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Nonlinear Tracking Control of Rigid Spacecraft under Disturbance using PD and PID Type $\mathcal{H}_\infty$ State Feedback

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***Abstract*— This study investigates six degrees-of-freedom nonlinear tracking control of rigid spacecraft under external disturbances. We propose two nonlinear tracking controllers having disturbance attenuation ability, namely, a proportional-derivative (PD)-type $\mathcal{H}_\infty$ state feedback controller and a proportional-integral-derivative (PID)-type $\mathcal{H}_\infty$ state feedback controller. Both these controllers have positive definite gain matrices whose conditions to be satisfied are given by linear matrix inequalities. The properties of these controller are compared and discussed through numerical studies.**

I. INTRODUCTION

Future space programs require agile relative position and attitude control technology of spacecraft. Rendezvous and docking, capturing of inoperative spacecraft, and formation flight in orbit are the typical scenarios where such systems can be used. A key component for controlling position and attitude is a tracking controller that controls the six degrees-of-freedom (six d.o.f) of spacecraft under the influence of external disturbances. For agility of the spacecraft, we treat above missions as nonlinear control problems where translation and rotation are dynamically coupled with each other.

Many studies have been carried out on the nonlinear attitude control of rigid spacecraft. Among these studies, the studies on passivity-based control [1], [2], [3] seem most promising, because this control technique is simple to implement, has robust stability against parameter uncertainties, and can be combined with an adaptive scheme. Attitude tracking using a proportional-derivative (PD)-type state feedback controller having positive scalar gains is proposed in [2] and it is extended to backstepping control [4]. However, these control methods ensure only the asymptotic stability of the relative attitude under a disturbance-free environment. For achieving tracking control under disturbance, most researchers have focused on the nonlinear $\mathcal{H}_\infty$ controller that makes $\mathcal{L}_2$ gain of closed-loop system from disturbance to controlled output less than $\gamma > 0$ [5], [6], [7]. They also employ PD-type scalar gain state feedback controllers. However, although these controllers generally require high feedback gains to achieve high disturbance attenuation ability, these control methods are not realizable because the maximum level of the control input is constrained by physical limitations. Therefore, we consider it is not necessarily only approach to the control purpose.

In light of the above facts, we first extend the results of [2] to a six d.o.f. PD-type tracking controller that has positive definite feedback gain matrices, and derive a condition to prove that this controller is a PD-type $\mathcal{H}_\infty$ controller. Then, we propose a proportional-integral-derivative (PID)-type state feedback controller that can effectively attenuate the constant signal under disturbance. Next, we derive a PID-type $\mathcal{H}_\infty$ state feedback controller. Finally, the properties of these controllers are compared and discussed via a numerical study.

The following notations are used throughout the paper. $a^\times \in \mathbb{R}^{3\times 3}$ is the skew symmetric matrix derived from vector $a \in \mathbb{R}^3$. $\|a\| = (a^T a)^{1/2}$ denotes vector 2-norm. $A > 0$ ($A \geq 0$) denotes A being positive (semi) definite, and $\lambda_A = \|A\|$ is the induced matrix 2-norm. I_n is a unit matrix of size $n \times n$. $O_{n\times m}$ is a zero matrix of size $n \times m$. Symbol $\star$ denotes a symmetric element.

II. MODELING AND PROBLEM DESCRIPTION

We consider a control problem in which a chaser spacecraft tracks a target point moving in the inertial frame under the influence of disturbances. Frames and vectors are defined in Fig. 1, where $\{i\}$ denotes the inertial frame, and $\{c\}$ and $\{t\}$ are the chaser and target spacecraft fixed frames, respectively. Our objective is to control the chaser so that its mass center C tracks point P and frame $\{c\}$ tracks the frame $\{t\}$.

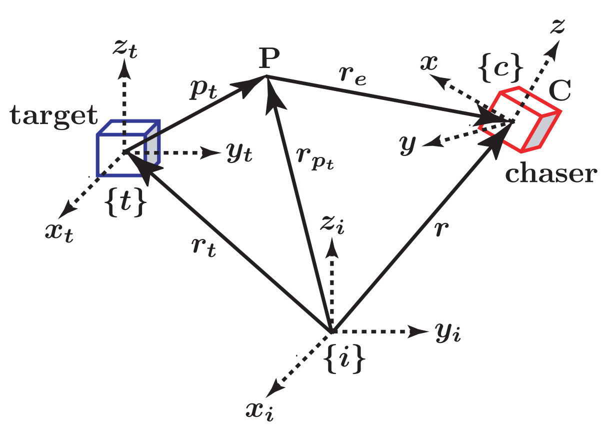

Fig. 1. Definitions of vectors and frames

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The translation and rotation dynamics of the chaser fixed frame $\{c\}$ are given by the following equations [8].

$$m\dot{v} + m\omega^{\times} v = f + d_f, \tag{1}$$

$$J\dot{\omega} + \omega^{\times} J\omega = \tau + d_\tau, \tag{2}$$

where variables $v, \omega \in \mathbb{R}^3$ are linear and angular velocities, $f, \tau \in \mathbb{R}^3$ are the control force and torque inputs, and $d_f, d_\tau \in \mathbb{R}^3$ are the disturbance force and torque inputs, respectively [1]. Constant coefficients $m \in \mathbb{R}$ and $J \in \mathbb{R}^{3\times 3}$ are the mass and inertia, respectively. The position of mass center C and the attitude of $\{c\}$ w.r.t $\{i\}$ are given by the following kinematics if a quaternion is used for attitude parametrization.

$$\dot{r} = v - \omega^{\times} r, \quad \dot{q} = E(q)\omega = \frac{1}{2}\begin{bmatrix} \eta I_3 + \epsilon^{\times} \\ -\epsilon^T \end{bmatrix}\omega, \tag{3}$$

where $r \in \mathbb{R}^3$ is the position and $q = [\ \epsilon^T\ \eta\ ]^T \in \mathbb{S}^3$ is the quaternion with the constraint $\|q\| = 1, \forall t \geq 0$.

On the other hand, the dynamics and kinematics of the target motion are described as follows.

$$m_t\dot{v}_t + m_t\omega_t^{\times} v_t = 0, \tag{4}$$

$$J_t\dot{\omega}_t + \omega_t^{\times} J_t\omega_t = 0, \tag{5}$$

$$\dot{r}_t = v_t - \omega_t^{\times} r_t, \quad \dot{q}_t = E(q_t)\omega_t = \frac{1}{2}\begin{bmatrix} \eta_t I_3 + \epsilon_t^{\times} \\ -\epsilon_t^T \end{bmatrix}\omega_t. \tag{6}$$

Then, the position and velocity of point P fixed in frame $\{t\}$ are given by

$$r_{p_t} = r_t + p_t, \quad v_{p_t} = v_t + \omega_t^{\times} p_t, \tag{7}$$

where the $p_t \in \mathbb{R}^3$ is a constant vector in fixed frame $\{t\}$.

