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Input–output finite–time stabilization of LTV systems via dynamic output feedback

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Abstract— **The problem of input–output finite-time stabilization of linear time-varying systems via dynamic output feedback is tackled in this paper. Sufficient conditions are provided in terms of Differential Linear Matrix Inequalities feasibility problems, which can be solved numerically in an efficient way by using off-the-shelf optimization tools, as illustrated by the proposed examples.**

Keywords: Input-output finite time stabilization; Dynamic Output feedback; DLMIs.

I. INTRODUCTION

The concept of input-output finite time stability (IO–FTS) has been introduced in [2]; roughly speaking, a system is defined to be IO–FTS if, given a class of norm bounded input signals defined over a specified time interval T, the outputs of the system do not exceed an assigned threshold during T.

In order to correctly frame the definition of IO-FTS in the current literature, we recall that a system is said to be IO $\mathcal{L}_p$-stable if for any input of class $\mathcal{L}_p$, the system exhibits a corresponding output which belongs to the same class [10, Ch. 5]. The main differences between *classic* IO stability and IO–FTS are that the latter involves signals defined over a finite time interval, does not necessarily require the inputs and outputs to belong to the same class, and that *quantitative* bounds on both inputs and outputs must be specified. Therefore, IO stability and IO–FTS are independent concepts. Indeed, while IO stability deals with the behavior of a system within a sufficiently long (in principle infinite) time interval, IO–FTS is a more practical concept, useful to study the behavior of the system within a finite (possibly short) interval. Furthermore, it should be mentioned that sufficient conditions for IO-FTS in the context of impulsive dynamical systems has been recently provided in [3].

It is important to remark that the definition of IO-FTS given in [2] is fully consistent with the definition of (state) FTS given in [6], where the state of a zero-input system, rather than the intput and the output, is involved.

For the sake of completeness, it should be mentioned that, with respect to the one given in [2], a different concept of IO–FTS for nonlinear systems has been given in [9] extending the definition of finite-time stability given in [4] to nonautonomous systems. In the latter works, the authors focus on the Lyapunov stability analysis of nonlinear systems whose trajectories converge to an equilibrium point in finite time and on the characterization of the associated *settling-time*. According to this definition of FTS, in [9] a different concept of *finite-time input-output stability* is introduced. In particular, the authors consider the case of nonautonomous system with a norm bounded input signal over the interval $[0 + \infty]$ and an initial condition $x(0) = x_0$. The finite time input-output stability is related to the property of a system to have a norm bounded output whose bound, after a finite time interval T, does not depend anymore on the initial state. Hence, we can conclude that the concept of IO-FTS introduced in [2] and the one in [9] are different.

In [2] sufficient conditions have been provided to check if a given linear time-varying (LTV) system is IO-FTS. Furthermore these conditions have been exploited to solve the problem of input-output finite-time stabilization by means of state feedback. In this paper we extend the work done [2] by providing sufficient conditions for the input-output finite-time stabilization of LTV systems via *dynamic output feedback*.

The results provided in this paper are stated in terms of Differential Linear Matrix Inequalities (DLMIs, [11]) feasibility problem, which can easily casted in the Linear Matrix Inequalities (LMIs, [5]) framework.

Our work is organized as follows. The next section introduces the definition of IO–FTS and the problem of input-output finite-time stabilization via dynamic output feedback. The results provided in [2] are recalled in Section III. The main contribution of this paper are given in Section IV, while the effectiveness of the proposed approach is illustrated by means of numerical examples in Section V. Eventually some conclusive remarks are given.

II. PROBLEM STATEMENT

In this section we introduce the concept of IO-FTS and we state the problem of IO finite-time stabilization via dynamic output feedback.

The following notation is adopted in this paper.
Given a vector $v \in \mathbb{R}^m$ we will denote with $|v|_q$ its q-norm. Given the set $\Omega = [t_0, t_0 + T]$, with $t_0 \in \mathbb{R}$ and $T > 0$, the symbol $\mathcal{L}_{p,q}(\Omega)$ denotes the space of vector-valued signals for which

$$v \in \mathcal{L}_{p,q}(\Omega) \implies \int_\Omega |v(t)|_q^p dt < +\infty ,$$

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moreover $\mathcal{L}_p(\Omega) = \mathcal{L}_{p,2}(\Omega)$.

