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Comparison of Two Nonlinear Model Predictive Control Methods and Implementation on a Laboratory Three Tank System

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Abstract—**Almost all industrial processes exhibit nonlinear dynamics, however most model predictive control (MPC) applications are based on linear models. Linear models do not always give a sufficiently adequate representation of the system and therefore Nonlinear Model Predictive Control (NMPC) techniques have to be used. In this article, two techniques of NMPC, namely successive linearization nonlinear model predictive control (SLNMPC) and wiener nonlinear model predictive control (WNMPC) are applied to nonlinear process systems. The major advantage of the two methods being that the NMPC problem is reduced to a linear model predictive control (LMPC) problem at each time step which thereafter allows the optimization problem to be solved using quadratic programming (QP) techniques. Another advantage of these methods is the reduced computational time in calculating the control effort which makes them suitable for online implementation.**

Both simulation and experimental results show the superiority of the SLNMPC over WNMPC in handling process nonlinearity. The work also shows the favourable performance of the NMPC over LMPC, as expected.

I. Introduction

MODEL Predictive Control (MPC) refers to a class of computer control algorithms in which a dynamic model of process is used to predict and optimize its performance. At each sample time a predictive controller takes measurement of the system output (or state if available), uses the internal model to predict the behaviour of the system over a prediction horizon and then computes a finite horizon control sequence that optimizes some open-loop performance objective while making sure that no constraints are violated. This control sequence is implemented until the next measurement becomes available. Then the optimisation problem is solved again.

Linear model predictive control (LMPC) is well established industry standard for controlling constrained multivariable processes Garcia *et al*. [1989]. In LMPC, the plant behaviour is described by linear dynamic models however most chemical processes are highly nonlinear hence the inadequacy of using linear models to represent them. This limitation of LMPC has brought about the development of NMPC in which a more accurate nonlinear model is used for prediction and optimization. This however does not come without its own problems. While in LMPC a convex optimisation problem is solved at each sample time, in NMPC the problem becomes non-convex in which an optimal solution cannot be guaranteed. In addition to this, NMPC optimisation problem tends to become too large to be solved online. To reduce the computational complexity, it has been proposed that the problem can be reduced to either linear program or quadratic problem.

Inspite of the above, there are systems where nonlinear effects are significant enough to justify the use of nonlinear model predictive control (NMPC). These include at least two broad categories of applications:

1. Regulatory control problems, where the process is highly nonlinear and subject to large frequent disturbances.
2. Servo control problems, where the operating points change frequently and span a sufficiently wide range of nonlinear process dynamics

To address this problem, quite a number of methods have been proposed by researchers for representing the internal model used in NMPC for predictive controller design, some mechanistic and some empirical. (Henson[1998], Kouvaritakis and Cannon [2001], Rossiter [2003], and Allgower *et al.* [2009]). Among these are Hammerstein, Wiener, Volterra, artificial neural network and successively linearized models. Camacho and Bordons [2004] give a review of these methods. In this work, NMPC by successive linearization and by the use of Wiener model are explored for controlling nonlinear process systems. Both techniques have been known to be computational less expensive because of the problem of having to solve a quadratic program at each sampling period.

In SLNMPC, the model used for output prediction is obtained by relinearizing the nonlinear model at every sampling instant and at the current operating point while the original nonlinear model can be used to compute the effect of past input moves. This reduces optimization problem to be solved at every sampling instant to a quadratic program. In WNMPC, since wiener model allows a nonlinear plant to be represented by a linear dynamic part and nonlinear static part, the problem is transformed to a linear one by employing the linear dynamic model for predictive controller design and using the inverse of the nonlinear part to compensate for the nonlinear effect.

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II. SUCCESSIVE LINEARIZATION NONLINEAR MODEL PREDICTIVE CONTROL (SLNMPC)

Model for a nonlinear process

It is a common practice to model a nonlinear system using (Richer and Lee [1995]):

$$\dot{\boldsymbol{x}} = f(x, u, d) + w^x \quad (1)$$

$$\dot{\boldsymbol{d}} = w^d \quad (2)$$

$$y = g(x, u, d) + v \quad (3)$$

Where x is the state vector, y is the output vector, u is the vector of manipulated variables, and w^x, w^d, and v are zero-mean, white noise with specified covariances. The w^x vector represents short-term disturbances having zero mean; d represents sustained disturbances – integrated white noise.

