2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Distributed Control of Multiple Wheeled Mobile Robots

Wenjie Dong[1], Mounir Ben Ghalia[1], Jay A. Farrell[2]
[1] Department of Electrical Engineering, The University of Texas - Pan American, Edinburg, TX, USA
[2] Department of Electrical Engineering, University of California, Riverside, CA, USA

Abstract— **This paper considers the cooperative control problem of multiple wheeled mobile robots. Cooperative control laws are proposed such that the state of each mobile robot asymptotically tracks a desired trajectory under the condition that the desired trajectory is only available to a portion of a group of mobile robots. Simulation results show the effectiveness of the proposed control laws.**

I. INTRODUCTION

Cooperative control of multiple robots has been an active research area due to many applications, e.g., rescue mission, large object moving, cooperative target pursuit, etc. Various methods have been proposed to solve cooperative control of multiple robots. Among these methods, graph theory based method is one of effective methods. With the aid of results from graph theory many cooperative control laws have been proposed for cooperative control of multiple linear systems in [1–6]. In those papers, the structure of the communication network between vehicles was described by Laplacian matrices. Each vehicle is treated as a vertex and the communication links between vehicles were treated as edges. The stability of the whole system was guaranteed by the stability of each modified individual linear system, where the modification to the linear system accounts for the structure of the communication network. Artificial potentials have also been applied to flocking of multi-agents with the aid of other techniques [7–9]. Article [7] discussed flocking of linear systems with the aid of the artificial potentials and virtual agents. In [8], cooperative control laws were proposed for fixed and switched communication networks. In [9], navigation functions were applied to design controllers for point-mass mobile agents without collision between robots. In [10], distributed swarm aggregation of multiple agents with collision avoidance was discussed with the aid of a repulsive potential field and an attractive potential field. In [11], distributed control laws were proposed such that multi-agents converge to a desired formation for both the cases of agents with single integrator and nonholonomic unicycle-type kinematics.

In cooperative control of multiple systems with a desired trajectory in literature, it is assumed that a desired trajectory is available to each system [7, 12–16]. However, this assumption is not realistic in practice if the number of robots is large. A more reasonable assumption is that a desired trajectory is available only to a portion of a group of systems. In [16], consensus algorithms were proposed for multiple first-order systems such that the state of each system converges to a time-varying reference trajectory under the condition that the reference trajectory is available to a portion of systems. Cooperative control laws were proposed with the aid of derivatives of neighbor's states. In [17], the consensus problem of multiple second-order linear systems with a reference system was considered, consensus algorithms were proposed with the aid of neighbors' acceleration information. In [18, 19], distributed tracking via a variable structure approach was considered for multiple first-order and second-order linear systems. Distributed discontinuous controllers were proposed such that the state of each system converge to a desired trajectory within finite time under the condition that the desired trajectory is available to a portion of the group of systems. In [20, 21], tracking control for multiple first-order linear systems with an active leader was discussed. Distributed dynamic controllers were proposed with the aid of distributed estimators. The designer can make the tracking error between the state of a system and the state of an active leader arbitrarily small by selecting a control parameter to be sufficiently large. In [22], a motion coordination problem was studied for achieving identical orientation and synchronous rotation for a group of rigid bodies. Distributed adaptive controllers were proposed such that the angular velocity of each rigid body converges to a desired angular velocity under the condition that the desired angular velocity is available only to the leader. In [23], a coordination problem was studied to steer a group of agents to a formation that translates with a prescribed reference velocity. An adaptive design was proposed such that each agent can reconstruct a reference velocity and recover a desired formation under the condition that a reference velocity information is only available to a leader.

In this paper we consider cooperative control of multiple nonholonomic wheeled mobile robots such that a group of robots converges to a desired geometric pattern whose centroid moves along a desired trajectory under the condition that the desired trajectory is available to only a portion of the group of systems. Toward this end, we apply Lyapunvov techniques and results from graph theory. Distributed control laws are proposed for each robot with the aid of neighbors' information. The contribution of this paper is that distributed control laws are proposed for multiple nonlinear systems such that they come into a desired formation under the condition that a desired trajectory is only available to a portion of robots.

The rest of the paper is organized as follows. In Section II, we formally state the control problem. In Section III, distributed control laws are proposed for the control problem.

Section V includes simulation results to show the effectiveness of the proposed results. The last section concludes this paper.