Given a symmetric positive definite matrix-valued function $R(\cdot)$, bounded on Ω, and a signal $s(\cdot) \in \mathcal{L}_p(\Omega)$, the weighted signal norm

$$\left(\int_\Omega \left[s(\tau)^T R(\tau) s(\tau) \right]^{\frac{p}{2}} d\tau \right)^{\frac{1}{p}} ,$$

will be denoted by $\|s(\cdot)\|_{p,R}$. If $p = \infty$ it is

$$\|s(\cdot)\|_{\infty,R} = \operatorname*{ess\,sup}_{t \in \Omega} \left[s^T(t) R(t) s(t) \right]^{\frac{1}{2}} .$$

Let us consider a linear time-varying (LTV) system in the form

$$\dot{x}(t) = A(t)x(t) + G(t)w(t) , \quad x(t_0) = 0 \tag{1a}$$

$$y(t) = C(t)x(t) \tag{1b}$$

where $A(\cdot) : \mathbb{R}_0^+ \mapsto \mathbb{R}^{n\times n}$, $G(\cdot) : \mathbb{R}_0^+ \mapsto \mathbb{R}^{n\times r}$, and $C(\cdot) : \mathbb{R}_0^+ \mapsto \mathbb{R}^{m\times n}$, are piecewise continuous matrix-valued functions.

Definition 1 (IO-FTS of Linear Systems): Given a positive scalar T, a class of input signals $\mathcal{W}$ defined over $\Omega = [t_0 , t_0 + T]$, a positive definite matrix-valued function $\Gamma(\cdot)$ defined in Ω, system (1) is said to be IO-FTS with respect to $\big(\mathcal{W}, \Gamma(\cdot), \Omega\big)$ if

$$w(\cdot) \in \mathcal{W} \ \Rightarrow \ y^T(t)\Gamma(t)y(t) < 1 , \quad t \in]t_0 , t_0 + T] .$$

▲

In particular, in this paper we consider two different classes of input signals, which will require different analysis and synthesis techniques, as it will be shown in Section IV. Hence, let consider the following two cases:

i) the set $\mathcal{W}$ coincides with the set of norm bounded square integrable signals over $\Omega = [t_0 , t_0 + T]$, defined as
$$\mathcal{W}_2\big(\Omega , R(\cdot)\big) := \left\{ w(\cdot) \in \mathcal{L}_2(\Omega) \, : \, \|w\|_{2,R} \le 1 \right\} .$$
ii) The set $\mathcal{W}$ coincides with the set of the uniformly bounded signals over $\Omega = [t_0 , t_0 + T]$, defined as
$$\mathcal{W}_\infty\big(\Omega , R(\cdot)\big) := \big\{ w(\cdot) \in \mathcal{L}_\infty(\Omega) \, : \, w^T(t)R(t)w(t) \le 1 \quad t \in \Omega \big\}.$$

where $R(\cdot)$ denotes a positive definite symmetric matrix-valued function.

Although the definitions of $\mathcal{W}_2\big(\Omega , R(\cdot)\big)$ and $\mathcal{W}_\infty\big(\Omega , R(\cdot)\big)$ depend on the choice of Ω and $R(\cdot)$, in the rest of the paper we will drop this dependency so as to simplify the notation.

In Section IV we provide sufficient conditions to prove IO-FTS for both the classes of input signals $\mathcal{W}_2$ and $\mathcal{W}_\infty$. These conditions will be then exploited to provide a solution to the following design problem, namely the problem of *input-output finite-time stabilization via dynamic output feedback*.