The objective of our tracking control problem is to find control laws such that

$$r = r_{p_t}, \quad q = q_t, \quad v = v_{p_t}, \quad \omega = \omega_t$$

when $t \to \infty$. To this end, an error system in $\{c\}$ is described as follows. Let the direction cosine matrix from $\{t\}$ to $\{c\}$ be

$$\mathcal{C} = \left(\eta_e^2 - \epsilon_e^T\epsilon_e\right) I_3 + 2\epsilon_e\epsilon_e^T - 2\eta_e\epsilon_e^{\times} \tag{8}$$

using the quaternion of relative attitude $q_e = [\ \epsilon_e^T\ \eta_e\ ]^T$, where ϵ_e and η_e are defined as

$$\epsilon_e = \eta_t\epsilon - \eta\epsilon_t + \epsilon^{\times}\epsilon_t, \quad \eta_e = \eta\eta_t + \epsilon^T\epsilon_t. \tag{9}$$

The relative position, linear velocity, and angular velocity are given in the same $\{c\}$ frame as

$$r_e = r - \mathcal{C}r_{p_t}, \quad v_e = v - \mathcal{C}v_{p_t}, \quad \omega_e = \omega - \mathcal{C}\omega_t. \tag{10}$$

Substitution of (10) into (1), (2), and (3) using the identity $\dot{\mathcal{C}} = -\omega_e^{\times}\mathcal{C}$ yields the following relative motion equations

$$\begin{aligned} m\dot{v}_e = &- m[(\omega_e + \mathcal{C}\omega_t)^{\times} v_e + \mathcal{C}\dot{v}_{p_t} \\ &+ (\mathcal{C}\omega_t)^{\times}\mathcal{C}v_{p_t}] + f + d_f, \end{aligned} \tag{11}$$

$$\begin{aligned} J\dot{\omega}_e = &- (\omega_e + \mathcal{C}\omega_t)^{\times} J(\omega_e + \mathcal{C}\omega_t) \\ &- J(\mathcal{C}\dot{\omega}_t - \omega_e^{\times}\mathcal{C}\omega_t) + \tau + d_\tau, \end{aligned} \tag{12}$$

$$\dot{r}_e = v_e - (\omega_e + \mathcal{C}\omega_t)^{\times} r_e, \tag{13}$$

$$\dot{q}_e = E(q_e)\omega_e = \frac{1}{2}\begin{bmatrix} \eta_e I_3 + \epsilon_e^{\times} \\ -\epsilon_e^T \end{bmatrix}\omega_e. \tag{14}$$

By transformation, the tracking control problem is reduced to a regulation problem to design control inputs f and τ such that

$$(r_e, \epsilon_e, \eta_e, v_e, \omega_e) \to (0, 0, 1, 0, 0)$$

when $t \to \infty$ under disturbances d_f and d_τ, according to (11)–(14).

III. CONTROLLER DESIGN

A. PD-Type $\mathcal{H}_\infty$ State Feedback Controller

First, we investigate PD-type state feedback controller. Its feature compared with conventional methods is that it allows matrix feedback gains. By the extension, the design of feedback gain parameters becomes very flexible. For this design, we further transform (11)–(14) as

$$\bar{v}_e = v_e - (\mathcal{C}\omega_t)^{\times} r_e, \tag{15}$$

$$f = \bar{f} + m\delta_r, \tag{16}$$

$$\tau = \bar{\tau} + \delta_q, \tag{17}$$

where $\bar{f}, \bar{\tau} \in \mathbb{R}^3$ are the new inputs, and

$$\begin{aligned} \delta_r = &2(\mathcal{C}\omega_t)^{\times}\bar{v}_e + (\mathcal{C}\omega_t)^{\times}(\mathcal{C}\omega_t)^{\times} r_e \\ &+ (\mathcal{C}\dot{\omega}_t)^{\times} r_e + \mathcal{C}\dot{v}_{p_t} + (\mathcal{C}\omega_t)^{\times}\mathcal{C}v_{p_t}, \end{aligned} \tag{18}$$

$$\begin{aligned} \delta_q = &\omega_e^{\times} J\mathcal{C}\omega_t + (\mathcal{C}\omega_t)^{\times} J(\omega_e + \mathcal{C}\omega_t) \\ &+ J(\mathcal{C}\dot{\omega}_t - \omega_e^{\times}\mathcal{C}\omega_t). \end{aligned} \tag{19}$$

Then, (11)–(14) are simplified as

$$m\dot{\bar{v}}_e = -m\omega_e^{\times}\bar{v}_e + \bar{f} + d_f, \tag{20}$$

$$J\dot{\omega}_e = -\omega_e^{\times} J\omega_e + \bar{\tau} + d_\tau, \tag{21}$$

$$\dot{r}_e = \bar{v}_e - \omega_e^{\times} r_e, \tag{22}$$

$$\dot{q}_e = E(q_e)\omega_e. \tag{23}$$

The above system (20)–(23) is passive w.r.t. inputs $u = [\ \bar{f}^T\ \bar{\tau}^T\ ]^T$ and outputs $y = [\ \bar{v}_e^T\ \omega_e^T\ ]^T$ (see Appendix A). By transformation (15)–(17), the control problem is now to regulate the system (20)–(23) in order to design control inputs $\bar{f}$ and $\bar{\tau}$ such that

$$(r_e, \epsilon_e, \eta_e, \bar{v}_e, \omega_e) \to (0, 0, 1, 0, 0).$$

when $t \to \infty$.

Now, let us consider the state feedback control law that has positive definite gain matrices as follows:

$$\bar{f} = -\frac{1}{a_2}(k_{p_1} r_e + K_{d_1}\bar{v}_e), \tag{24}$$

[1] Practically, the resultant inputs f and τ are given by $f = f_c$ and $\tau = \rho_c^{\times} f_c + \tau_c$, respectively, where $f_c, \tau_c \in \mathbb{R}^3$ are the outputs of force and torque actuators, respectively, and $\rho_c \in \mathbb{R}^3$ is the arm length from the mass center to the point of application of force. It should be noted that f_c and τ_c are uniquely determined from f and τ, respectively, if ρ_c is given. Disturbances d_f and d_τ are in the same location.

$$\bar{\tau} = -\frac{1}{b_2}(K(q_e)\epsilon_e + K_{d_2}\omega_e), \tag{25}$$
$$K(q_e) = (\eta_e I_3 - \epsilon_e^\times)K_{p_2} + k_{p_3}(1-\eta_e)I_3,$$

where $a_2, b_2 \in \mathbb{R}$ are the positive design parameters,

$$k_{p_1} > 0,\ K_{p_2} = K_{p_2}^T > 0,\ k_{p_3} > 0, \tag{26}$$
$$K_{d_1} = K_{d_1}^T > 0,\ K_{d_2} = K_{d_2}^T > 0, \tag{27}$$
$$k_{p_1}, k_{p_3} \in \mathbb{R},\ K_{p_2}, K_{d_1}, K_{d_2} \in \mathbb{R}^{3\times 3},$$

and the output to be controlled is defined as $z = \Sigma\zeta$, where Σ is the weighting matrix,

$$\Sigma = \mathrm{diag}\{\sigma_r, \sigma_v, \sigma_\eta, \sigma_\omega\},$$
$$\sigma_r, \sigma_v, \sigma_\omega \in \mathbb{R}^{3\times 3},\ \sigma_\eta \in \mathbb{R},$$
$$\zeta = [\ r_e^T\ \bar{v}_e^T\ 2\cos^{-1}(|\eta_e|)\ \omega_e^T\ ]^T.$$

From the definition of quaternion, $2\cos^{-1}(|\eta_e|)$ of the element of ζ represents the eigen-angle around the unit vector (eigen-axis) with respect to relative attitude [6]. In addition, regarding the target states, the following assumption is made.