II. Problem Statement

Consider a group of m nonholonomic wheeled mobile robots which move on a horizontal plane. The motion of system j (i.e., robot j) is described by

$$\dot{x}_j = v_j \cos\theta_j, \quad \dot{y}_j = v_j \sin\theta_j, \quad \dot{\theta} j = \omega_j \tag{1}$$

where (x_j, y_j) is the position of robot j in a coordinate system, θ_j is the orientation of the robot j, v_j is the speed of the robot j, and ω_j is angular velocity of the robot j. The control signals are v_j and ω_j

Each system knows its own state and the states of some of the other systems by communication and/or sensors. For simplicity, we assume that the communication between the systems are bidirectional. If each system is considered as a node, the communication between the systems can be described by a (undirected) graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, m\}$ is a node set, and $\mathcal{E}$ is an edge set with unordered pair (i, j) which describes the communication between node i and node j. If the state of node i is available to node j, node i is called a neighbor of node j. The set of all neighbors of node j is denoted by $\mathcal{N}_j$. A graph is called connected if for any two nodes there exists a set of edges which connect the two nodes. For more information on graph theory, interested readers may refer to [24].

We describe a desired geometric pattern $\mathcal{P}$ with m vertexes. The pattern $\mathcal{P}$ can be described by orthogonal coordinates (p_{jx}, p_{jy}). Without loss of generality, we assume that $\sum_{j=1}^{m} p_{jx} = 0$ and $\sum_{j=1}^{m} p_{jy} = 0$, i.e., the center of the geometric pattern $\mathcal{P}$ is at the origin of a local orthogonal coordinate system. Given a reference trajectory $q_0(t) = (x_0(t), y_0(t), \theta_0(t))$ which satisfies

$$\dot{x}_0 = v_0 \cos\theta_0, \quad \dot{y}_0 = v_0 \sin\theta_0, \quad \dot{\theta}_0 = \omega_0 \tag{2}$$

where v_0 and ω_0 are known time-varying functions. We assume that q_0 is available to a portion of the m wheeled mobile robots.

The desired trajectory satisfies the following assumptions.

Assumption 1: The $\frac{d^i\theta_0}{dt^i}$ $(0 \le i \le 2)$ are bounded and $\int_t^{t+T} \dot{\theta}_0^2(\tau) d\tau > \delta > 0$ for a finite time $T > 0$ and any $t \ge 0$.

Assumption 2: x_0, y_0, and v_0 are bounded.

Assumption 1 means that the signal $\dot{\theta}_0$ is a persistent excited signal (P. E. signal). For a P. E. signal, the following lemma is useful in this paper.

Lemma 1: For the system

$$\dot{\zeta} = -\eta_1(t)^2 \zeta + \eta_2(t) \tag{3}$$

if $\int_t^{t+T} \eta_1^2(\tau) d\tau > \delta > 0$ for a finite time $T(>0)$ and any time t and $\eta_2(t)$ converges to zero, then ζ converges to zero.

The control problem discussed in this article is defined as follows.

Control Problem: Design a control laws v_j and ω_j for system j using $(q_j, \dot{q}_j)$, $(q_l, \dot{q}_l)$ and (p_{lx}, p_{ly}) for $l \in \mathcal{N}_j$ such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}, \ 1 \le i \ne j \le m \tag{4}$$

$$\lim_{t\to\infty} (\theta_i - \theta_0) = 0 \tag{5}$$

$$\lim_{t\to\infty} \left(\sum_{j=1}^{m} \frac{x_j}{m} - x_0 \right) = 0, \tag{6}$$

$$\lim_{t\to\infty} \left(\sum_{j=1}^{m} \frac{y_j}{m} - y_0 \right) = 0. \tag{7}$$

Remark 1: In the control problem, the control laws for system j are designed based on the state of system j, (p_{lx}, p_{ly}) and the state of system l for $l \in \mathcal{N}_j$, and the desired trajectory q_0 if it is available to system j. Eqn. (4) ensures that the group of robots converge to the desired geometric pattern $\mathcal{P}$. Eqn. (5) ensures that the orientation of each robot converges to a desired value. Eqns. (6)-(7) ensure that the geometric centroid of the group of robots asymptotically converges to the desired trajectory (x_0, y_0).

III. Cooperative Controller Design

Define the change of variables

$$\left\{ \begin{array}{lcl} z_{1j} &=& \theta_j \\ z_{2j} &=& (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j \\ && +k_3 u_{1j}\left((x_j - p_{jx})\sin\theta_j - (y_j - p_{jy})\cos\theta_j\right) \\ z_{3j} &=& (x_j - p_{jx})\sin\theta_j - (y_j - p_{jy})\cos\theta_j \\ u_{1j} &=& \omega_j \\ u_{2j} &=& v_j - u_{1j} z_{3j} \end{array} \right. \tag{8}$$

for $0 \le j \le m$, where $k_3 > 0$ and $p_{0x} = p_{0y} = 0$. The transformed state space model is

$$\dot{z}_{1j} = u_{1j} \tag{9}$$

$$\dot{z}_{2j} = u_{2j} + k_3 \dot{u}_{1j} z_{3j} + k_3 u_{1j}^2 z_{2j} - k_3^2 u_{1j}^3 z_{3j} \tag{10}$$

$$\dot{z}_{3j} = -k_3 u_{1j}^2 z_{3j} + u_{1j} z_{2j}. \tag{11}$$

With the aid of Lemma 1, we have the following results.