Problem 1: Consider the LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + G(t)w(t) , \quad x(t_0) = 0 \tag{2a}$$

$$y(t) = C(t)x(t) \tag{2b}$$

where $u(\cdot)$ is the control input and $w(\cdot)$ is the exogenous input. Given a class of disturbances $\mathcal{W}$ defined over Ω, and a positive definite matrix-valued function $\Gamma(\cdot)$ defined over Ω, find a dynamic output feedback controller in the form

$$\dot{x}_c(t) = A_K(t)x_c(t) + B_K(t)y(t) , \tag{3a}$$

$$u(t) = C_K(t)x_c(t) + D_K(t)y(t) \tag{3b}$$

where $x_c(t)$ has the same dimension of $x(t)$, such that the closed loop system obtained by the connection of (2) and (3) is IO-FTS with respect to $\big(\mathcal{W}, \Gamma(\cdot), \Omega\big)$. In particular, the closed loop system is in the form

$$\begin{pmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{pmatrix} = \begin{pmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{pmatrix} \begin{pmatrix} x(t) \\ x_c(t) \end{pmatrix} + \begin{pmatrix} G \\ 0 \end{pmatrix} w(t)$$
$$=: A_{\mathrm{CL}}(t)\, x_{\mathrm{CL}}(t) + G_{\mathrm{CL}}(t)\, w(t) \tag{4a}$$

$$y(t) = \begin{pmatrix} C & 0 \end{pmatrix} x_{\mathrm{CL}}(t) =: C_{\mathrm{CL}}(t)\, x_{\mathrm{CL}}(t) \tag{4b}$$

where all the considered matrices depends on time, even when not explicitly written. ▲

III. PRELIMINARY RESULTS

The sufficient conditions for the IO-FTS of system (1) originally presented in [2] are recalled in this section. These conditions are then exploited in Section IV to solve Problem 1. As for the proofs of the following two theorems, the reader can refer to [2].

Theorem 1 (IO-FTS of LTV systems for $\mathcal{W}_2$ inputs): If there exists a continuously differentiable positive definite solution $P(\cdot)$ such that

$$\dot{P}(t) + A(t)^T P(t) + P(t)A(t) + P(t)G(t)R^{-1}(t)G(t)^T P(t) < 0 \tag{5a}$$

$$P(t) \ge C(t)^T \Gamma(t) C(t) \tag{5b}$$

are satisfied in the time interval Ω, then the LTV system (1) is IO-FTS with respect to $\big(\mathcal{W}_2 , \Gamma(\cdot), \Omega\big)$. ■

Theorem 2 (IO-FTS of LTV systems for $\mathcal{W}_\infty$ inputs): Let $\widetilde{\Gamma}(t) = t\,\Gamma(t)$. If there exists a continuously differentiable positive definite solution $P(\cdot)$ such that (5a) and

$$P(t) \ge C(t)^T \widetilde{\Gamma}(t) C(t) , \quad \forall\ t \in \Omega \tag{6}$$

are satisfied in the time interval Ω, then LTV system (1) is IO-FTS with respect to $\big(\mathcal{W}_\infty , \Gamma(\cdot), \Omega\big)$. ■

It is worth to notice that the sufficient conditions stated in theorems 1 and 2 can be readily casted in the DLMI framework by applying Schur complements to (5a), as it is shown in [2].

IV. MAIN RESULTS

In this section, we state the principal contribution of this paper, i.e., two sufficient conditions for the IO finite-time stabilization via dynamic output feedback of LTV systems, when the two input classes $\mathcal{W}_2$ and $\mathcal{W}_\infty$ are considered.

Theorem 3: Given the exogenous input $w(t) \in \mathcal{W}_2$, Problem 1 is solvable if there exist two continuously differentiable symmetric matrix-valued functions $Q(\cdot)$, $S(\cdot)$, a nonsingular matrix-valued function $N(\cdot)$ and matrix-valued functions $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that the following DLMIs are satisfied (the time argument is omitted for brevity)

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} & 0 \\ \Theta_{12}^T & \Theta_{22} & SG \\ 0 & G^T S & -R \end{pmatrix} < 0\,, \quad t \in \Omega \tag{7a}$$

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & 0 \\ \Psi_{12}^T & Q & QC^T \\ 0 & CQ & \Gamma^{-1} \end{pmatrix} \geq 0\,, \quad t \in \Omega \tag{7b}$$

where

$$\Theta_{11} = -\dot{Q} + AQ + QA^T + B\hat{C}_K + \hat{C}_K^T B^T + GR^{-1}G^T$$
$$\Theta_{12} = A + \hat{A}_K^T + BD_K C + GR^{-1}G^T S$$
$$\Theta_{22} = \dot{S} + SA + A^T S + \hat{B}_K C + C^T \hat{B}_K^T$$
$$\Psi_{11} = S - C^T \Gamma C$$
$$\Psi_{12} = I - C^T \Gamma C Q$$