Estimation of state

By linearization and discretization of equations (1) – (3) for a sampling period of t_s time units, an optimal estimate predictions of state and output can be made using the Kalman Filter. Accordingly, let $x_{k-1|k-1}$, $y_{k-1|k-1}$, and $d_{k-1|k-1}$ represents the estimates of the state, the output and the disturbance at time $k-1$ given information up to $k-1$, then

$$\begin{bmatrix} x_{k|k} \\ d_{k|k} \end{bmatrix} = \begin{bmatrix} Ax_{k-1|k-1} + Bu_{k-1|k-1} \\ d_{k-1|k-1} \end{bmatrix} + L_k(y_k - \hat{y}_{k|k-1}) \quad (4)$$

$$\hat{y}_{k|k-1} = Cx_{k|k-1}$$

Where y_k is the measured output, $\hat{y}_{k|k-1}$ is estimated output, and L_k is Kalman Filter gain matrix. It is assumed to be linear time-invariant in this work as this does not introduce serious error in state estimation.

Linearization for prediction/control

With the availability of the estimates of $x_{k|k}$ and $d_{k|k}$ at time t_k from equation (4) and the current inputs being u_{k-1}, the problem is to compute u_k, which will be sent to the plant (to be implemented from t_k to t_{k+1}. Also, with the expectations that w^x will be zero and that d will be constant:

$$d_{k+j|k} = d_{k|k},\ j \geq 1 \quad (5)$$

Equations (1) and (2) can be linearized with respect to x and u to obtain:

$$\dot{x} \approx f(x_{k|k}, u_{k-1}, d_{k|k}) + A_k(x - x_{k|k}) + B_k(u - u_{k-1}) \quad (6)$$

$$y \approx g(x_{k|k}, u_{k-1}, d_{k|k}) + C_k(x - x_{k|k}) + D_k(u - u_{k-1}) \quad (7)$$

where

$$A_k = \left.\frac{\partial f}{\partial x}\right|_{x_{k|k}, u_{k-1}, d_{k|k}},\ B_k = \left.\frac{\partial f}{\partial u}\right|_{x_{k|k}, u_{k-1}, d_{k|k}}$$

$$C_k = \left.\frac{\partial g}{\partial x}\right|_{x_{k|k}, u_{k-1}, d_{k|k}} \quad (8)$$

are matrices of appropriate sizes.

The next step is to discretize the linearized model. However, a complication arises in that the reference point for linearization is the current state, i.e. an unsteady-state. A convenient way of dealing with this is to include the initial condition

$$f_{ok} = f(x_{k|k}, u_{k-1}, d_{k|k})$$

as a column of the B matrix corresponding to a constant input. We can then write equation (6) as

$$\dot{x} = A_k(x - x_{k|k}) + [B_k \quad f_{ok}] \begin{bmatrix} u - u_{k-1} \\ 1 \end{bmatrix} \quad (9a)$$

or $\dot{\boldsymbol{x}}^* = A_k x^* + [B_k \quad f_{ok}] \begin{bmatrix} u^* \\ 1 \end{bmatrix}$ (9b)

where $x^* = x - x_{k|k}$, $u^* = u - u_{k|k}$ are deviation variables

We then discretize equations (9b) and (7) to give:

$$x^*_{k+j+1} = \Phi_k x^*_{k+j} + \Gamma_k u^*_{k+j} + \Gamma_{ok} \quad (10)$$

$$y^*_{k+j} = C_k x^*_{k+j} \quad (11)$$

where for $j \geq 0$:

$$x^*_{k+j} = x_{k+j|k} - x_{k|k},$$

$$y^*_{k+j} = y_{k+j|k} - g(x_{k|k}, u_{k-1}, d_{k|k})$$

$$u^*_{k+j} = u_{k+j|k} - u_{k-1}$$

Where Φ_k and Γ_k are discrete state space matrices obtained from A_k, B_k and the sampling time, t_s.

Linear prediction of future outputs

The above equations are used to develop a linear prediction of future outputs for onward computation of control actions as is found in the usual MPC formulations (Richer and Lee [1995]).