Lemma 2: If

$$\lim_{t\to\infty} (z_{1j} - z_{10}) = 0 \tag{12}$$

$$\lim_{t\to\infty} (z_{2j} - z_{20}) = 0 \tag{13}$$

$$\lim_{t\to\infty} (z_{3j} - z_{30}) = 0 \tag{14}$$

$$\lim_{t\to\infty} (u_{1j} - u_{10}) \overset{exp.}{=} 0 \tag{15}$$

for $1 \le j \le m$, then (4)-(7) hold, where $\overset{exp.}{=}$ means "exponentially converges to".

The lemma can be proved by simple calculation and applying Lemma 1. So, it is omitted here.

With the aid of Lemma 2, a tracking controller can be designed for each system if the information of system 0 is available to each system. However, the information of system 0 is not available to each system in the defined problem.

Since $z_{*0} = [z_{10}, z_{20}, z_{30}]^\top$ may not be available to system j ($1 \le j \le m$), we build an observer for system j as follows

$$\begin{aligned}\dot{\hat{z}}_{1j} &= \hat{u}_{1j} &(16)\\ \dot{\hat{z}}_{2j} &= \hat{u}_{2j} &(17)\\ \dot{\hat{z}}_{3j} &= -k_3\hat{u}_{1j}^2\hat{z}_{3j} + \hat{u}_{1j}\hat{z}_{2j}. &(18)\end{aligned}$$

where $\hat{u}_{1j}$ and $\hat{u}_{2j}$ will be chosen such that

$$\lim_{t\to\infty}(\hat{z}_{*j} - z_{*0}) = 0, \quad 1 \le j \le m \qquad (19)$$

with $\hat{z}_{*j} = [\hat{z}_{1j}, \hat{z}_{2j}, \hat{z}_{3j}]^\top$.

Before proposing $\hat{u}_{1j}$ and $\hat{u}_{2j}$, some preparation is needed. For the group of m systems, the information flow between systems is described by a graph $\mathcal{G} = \{\mathcal{A}, \mathcal{E}\}$. Given an $m \times m$ symmetric constant matrix $B = [b_{ji}]$ with $b_{ji} = b_{ij} > 0$, the Laplacian matrix $\mathcal{L} = [L_{ji}]$ of the graph $\mathcal{G}$ with weight matrix B is defined by

$$L_{ji} = \begin{cases} -b_{ji}, & \text{if } i \in \mathcal{N}_j \text{ and } i \ne j \\ 0, & \text{if } i \notin \mathcal{N}_j \text{ and } i \ne j \\ \sum\limits_{l\ne j, l\in\mathcal{N}_j} b_{jl}, & \text{if } j = i. \end{cases}$$

Because communications are bidirectional, $\mathcal{L}$ is a symmetric matrix with real eigenvalues. For the Laplacian matrix, the following result is useful.

Lemma 3: If the information interchange graph $\mathcal{G}$ is connected, then $(L + \mathrm{diag}(\mu))$ is a positive definite symmetric matrix, where constant vector $\mu = [\mu_1, \mu_2, \ldots, \mu_m]^\top$, $\mu_i \ge 0$ ($1 \le i \le m$) and at least one of the elements of μ is nonzero.

Proof: We consider the m systems and a virtual system that generates the desired trajectory q_0 as a new group of $(m+1)$ systems. The virtual system is labeled as system 0. The information interchange between the $(m+1)$ systems can be described by a directed graph $\mathcal{G}^e$. The matrix

$$\mathcal{L}^e = \begin{bmatrix} \mathcal{L} + \mathrm{diag}(\mu) & -\mu \\ \mathbf{0} & 0 \end{bmatrix}$$

can be considered as a Laplacian matrix of the directed graph $\mathcal{G}^e$ with appropriate weights. Since μ is nonzero, the information interchange directed graph $\mathcal{G}^e$ of the $(m+1)$ systems has a spanning tree. Therefore, $\mathcal{L}^e$ has one zero eigenvalue and m eigenvalues with positive real parts [25]. Noting the structure of $\mathcal{L}^e$, the eigenvalues of the matrix $(\mathcal{L} + \mathrm{diag}(\mu))$ are the m nonzero eigenvalues of $\mathcal{L}^e$. Since $(\mathcal{L} + \mathrm{diag}(\mu))$ is symmetric, the eigenvalues of $(\mathcal{L} + \mathrm{diag}(\mu))$ are real positive numbers. Therefore, $(\mathcal{L} + \mathrm{diag}(\mu))$ is positive definite. ■

With the aid of Lemma 3, we have the following lemma.