Proof. From Theorem 1 it readily follows that system (4) is IO-FTS wrt $(\mathcal{W}, \Gamma(\cdot), \Omega)$ if there exists a continuously differentiable symmetric matrix-valued function $P(\cdot)$ such that

$$\dot{P}(t) + A_{\mathrm{CL}}(t)^T P(t) + P(t) A_{\mathrm{CL}}(t) + P(t) G_{\mathrm{CL}}(t) R^{-1} G_{\mathrm{CL}}(t)^T P(t) < 0\,,\ t \in \Omega \tag{8a}$$

$$P(t) \geq C(t)^T \Gamma(t) C(t)\,, \quad t \in \Omega \tag{8b}$$

Now let us define, according to [7],

$$P(t) = \begin{pmatrix} S(t) & M(t) \\ M^T(t) & U(t) \end{pmatrix}, \ P^{-1}(t) = \begin{pmatrix} Q(t) & N(t) \\ N^T(t) & \star \end{pmatrix},$$
$$\Pi_1(t) = \begin{pmatrix} Q(t) & I \\ N^T(t) & 0 \end{pmatrix} \quad \Pi_2(t) = \begin{pmatrix} I & S(t) \\ 0 & M^T(t) \end{pmatrix}.$$

Note that, by definition, it is

$$S(t)Q(t) + M(t)N^T(t) = I \tag{9a}$$

$$Q(t)\dot{S}(t)Q(t) + N(t)\dot{M}^T(t)Q(t) + Q(t)\dot{M}(t)N^T(t) + N(t)\dot{U}(t)N^T(t) = -\dot{Q}(t) \tag{9b}$$

$$P(t)\Pi_1(t) = \Pi_2(t) \tag{9c}$$

where equality (9b) can be easily derived by noticing that

$$\dot{P}^{-1}(t) = -P^{-1}(t)\dot{P}(t)P^{-1}(t)\,.$$

We now prove that, with the given choice of $P(t)$, conditions (8) are equivalent to (7). Indeed, by pre- and post-multiplying (8a)–(8b) by $\Pi_1^T(t)$ and $\Pi_1(t)$ respectively, taking into account (9) and [1, Lemma 5.1], the proof follows once we let[1]

$$\begin{pmatrix} Q & I \\ I & S \end{pmatrix} > 0 \tag{10a}$$

$$\hat{B}_K = MB_K + SBD_K \tag{10b}$$

$$\hat{C}_K = C_K N^T + D_K C Q \tag{10c}$$

$$\hat{A}_K = \dot{S}Q + \dot{M}N^T + MA_K N^T + SBC_K N^T + MB_K CQ + S\big(A + BD_K C\big)Q\,. \tag{10d}$$

[1]Time argument is omitted for brevity.

Note that (10a) does not need to be explicitly imposed since it is implied by (7b). Furthermore, in order to invert (10) to get the feedback system matrices, we need to preliminary choose the value of $N(t)$. The only constraint for $N(t)$ to be a non singular matrix. ■

Remark 1 (Controller design): Assuming that the hypotheses of Theorem 3 are satisfied; in order to design the controller, the following steps have to be followed:

i) Find $Q(\cdot)$, $S(\cdot)$, $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that (7) are satisfied.
ii) Let $M(t) = (I - S(t)Q(t))N^{-T}(t)$.
iii) Obtain $A_K(\cdot)$, $B_K(\cdot)$ and $C_K(\cdot)$ by inverting (10).

▲

Note that Theorem 3 cannot be used for optimal designs. Indeed, as far as IO finite–time stability and stabilization are concerned, the $\Gamma(\cdot)$ matrix is an input of the problem.

Since the sufficient conditions for the IO-FTS of LTV systems in Theorem 2 readily follow from that of Theorem 1, *mutatis mutandis*, by letting $\widetilde{\Gamma}(t) = t\,\Gamma(t)$, the following result can be easily derived.

