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About Friction Modeling For Observer-Based Leak Estimation In Pipelines

J-F. Dulhoste, G. Besançon, L. Torres, O. Begovich and A. Navarro

***Abstract*— A study of the friction model for the leak detection with a nonlinear observer is presented. Classically the friction model in leak detection algorithms only relies on a constant parameter. In the present work, the use of a more elaborate friction model is considered, with a coefficient explicitly depending on the flow, either algebraically, or even differentially. Those friction formulations are implemented within a nonlinear-observer-based algorithm for detection and location of leaks, which is analyzed, and tested with real measurements on some experimental pipeline prototype. Some conclusions and recommendations about this extended friction modeling are finally given.**

***Keywords:* Leak Detection and Isolation, Nonlinear model, Friction models, Nonlinear observer, High gain.**

I. Introduction

Leak Detection and Isolation (LDI) in real pipelines remains an important problem today. Various algorithms providing solutions can be found as in [1], [2], [3], [4], but most of them are based on the assumption of a *constant* friction coefficient in the dynamics, while this friction coefficient in a pipeline depends of the flow. This means that corresponding LDI algorithms are only valid in a particular operation condition, and small changes of the flow can make inoperative the algorithm.

In order to tackle this problem, a Gradient algorithm is implemented to estimate the friction coefficient in [2] and [6], while it is estimated with a Kalman Observer approach in [5].

However in the fluid mechanics literature [7],[8],[9] various models of friction coefficients have been investigated, allowing to directly calculate this parameter in function of the flow. The basis of those friction models is the Moody diagram and one of the first formulations is the Colebrook equation [7], which is an implicit equation not easily usable in an algorithm. Some explicit formulations of the friction were made to approximate Colebrook implicit equation, one of those being the so-called Swamee-Jain equation [7], and others including Hazen-Williams and Chezy-Manning models [7].

On the other hand, various studies have also been driven about *non-stationary* models of friction in pipes [10],[11],[12],[13]. A non-stationary formulation for a one-dimensional model includes an additional term in the momentum equation, this additional term being a function of the Reynolds number and some coefficient in general not easily obtained. In [13] a non-stationary formulation of friction is presented based on well-known coefficients.

The purpose of the present paper is thus to study the effect of using such more elaborate models for friction in pipeline dynamics, for the purpose of LDI. The chosen LDI approach is that of direct identification of leak coefficients by means of an observer, in a similar way as in [5], here using a nonlinear *high gain* observer [14].

The paper continues as follows: section II provides the pipeline modeling for constant friction coefficient, friction coefficient obtained by Swamee-Jain equation and for non stationary friction model. Section III then briefly describes the considered nonlinear observer-based LDI, used to compare friction models. Section IV subsequently presents some experimental tests made on a real pipeline prototype, while section V finally concludes the paper.

II. Pipeline and Friction Modeling

A. Partial Differential Equations and Friction Model

For incompressible fluid flow, the classical model that describes the non-stationary phenomena in a pipeline is the set of well-known *water hammer* equations. They correspond to a couple of partial differential equations - the continuity equation and the momentum equation, that can be summarized as follows:

$$\begin{aligned} &\frac{\partial H(z,t)}{\partial t}+\frac{c^2}{gA}\frac{\partial Q(z,t)}{\partial z}=0 \\ &\frac{1}{A}\frac{\partial Q(z,t)}{\partial t}+g\frac{\partial H(z,t)}{dz}+J(Q(z,t))=0 \end{aligned} \tag{1}$$

where z denotes the one dimension spatial coordinate $[m]$, t the time $[s]$, $H(z,t)$ stands for the hydraulic head $[m]$, $Q(z,t)$ for the fluid flow $[m^3/s]$, A for the cross-section area $[m^2]$, g for the gravity acceleration $[m/s^2]$, c for the

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fluid pressure wave speed $[m/s]$, and J for the friction losses affecting the fluid dynamics within the pipe.