Lemma 4: For the $(m+1)$ systems in eqn. (9) ($0 \le j \le m$), if the information flow graph $\mathcal{G}$ is connected and the system 0 is at least available to one of the m systems, the control laws

$$\begin{aligned}\hat{u}_{1j} &= \zeta_{1j} - \sum_{i\in\mathcal{N}_j} b_{ji}(\hat{z}_{1j} - \hat{z}_{1i}) - a_j(\hat{z}_{1j} - z_{10}) &(20)\\ \dot{\zeta}_{1j} &= -\sum_{i\in\mathcal{N}_j} b_{ji}(\zeta_{1j} - \zeta_{1i}) - a_j(\zeta_{1j} - \zeta_{10}) \\ &\quad -\rho_1 \,\mathrm{sign}\Bigg(\sum_{i\in\mathcal{N}_j} b_{ji}(\zeta_{1j} - \zeta_{1i}) \\ &\quad -a_j(\zeta_{1j} - \zeta_{10})\Bigg) &(21)\\ \hat{u}_{2j} &= -\sum_{i\in\mathcal{N}_j} b_{ji}(\hat{z}_{2j} - \hat{z}_{2i}) - a_j(\hat{z}_{2j} - z_{20}) \\ &\quad -\rho_2 \,\mathrm{sign}\Bigg(\sum_{i\in\mathcal{N}_j} b_{ji}(\hat{z}_{2j} - \hat{z}_{2i}) \\ &\quad -a_j(\hat{z}_{2j} - z_{20})\Bigg) &(22)\end{aligned}$$

for $1 \le j \le m$ guarantee that $\lim_{t\to\infty}(\hat{z}_{*j} - z_{*0}) \overset{exp.}{=} 0$ and $\lim_{t\to\infty}(\hat{u}_{1j} - u_{10}) \overset{exp.}{=} 0$, where $\zeta_{10} = \dot{z}_{10} = \dot{\theta}_0$, and ρ_1 and ρ_2 are sufficiently large. The parameter $a_j > 0$ if system 0 is available to system j and $a_j = 0$ if system 0 is not available to system j.

Proof: With (20)-(21), we have

$$\begin{aligned}\dot{\zeta}_{1*} &= -\mathcal{L}\zeta_{1*} - A(\zeta_{1*} - \mathbf{1}\zeta_{10}) \\ &\quad -\rho_1 \,\mathrm{sign}\left(\mathcal{L}\zeta_{1*} + A(\zeta_{1*} - \mathbf{1}\zeta_{10})\right) &(23)\\ \dot{\hat{z}}_{1*} &= \zeta_{1*} - \mathcal{L}\hat{z}_{1*} - A(\hat{z}_{1*} - \mathbf{1}z_{10}) &(24)\end{aligned}$$

where $\zeta_{1*} = [\zeta_{11}, \zeta_{12}, \ldots, \zeta_{1m}]^\top$, $\hat{z}_{1*} = [\hat{z}_{11}, \hat{z}_{12}, \ldots, \hat{z}_{1m}]^\top$, and $A = \mathrm{diag}[a_1, a_2, \ldots, a_m]$.

Let $\bar{\zeta}_{1*} = \zeta_{1*} - \mathbf{1}\zeta_{10}$, then

$$\dot{\bar{\zeta}}_{1*} = -(\mathcal{L} + A)\bar{\zeta}_{1*} - \rho_1 \mathrm{sign}\left((\mathcal{L} + A)\bar{\zeta}_{1*}\right) - \mathbf{1}\dot{\zeta}_{10} \qquad (25)$$

where we apply the fact that $\mathcal{L}\zeta_{10} = \mathbf{0}$. Choose a Lyapunov function

$$V = \frac{1}{2}\bar{\zeta}_{1*}^\top\bar{\zeta}_{1*} \qquad (26)$$

and differentiate it along the solution of (25), we get

$$\begin{aligned}\dot{V} &= -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} - \rho_1\bar{\zeta}_{1*}^\top\mathrm{sign}\left((\mathcal{L} + A)\bar{\zeta}_{1*}\right) \\ &\quad -\bar{\zeta}_{1*}^\top\mathbf{1}\dot{\zeta}_{10} \\ &= -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} - \bar{\zeta}_{1*}^\top\mathbf{1}\dot{\zeta}_{10} - \rho_1[(\mathcal{L} \\ &\quad +A)\bar{\zeta}_{1*}]^\top(\mathcal{L} + A)^{-\top}\mathrm{sign}\left((\mathcal{L} + A)\bar{\zeta}_{1*}\right) \\ &\le -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} - \rho_1[(\mathcal{L} + A)\bar{\zeta}_{1*}]^\top\lambda_m I \times \\ &\quad \mathrm{sign}\left((\mathcal{L} + A)\bar{\zeta}_{1*}\right) - \bar{\zeta}_{1*}^\top\mathbf{1}\dot{\zeta}_{10} \\ &= -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} - [(\mathcal{L} + A)\bar{\zeta}_{1*}]^\top(\mathcal{L} + A)^{-\top}\mathbf{1}\dot{\zeta}_{10} \\ &\quad -\rho_1\lambda_m[(\mathcal{L} + A)\bar{\zeta}_{1*}]^\top\mathrm{sign}\left((\mathcal{L} + A)\bar{\zeta}_{1*}\right) \\ &\le -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} - \rho_1\lambda_m\sum_{i=1}^m |s_i| - \sum_{i=1}^m d|s_i| \\ &\le -\bar{\zeta}_{1*}^\top(\mathcal{L} + A)\bar{\zeta}_{1*} \le -\lambda_m\bar{\zeta}_{1*}^\top\bar{\zeta}_{1*} &(27)\end{aligned}$$