Theorem 4: Given the exogenous input $w(t) \in \mathcal{W}_\infty$, Problem 1 is solvable if there exist two continuously differentiable symmetric matrix-valued functions $Q(\cdot)$, $S(\cdot)$, a nonsingular matrix-valued function $N(\cdot)$ and matrix-valued functions $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that the following DLMIs are satisfied

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} & 0 \\ \Theta_{12}^T & \Theta_{22} & SG \\ 0 & G^T S & -R \end{pmatrix} < 0\,, \quad t \in \Omega \tag{11a}$$

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & 0 \\ \Psi_{12}^T & Q & QC^T \\ 0 & CQ & \widetilde{\Gamma}^{-1} \end{pmatrix} \geq 0\,, \quad t \in \Omega \tag{11b}$$

where the variable Θ_{ij} have the same expression as in Theorem 3 and

$$\Psi_{11} = S - C^T \widetilde{\Gamma} C\,,$$
$$\Psi_{12} = I - C^T \widetilde{\Gamma} C Q\,.$$

■

V. NUMERICAL EXAMPLES

Two numerical examples are presented in this section in order to show the effectiveness of the proposed approach. IO finite-time stabilization via output feedback of LTV system with respect to the $\mathcal{W}_\infty$ and $\mathcal{W}_2$ input classes is tackled.

In both cases we consider the second order unstable LTV system defined by

$$A = \begin{pmatrix} 0.5 + t & 0.1 \\ 0.4 & -0.3 + t \end{pmatrix}, B = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$
$$G = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \end{pmatrix}. \tag{12}$$

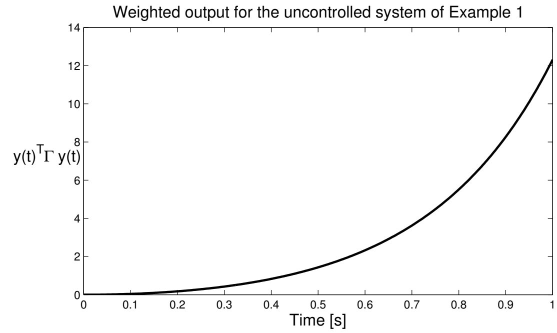

Fig. 1. Weighted output for the uncontrolled LTV system (12) with parameters in (13).

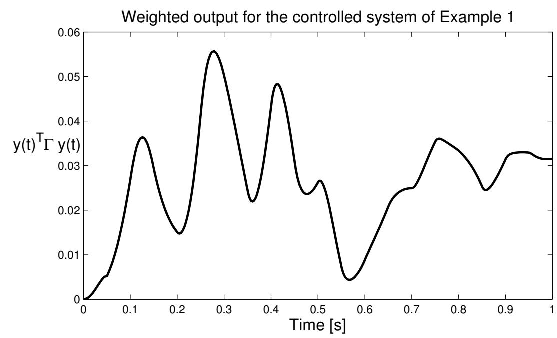

Fig. 2. Weighted output for the controlled LTV system (12) with parameters in (13).

Example 1: Let us first present the case of $\mathcal{W}_\infty$ input class, with the following list of parameters

$$R = 1\,,\ \Gamma = 1\,,\ \Omega = \big[0\,,1\big]\,. \tag{13}$$

Given the considered unstable LTV system, by means of simulation among the all the possible real impulses in the considered time interval, it turned out that the worst inputs in the class $\mathcal{W}_\infty$ are given by $w(t) = \pm 1$ for all $t \in \Omega$. Furthermore, given the parameters specified in (13), it turns out that system (12) is IO finite-time unstable. The weighted output for the uncontrolled system is shown in Fig. 1 when $w(t) = 1$ for all $t \in \Omega$ is considered.

In order to recast the DLMI condition provided in Theorem 4 in terms of LMIs, the matrix-valued functions $Q(\cdot)$ and $S(\cdot)$ have been assumed piecewise linear. In particular, the time interval Ω is divided in $n = T/T_s$ subintervals, hence the time derivatives of $Q(t)$ and $S(t)$ have been considered constant in each subinterval. It is straightforward to recognize that such a piecewise linear functions can approximate a generic continuous matrix-valued functions with adequate accuracy, provided that the length of T_s is sufficiently small.