Classically, J is expressed in a *stationary* form given by:

$$J = J_s = \frac{fQ|Q|}{2DA^2} \tag{2}$$

where f corresponds to the *Darcy-Weisbach friction coefficient.* In most of model-based LDI approaches, this coefficient is considered as a constant, even if it is sometimes updated when a leak is detected.

But this coefficient is actually known to depend on the so-called Reynolds number (Re) and the roughness coefficient of the pipe (e). The Colebrook implicit equation even describes this coefficient value for a pipe with a circular section of diameter (D) as:

$$\frac{1}{\sqrt{f}} = 0.86 \ln\left(\frac{1}{3.7}\frac{e}{D} + \frac{2.51}{Re\sqrt{f}}\right) \tag{3}$$

where the Reynolds number can be calculated with:

$$Re = \frac{\rho V D}{\mu} = \frac{4\rho Q}{\pi D \mu} \tag{4}$$

for ρ the fluid density and μ the fluid viscosity.

This equation cannot easily be implemented, and an approximate explicit formulation, known as *Swamee-Jain equation*, can be used instead:

$$f = 1.325\left\{\ln\left[0.27\left(\frac{e}{D}\right) + 5.74\left(\frac{1}{Re}\right)^{0.9}\right]\right\}^{-2} \tag{5}$$

This equation is valid for $10^{-8} < e/D < 0.01$ and $5000 < Re < 10^8$.

For an even more complete friction modeling, some *non-stationary* friction losses J_u can also be added (namely $J = J_s + J_u$ in equation (1)), according to the model below:

$$J_u = \frac{k}{2A}\left(\frac{\partial Q}{\partial t} + c\Phi_A\left|\frac{\partial Q}{\partial z}\right|\right) \tag{6}$$

where $\Phi_\mathrm{A} = \mathrm{sgn}(Q)$ and k denotes the *Brunone coefficient*, which can be calculated with:

$$k = \sqrt{0.0476}/2 \quad \text{for laminar flow and}$$
$$k = \left(\sqrt{\frac{7.41}{Re^{log(14.3/Re^{0.05})}}}\right)/2 \quad \text{for turbulent flow.} \tag{7}$$

With the two terms of friction losses the pipeline equations become:

$$\begin{aligned} \frac{\partial H}{\partial t} &= -\frac{c^2}{gA}\frac{\partial Q}{\partial z} \\ \frac{\partial Q}{\partial t} &= \frac{1}{(2+k)}\left(-2gA\frac{\partial H}{dz} - c\Phi_A k\left|\frac{\partial Q}{\partial z}\right| - \frac{fQ|Q|}{DA}\right) \end{aligned} \tag{8}$$

where coefficients f and k can be calculated at any time. Omitting non-stationary friction losses just means $k = 0$.

B. Discretized Equations

In order to use the above equations for simulation as well as model-based LDI, we can discretize them in space and include a leak equation, as this was done for instance in [3], [4] in the case of a simple friction model. This means using a finite difference method for space discretization, and the following balance equation in a leak point:

$$Q_i = Q_{i^*} + \lambda_{fi}\sqrt{H_i} \tag{9}$$

where Q_i is the flow in point i before the leak and Q_i^* is the flow in point i after the leak, while λ_{fi} denotes the leak coefficient.

With this approach and the model of previous section, we obtain discretized equations as follows:

$$\frac{\partial H_i}{\partial t} = \frac{c^2}{gA}\frac{Q_{i-1} - Q_{i^*} - \lambda_{fi}\sqrt{H_i}}{z_i - z_{i-1}} \tag{10}$$

$$\frac{\partial Q_i}{\partial t} = \frac{1}{(2+k_i)}\left(\begin{array}{c} 2gA\dfrac{H_i - H_{i+1}}{z_{i+1} - z_i} \\ +c\Phi_{A_i}k_i\left|\dfrac{Q_i - Q_{i+1}}{z_{i+1} - z_i}\right| - \dfrac{f_i Q_i|Q_i|}{DA} \end{array}\right) \tag{11}$$

where index i is used for the value of the corresponding variable at position z_i.