where λ_m is the smallest eigenvalue of the symmetric matrix $(\mathcal{L} + A)^{-1}$, $s = [s_1, \ldots, s_m]^\top = (\mathcal{L} + A)\bar{\zeta}_{1*}$, and d is a bounded positive number which depends on $(\mathcal{L}+A)$ and $\dot{\zeta}_{10}$.

We also apply the fact that ρ_1 is sufficiently large to derive the inequality (27). Since $(\mathcal{L}+A)$ is positive definite by Lemma 3, λ_m is a positive number. Therefore, V exponentially converges to zero, which means that $\bar{\zeta}_{1*}$ exponentially converges to zero. So, ζ_{1j} exponentially converge to ζ_{10} for $1 \le j \le m$.

Let $\bar{z}_{1*} = \hat{z}_{1*} - \mathbf{1}z_{10}$, then

$$\dot{\bar{z}}_{1*} = -(\mathcal{L}+A)\bar{z}_{1*} + \bar{\zeta}_{1*}. \quad (28)$$

Eqn. (28) can be considered as a stable system subject to disturbance which exponentially converges to zero. Therefore, $\bar{z}_{1*}$ exponentially converges to zero and $\hat{z}_{1j}$ exponentially converges to z_{10} for $1 \le j \le m$. Noting (20), it is obvious that $\hat{u}_{1j}$ exponentially converges to u_{10}.

Similarly, it can be shown that $\hat{z}_{2j}$ exponentially converges to z_{20} for $1 \le j \le m$. Let $\bar{z}_{3j} = \hat{z}_{3j} - z_{30}$, it can be proved that $\bar{z}_{3j}$ exponentially converges to zero with the aid of Lemma 1. ∎

With the aid of Lemmas 1-4, we get the following results.

Theorem 1: For the m systems in eqn. (1) ($1 \le j \le m$), under Assumptions 1-2, if the information flow graph $\mathcal{G}$ is connected, the control laws

$$\begin{aligned} u_{1j} &= -k_1(z_{1j} - \hat{z}_{1j}) + \hat{u}_{1j} & (29)\\ u_{1j} &= -k_2(z_{2j} - \hat{z}_{2j}) + \hat{u}_{2j} - k_3\dot{u}_{1j}z_{3j} \\ &\quad -k_3u_{1j}^2 z_{2j} + k_3^2 u_{1j}^3 z_{3j} & (30)\end{aligned}$$

for $1 \le j \le m$ guarantee that (4)-(7) hold, where $k_1 > 0$, and $k_2 > 0$.

Proof: With the control laws (29)-(30), it can be shown that $\lim_{t\to\infty}(z_{1j} - \hat{z}_{1j}) \overset{exp.}{=} 0$, $\lim_{t\to\infty}(z_{2j} - \hat{z}_{2j}) \overset{exp.}{=} 0$, and $\lim_{t\to\infty}(u_{1j} - \hat{u}_{1j}) \overset{exp.}{=} 0$. Furthermore, it can be shown that $\lim_{t\to\infty}(z_{3j} - \hat{z}_{3j})$=0. With the aid of Lemma 4 and Lemma 2, (4)-(7) hold. ∎

Remark 2: In Theorem 1, ρ_1 and ρ_2 should be chosen such that $\rho_1 \ge M_1 = \max_{t\in[0,\infty)} |\ddot{\theta}_0|$ and $\rho_2 \ge M_2 = \max_{t\in[0,\infty)} |\dot{z}_{20}|$. Since $\ddot{\theta}_0$ and $\dot{z}_{20}$ are not available to each system, each system does no know M_1 and M_2. Therefore, ρ_1 and ρ_2 should be chosen large enough. It is possible to estimate ρ_1 and ρ_2 with the aid of neighbors' information. Due to space limitation, we will not discuss them.

IV. Cooperative Control Laws for Switching Communication Graph

In the previous section, the communication graph is assumed to be fixed. If the communication graph is not fixed, the proposed results also hold. An extension of Theorem 1 is as follows.