For a 'prediction horizon' of P sampling periods $(P \geq 1)$ we obtain:

$$Y_{k+1|k} = Y^o_{k+1|k} + S^u_k \Delta U_k \quad (12)$$

where

$$Y_{k+1|k} = \begin{bmatrix} y_{k+1|k} \\ y_{k+1|k} \\ \vdots \\ y_{k+P|k} \end{bmatrix},\ \Delta U = \begin{bmatrix} \Delta u_k = u_k - u_{k-1} \\ \Delta u_{k+1} = u_{k+1|k} - u_{k|k} \\ \vdots \\ \Delta u_{k+P+1} \end{bmatrix} \quad (13)$$

$$Y^o_{k+1|k} = \begin{bmatrix} C_k\Gamma_{ok} + g(x_{k|k}, u_{k-1}, d_{k|k}) \\ C_k(\Phi_k + I)\Gamma_{ok} + g(x_{k|k}, u_{k-1}, d_{k|k}) \\ \vdots \\ C_k \sum_{i=1}^{P} \Phi_k^{P-i} \Gamma_{ok} + g(x_{k|k}, u_{k-1}, d_{k|k}) \end{bmatrix} \quad (14)$$

S^u_k is the matrix of step response coefficients used only to represent the effect of future manipulated variables, and it can be obtained from

$$S^u_k = \sum_{i=1}^{P} C_k \Phi_k^{P-i} \Gamma_k \quad (15)$$

S^u_k is of dimension $n_o \times n_i$, where n_o is the number of outputs and n_i is the number of inputs

Calculation of control action

The control signal is calculated by solving the quadratic program problem:

$$\min_{\Delta U} \left\{ \left\| \left(R_{k+1|k} - Y_{k+1|k}\right) \right\|^2_{\Lambda^y} + \|\Delta U\|^2_{\Lambda^u} \right\} \quad (16)$$

Subject to

$$u^{low}_{k+j} \leq u_{k+j} \leq u^{high}_{k+j} \quad j = 0, m-1 \quad (17)$$

$$-\Delta u^{max}_{k+j} \leq \Delta u_{k+j} \leq \Delta u^{max}_{k+j} \quad j = 0, m-1 \quad (18)$$

$$y^{low}_{k+j|k} \leq y_{k+j|k} \leq y^{high}_{k+j|k} \quad j = 1, P \quad (19)$$

Where $R_{k+1|k}$ is the vector of future set-points (corresponding to the predicted output $Y_{k+1|k}$), and Λ^y and Λ^u are tuning parameters.

III. WIENER MODEL BASED NONLINEAR MODEL PREDICTIVE CONTROL (WNMPC)

A. Wiener Model Structure

For the purposes of control system design, any nonlinear system as represented using equations (1) – (3), can be approximated using Wiener model. A Wiener model consists of a dynamic linear element (LDE) in cascade with a static nonlinear part (NL), as shown in fig. 1.

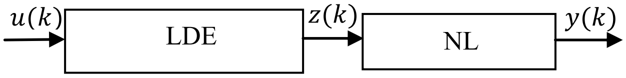

Fig.1: Block representation of Wiener model

For the linear element, a state space description is used as follows:

$$x(k+1) = Ax(k) + Bu(k)$$
$$z(k) = Cx(k) + Du(k) \quad (20)$$

For the static nonlinear element, the polynomial function can be used

$$y(k) = h\big(z(k)\big) = \sum_{j=0}^{N} \alpha_j z^j(k) \quad (21)$$

It will be assumed here that the function h has an inverse and thus can be approximated too using a polynomial function.

B. **Wiener based nonlinear model predictive control Algorithm**

In LMPC, the general optimization problem to be solved at every sampling instant is posed as equations (16) – (19). If the state vector at the present time and the future behaviour of the variables are assumed to be known, they can be written in a matrix form:

$$z(k) = [z^T(k+1) \ldots z^T(k+P)]^T$$
$$\Delta U(k) = [\Delta u^T(k+1) \ldots \Delta u^T(k+M)]^T$$
$$y(k) = [y^T(k+1) \ldots y^T(k+P)]^T$$
$$r(k) = [r^T(k+1) \ldots r^T(k+P)]^T$$