For the leak detection and isolation problem that we study here, we use a three-point discretization scheme with hydraulic heads as controlled boundary conditions, as shown in fig. 1.

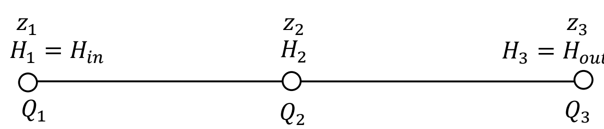

Fig. 1. Three-point discretization scheme.

With this scheme, the discretized model becomes:

$$\begin{aligned} \frac{\partial Q_1}{\partial t} &= \frac{1}{(2+k_1)}\left(\begin{array}{c} 2gA\dfrac{H_1 - H_2}{z_2 - z_1} \\ +c\Phi_{A_1}k_1\left|\dfrac{Q_1 - Q_2}{z_2 - z_1}\right| - \dfrac{f_1 Q_1|Q_1|}{DA} \end{array}\right) \\ \frac{\partial H_2}{\partial t} &= \frac{c^2}{gA}\frac{Q_1 - Q_2 - \lambda_{f2}\sqrt{H_2}}{z_2 - z_1} \\ \frac{\partial Q_2}{\partial t} &= \frac{1}{(2+k_2)}\left(\begin{array}{c} 2gA\dfrac{H_2 - H_3}{z_3 - z_2} \\ +c\Phi_{A_2}k_2\left|\dfrac{Q_2 - Q_3}{z_3 - z_2}\right| - \dfrac{f_2 Q_2|Q_2|}{DA} \end{array}\right) \end{aligned} \tag{12}$$

Observe that equations (12) can be declined in three different models:

Model 1: if we take f as a constant and $k = 0$, the model only takes into account *stationary* friction losses with a constant friction coefficient. This is the classical model used for leak detection and isolation. In this case f has to be identified, classically by using steady-state measurements and equation (2) in the absence of any leak.

Model 2: if we take $f = F(Re, e/D)$ (as in Swamee-Jain equation) and $k = 0$, the model has a friction coefficient that explicitly depends on the flow and roughness coefficient and no unstationary friction loss. In this case the roughness coefficient (e) has to be determined from the pipeline characteristics, or by identification in the same way as f for model 1, but here using equations (2) and (5). Notice that in the latest, the value of e represents some equivalent roughness coefficient and not a theoretical one as it is presented in fluid mechanics books. The reason is that this equivalent coefficient takes into account the actual pipeline equipment, which can make the identification more reliable.

Model 3: if we take $f = F(Re, e/D)$ (as in model 2), and $k \neq 0$ calculated by Brunone formulation (7), then the model includes a friction coefficient explicitly depending on the flow and roughness coefficient, and unstationary friction losses as well. This makes the model more precise than the two former ones, and in that case e has to be determined as for model 2.

III. Observer based LDI

In order to detect and locate a leak in a pipeline from a limited number of measurements (typically flow and pressures upstream and downstream) an approach can be to directly estimate leak coefficients by an observer, as in [4], [5] for instance. To that end, model (12) can be extended with the "dynamics" of such parameters. Assuming that water flows at the pipe ends are directly measured, one gets the following system states, inputs and outputs (with notations of Fig.1):

$$x = \left[Q_1, H_2, Q_2, z_2 - z_1, \lambda_{f2}\right]^T \tag{13}$$

$$u = [H_1, H_3]^T \tag{14}$$

$$y = [Q_1, Q_3]^T \tag{15}$$

where x is composed by elements x_i for $i = 1, \ldots, 5$.

Notice that under constant downstream pressure operation, as this will be considered from now on (namely $H_{out} = cst$),we get from the dynamical model that $Q_3 = Q_2$.