Assumption 1: The target states r_t, ϵ_t, η_t, v_t, ω_t, $\dot{v}_t$, and $\dot{\omega}_t$ are directly measurable, uniformly continuous, bounded, and known for all $t \in [0, \infty)$.

Then, the following theorem can be obtained.

Theorem 1:

Given a_1, a_2, b_1, b_2, and γ, where $a_1, b_1 \in \mathbb{R}$ are the positive design parameters, the closed-loop system of (20)–(23) with (24) and (25) satisfies the $\mathcal{L}_2$ gain less than or equal to γ from disturbance input $d = [\ d_f^T\ d_\tau^T\ ]^T \in \mathcal{L}_2[0, T]$ to controlled outputs z if feedback gains satisfy following conditions

$$\mathcal{F} > 0,\ 2k_{p_3}I_3 > K_{p_2} > k_{p_3}I_3,\ \mathcal{R} > 0, \tag{28}$$
$$\mathcal{R} - \bar{\Sigma}^T\bar{\Sigma} - \frac{1}{4\gamma^2}W^TW \geq 0, \tag{29}$$

$$\mathcal{F} = \mathrm{diag}\{\mathcal{F}_1, \mathcal{F}_2\},\ \mathcal{R} = \mathrm{diag}\{\mathcal{R}_1, \mathcal{R}_2\},$$

$$\mathcal{F}_1 = \begin{bmatrix} k_{p_1}I_3 & a_1 m I_3 \\ \star & a_2 m I_3 \end{bmatrix},\ \mathcal{F}_2 = \begin{bmatrix} 2K_{p_2} & b_1 J \\ \star & b_2 J \end{bmatrix},$$

$$\mathcal{R}_1 = \begin{bmatrix} \frac{a_1}{a_2}k_{p_1}I_3 & \frac{a_1}{2a_2}K_{d_1} \\ \star & K_{d_1} - a_1 m I_3 \end{bmatrix},$$

$$\mathcal{R}_2 = \begin{bmatrix} \frac{b_1}{b_2}(2k_{p_3}I_3 - K_{p_2}) & \frac{b_1}{2b_2}K_{d2} \\ \star & K_{d_2} - \frac{3}{2}b_1\lambda_J I_3 \end{bmatrix},$$

$$W = \begin{bmatrix} a_1 I_3 & a_2 I_3 & O_{3\times 3} & O_{3\times 3} \\ O_{3\times 3} & O_{3\times 3} & b_1 I_3 & b_2 I_3 \end{bmatrix},$$

$$\bar{\Sigma} = \mathrm{diag}\{\sigma_r, \sigma_v, \pi\sigma_\eta I_3, \sigma_\omega\}.$$

Moreover, the state variable of the closed-loop system becomes

$$(r_e, \epsilon_e, \eta_e, \bar{v}_e, \omega_e) \to (0, 0, 1, 0, 0).$$

as $t \to \infty$ for arbitrary initial state when $d = 0$.

Proof: See Appendix B. ■

Remark 1:

1) The obtained conditions (26)–(29) are linear matrix inequalities (LMIs) w.r.t. feedback gains that are effectively solved using convex optimization tools [9].
2) When only the asymptotic stability is required, the positive definiteness of feedback gains automatically satisfy (28). This is proved by letting design parameters $a_1 = b_1 = 0$ and $a_2 = b_2 = 1$ in the proof.
3) If we set $K_{p2} = k_{p3}I_3$, matrix $K(q_e)$ becomes positive scalar constant k_{p_3}.

B. PID-Type $\mathcal{H}_\infty$ State Feedback Controller

The PD-type $\mathcal{H}_\infty$ state feedback controller has disturbance attenuation ability in that it minimizes the $\mathcal{L}_2$ norm of the control output. However, the magnitude of the feedback control input generally becomes larger when the attenuation requirement increases. In the classical linear control theory, the integral compensation is usually used to eliminate the offset errors caused by the constant step disturbance. This section proposes a nonlinear PID-type state feedback controller.

We consider the following control law:

$$\begin{cases} \bar{f} = -\frac{1}{a_2}\left(k_{p_1}r_e + K_{d_1}\bar{v}_e\right) - k_{i_1}\xi_1 \\ \xi_1 = \int_0^t \left(r_e + \frac{a_2}{a_1}\omega_e^\times r_e\right)dt \end{cases}, \tag{30}$$

$$\begin{cases} \bar{\tau} = -\frac{1}{b_2}\left(K(q_e)\epsilon_e + K_{d_2}\omega_e\right) - k_{i_2}\xi_2 \\ \xi_2 = \int_0^t \left(\epsilon_e + \frac{b_2}{2b_1}\{(2-\eta_e)I_3 - \epsilon_e^\times\}\omega_e\right)dt \end{cases}, \tag{31}$$

where

$$k_{i_1}, k_{i_2} \in \mathbb{R},\ k_{i_1}, k_{i_2} > 0, \tag{32}$$

and other feedback gains and design parameters are the same as defined in (24) and (25). Then, the following theorem can be obtained.