Then, the predicted output for the linear model is

$$\hat{z}(k) = \beta\Delta U + \xi x(k) + d(k) \quad (22)$$

Where

$$\beta = \begin{bmatrix} C^TB & D & 0 & 0 & 0 \\ C^TAB & C^TB & D & 0 & 0 \\ C^TA^2B & C^TAB & C^TB & D & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C^TA^{P-1}B & C^TA^{P-2}B & C^TA^{P-3}B & \ldots & C^TA^{P-M}B \end{bmatrix},$$

$$\xi = \begin{bmatrix} C^TA \\ C^TA^2 \\ C^TA^3 \\ \vdots \\ C^TA^{P-1} \end{bmatrix}, \text{ And } d(k) = [d(k+1|k) \ldots d(k+P|k)]$$

Then, the predicted output for the complete model is

$$\hat{y}(k) = \begin{bmatrix} h\big(\hat{z}(k+1)\big) \\ h\big(\hat{z}(k+2)\big) \\ h\big(\hat{z}(k+3)\big) \\ \vdots \\ h\big(\hat{z}(k+P)\big) \end{bmatrix} = h(\hat{z}(k)) \quad (23)$$

We now define some points related to the MPC structure:

(1) Since the polynomial function h was assumed to be invertible, it is possible to write the desired signal referred to the output of linear model as transformation of the set point $r(k)$ as,

$$r^*(k) = h^{-1}(r(k)) \quad (24)$$

(2) If $y_{k+j|k}^{high}$ and $y_{k+j|k}^{low}$ are the upper and lower bounds for the output variables $y_{k+j|k}$, then these magnitudes can be translated to the linear model as

$$z_{k+j|k}^{high} = h^{-1}\big(y_{k+j|k}^{high}\big)$$
$$z_{k+j|k}^{low} = h^{-1}\big(y_{k+j|k}^{low}\big) \quad (25)$$

(3) Disturbances are typically handled by assuming that a step signal has entered at the output and that it will remain constant for all future time $(d(k) = d(k+j), j = 1, \ldots, P)$. In this case the step disturbance is computed:

$$d(k) = h^{-1}(y^m(k)) - \hat{z}(k)) \quad (26)$$

where $\hat{z}(k)$ is the current predicted output for the linear model and $y^m(k)$ is the current measured output for the process. It is straightforward that introducing this bias in the error, as a perturbation, allows to remove any model errors offset in steady-state.

Finally, the WNMPC can be posed as quadratic optimization problem (QP),

$$\min_{\Delta U}\{\|(\hat{z}(k) - r^*(k))\|_{\Lambda^y}^2 + \|\Delta U\|_{\Lambda^u}^2\} \quad (27)$$

subject to equation(22) and constraints (17), (18) and (28)

$$z_{k+j|k}^{low} \le z_{k+j|k} \le z_{k+j|k}^{high} \quad j = 1, P \quad (28)$$

IV. EXAMPLES

A. STIRRED-TANK REACTOR (CSTR)

Process Description and modeling

The process under consideration here is the constant volume continuous stirred-tank reactor in which an irreversible, exothermic reaction $A \rightarrow B$ occurs that is cooled by single coolant stream (see fig.2). Interested reader should consult Cervantes *et al*. (2003), and Prakash and Srinivasan (2009) for more information about this process.

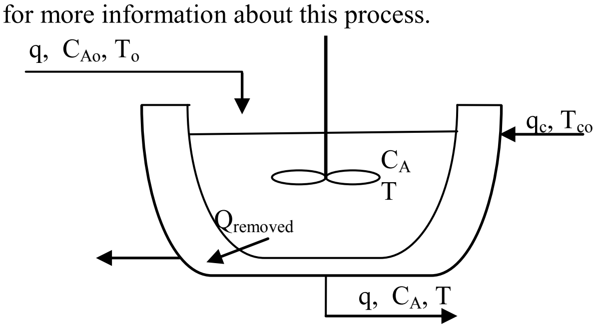

Fig.2: Continuous stirred tank reactor

The process is modeled by the following equations:

$$\frac{dC_A(t)}{dt} = \frac{q(t)}{V}(C_{AO}(t) - C_A(t)) - k_O C_A(t) exp\left(\frac{-E}{RT(t)}\right) \quad (29)$$

$$\frac{dT(t)}{dt} = \frac{q(t)}{V}\big(T_O(t) - T(t)\big) - \frac{(-\Delta H)k_O C_A(t)}{\rho c_p} exp\left(\frac{-E}{RT(t)}\right) + \frac{\rho_c c_{pc}}{\rho c_p V} q_c(t)\left[1 - exp\left(\frac{-hA}{q_c(t)\rho c_p}\right)\right](T_{co}(t) - T(t)) \quad (30)$$

The measured concentration has a time delay $t_d = 0.5\, min$ modelled by

$$C_{A_{meas}}(t) = C_A(t - t_d) \quad (31)$$

The objective of the control system is to control the measured concentration of A, C_A by manipulating the coolant flow rate q_c. Process nominal operating data is displayed in Table 1.