The full system finally takes the form of the following nonlinear state-space representation:

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{(2+k_1)}\left(\begin{array}{c} 2gA\dfrac{u_1 - x_2}{x_4} \\ +c\Phi_{A_1}k_1\left|\dfrac{x_1 - x_3}{x_4}\right| - \dfrac{f_1 x_1|x_1|}{DA}\end{array}\right) \\
\dot{x}_2 &= \frac{c^2}{gA}\frac{x_1 - x_3 - x_5\sqrt{x_2}}{x_4} \\
\dot{x}_3 &= \frac{1}{(2+k_2)}\left(2gA\frac{x_2 - u_2}{L - x_4} - \frac{f_2 x_3|x_3|}{DA}\right) \\
\dot{x}_4 &= 0 \\
\dot{x}_5 &= 0 \\
y &= [Q_1 \;\; Q_2]^T
\end{aligned}$$

where L is the length of the pipe,
or, in a more compact form:

$$\begin{aligned}\dot{x}(t) &= F\big(x(t)\big) + G\big(x(t)\big)u(t) \\ y(t) &= h\big(x(t)\big)\end{aligned} \tag{16}$$

If the unsteady friction losses are omitted ($k = 0$) for the sake of a simpler presentation, and under operation with constant pressure heads $u(t) = u_0$, the model can be re-written as:

$$\begin{aligned}\dot{x}(t) &= F_{u0}\big(x(t)\big) + J\big(y(t)\big) \\ y(t) &= h\big(x(t)\big)\end{aligned} \tag{17}$$

where vector $J(y(t))$ gathers the friction terms (with components J_i), and its observability can be checked to be satisfied as follows:
define the following transformation:

$$\begin{aligned}\Phi &: \Re^5 \to \Re^5 \\ x &\to \xi = \Phi(x)\end{aligned} \tag{18}$$

with

$$\Phi(x) = \left[h_1(x) \;\; L_{F_{u0}}h_1(x) \;\; h_2(x) \;\; L_{F_{u0}}h_2(x) \;\; L^2{}_{F_{u0}}h_2(x)\right] \tag{19}$$

where $L_{F_{u0}}h_i(x)$ as usual means the Lie derivative of h_i along F_{u0}.

Then its jacobian matrix has a determinant given by:

$$\Delta_\Phi = \frac{(cgA)^2\sqrt{x_2}(x_4u_2 + u_1L - u_1x_4 - x_2L)}{x_4^4(L - x_4)^2} \tag{20}$$

which only vanishes whenever:

$$x_4(u_1 - u_2) = L(u_1 - x_2) \tag{21}$$

This condition appears to be inconsistent with equations (16) in steady state in the presence of a leak, which means that for any constant input, Φ defines a change of coordinates.

With the following notations:

$$\Phi: x \to \begin{cases}\xi^1 = \left[h_1(x) \;\; L_{F_{u0}}h_1(x)\right]^T \\ \xi^2 = \left[h_2(x) \;\; L_{F_{u0}}h_2(x) \;\; L^2{}_{F_{u0}}h_2(x)\right]^T\end{cases} \tag{22}$$

where ξ^1 and ξ^2 are composed by elements ξ_{1i} and ξ_{2j}

respectively, with $i = 1,2$ and $j = 1,2,3$, the system in ξ coordinates becomes:

$$\begin{array}{ll} \dot{\xi}_{11} = \xi_{12} + J_1(y_1) & \dot{\xi}_{21} = \xi_{22} + J_2(y_2) \\ \dot{\xi}_{12} = \phi_{12}(\xi, u_0) & \dot{\xi}_{22} = \xi_{23} \\ y_1 = \xi_{11} & \dot{\xi}_{23} = \phi_{22}(\xi, u_0) \\ & y_2 = \xi_{21} \end{array} \tag{23}$$

The system is under the form of a *uniformly observable* one, for which some *high gain* observer (up to output injection) can be designed adapting the classical single-output result of [14], or the multi-output result of [15], for instance.