Theorem 2:

Given a_1, a_2, b_1, b_2, and γ, the closed-loop system of (20)–(23) with (30) and (31) satisfies the $\mathcal{L}_2$ gain less than or equal to γ from d to z if feedback gains satisfy following conditions

$$\hat{\mathcal{F}} > 0,\ 2k_{p_3}I_3 > K_{p_2} > k_{p_3}I_3,\ \hat{\mathcal{R}} > 0, \tag{33}$$
$$\hat{\mathcal{R}} - \bar{\Sigma}^T\bar{\Sigma} - \frac{1}{4\gamma^2}W^TW \geq 0, \tag{34}$$

$$\hat{\mathcal{F}} = \mathrm{diag}\{\hat{\mathcal{F}}_1, \hat{\mathcal{F}}_2\},\ \hat{\mathcal{R}} = \mathrm{diag}\{\hat{\mathcal{R}}_1, \hat{\mathcal{R}}_2\},$$

$$\hat{\mathcal{F}}_1 = \begin{bmatrix} k_{p_1} I_3 & a_1 m I_3 & a_2 k_{i_1} I_3 \\ \star & a_2 m I_3 & 0 \\ \star & \star & a_1 k_{i_1} I_3 \end{bmatrix},$$

$$\hat{\mathcal{F}}_2 = \begin{bmatrix} 2K_{p_2} & b_1 J & b_2 k_{i_2} I_3 \\ \star & b_2 J & 0 \\ \star & \star & b_1 k_{i_2} I_3 \end{bmatrix},$$

$$\hat{\mathcal{R}}_1 = \begin{bmatrix} \frac{a_1}{a_2} k_{p_1} I_3 - a_2 k_{i1} I_3 & \frac{a_1}{2a_2} K_{d_1} \\ \star & K_{d_1} - a_1 m I_3 \end{bmatrix},$$

$$\hat{\mathcal{R}}_2 = \begin{bmatrix} \frac{b_1}{b_2}(2k_{p_3} I_3 - K_{p_2}) - (b_2 + \frac{b_2^2}{4b_1}) k_{i2} I_3 & \frac{b_1}{2b_2} K_{d2} \\ \star & K_{d2} - \frac{3}{2} b_1 \lambda_J I_3 - \frac{b_2^2}{16 b_1} k_{i_2} I_3 \end{bmatrix}.$$

Moreover, the state variable of the closed-loop system becomes

$$(r_e, \epsilon_e, \eta_e, \bar{v}_e, \omega_e, \xi_1, \xi_2) \to (0, 0, 1, 0, 0, 0, 0).$$

as $t \to \infty$ for arbitrary initial state when $d = 0$.

Proof: See Appendix C. ∎

Remark 2:

1) As in Theorem 1, the obtained conditions (26), (27), and (32)–(34) are LMIs w.r.t. feedback gains, and controllers (30) and (31) have a scalar proportional feedback gain k_{p_3} by letting $K_{p_2} = k_{p_3} I$.
2) It can be shown that the position and attitude can track their targets without offset errors when the disturbance is constant. At the steady state, $\bar{v}_e = 0$ and $\omega_e = 0$ hold. Therefore, as

$$K(q_e) = K_{p_2},\ \xi_1 = \int_0^t r_e dt,\ \xi_2 = \int_0^t \epsilon_e dt,$$

the closed-loop system is

$$\frac{1}{a_2} k_{p_1} r_e + k_{i_1} \int_0^t r_e dt - d_f = 0, \tag{35}$$

$$\frac{1}{b_2} K_{p_2} \epsilon_e + k_{i_2} \int_0^t \epsilon_e dt - d_\tau = 0. \tag{36}$$

If we define

$$e_1 = \left(\int_0^t r_e dt - \frac{1}{k_{i1}} d_f \right),$$

$$e_2 = \left(\int_0^t \epsilon_e dt - \frac{1}{k_{i_2}} d_n \right),$$

then (35) and (36) become

$$\dot{e}_1 = -\frac{k_{i_1}}{k_{p_1}} e_1,\ \dot{e}_2 = -k_{i_2} K_{p_2}^{-1} e_1. \tag{37}$$

Since $a_2, b_2, k_{p_1}, k_{i_1}, k_{i_2} > 0$ and $K_{p_2} = K_{p_2}^T > 0$, $e_i \to 0$ when $t \to \infty$. Therefore, from (35) and (36), it can be said that $r_e \to 0$ and $\epsilon_e \to 0$ when $t \to \infty$.

IV. NUMERICAL STUDY

The properties of the proposed controllers are compared and discussed in this numerical study. For this purpose, we set the physical parameters of the target and the chaser spacecraft as

$$m_t = 300 \text{ [kg]}, \quad J_t = \text{diag}\{50, 275, 275\} [\text{kgm}^2].$$

$$m = 200 \text{ [kg]}, \quad J = \begin{bmatrix} 75.0 & -28.1 & -28.1 \\ \star & 75.0 & -28.1 \\ \star & \star & 75.0 \end{bmatrix} [\text{kgm}^2].$$

The target position in the $\{t\}$ frame is given as $p_t = [0\ 5\ 0]^T$. The initial conditions for the chaser spacecraft are

$$r(0) = [10\ 10\ 10]^T \text{ [m]},\ v(0) = [0\ 0\ 0]^T \text{ [m/s]},$$

$$q(0) = [0.06\ 0.69\ 0.06\ 0.72]^T \text{ [-]},\ \omega(0) = [0\ 0\ 0]^T \text{ [rad/s]},$$

and those for the target are [2]

$$r_t(0) = [3\ 3\ 3]^T \text{ [m]},\ v_t(0) = [0\ 0\ 0]^T \text{ [m/s]}$$

$$q_t(0) = [0\ 0\ 0\ 1]^T \text{ [-]},\ \omega_t(0) = [0.2\ 0.2\ 0.2]^T \text{ [rad/s]}.$$

A. Tracking Performance under Disturbance-Free Environment

First, we show the six d.o.f. tracking ability of the PD- and PID-type controllers when $d = 0$. The PD and PID controller gains are chosen to satisfy only the conditions (28) and (33), respectively. The controller gains are set as

$$\text{PD} : a_1 = b_1 = 0,\ a_2 = b_2 = 1,$$

$$\text{PID} : a_1 = 0.2,\ b_1 = 0.1,\ a_2 = b_2 = 1,$$

$$k_{p_1} = 15 I_3,\ K_{p_2} = 10 I_3,\ k_{p_3} = 12,$$

$$K_{d_1} = 120 I_3,\ K_{d_2} = 40 I_3,\ k_{i_1} = 0.8,\ k_{i_2} = 0.3.$$

Figs. 2 and 3 show the responses of positions, quaternions of chaser spacecraft, and target of the PD and PID controllers, respectively. For both controllers, the chaser tracks the target. However, the performance of the PID controller degrades because of the time delay introduced by the integral compensation.

B. Tracking under Constant Disturbance

The following three controllers are applied under the same initial conditions, when constant disturbances

$$d_f = [3\ 3\ 3]^T \text{ [N]}, \quad d_n = [3\ 3\ 3]^T \text{ [Nm]}$$

are added:

- Case 1 : PD controller,
- Case 2 : PD $\mathcal{H}_\infty$ controller when $\gamma = 0.2$,
- Case 3 : PID controller.

In Case 2, the design parameters are set as

$$a_1 = 8,\ b_1 = 4,\ a_2 = b_2 = 40,$$

[2]The attitudes described by quaternion $q(0)$ and $q_t(0)$ correspond to Euler angles of 3-2-1 system of $(\phi(0), \theta(0), \psi(0)) = (80, 80, 80)$ [deg] and $(\phi_t(0), \theta_t(0), \psi_t(0)) = (0, 0, 0)$ [deg], respectively. However, all simulations are performed using the parameterization of quaternion.