Table 1: CSTR process data

Product concentration	C_A	0.1 mol/l
Reactor temperature	T	438.54 K
Coolant flow rate	q_c	103.41 l/min
Process flow rate	q	100 l/min
Feed concentration	C_{Ao}	1 mol/l
Feed temperature	T_o	350 K
Inlet coolant temperature	T_{co}	350 K
CSTR volume	V	100 l
Heat transfer term	HA	7 x 10^5cal/min K
Reaction rate constant	K_o	7.2 x 10^{10} l/min
Activation energy term	E/R	1 x 10^4 K
Heat of reaction	ΔH	- 2 x 10^5cal/mol
Liquid densities	ρ , ρ_c	1 x 10^5 g/l
Specific heats	c_p ,c_{pc}	1 cal/g K

Identification of the process

The results of the identification experiment performed gives the linear dynamic element (LDE) of the wiener model as:

$$\begin{bmatrix} \frac{dC_A}{dt} \\ \frac{dT}{dt} \end{bmatrix} = \begin{bmatrix} -9.9839 & -0.0467 \\ 1796.8 & 7.3112 \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ -0.8772 \end{bmatrix} q_c \quad (32)$$

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} \quad (33)$$

The nonlinear static element and its inverse are plotted in fig.3

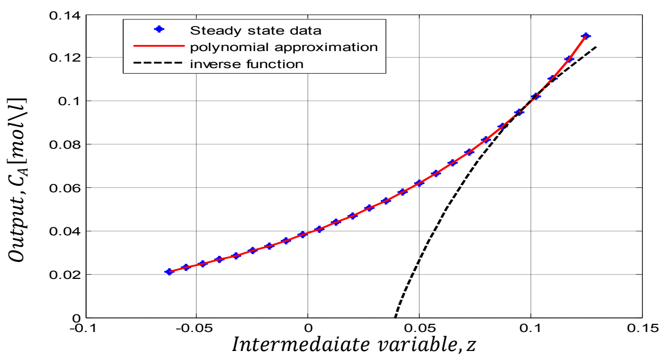

Fig.3: Polynomial approximation for the CSTR

Nonlinear predictive control system design

The two techniques (SLNMPC and WNMPC) for designing nonlinear model predictive control system discussed in sections 2 and 3 respectively are employed to design predictive controllers for the CSTR. For the purpose of making comparison, linear model predictive controller is also designed for the same system. The designed controllers are implemented on a simulation model of the CSTR developed in MATLAB/SIMULINK. The MPC tuning parameters are shown in table 2.

The simulation results as depicted in fig.4 show that SLNMPC has the best performance with fastest settling time both for the positive and negative set-point changes. WNMPC is the next in performance by exhibiting faster response than LMPC for a negative set-point change. However, LMPC response for a positive set-point change is better than that of a WNMPC. This is due to the fact that the system is being driven to higher gain region.

The performance indices computed for each of the techniques as displayed in table 3 also indicates the superiority of SLNMPC over WNMPC and LMPC, having the least error.

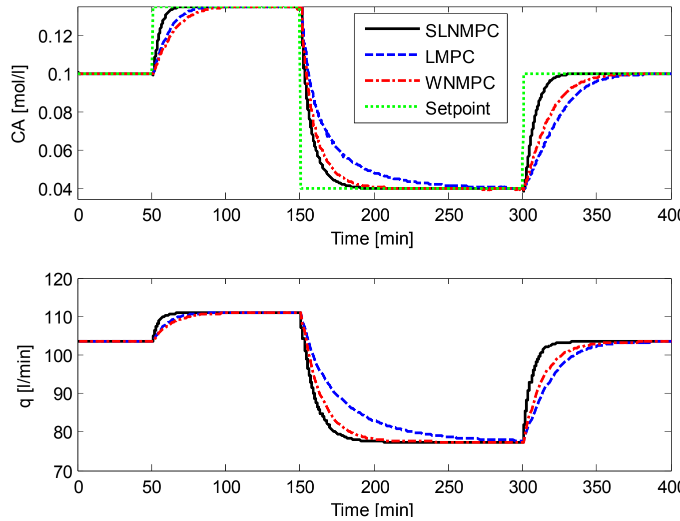

Fig.4: Process output and control effort for a set-point change

Table 2: MPC tuning parameters [CSTR]