Let us here consider two subsystems of the form:

$$\dot{\xi}^i = A_i \xi^i + \phi_i(\xi, u_0), y_i = C_i \xi^i \tag{24}$$

for $i = 1,2$ and appropriate A_i, C_i, ϕ_i resulting from (24). Then the observer design can be achieved on the basis of two separate single-output designs as follows:

$$\dot{\hat{\xi}}^i = A_i \hat{\xi}^i + \phi_i(\hat{\xi}, u_0) - S_i^{-1} C_i^T (C_i \hat{\xi}^i - y_i), i = 1,2 \tag{25}$$

(26)

with S_i classically given by [16]:

$$S_1 = \begin{bmatrix} \frac{1}{\lambda_1} & -\frac{1}{\lambda_1^2} \\ -\frac{1}{\lambda_1^2} & \frac{2}{\lambda_1^3} \end{bmatrix}; \; S_2 = \begin{bmatrix} \frac{1}{\lambda_2} & \frac{-1}{\lambda_2^2} & \frac{1}{\lambda_2^3} \\ \frac{-1}{\lambda_2^2} & \frac{2}{\lambda_2^3} & \frac{-3}{\lambda_2^4} \\ \frac{1}{\lambda_2^3} & \frac{-3}{\lambda_2^4} & \frac{6}{\lambda_2^5} \end{bmatrix} \tag{27}$$

for $\lambda_1, \lambda_2 > 0$.

Notice that the observer can be implemented in original coordinates as follows:

$$\dot{\hat{x}} = F(\hat{x}) + G(\hat{x})u - \left(\frac{\partial \Phi(\hat{x})}{\partial x}\right)^{-1} \begin{bmatrix} S_1^{-1} C_1^T \\ S_2^{-1} C_2^T \end{bmatrix} (h(\hat{x}) - y) \tag{28}$$

Remark: The same type of approach can also be adopted when using varying pressure heads u and unsteady friction model with $k \neq 0$.

IV. Experimental Tests

In this section, we present some experimental results in order to validate the proposed modeling in the context of LDI, and compare the performances on the achieved leak detection and isolation of three proposed observers based on the three friction models which have been commented: observer 1 for model 1 with constant friction coefficient, observer 2 for model 2 with variable friction coefficient calculated with Swamee-Jain equation and observer 3 for model 3 with variable friction coefficient and non-stationary friction losses.

The experiments have been realized on a pipeline with the following physical parameters:

TABLE I
Test Pipeline Parameters

g	L	c	D
9.81 $[m/s]$	85$[m]$	407.75 $[m/s]$	0.0635 $[m]$

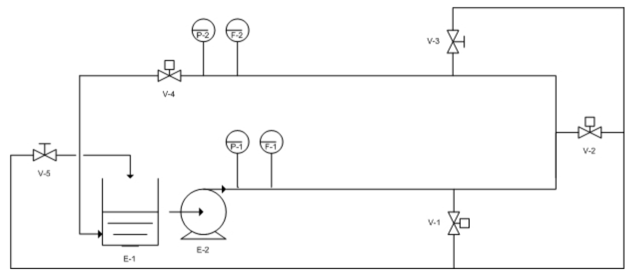

Fig. 2. Experimental pipeline.

This pipeline is available at the CINVESTAV (Research and Advanced Studies Center) in Guadalajara, Mexico. It is equipped with water flow (F) and pressure head (P) sensors at some input and output points, as well as various valves allowing to simulate leaks, as shown in Fig. 2. The pipe is made of PolyPropylene Random copolymer (PP-R). More details about its full physical composition can be found in [6].

A large number of experiments were made on the pipeline, and one of them is reported here for the purpose of illustration, and support for conclusions on the study.

In the considered experiment, a leak with an average flow of about $Q_f = 1.8971 \times 10^{-4}$ $[m^3/s]$ is produced at position $\Delta z_f = z_2 - z_1 = 20$ m and at an initial time $t = 130$ s. Figure 3 shows the measured values of flows and pressures at both ends of the pipe.