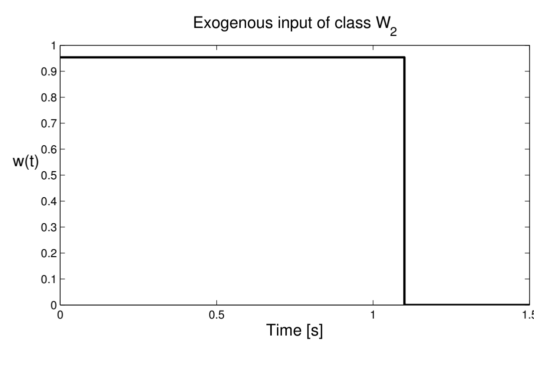

Fig. 3. Worst case exogenous input of class $\mathcal{W}_2$ for system (12) with parameters in (14).

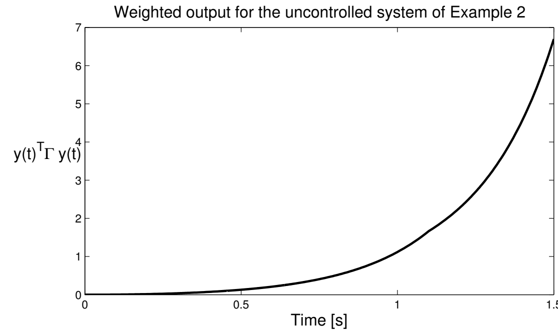

Fig. 4. Weighted output for the uncontrolled LTV system (12) with parameters in (14).

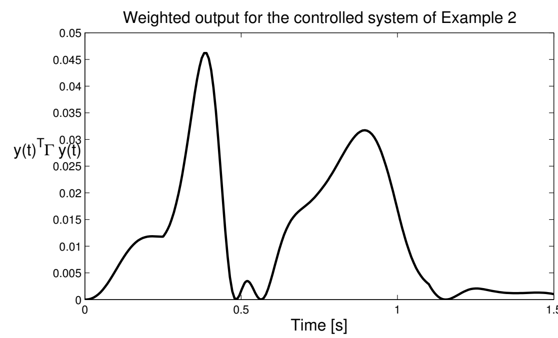

Fig. 5. Weighted output for the controlled LTV system (12) with parameters in (14).

Exploiting standard optimization tools such as the Matlab LMI Toolbox® ([8]) or TOMLAB® ([12]), it is possible to find the matrix functions $A_k(\cdot)\,, B_k(\cdot)\,, C_k(\cdot)\,, D_k(\cdot)$ that verify the conditions of Theorem 4 and make the closed loop system (4) IO–FTS wrt $\big(\mathcal{W}_\infty\,, 1\,, [0\,,1]\big)$. Fig. 2 shows the weighted output of the controlled system.

Example 2: Let us consider again system (12) and the following IO-FTS parameters:

$$R = 1\,,\ \Gamma = 0.1\,,\ \Omega = \big[0\,,1.5\big]\,. \tag{14}$$

Furthermore, the input signal $w(\cdot)$ reported in Fig. 3, is the one that in simulation turned out to be the worst $\mathcal{W}_2$ input signal[2].

Fig. 4 shows that the uncontrolled system is not IO-FTS. Exploiting Theorem 3 and making the same assumptions on the matrix-valued functions $Q(\cdot)$ and $S(\cdot)$ as in Example 1, it is possible to IO finite-time stabilize system (12)

[2]As in Example 1, exploiting system linearity, there are two *worst inputs signals*, since the signal $\tilde{w}(t) = -w(t)$ gives the same weighted output as $w(t)$.

wrt $(\mathcal{W}_2, 0.1, [0, 1.5])$ via dynamic output feedback. The correspondent weighted output is shown in Fig. 5.

CONCLUSIONS

In this paper sufficient conditions for IO finite-time stabilization of LTV systems via dynamic output feedback have been provided. The proposed results are stated in terms of DLMI feasibility problems; such conditions can be efficiently solved by using off–the–shelf optimization tools, as it has been illustrated by means of numerical examples.

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