$\Lambda^y - \Lambda^u$	5I – 0
$P - m$	200 – 10
$y^{low}_{k+j\|k} - y^{high}_{k+j\|k}$	0 – 0.2
$u^{low}_{k+j} - u^{high}_{k+j}$	60 – 120

Table 3: Performance indices [CSTR]

	SLNMPC	WNMPC	LMPC
IAE	1.3	2.4	3.4
ITAE	263.4	500.7	749.0
ITSE	9.8	17.3	25.4
ISE	0.050	0.084	0.12

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B. THREE-TANK-SYSTEM

Process description and modeling

The principal structure of the three tank plant considered here for study is shown in fig. 5 below (See Amira (2002) for detailed description). The controlled variables are the levels, h_1 and h_2 inside tanks 1 and 2. The level h_3, in tank 3, though not being controlled is to be maintained not to overflow or run dry. The maximum level of each tank is 62 cm(+/- 1cm). Inflow, q_1 of tank 1 and inflow q_2 of tank 2 are considered as the manipulated variables.

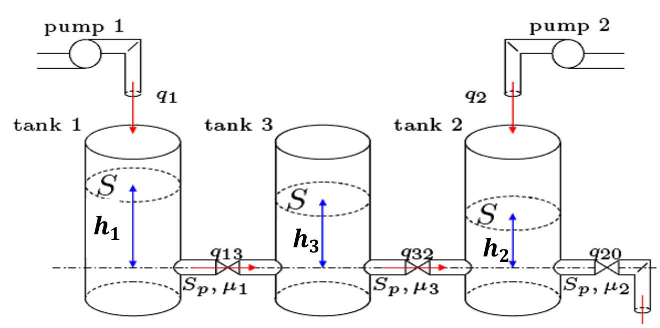

Fig.5: Laboratory three- tank-system

The transient balance equations for all the tanks are:

$$S\frac{dh_1}{dt} = q_1 - q_{13} \quad (34)$$

$$S\frac{dh_3}{dt} = q_{13} - q_{32} \quad (35)$$

$$S\frac{dh_2}{dt} = q_2 + q_{32} - q_{20} \quad (36)$$

where $q_{ij} = \mu_i . S_p . sign\left(h_i - h_j\right) . \sqrt{2g\left|h_i - h_j\right|}$ (37)

$$q_{20} = \mu_2 . S_p . \sqrt{2gh_2} \quad (38)$$

The state space model of the three tank system around the operating point shown in table 8 is:

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (39)$$

where

$$A = \begin{bmatrix} a_{11} & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, B = \begin{bmatrix} 1/_S & 0 \\ 0 & 1/_S \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$a_{11} = \frac{(-1/_S).\mu_1.S_p.\sqrt{2g}.sign\,(h_{1o} - h_{3o})}{2\sqrt{|h_{1o} - h_{3o}|}},$$

$$a_{13} = \frac{(1/_S).\mu_1.S_p.\sqrt{2g}.sign\,(h_{1o} - h_{3o})}{2\sqrt{|h_{1o} - h_{3o}|}}$$

$$a_{22} = \frac{(-1/_S).\mu_3.S_p.\sqrt{2g}.sign(h_{3o} - h_{2o})}{2\sqrt{|h_{3o} - h_{2o}|}} - \frac{(1/_S).\mu_2.S_p.\sqrt{2g}.sign(h_{2o})}{2\sqrt{|h_{2o}|}}$$

$$a_{23} = \frac{(1/_S).\mu_3.S_p.\sqrt{2g}.sign(h_{3o} - h_{2o})}{2\sqrt{|h_{3o} - h_{2o}|}}$$

$$a_{31} = \frac{(1/_S).\mu_1.S_p.\sqrt{2g}.sign\,(h_{1o} - h_{3o})}{2\sqrt{|h_{1o} - h_{3o}|}},$$

$$a_{32} = \frac{(1/_S).\mu_3.S_p.\sqrt{2g}.sign\,(h_{3o} - h_{2o})}{2\sqrt{|h_{3o} - h_{2o}|}}$$

$$a_{33} = \frac{(-1/_S).\mu_1.S_p.\sqrt{2g}.sign(h_{1o} - h_{3o})}{2\sqrt{|h_{1o} - h_{3o}|}} - \frac{(1/_S).\mu_3.S_p.\sqrt{2g}.sign(h_{3o} - h_{2o})}{2\sqrt{|h_{3o} - h_{2o}|}}$$