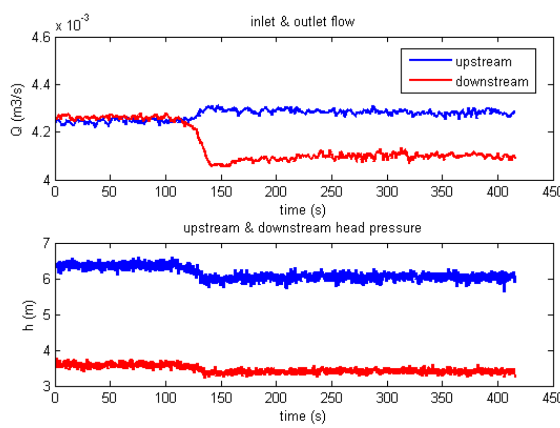

Fig. 3. Flow and pressure measurements in the pipe.

For this set of measured values, the estimated friction coefficient is $f = 2.2566 \times 10^{-2}$ and the estimated equivalent roughness coefficient is $e = 6.01 \times 10^{-5}$ $[m]$ (for a reference value, the PP-R theoretical roughness coefficient is $e = 7 \times 10^{-6}$ $[m]$).

For the estimation, the observer was tuned with $\lambda_1 = \lambda_2 = 1$, and initialized with $\hat{x}_1 = 4.256 \times 10^{-3}$ $[m^3/s]$, $\hat{x}_2 = 5.4607$ $[m]$, $\hat{x}_3 = 4.256 \times 10^{-3}$ $[m^3/s]$, $\hat{x}_4 = 35$ $[m]$ and $\hat{x}_5 = 0$ $[m^2]$.

Figure 4 shows the resulting estimation values of the three observers, observer 1 with constant friction coefficient (f const), observer 2 with variable friction coefficient calculated with Swamee-Jain equation (f var) and observer 3 with variable friction coefficient and non-stationary friction losses (f var and NS).

It is important to note that from the way they are built, the observers only work properly when a leak does exist. This means that values obtained before time $t = 150$ s are not meaningful and results are thus presented for $t \geq 100$ s.

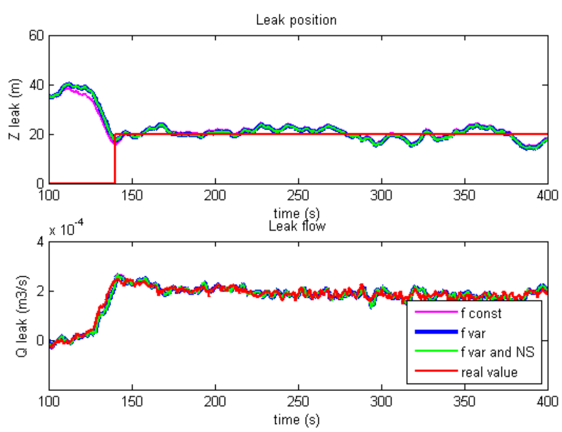

Fig. 4. Estimated values of leak flow and leak position for a calculated constant friction parameter (fconst), variable friction parameter (f var), and non stationary friction losses (f var and NS) respectively, compared with the real value.

Table II below then summarizes the obtained mean estimated values of leak flow and leak position respectively, when only considering $t \geq 150$ s.

TABLE II
MEAN VALUES OF LEAK AND LEAK POSITION

	real	f const	f var	f var NS
Q_f $[m^3/s]$ $\times 10^{-4}$	1.8971	1.9003	1.8991	1.8991
Δz_f $[m]$	20	20.560	19.909	19.914

From those data, it can be seen that the three observers produce good and quite similar results.

The difference between them is very small and do not allow to directly conclude about the approach that would be better.

For this reason, we propose some additional simulation study on the sensitivity of the results w.r.t. parameters f and e.

Figures 5 and 6 below show some estimation results with the three observers in this context, namely when the values of parameters f and e are modified from the one initially calculated of about 1% more (fig. 5) and 1% less (fig. 6) respectively.