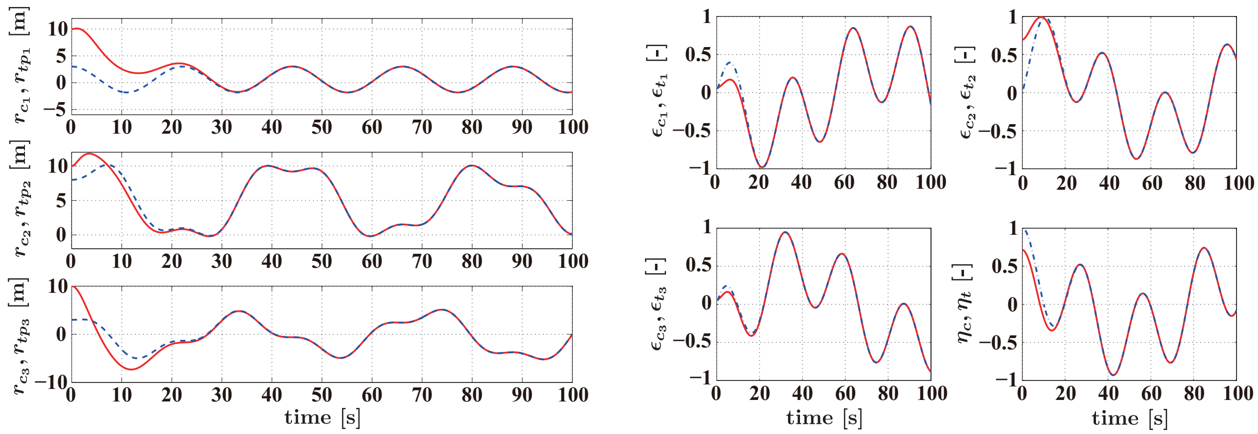

Fig. 2. PD controller: responses of position (left) and attitude (right) of chaser (solid line) and target (dashed line).

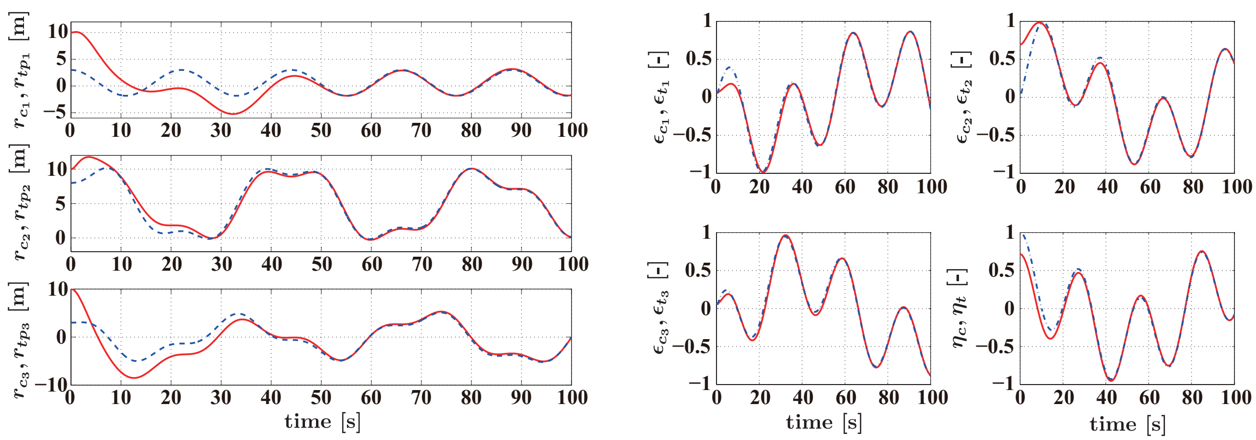

Fig. 3. PID controller: responses of position (left) and attitude (right) of chaser (solid line) and target (dashed line).

$$\sigma_r = 6I_3,\ \sigma_v = 1I_3,\ \sigma_\eta = 3,\ \sigma_\omega = I_3,$$

and feedback gains are derived by solving LMIs (26)–(29). In addition, to prevent relative error from vibrating, the following conditions are added:

$$K_{d_1} > 8k_{p_1}I_3,\ K_{d_2} > 8K_{p_2},\ K_{d_2} > 8k_{p_3}I_3.$$

The results are shown in Figs. 4, 5, and 6. The PD controller fails in tracking, but it succeeds when the $\mathcal{H}_\infty$ property is added. However, finally, tracking errors are retained, although their values are very small. On the other hand, PID controller achieves the best performance in the steady state, even though it does not exhibit the $\mathcal{H}_\infty$ property. However, near the initial time $t = 0$, the performance degrades because of the time delay caused by integral compensation.

C. Disturbance Attenuation Ability

Finally, we examine the disturbance attenuation ability. We compare the PD and PID controllers, both of which exhibit the $\mathcal{H}_\infty$ property when $\gamma = 0.8,\ 0.4$. The design parameters are set as

$$a_1 = 8,\ b_1 = 4,\ a_2 = b_2 = 40,$$
$$\sigma_r = 6I_3,\ \sigma_v = 1I_3,\ \sigma_\eta = 3,\ \sigma_\omega = I_3,$$

and feedback gains of the PD controller are derived by solving LMIs (26)–(29) whereas those of the PID controller are derived by solving LMIs (26), (27), and (32)–(34). In addition, to prevent relative error from vibrating and integral gains k_{i1}, k_{i2} from becoming very small, the following conditions are applied:

$$K_{d_1} > 8k_{p_1}I_3,\ K_{d_2} > 8K_{p_2},\ K_{d_2} > 8k_{p_3}I_3,$$
$$k_{i1} > 0.8,\ k_{i2} > 0.5.$$

The disturbance input $d \in \mathcal{L}_2[0, T]$ is

$$d_f = 3\sin\left(\frac{\pi}{40}t\right)[1\ 1\ 1]^T\ [\mathrm{N}],$$
$$d_\tau = 3\sin\left(\frac{\pi}{40}t\right)[1\ 1\ 1]^T\ [\mathrm{Nm}],$$

where $t \in [0, 100]$. For the same initial conditions, the responses of norms of relative positions and attitude described by Euler angles of 3-2-1 system are obtained. They are shown in Figs. 7 and 8. It is concluded that the transient property of the PID $\mathcal{H}_\infty$ controller is better than that of the PD $\mathcal{H}_\infty$ controller. However, the convergence for the sinusoidal disturbance degrades.

