (26) yields a simple criterion

$$\frac{b_i C^2}{(\hat{x}_i + g_i)^3} > L(N-2), \quad i = 1, 2, ..., N. \tag{32}$$

It can be verified that this inequality guarantees $\rho(Z) < 1$ and achieves (31) with $\zeta_1 = ... = \zeta_N = 1$.

For simulations, we use the load curve plotted in Fig.1 as the non-PEV demand. The population of Kyushu region is about 13 million, while the number of registered family cars is about 6 million. The number is over 9 million if heavy vehicles are included. The proposed control (4) is decentralized so that it tolerates the PEV population N of order 10^7 and more. For the sake of simulations using a single PC, we scale down the size by 10^{-5} as

$$N = 60, \ C = 185. \tag{33}$$

We use the following parameters:

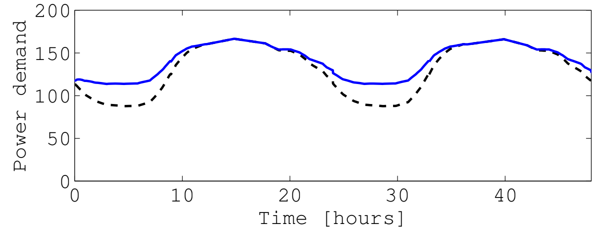
$$L = 260, \ \lambda_i = 1, \ b_i = 15, \ g_i = 2.5, \ w_i = 1, \ h_i = 1, \ \forall i \tag{34}$$

These choices fulfill (29). Figure 2 shows the non-PEV demand $D(t)$ and the total demand which is the sum of $D(t)$ and the total PEV demand. The initial charging rates are set to $x_i(\tau) = 0.02(i \bmod 5)$ for $\tau \in [-\max_i T_i, 0]$. The initial time corresponds to the midnight. The time delays are set to $T_i = 0.25[h]$, $i = 1, 2, ..., N$. Figure 2 shows that the PEV charging uses the valley of the off-peak hours. Figure 3 plots the demands for b_i set to

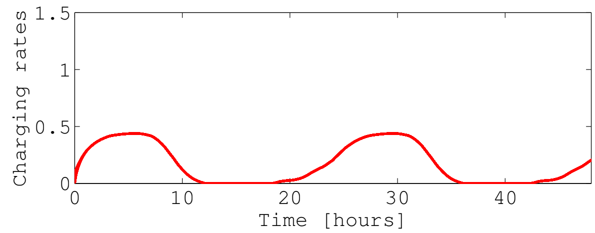
$$b_i = \begin{cases} 15.0, & 1 \leq i \leq 15 \quad , \quad 16.5, \ 16 \leq i \leq 30 \\ 18.0, & 31 \leq i \leq 45 \quad , \quad 19.5, \ 46 \leq i \leq 60 \end{cases} . \tag{35}$$

The PEV charging for $46 \leq i \leq 60$ assigned the largest b_i becomes the most aggressive (overlapped dotted black lines in (b) Fig.3) and raises the peak demand slightly. Even in the case of $h_i = 0$, $\forall i$, the condition (32) with $D^* = 150$ is also met for (33)-(34) and b_i's in (35) as well. Figure 4 shows the total demand and the charging rates in the presence of quicker charging stations designed with smaller w_i's as well as larger b_i's:

$$b_i = \begin{cases} 15, & 1 \leq i \leq 40 \\ 19.5, & 41 \leq i \leq 60 \end{cases}, \ w_i = \begin{cases} 1, & 1 \leq i \leq 40 \\ 0.33, & 41 \leq i \leq 60 \end{cases} . \tag{36}$$

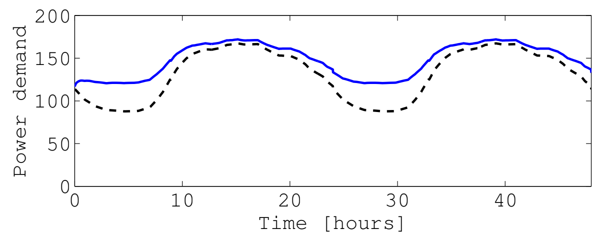


(a) Non-PEV demand (broken) and total demand (solid)

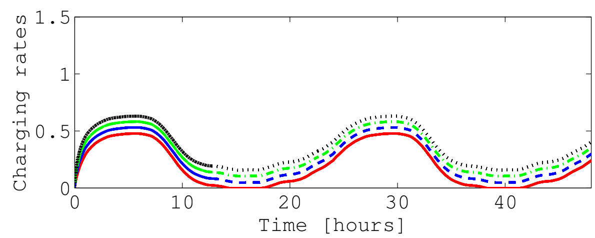


(b) Local PEV charging rates

Fig. 2. Evolution of demand and charging rate: Homogeneous charging.



(a) Non-PEV demand (broken) and total demand (solid)



(b) Local PEV charging rates

Fig. 3. Evolution of demand and charging rate: Nonhomogeneous charging.

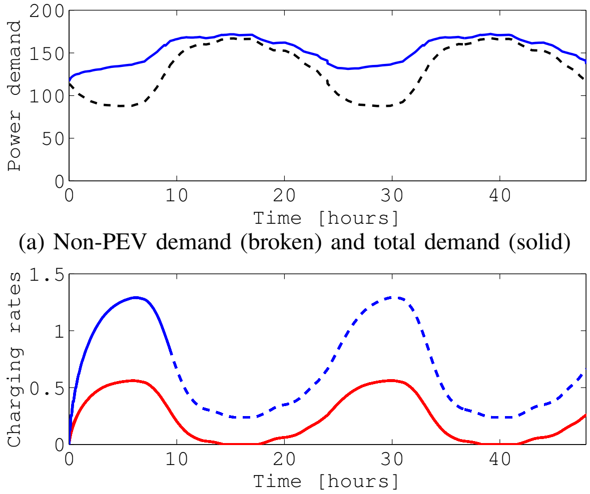
These parameters also satisfy (29).

