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Asynchronous Dynamic Multi-Group Formation for Swarm Robots

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Abstract— **This paper addresses the asynchronous dynamic formation problem for swarm robots based on multi-group shape transformation. Considering robot swarm as a whole will make achieving the adaptation to the environment very difficult. Decomposition of multi-robot system into smaller groups gives capability and flexibility in complex pattern transformation which remarkably increases environmental adjustability. Unlike existing methods in dynamic formation control of swarm robots which is limited to synchronous shape transformation of the whole system, and is causing unnecessary repositioning of some robots; the proposed method can deal with asynchronous shape transformation which leads to efficient dynamic formation control. The stability of the system is examined by introducing a Lyapunov-like function. Simulation results are presented to illustrate the performance of the proposed method.**

I. INTRODUCTION

Formation control has gained wide applications in mobile autonomous agents [1], unmanned aerial vehicles [2] and mobile sensor networks [3],[4]. Several approaches have been developed to solve formation control problem which can be mainly categorized as behavior-based approach [5],[6], leader-follower approach [7] and virtual structure strategy [8].

In formation navigation, two types of formation structure are possible: fixed and dynamic formation. Fixed formation is more suitable for obstacle-free environments in which formation can be preplanned. Formation control with fixed structure has been studied extensively in the literature. Hendrickx et al. [9] introduced the persistent directed graph concept to analyze the rigidity of the shape formations. Some applications of rigid formation can be listed as: cooperative object transportation [10], automated highway systems [11] and satellite formation flying [12], [13]. For large number of robots, rigid formation can be interpreted as fixed shape formation, such that group of robots maintains a consistent shape during movement. Hsieh and Kumar [14] addressed 2D pattern formation by developing communication-less decentralized controllers. Cheah at al. [15] proposed a region-based shape controller for swarm of robots such that the robots move inside a desired moving region.

Despite of the above mentioned applications of the fixed formation, it is too restricted in utilization due to limitations in operating versatility. In real implementation, the group formation must have the ability to adapt to the environment and avoid obstacles. For small number of robots, dynamic formation is achievable by reconfiguration of robots' positions. Desai [16] studied the formation reconfiguration problem based on the transition from one control graph to another. Defoort et al. [17] addressed time-varying leader-follower formations. For large number of robots, dynamic formation can be obtained by changing the shape of the group. Belta and Kumar [18] introduced an abstraction manifold; such that shape variables can be controlled independently, which can lead to change in the shape of the formation. Hou et al. [19] addressed the dynamic region formation problem based on shape transformation. Sun et al. [20] considered generalized superellipse formation with time-varying parameters. Varghese and Mckee [21] addressed pattern transformation for swarm of robots based on Moebius transformation.

Existing methods on dynamic formation is based on synchronous shape transformation, in the sense that overall shape of the group is deformed simultaneously which leads to repositioning of all robots. In most applications repositioning of all robots is unnecessary and inefficient; therefore to eliminate the superfluous movements, asynchronous shape transformation must be applied. In this paper, we propose multi-group shape transformation to overcome asynchronous dynamic shape formation problem for swarm of robots. Decomposition of multi-robot system into smaller groups and independent control of each group gives capability in complex dynamic formation, such as group separation which is advantageous in obstacle avoidance, multi-group reconfiguration, and asynchronous shape transformation. Asynchronous shape transformation is defined as an independent shape formation control of each group. Here, asynchronous is considered from two different aspects: non-simultaneity in time such that deformation of groups can happen at the different moments, therefore local transformation can be achieved. The second aspect is non-consensusness in occurred transformations which means groups can deform with different transformation model like expansion in one group and contraction in another group. The main contributions of this paper can be summarized as follows: (i) introducing the idea of multi-group coordination to solve dynamic formation problem for swarm of robots; and simplifying the complex pattern transformation into multi-group shape transformation. (ii) Independent control of each group gives capability and flexibility in group separation and asynchronous shape transformation, which remarkably enhances the ability of environmental adjustability.

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The paper is organized as follows. Section 2 presents the equations describing the motion of multi-group of robots. Section 3 presents a multi-group shape transformation strategy. Social interactions which are decomposed to intra-group interactions and inter-group interactions, are also introduced. An adaptive interaction force is developed to deal with inter-group interactions. A control law for dynamic group formation is presented in Section 4. Section 5 presents the simulation results and section 6 concludes this paper.