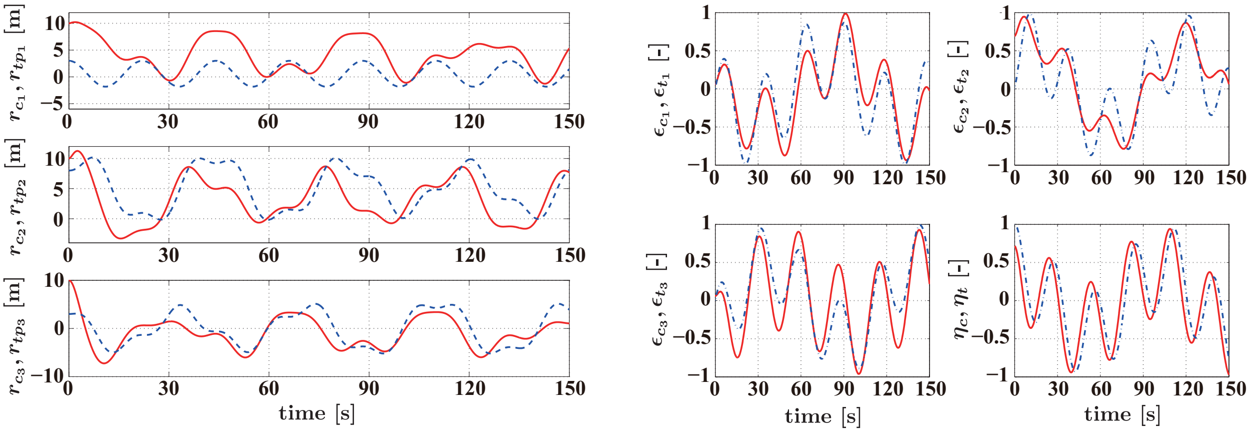

Fig. 4. PD controller (Case 1) : responses of position (left) and attitude (right) of chaser (solid line) and target (dashed line) under constant disturbance.

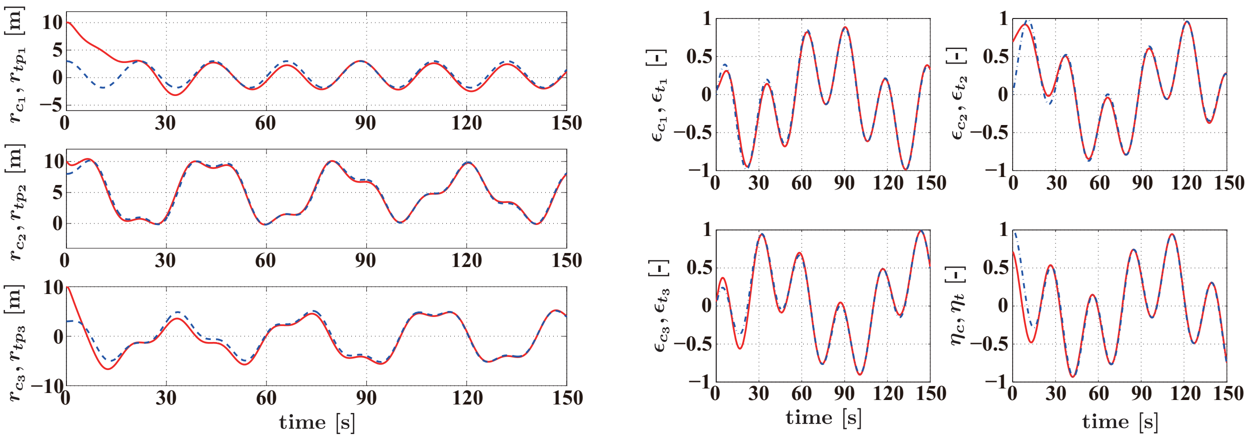

Fig. 5. PD $\mathcal{H}_\infty$ controller (Case 2) : responses of position (left) and attitude (right) of chaser (solid line) and target (dashed line) under constant disturbance.

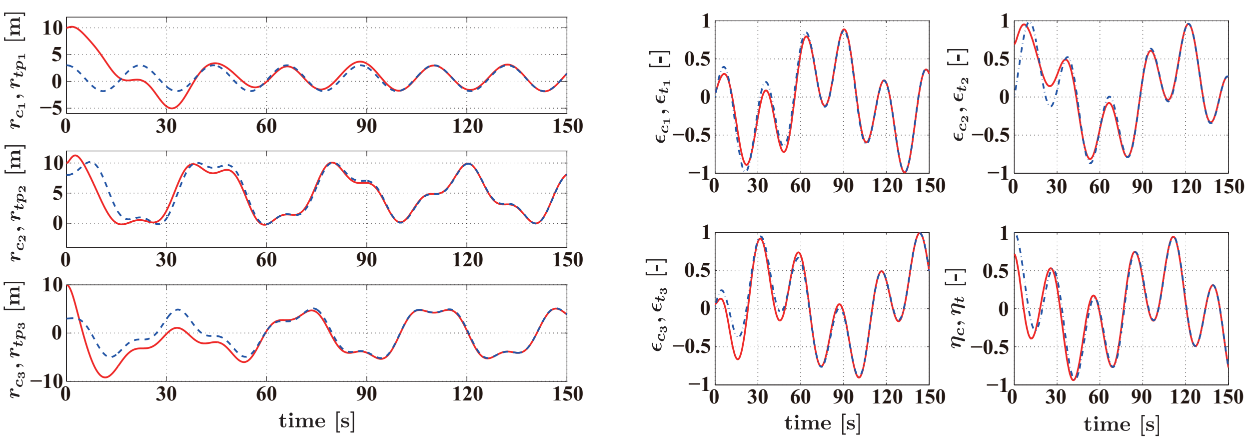

Fig. 6. PID controller (Case 3) : responses of position (left) and attitude (right) of chaser (solid line) and target (dashed line) under constant disturbance.

V. CONCLUSION

We have investigated six d.o.f nonlinear tracking control technologies of spacecraft under external disturbance in order to prepare for the future space missions. To this end, a PID-type state feedback controller as well as a PD-type state feedback controller was proposed. The conditions of the asymptotic stability of error systems and the $\mathcal{L}_2$ gain properties of a closed-loop system were obtained. The performances of the above-mentioned controller have been compared and discussed through numerical studies. These controllers require the accurate values of mass and inertia. However, they can be extended to include parameter adaptive schemes, as shown in [10].