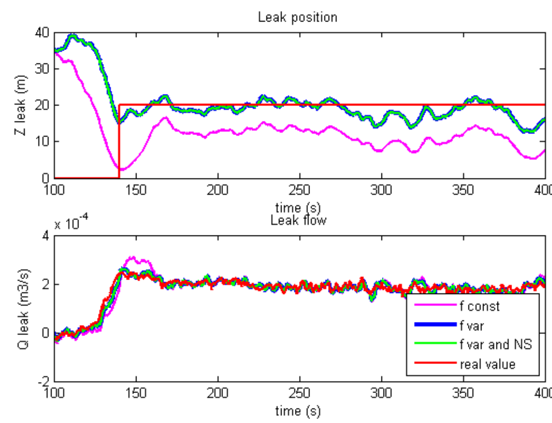

Fig. 5. Estimated values of leak flow and leak position for a calculated value **plus 1%** for constant friction parameter (fconst), variable friction parameter (f var), and non stationary friction losses (f var and NS) respectively, compared with the real value.

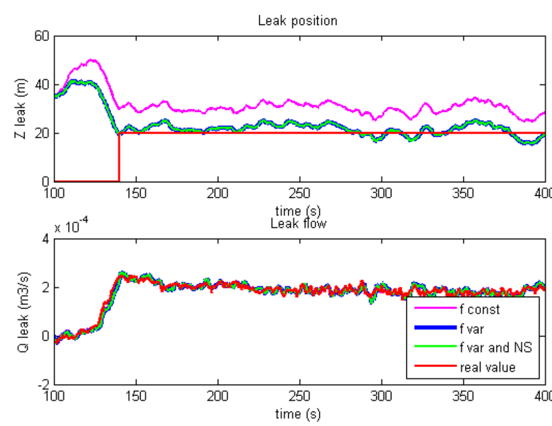

Fig. 6. Estimated values of leak flow and leak position for a calculated value **less 1%** for constant friction parameter (fconst), variable friction parameter (f var), and non stationary friction losses (f var and NS) respectively, compared with the real value.

It can clearly be concluded from those results that the sensitivity w.r.t. parameter e in observers 2 and 3, with a variable friction coefficient, is very small for leak location and leak flow. But the sensitivity w.r.t. parameter f used for observer 1, with constant friction coefficient, is quite large for leak location: a 1% -variation of f produces about 50% of error in leak location estimation (Δz_f).

This fact is very important for practical application because the real value of f depends on the flow in the pipe, which means that small variations of the flow can induce large errors in leak location estimations. For this reason it is strongly recommended to use a variable formulation of friction coefficient and not a constant one.

On the other hand, if we compare the results of observers 2 and 3 (without and with non-stationary formulation), we can see almost identical estimations. More precisely the maximum difference between the leak position estimation of the two observers is about 5.02×10^{-3} $[m]$, that is about 0.025%. From this one can conclude that it is not necessary to use a non-stationary formulation for leak detection application, because this just makes the model more complex without providing better results.

V. Conclusion

In this paper, the problem of leak detection and isolation in pipelines by means of nonlinear observers has been studied, in particular w.r.t. the friction modeling. Three dynamical models have been considered corresponding to three friction formulations with an increasing complexity: the first one with a constant friction coefficient, the second one with a variable friction coefficient and the last with variable friction coefficient and non stationary friction losses.

It has been emphasized how for the three cases a nonlinear high gain observer can be derived for the purpose of leak estimation and location, and corresponding experimental results have been provided for validations and comparisons.

From the results, it clearly appears that the better formulation is the use of a variable friction coefficient without non-stationary losses. The model with a constant friction coefficient is limited to nearly constant flow applications, and therefore is not recommended for real systems, while the non-stationary formulation makes the observer structure more complex, without significantly improving leak estimations.

Finally the Swamee-Jain equation appears to be a good explicit formulation to calculate the friction coefficient value in the variable friction coefficient formulation.

Future works will be dedicated to studying the applicability of the method to estimate and locate more than one leak.

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