Table 4: Steady state operation table

h_{1o}, h_{3o}, h_{2o} in cm	41.5, 26.5, 11.5
Outflow coefficients, μ_1, μ_2, μ_3	(0 – 1)
Area of tank (S_1 to S_3) in cm^2	149
Area of connecting pipes in cm^2, S_P	0.5

Identification of the process

The results of the identification experiment performed gives the linear dynamic element (LDE) of the wiener model as:

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} -0.0112 & 0 & 0.0112 \\ 0 & -0.0404 & 0.0112 \\ 0.0112 & 0.0112 & -0.0224 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} 0.0067 & 0 \\ 0 & 0.0067 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \quad (40)$$

The nonlinear element and its inverse are plotted in fig. 6.

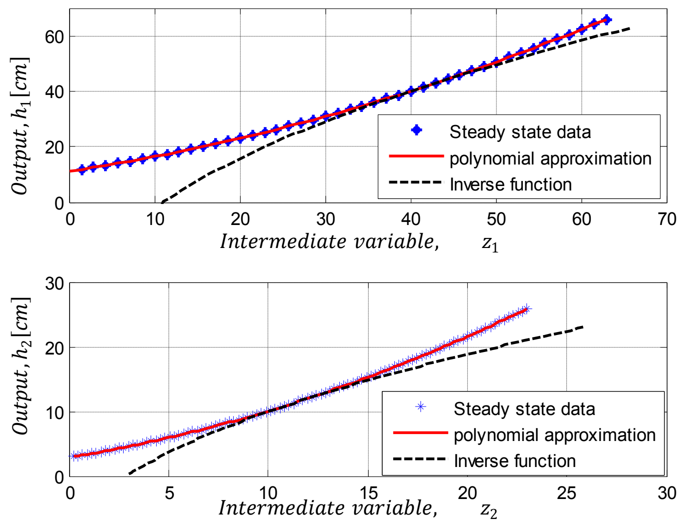

Fig.6: polynomial approximation for h_1 and h_2

Simulation and Experimental Results

Again, predictive controllers were designed for the three-tank plant by the three techniques, SLNMPC, WNMPC and LMPC, and implemented both on a simulated model of the three-tank and on an experimental set-up of the tank housed in our Process System Engineering Laboratory. The tuning parameters are displayed in Table 5. Figs.7-10 show the response of the process when the second output set-points have changed, while the first output is set to the nominal value 41.5cm. SLNMPC and WNMPC show similar and better performance than LMPC with shorter settling time. In addition to this, H_1 reaction to set-point change in H_2 is least for SLNMPC followed by WMPC. Performances indices are summarized in Tables 6 and 7.

Table 5: MPC tuning parameters [three-tank-system]

$\Lambda^y - \Lambda^u$	10I – 0.01
$P - m$	8 – 1
$y^{low}_{k+j\|k} - y^{high}_{k+j\|k}$	0 – 63
$u^{low}_{k+j} - u^{high}_{k+j}$	0 – 100