Theorem 2: For the m systems in eqn. (1) ($1 \le j \le m$), under Assumptions 1-2, if the information flow graph $\mathcal{G}$ is connected at each time, the control laws (29)-(30) guarantee that (4)-(7) hold.

The theorem can be proven by following the proofs of Lemma 4 and Theorem 1. Therefore, it is omitted here.

Remark 3: In the control law for robot j, the required information are the state of robot j, (p_{ix}, p_{iy}) and the state of robot i for $i \in \mathcal{N}_j$. In Theorem 1, the state of system 0 is not required to be known to each system. In contrast to leader-follower approach in [26], the leader-follower communication pattern has been pre-defined before the controller design. In Theorem 1, there is no pre-defined patterns for the communication between systems. Instead, the requirement is that at each time the m robots are connected and the desired trajectory is available to only one of the m robots.

V. Simulations

To show the effectiveness of the proposed results, simulation has been done for five robots. The desired geometric pattern $\mathcal{P}$ is shown in Fig. 1. The pattern $\mathcal{P}$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (2.00, 0.00)$, $(p_{2x}, p_{2y}) = (0.62, 1.90)$, $(p_{3x}, p_{3y}) = (-1.62, 1.18)$, $(p_{4x}, p_{4y}) = (-1.62, -1.18)$, and $(p_{5x}, p_{5y}) = (0.62, -1.90)$. Assume the reference trajectory is $(x_0, y_0, \theta_0) = (12\sin(0.5t), -12\cos(0.5t), 0.5t)$, by (2) $v_0 = 6$ and $\omega_0 = 0.5$. So, Assumption 1 is satisfied.

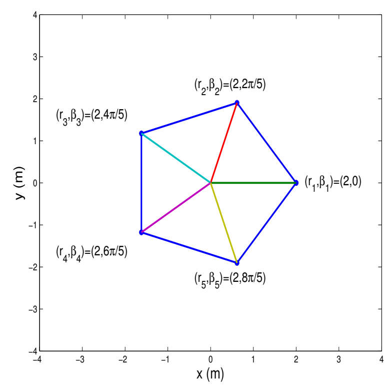

Fig. 1. Desired geometric pattern.

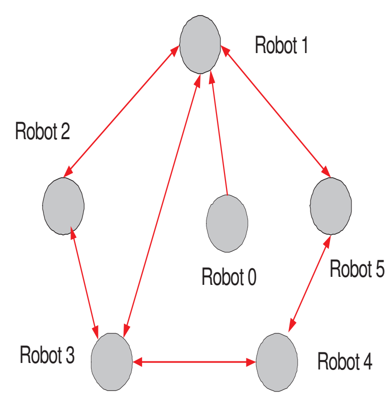

Fig. 2. Information interchange graph $\mathcal{G}$

Assume the communication graph is shown in Fig. 2. The cooperative controllers can be obtained by Theorem 1. We choose the control parameters $b_{ji} = 2$, $k_1 = k_2 = k_3 = 2$, $a_1 = 2$, $\rho_1 = 2$, and $\rho_2 = 2$. Fig. 3 shows the centroid of x_i ($1 \le i \le 5$) and x_0. Fig. 4 shows the centroid of y_i ($1 \le i \le 5$) and y_0. Fig. 5 shows $(\theta_i - \theta_0)$ ($1 \le i \le 5$). From the simulation (5)-(7) are satisfied. Fig. 6 shows the path of the centroid of the five robots and its desired path. From the simulation (5)-(7) are satisfied. Eqn. (4) is also verified and the response of them is omitted here.

If the information interchange graph is time-varying, the control laws in Theorem 2 also solve the defined control problem. Assume the information interchange graph switches according to the following logic.

$$\mathcal{G} = \begin{cases} \mathcal{G} \text{ in Fig. 2}, & \text{if } t - \text{round}(t) \geq 0 \\ \mathcal{G} \text{ in Fig. 7}, & \text{if } t - \text{round}(t) < 0 \end{cases}$$

Fig. 8 shows the centroid of x_i $(1 \leq i \leq 5)$ and x_0. Fig. 9 shows the centroid of y_i $(1 \leq i \leq 5)$ and y_0. Fig. 10 shows $(\theta_i - \theta_0)$ $(1 \leq i \leq 5)$. From the simulation (5)-(7) are satisfied. Fig. 11 shows the path of the centroid of the five robots and its desired path. From the simulation (5)-(7) are satisfied. Eqn. (4) is also verified and the response of them is omitted here.

VI. Conclusion

In this paper, the formation control of multiple wheeled mobile robots were considered under the condition that a desired trajectory is available to only a portion of the systems. Distributed control laws were proposed with the aid of Lyapunov techniques and results from graph theory. Simulation results show the effectiveness of the proposed control laws.