V. Conclusions

In this paper, a control system design based on static noncooperative games has been investigated. Sufficient conditions for the stability and robustness of the control system with respect to disturbance and time delay have been derived in the iISS framework.

The proposed methodology has been applied to the problem PEV charging in the electrical grid. A decentralized charging scheme for allocating the PEV load to off-peak hours of the background demand is formulated as a gradient algorithm associated with a static noncooperative game. This paper has proved its iISS property with respect to the variation of the background demand and the time-delays in communication and processing. The methodology developed in this paper can be considered as a theoretical extension of the recent result on CDMA power control presented in [6].

To improve the valley-filling performance in the PEV charging, it is worth investigating the incorporation of an



(a) Non-PEV demand (broken) and total demand (solid)

(b) Local PEV charging rates

Fig. 4. Evolution of demand and charging rate: Bipolarized charging.

internal model into the gradient algorithm maintaining robustness. The dynamic pricing introduced in [3] will be useful in such a direction. Developing an advanced stability criterion allowing non-constant ζ_i's by expanding the idea in [5] is an interesting topic of future study.

APPENDIX

A. Proof of Theorem 1

In this proof, a function $\omega : \mathbb{R}_+ \to \mathbb{R}_+$ is written as $\omega \in \mathcal{K}$ if it is continuous, strictly increasing and satisfies $\omega(0) = 0$. We write $\omega \in \mathcal{K}_\infty$ if $\omega \in \mathcal{K}$ and $\lim_{s\to\infty} \omega(s) = \infty$. First, it is verified that the existence of $c > 1$ achieving (6) implies the existence of $\mu \in (0, 1)$ and $\epsilon, \xi \in (0, \infty)$ satisfying

$$\sum_{i=1}^{N} \frac{\zeta_i}{2}(1+\epsilon)(1+\xi)\left(B_i(x) - B_i(x^*)\right)^2 \le \sum_{i=1}^{N} \frac{\zeta_i(1-\mu)}{2}\left(A_i(x_i) - A_i(x_i^*)\right)^2, \ \forall x \in \mathbb{R}_+^N. \quad (37)$$

Define

$$\mathcal{F}_i(\tau) = \frac{-\tau}{T_i} + (1+\epsilon)\frac{\tau + T_i}{T_i}, \ \tau \in [-\bar{T}, 0]$$
$$V_i(x_i - x_i^*) = \frac{1}{\lambda_i}\int_{x_i^*}^{x_i} \left(A_i(s) - A_i(x_i^*)\right) ds,$$

where $\bar{T} = \max_i T_i$ is the maximum involved delay. Consider

$$V(\phi) = \sum_{i=1}^{N} \zeta_i V_i(\phi_i(0)) + \frac{\zeta_i}{2}\int_{-T_i}^{0} \mathcal{F}_i(\tau)\psi_i(\tau)^2 d\tau, \quad (38)$$

where $\phi_i(\tau) = \tilde{x}_i(t+\tau)$ and $\psi_i(\tau) = B_i(x(t+\tau)) - B_i(x^*)$. The property (5) implies that V_i is positive definite and radially unbounded. The property (5) gives $A_i(0) - A_i(x_i^*) \le 0$. Hence, from (4) and the projection, we obtain

$$\dot{V}_i \le -\left(A_i(x_i) - A_i(x_i^*)\right) \cdot \left(A_i(x_i(t)) + B_i(x(t - T_i)) + d_i(t)\right)$$

along the solution $x_i(t)$ of (4). Using Young's inequality with $0 < \mu < 1$, along the solution $x(t)$ of (4), we arrive at

$$\dot{V} \le \sum_{i=1}^{N} \zeta_i \Bigg\{ -\frac{1-\mu}{2}\left(A_i(x_i(t)) - A_i(x_i^*)\right)^2 - \frac{\epsilon}{2T_i}\int_{-T_i}^{0} \psi_i(\tau)^2 d\tau + \frac{1}{2}(1+\epsilon)\left(B_i(x(t)) - B_i(x^*)\right)^2 + \frac{1}{2\mu} d_i(t)^2 \Bigg\}. \quad (39)$$

Hence, due to (37), there exists $\alpha \in \mathcal{K}$ such that

$$\dot{V} \le -\alpha(V(\phi)) + \sum_{i=1}^{M} \frac{\zeta_i}{2\mu} d_i(t)^2.$$

Thus, V is an iISS Lyapunov-Krasovskii functional with respect to input d and state $\tilde{x}$ [10], [5]. The property $\lim_{x_i\to\infty} A_i(x_i) = \infty$ in (39) guarantees the existence of $\alpha \in \mathcal{K}_\infty$ qualifying V as an ISS Lyapunov-Krasovskii functional.

B. Proof of Theorem 2

An application of the Perron-Frobenius theorem implies that $\rho(Z) < 1$ ensures the existence of $c > 1$ and $\zeta_i > 0$, $i = 1, 2, ..., N$, achieving (6).

C. Proof of Theorem 3

The following V is a Lyapunov-Krasovskii functional:

$$V(\phi) = \sum_{i=1}^{N} \zeta_i (1 - U_i) V_i(\phi_i(0)) + \frac{\zeta_i}{2}\int_{-T_i(t)}^{0} \psi_i(\tau)^2 d\tau \quad (40)$$

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