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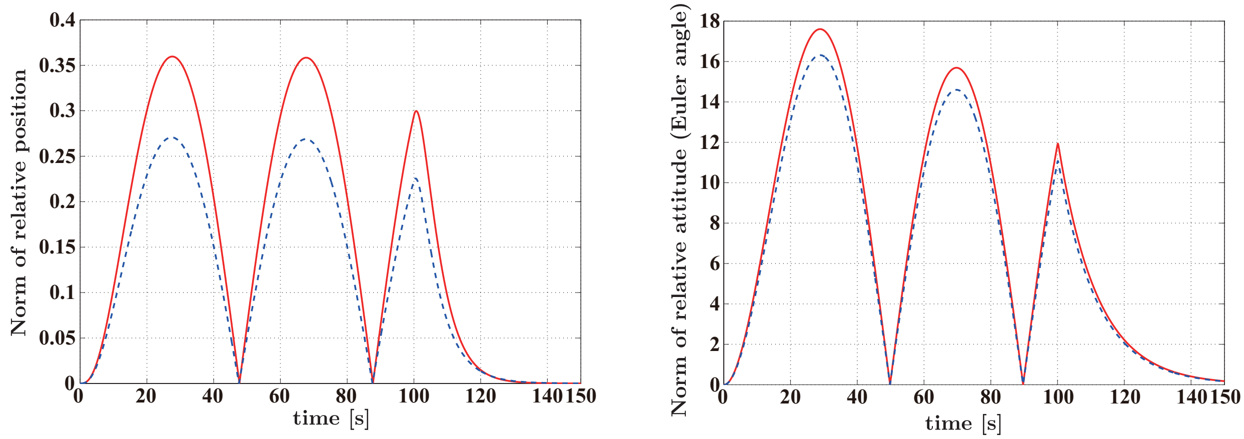

Fig. 7. PD $\mathcal{H}_\infty$ controller: responses of norms of relative position (left) and attitude (right) for $\gamma = 0.8$ (solid line) and $\gamma = 0.4$ (dotted line).

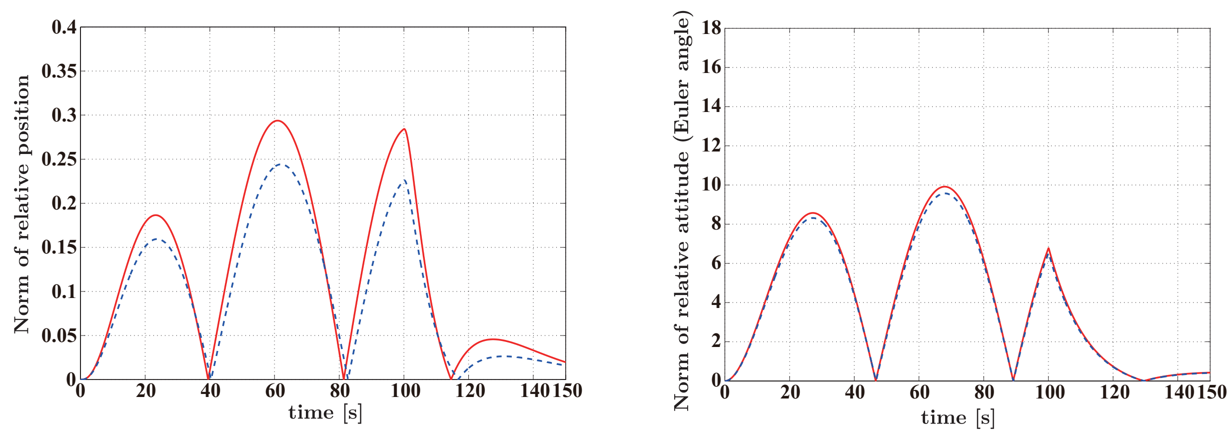

Fig. 8. PID $\mathcal{H}_\infty$ controller: responses of norms of relative position (left) and attitude (right) for $\gamma = 0.8$ (solid line) and $\gamma = 0.4$ (dotted line).

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APPENDIX

A. Passivity of Systems (20)–(23)

Let us define the storage function as

$$\mathcal{E} = \frac{m}{2}\|\bar{v}_e\|^2 + \frac{1}{2}\omega_e^T J\omega_e. \tag{38}$$

The time derivatives of (38) along the trajectories of system (20)–(23) with $d_f = d_\tau = 0$ become

$$\begin{aligned}\dot{\mathcal{E}} &= m\bar{v}_e^T(-m\omega_e^\times \bar{v}_e + \bar{f}) + \omega_e^T(-\omega_e^\times J\omega_e + \bar{\tau})\\ &= \bar{v}_e^T\bar{f} + \omega_e^T\bar{\tau}\\ &= y^T u.\end{aligned}$$

Therefore, system (20)–(23) is passive w.r.t. input u and output y.

B. Proof of Theorem 1

Let us define the candidate of Lyapunov function as

$$\begin{aligned}\mathcal{V} =& \frac{a_2}{2}m\|\bar{v}_e\|^2 + \frac{k_{p_1}}{2}\|r_e\|^2 + a_1 m r_e^T\bar{v}_e + \frac{b_2}{2}\omega_e^T J\omega_e\\ &+ \epsilon_e^T K_{p_2}\epsilon_e + k_{p_3}(\eta_e - 1)^2 + b_1\epsilon_e^T J\omega_e\\ =& \frac{1}{2}\chi^T\mathcal{F}\chi + k_{p_3}(\eta_e - 1)^2,\\ &\chi = [r_e^T\ \bar{v}_e^T\ \epsilon_e^T\ \omega_e^T]^T.\end{aligned} \tag{39}$$

Therefore, $\mathcal{V} > 0$ if $\mathcal{F} > 0$. The time derivative of (39) along the trajectories of the closed-loop system become [3]

$$\begin{aligned}\dot{\mathcal{V}} =& (a_1 r_e + a_2 \bar{v}_e)^T(\bar{f} + d_f) + k_{p_1} r_e^T \bar{v}_e + a_1 m\|\bar{v}_e\|^2 \\ &+ (b_1\epsilon_e + b_2\omega_e)^T(\bar{\tau} + d_\tau) + \omega_e^T K(q_e)\epsilon_e \\ &+ \frac{b_1}{2}\omega_e^T JT(q_e)\omega_e - b_1\epsilon_e^T\omega_e^\times J\omega_e \\ =& -\frac{a_1}{a_2}k_{p_1}\|r_e\|^2 - \frac{a_1}{a_2}r_e^T K_{d_1}\bar{v}_e - \bar{v}_e^T(K_{d_1} - a_1 m I_3)\bar{v}_e \\ &- \frac{b_1}{b_2}\epsilon_e^T K(q_e)\epsilon_e - \frac{b_1}{b_2}\epsilon_e^T K_{d_2}\omega_e - \omega_e^T K_{d_2}\omega_e \\ &+ \frac{b_1}{2}\omega_e^T JT(q_e)\omega_e - b_1\epsilon_e^T\omega_e^\times J\omega_e + (a_1 r_e + a_2\bar{v}_e)^T d_f \\ &+ (b_1\epsilon_e + b_2\omega_e)^T d_\tau, \end{aligned} \tag{40}$$

where $T(q_e) = \eta_e I_3 + \epsilon_e^\times$. In (40), as

$$\|T(q_e)\| = 1,\ \|\epsilon_e\| \le 1,\ \|\omega_e^\times\| = \|\omega_e\|$$

$$\left|\frac{b_1}{2}\omega_e^T JT\omega_e - b_1\epsilon_e^T\omega_e^\times J\omega_e\right| \le \frac{3}{2}b_1\lambda_J\|\omega_e\|^2,$$

and the identity $\epsilon_e^T(\eta_e I_3 - \epsilon_e^\times) = \epsilon_e^T\eta_e$ yields

$$\begin{aligned}-\epsilon_e^T K(q_e)\epsilon_e &= -\epsilon_e^T\{\eta_e(K_{p_2} - k_{p_3}I_3) + k_{p_3}I_3\}\epsilon_e \\ &= -\epsilon_e^T\mathcal{G}(\eta_e)\epsilon_e,\end{aligned} \tag{41}$$

where $\mathcal{G}(\eta_e)$ is described as follows according to η_e.