References

[1] A. K. Das, R. Fierro, and V. Kumar, *Cooperative Control and Optimization of Applied Optimization, in Control Graphs for Robot Networks*. Norwell, MA: Kluwer, 2002.

[2] T. Eren, P. N. Belhumeur, and A. S. Morse, "Closing ranks in vehicle formations based on rigidity," *Proc. IEEE Conf. Decision and Control*, pp. 2959–2964, 2002.

[3] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. on Auto. Contr.*, vol. 49, pp. 1465–1476, 2004.

[4] G. Lafferriere, J. Caughman, and A. Williams, "Graph theoretic methods in the stability of veicle formations," *Proc. of Ameircan Control Conf.*, pp. 3729–3734, 2004.

[5] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veermany, "Decentralized control of vehicle formations," *Systems and Control Letters*, vol. 53, pp. 899–910, 2005.

[6] R. Olfati-Saber and R. M. Murray, "Distributed structural stabilization and tracking for formations of dynamic multiagents," *Proc. IEEE Conf. Decision and Control*, pp. 209–215, 2002.

[7] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Trans. on Auto. Contr.*, vol. 51, no. 3, pp. 401–420, 2006.

[8] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[9] H. G. Tanner and A. Kumar, "Decentralized formation stabilization and collision avoidance using navigation functions," *Transactions on Robotics and Automation (submitted)*, 2007.

[10] D. V. Dimarogonas and K. J. Kyriakopoulos, "Connectedness preserving distributed swarm aggregation for multiple kinematic robots," *IEEE Transactions on Robotics*, vol. 24, pp. 1213–1223, 2008.

[11] ——, "A connection between formation infeasibility and velocity alignment in kinematic multi-agent systems," *Automatica*, vol. 44, pp. 2648–2654, 2008.

[12] D. Sun, C. Wang, W. Shang, and G. Feng, "A synchronization approach to trajectory tracking of multiple mobile robots while maintaining time-varying formations," *IEEE Trans. on Robotics*, vol. 25, no. 5, pp. 1074–1086, 2009.

[13] S. Chung and J. E. Slotine, "Cooperative robot control and concurrent synchronization of lagrangian systems," *IEEE Trans. on Robotics*, vol. 25, no. 3, pp. 686–700, 2009.

[14] W. Dong and J. A. Farrell, "Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty," *Automatica*, vol. 45, pp. 706–710, 2009.

[15] ——, "Cooperative control of multiple nonholonomic mobile agents," *IEEE Trans. on Auto. Contr.*, vol. 53, no. 6, pp. 1434–1448, 2008.

[16] W. Ren, "Multi-vehicle consensus with a time-varying reference state," *Systems & Control Letters*, vol. 56, pp. 474–483, 2007.

[17] ——, "On consensus algorithms for double-inegrator dynamics," *IEEE Trans. on Automatic Control*, vol. 53, no. 6, pp. 1503–1509, 2008.

[18] Y. Cao and R. Wei, "Distributed coordinated tracking via a variable structure approach - part i: Consensus tracking," *American Control Conference*, pp. 4744–4749, 2010.

[19] ——, "Distributed coordinated tracking via a variable structure approach - part ii: Swarm tracking," *American Control Conference*, pp. 4750–4755, 2010.

[20] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

[21] Y. Hong, G. R. Chen, and L. Bushnell, "Distributed observers design for leader-following control of multi-agent," *Automatica*, vol. 44, no. 3, pp. 846–850, 2008.

[22] H. Bai, M. Arcak, and J. T. Wen, "Rigid body attitude coordination without inertial frame information," *Automatica*, vol. 44, pp. 3170–3175, 2008.

[23] ——, "Adaptive design for reference velocity recovery in motion coordination," *Systems and Control Letters*, vol. 57, no. 8, pp. 602–610, 2008.

[24] F. R. K. Chung, *Spectral Graph Theory*. Regional Conf. Series in Mathematics of Amer. Mathematical Soc., 1997, vol. 92.

[25] W. Ren and R. W. Beard, "Consensus seeking in multi-agent systems under dynamically changing interaction topologies," *IEEE Trans. on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[26] J. P. Desai, J. P. Ostrowski, and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots," *IEEE Trans. Robot. Automat.*, vol. 17, pp. 905–908, 2001.

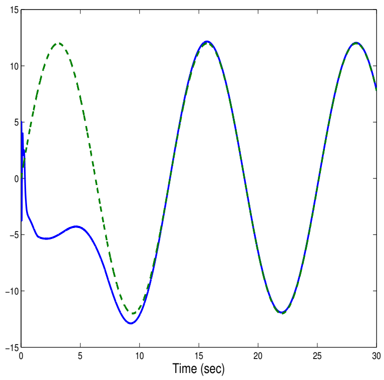

Fig. 3. Response of the centroid of x_i (solid) for $1 \leq i \leq 5$ and x_0 (dashed).