II. Dynamic Equations of Multi-Group Robotic System

The dynamic equations of N groups of robots which each group has n_k members can be represented as follows :

$$\begin{cases} G_1 : M_{1i}\,(x_{1i})\,\ddot{x}_{1i} \;+\; C_{1i}\,(x_{1i},\dot{x}_{1i})\,\dot{x}_{1i} \;+\; D_{1i}\,(x_{1i})\,\dot{x}_{1i} \\ \qquad + g_{1i}(x_{1i}) = u_{1i} \\ G_2 : M_{2i}\,(x_{2i})\,\ddot{x}_{2i} \;+\; C_{2i}\,(x_{2i},\dot{x}_{2i})\,\dot{x}_{2i} \;+\; D_{2i}\,(x_{2i})\,\dot{x}_{2i} \\ \qquad + g_{2i}(x_{1i}) = u_{2i} \\ \vdots \\ G_N : M_{Ni}(x_{Ni})\ddot{x}_{Ni} + C_{Ni}(x_{Ni},\dot{x}_{Ni})\dot{x}_{Ni} + D_{Ni}(x_{Ni})\dot{x}_{Ni} \\ \qquad + g_{Ni}(x_{Ni}) = u_{Ni} \end{cases} \tag{1}$$

where G_k represent group k, the subscript ki refers to member i of group k, $k = 1,2,...,N$, $i = 1,2,...,n_k$, $x_{ki} \in \mathbb{R}^{\mathbb{N}}$ are generalized coordinates, $M_{ki}(x_{ki}) \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ are symmetric and positive definite inertia matrices, $C_{ki}(x_{ki},\dot{x}_{ki}) \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ are matrices of Coriolis and centripetal terms which together with inertia matrices satisfy skew-symmetric property, $D_{ki}(x_{ki}) \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ represent the positive definite damping matrices, $g_{ki}(x_{ki}) \in \mathbb{R}^{\mathbb{N}}$ denote gravitational force vectors, and $u_{ki} \in \mathbb{R}^{\mathbb{N}}$ denote the control inputs.

By linear parameterization, the dynamic equation of each robot can be written as [22]:

$$\begin{aligned} M_{ki}(x_{ki})\ddot{x}_{ki} + C_{ki}(x_{ki},\dot{x}_{ki})\dot{x}_{ki} + D_{ki}(x_{ki})\dot{x}_{ki} + g_{ki}(x_{ki}) \\ = \mathrm{Y}_{ki}(x_{ki},\dot{x}_{ki},\dot{x}_{ki},\ddot{x}_{ki})\theta_{ki} \end{aligned} \tag{2}$$

where $\mathrm{Y}_{ki}(x_{ki},\dot{x}_{ki},\dot{x}_{ki},\ddot{x}_{ki})$ are known regressor matrices and θ_{ki} are unknown parameter vectors.

III. Multi-Group Shape Transformation

In region based shape control, a controller is designed such that all robots move inside a specified region. The restriction of this method is the difficulty in representing complex shapes. By formulating a group coordination problem, we can decompose every complex shape to several simple regions. In this case, complex shapes can be formed by using some basic building blocks of regions such as circle, ellipse and rectangle. An example is alphabetic letters. As illustrated in figure 1, a mathematical description of letter "R" is difficult, but by dividing into simple regions the complex formation is formulated into group coordination problem of several simple regions.

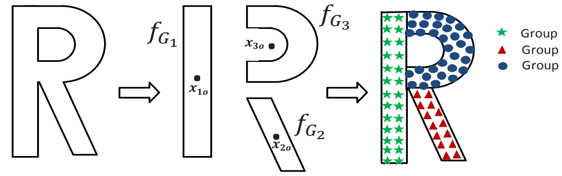

Fig. 1: Decomposition of R into simple regions.

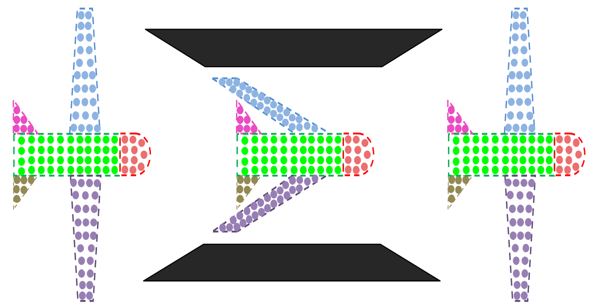

Fig. 2: An example of asynchronous dynamic formation of multi-group system.