Table 6: Performance indices, three-tank, for the simulation results

	SLNMPC	WNMPC	LMPC
IAE	188.6	218.1	383.1
ITAE	5.8e4	7.1e4	1.7e5
ITSE	2.4e5	2.8e5	5.4e5
ISE	772.4	898.7	1309

Table 7: Performance indices, three-tank, for the experimental results

	SLNMPC	WNMPC	LMPC
IAE	213.3	213.3	217.0
ITAE	6.9e4	6.9e4	7.0e4
ITSE	2.5e5	2.5e5	2.6e5
ISE	760.4	760.4	797.5

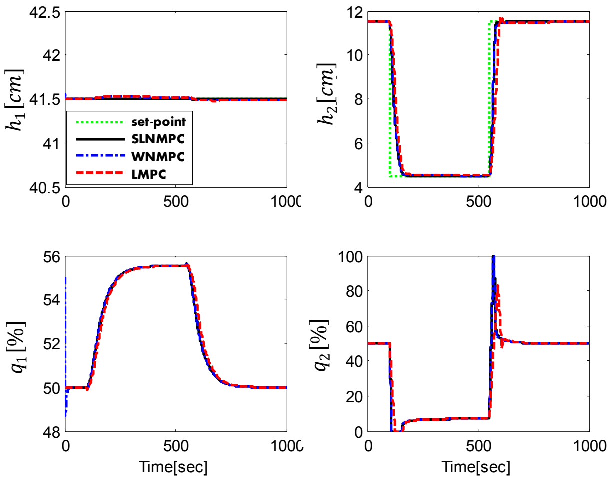

Fig.7: Simulation result for set-point change in h_2

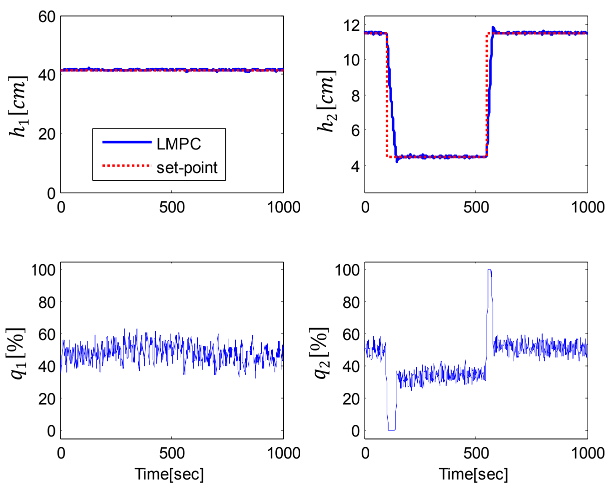

Fig.8: Experimental result for set-point change in h_2

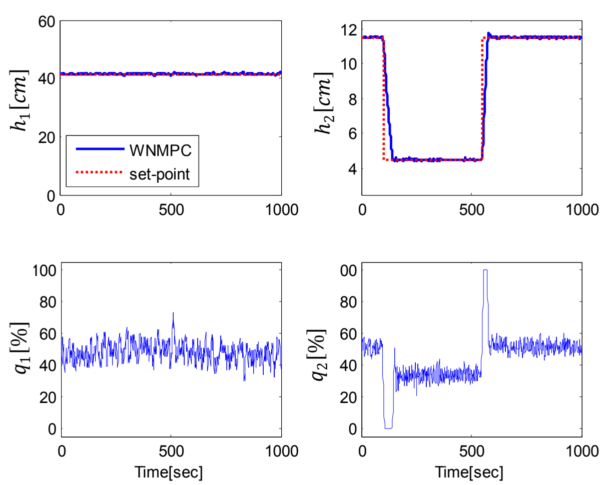

Fig.9: Experimental result for set-point change in h_2

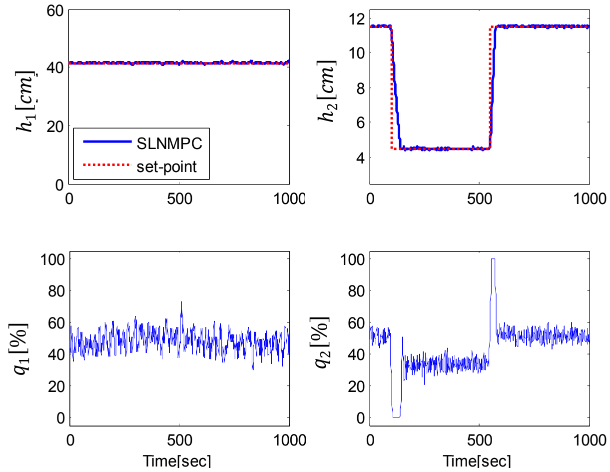

Fig.10: Experimental result for set-point change in h_2

V. CONCLUSION

In this paper two techniques of NMPC have been applied to control nonlinear processes and compared with that of LMPC. These two methods enjoy the advantage of having to solve a quadratic programming problem at each sampling instant compared to the original formulation of NMPC which results to non-convex nonlinear programming. Both the simulation results and results of real time implementation carried out on our experimental three-tank system show that there could be improvement in the performance of a process plant by appropriately applying NMPC. In all of the cases studied, NMPC outperforms the LMPC without excessive control actions.

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