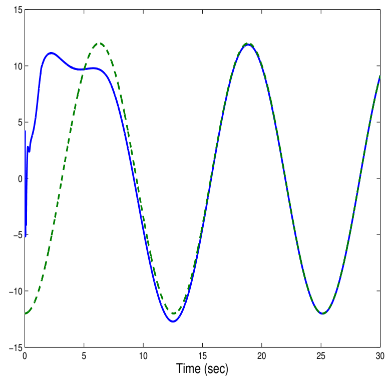

Fig. 4. Response of the centroid of y_i (solid) for $1 \leq i \leq 5$ and y_0 (dashed).

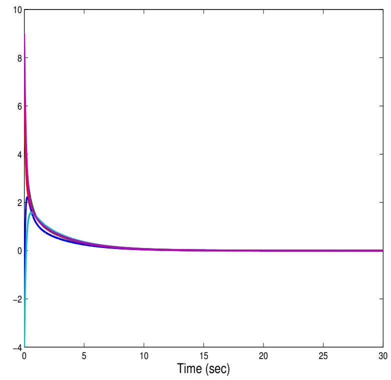

Fig. 5. Response of $(\theta_i - \theta_0)$ for $1 \leq i \leq 5$.

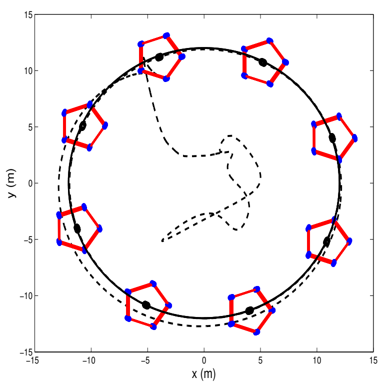

Fig. 6. The path of the centroid of the five robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the five robots at several moments (red pentagons).

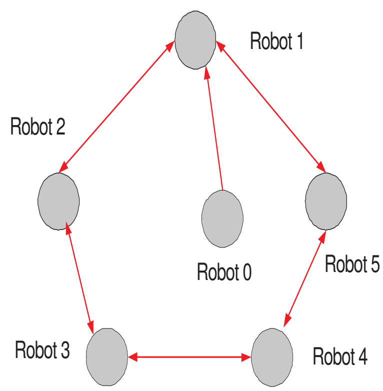

Fig. 7. Information interchange graph $\mathcal{G}$

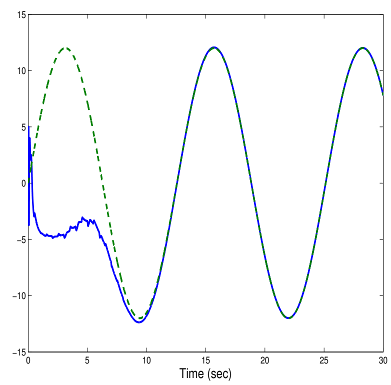

Fig. 8. Response of the centroid of x_i (solid) for $1 \leq i \leq 5$ and x_0 (dashed).

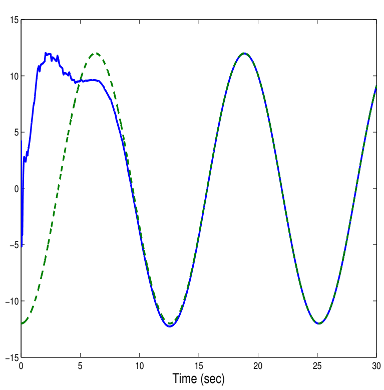

Fig. 9. Response of the centroid of y_i for $1 \leq i \leq 5$ and y_0.

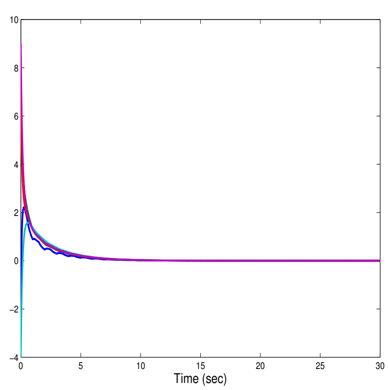

Fig. 10. Response of $(\theta_i - \theta_0)$ for $1 \leq i \leq 5$.

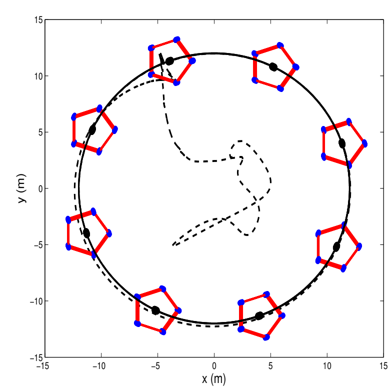

Fig. 11. The path of the centroid of the five robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the five robots at several moments (red pentagons).