The idea of multi-group coordination gives advantage not only in complex formation problem but also in complex pattern transformation. Independent control of each group make easy the achievement of group separation, multiple group reconfiguration and asynchronous shape transformation. An illustration of asynchronous dynamic formation of multi-group system is shown in figure 2. This example considers the movement of swarm of robots in plane-shaped formation through narrow passageway. Existing methods deal with this problem by contracting or rotating the group as a whole. However, this leads to unnecessary movements of robots located at the middle of the group. An efficient solution for this problem is the transformation of involved robots which in this case are those that formed wings of the plane. It can be seen that asynchronous dynamic formation not only increase environmental adjustability, but is also more efficient than synchronous transformation.

The desired shape functions that define time-varying shapes for each group can be specified as follows:

$$\begin{cases} f_{G_1} = [f_{G_{11}}(\Delta X_{1io1}),\, f_{G_{12}}(\Delta X_{1io2}), ...,\, f_{G_{1m_1}}(\Delta X_{1iom_1})]^T \leq 0 \\ f_{G_2} = [f_{G_{21}}(\Delta X_{2io1}),\, f_{G_{22}}(\Delta X_{2io2}), ...,\, f_{G_{2m_2}}(\Delta X_{2iom_2})]^T \leq 0 \\ \vdots \\ f_{G_N} = [f_{G_{N1}}(\Delta X_{Nio1}),\, f_{G_{N2}}(\Delta X_{Nio2}), ...,\, f_{G_{Nm_N}}(\Delta X_{Niom_N})]^T \leq 0 \end{cases} \tag{3}$$

where $\Delta X_{kiol} = (R_k S_k)^{-1} \Delta x_{kiol}$, such that $R_k(t)$ and $S_k(t)$ represent the time-varying rotation matrices and the time-varying scaling (or shearing) matrices which at least belong to class $\mathcal{C}^2$; $\Delta x_{kiol} = x_{ki} - x_{kol}$, $x_{kol}(t)$ is a reference point of the l^{th} desired region of group k, $l = 1,2,...,m_k$. $f_{G_{kl}}(\Delta X_{kiol})$ are shape functions such that the boundedness of $f_{G_k}(\Delta X_{kiol})$ ensure the boundedness of $\frac{\partial f_{G_k}(\Delta X_{kiol})}{\partial \Delta X_{kiol}}$. Since all the specified subregions for each group move with the same speed; therefore, x_{kol} is a constant offset of x_{ko}.

We define the following potential energy functions for each group

$$\begin{cases} P_{G_{1_i}}(\Delta X_{1i}) = \sum_{l=1}^{m_1} P_{G_{1l}}(\Delta X_{1iol}) \\ P_{G_{2_i}}(\Delta X_{2i}) = \sum_{l=1}^{m_2} P_{G_{2l}}(\Delta X_{2iol}) \\ \quad \vdots \\ P_{G_{N_i}}(\Delta X_{Ni}) = \sum_{l=1}^{m_N} P_{G_{Nl}}(\Delta X_{Niol}) \end{cases} \tag{4}$$

where $P_{G_{k_i}}(\Delta X_{kiol})$ is potential function for robot i of group k. The functions $P_{G_{kl}}(\Delta X_{kiol})$ are defined as follows

$$\begin{aligned} P_{G_{kl}}(\Delta X_{kiol}) &= \frac{1}{n} k_{kl} \left[\max\left(0, f_{G_{kl}}(\Delta X_{kiol})\right)\right]^n \\ &= \begin{cases} 0, & f_{G_{kl}}(\Delta X_{kiol}) \le 0 \\ \frac{k_{kl}}{n} f^n_{G_{kl}}(\Delta X_{kiol}), & f_{G_{kl}}(\Delta X_{kiol}) > 0 \end{cases} \end{aligned} \tag{5}$$

where k_{kl} are positive constants. n is positive constant, and is defined in a way that the potential functions at least belong to class $\mathcal{C}^2$. The shape control forces for whole swarm to form the specified region can be obtained by partial differentiation of potential functions with respect to ΔX_{kiol} as

$$\begin{cases} \Delta\xi_{1i} = \sum_{l=1}^{m_1} k_{kl} \left[\max\left(0, f_{G_{1l}}(\Delta X_{1iol})\right)\right]^{n-1} \left(\frac{\partial f_{G_{1l}}(\Delta X_{1iol})}{\partial \Delta X_{1iol}}\right)^T \\ \Delta\xi_{2i} = \sum_{l=1}^{m_2} k_{kl} \left[\max\left(0, f_{G_{2l}}(\Delta X_{2iol})\right)\right]^{n-1} \left(\frac{\partial f_{G_{2l}}(\Delta X_{2iol})}{\partial \Delta X_{2iol}}\right)^T \\ \quad \vdots \\ \Delta\xi_{Ni} = \sum_{l=1}^{m_N} k_{kl} \left[\max\left(0, f_{G_{Nl}}(\Delta X_{Niol})\right)\right]^{n-1} \left(\frac{\partial f_{G_{Nl}}(\Delta X_{Niol})}{\partial \Delta X_{Niol}}\right)^T \end{cases} \tag{6}$$