$$\mathcal{G}(\eta_e) = \begin{cases} \eta_e(K_{p2} - k_{p3}I_3) + k_{p3}I_3, & 1 \ge \eta_e > 0 \\ k_{p3}I_3, & \eta_e = 0 \\ -|\eta_e|(K_{p2} - k_{p3}I_3) + k_{p3}I_3, & 0 > \eta_e \ge -1 \end{cases}$$

From the above equation, $\mathcal{G}(\eta_e) > 0$ for all η_e if $2k_{p_3}I_3 > K_{p_2} > k_{p_3}I_3$. Furthermore, minimum value of $\mathcal{G}(\eta_e)$ is

$$\min_{\eta_e}\mathcal{G}(\eta_e) = \mathcal{G}(-1) = 2k_{p3}I_3 - K_{p2}.$$

Therefore, if $2k_{p_3}I_3 > K_{p_2} > k_{p_3}I_3$, then (41) becomes

$$-\epsilon_e^T K(q_e)\epsilon_e \le -\epsilon_e^T(2k_{p_3}I_3 - K_{p_2})\epsilon_e,$$

and (40) satisfies

$$\begin{aligned}\dot{\mathcal{V}} \le& -\frac{a_1}{a_2}k_{p_1}\|r_e\|^2 - \frac{a_1}{a_2}r_e^T K_{d_1}\bar{v}_e - \bar{v}_e^T(K_{d_1} - a_1 m I_3)\bar{v}_e \\ &- \frac{b_1}{b_2}\epsilon_e^T(2k_{p_3}I_3 - K_{p_2})\epsilon_e - \frac{b_1}{b_2}\epsilon_e^T K_{d_2}\omega_e \\ &- \omega_e^T\Big(K_{d_2} - \frac{3}{2}b_1\lambda_J I_3\Big)\omega_e + (a_1 r_e + a_2\bar{v}_e)^T d_f \\ &+ (b_1\epsilon_e + b_2\omega_e)^T d_\tau \\ =& -\chi^T\mathcal{R}\chi + \chi^T W^T d.\end{aligned} \tag{42}$$

By the completion of the square, we obtain

$$\begin{aligned}\dot{\mathcal{V}} + \|z\|^2 - \gamma^2\|d\|^2 \le& -\chi^T\mathcal{R}\chi + \frac{1}{4\gamma^2}\chi^T W^T W\chi \\ &- \gamma^2\|d - \frac{1}{2\gamma^2}W\chi\|^2 + \zeta^T\Sigma^T\Sigma\zeta.\end{aligned}$$

If we note that

$$d^* = \frac{1}{2\gamma^2}W\chi$$

is the worst-case disturbance and that

$$\zeta^T\Sigma^T\Sigma\zeta \le \chi^T\bar{\Sigma}^T\bar{\Sigma}\chi$$

from $2\cos^{-1}(|\eta_e|) \le \pi\|\epsilon_e\|$ [6], the inequality

$$\dot{\mathcal{V}} + \|z\|^2 - \gamma^2\|d\|^2 \le -\chi^T\Big(\mathcal{R} - \bar{\Sigma}^T\bar{\Sigma} - \frac{1}{4\gamma^2}W^T W\Big)\chi$$

holds $\forall d \in \mathcal{L}_2[0, T]$. Therefore, condition (29) implies

$$\dot{\mathcal{V}} \le \gamma^2\|d\|^2 - \|z\|^2$$

indicating that the $\mathcal{L}_2$ gain of the closed-loop is less than or equal to γ [11].

With regard to the asymptotic stability, when $d = 0$, (42) becomes

$$\dot{\mathcal{V}} = -\chi^T\mathcal{R}\chi$$

and $\dot{V} \le 0$ if $\mathcal{R} > 0$. Therefore,

$$\mathcal{V}(x(t)) \le \mathcal{V}(x(0)),\ \forall t \ge 0.\ (x = [\chi^T\ \ \eta_e]^T)$$

and x is bounded because $\mathcal{V}$ is radially unbounded. Furthermore, control inputs (24) and (25) are bounded by the conditions in Assumption 1. Since $\dot{x}$ is also bounded,

$$\ddot{\mathcal{V}} = -2\chi^T\mathcal{R}\dot{\chi}$$

is bounded and $\dot{\mathcal{V}}$ is uniformly continuous w.r.t. t. Additionally, since $\mathcal{V}$ is lower bounded from $\mathcal{V} \ge 0$,

$$\dot{\mathcal{V}} \to 0 \Rightarrow \chi \to 0$$

when $t \to \infty$ from Lyapunov-like lemma [12]. Therefore, $\eta_e = 1$ when $\mathcal{V} = 0$; that is, the closed-loop system is asymptotically stable for all initial states.

C. Proof of Theorem 2

Let us define the candidate of Lyapunov function as

$$\begin{aligned}\mathcal{V} =& \frac{a_2}{2}m\|\bar{v}_e\|^2 + \frac{k_{p_1}}{2}\|r_e\|^2 + a_1 m r_e^T\bar{v}_e + a_2 k_{i1}\xi_1^T r_e \\ &+ \frac{a_1}{2}k_{i1}\|\xi_1\|^2 + \frac{b_2}{2}\omega_e^T J\omega_e + \epsilon_e^T K_{p_2}\epsilon_e + k_{p_3}(\eta_e - 1)^2 \\ &+ b_1\epsilon_e^T J\omega_e + b_2 k_{i_2}\xi_2^T\epsilon_e + \frac{b_1}{2}k_{i_2}\|\xi_2\|^2 \\ =& \frac{1}{2}\hat{\chi}^T\hat{\mathcal{F}}\hat{\chi} + k_{p_3}(\eta_e - 1)^2, \\ &\hat{\chi} = [r_e^T\ \bar{v}_e^T\ \xi_1^T\ \epsilon_e^T\ \omega_e^T\ \xi_2^T]^T.\end{aligned}$$

This is constructed from (39) by adding terms of ξ_1 and ξ_2. Therefore, using the same steps those used in the proof of Theorem 1, we obtain its time derivative as

$$\dot{\mathcal{V}} \le -\chi^T\hat{\mathcal{R}}\chi + \chi^T W^T d.$$

Along the line of the above discussion, the $\mathcal{L}_2$ gain property of a closed-loop system and the asymptotic stability are concluded.

[3] In the manipulation of equations, following relations are used: $\forall a, b, c \in \mathbb{R}^3$, $a^\times a = 0$, $b^T a^\times b = 0$, and $b^T a^\times c + c^T a^\times b = 0$.