After defining the control force for shape formation, we define the interaction force between robots. In this regard two potential functions are defined; the first one is related to intra-group interactions which occur between members of a group to maintain minimum distance between them as well as keeping group unity. This potential function decomposes environment around each robot to four areas. Separation area, in order to keep minimum distance between robots; Neutral area, which robots can choose a desired range with respect to each other in order to increase the flexibility of movement; Attractive area in order to keep the group unity during movement; Inactive area, which appears by increasing distance among robots and vanishing in attractive force. The second potential function is related to inter-group interactions which occur between members of one group with other groups and only need to ensure minimum distance between them. The first proposed interaction potential functions for members of group k are defined as follows:

$$Q(\Delta X_{kikj}) = \sum_{kj \in N_{ki}} \frac{k_{kikj}}{n} \left[max\left(0, g(\Delta X_{kikj})\right)\right]^n \tag{7}$$

where N_{ki} refers to neighbors of robot i of group k; and for each ki and kj we have $\Delta X_{kikj} = (R_k S_k)^{-1} \Delta x_{kikj}$ such that $\Delta x_{kikj} = x_{ki} - x_{kj}$, k_{kikj} are positive constants and

$$g\left(\Delta X_{kikj}\right) = \left(1 - e^{-a_1(\|\Delta X_{kikj}\|^2 - d_1^2)}\right)\left(1 - e^{-a_2(\|\Delta X_{kikj}\|^2 - d_2^2)}\right) \le 0 \tag{8}$$

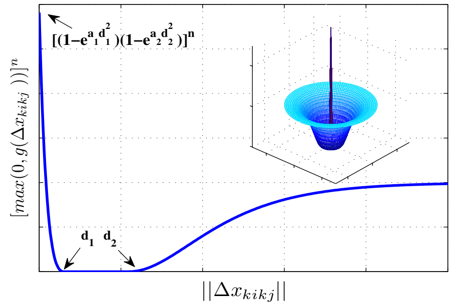

Fig. 3: Function $[max(0, g(.))]^n$ and potential function Q

where a_1, a_2, d_1 and d_2 are positive constants, such that, specify the width of each area. The function $[max(0, g(.))]^n$ and potential function Q is shown in figure 3. It can be seen that interaction forces only exert between one robot and its neighbors and by increasing the distance, the force goes to zero. In this case each robot only needs to know its neighboring robots. From (8) it can be seen that

$$g(\Delta X_{kikj}) = g(\Delta X_{kjki}) \tag{9}$$

$$\frac{\partial g(\Delta X_{kikj})}{\partial \Delta X_{kikj}} = -\frac{\partial g(\Delta X_{kjki})}{\partial \Delta X_{kjki}} \tag{10}$$

The interaction force between members of group k can be obtained by partial differentiation of the potential function with respect to ΔX_{kikj} as follows

$$\Delta\rho_{ki-kj} = \sum_{kj \in N_{ki}} \underbrace{k_{kikj} \left[\max\left(0, g(\Delta X_{kikj})\right)\right]^{n-1} \left(\frac{\partial g(\Delta X_{kikj})}{\partial \Delta X_{kikj}}\right)^T}_{\Delta\bar{\rho}_{ki-kj}} \tag{11}$$

To maintain a minimum distance between the members of neighboring groups, the following objective function is proposed

$$h(\Delta X_{ki\bar{k}j}) = -\left(1 - e^{-a(\|\Delta X_{ki\bar{k}j}\|^2 - d^2)}\right) \le 0 \tag{12}$$

where $\Delta X_{ki\bar{k}j} = (R_k S_k)^{-1} \Delta x_{ki\bar{k}j}$ such that $\Delta x_{ki\bar{k}j} = x_{ki} - x_{\bar{k}j}$ and subscript $\bar{k}j$ refers to those members which are neighbors of robot i of group k but don't belong to group k; a and d are positive constants. The following potential energy function can be defined

$$H(\Delta X_{ki\bar{k}j}) = \sum_{\bar{k}j \in N_{ki}} \frac{k_{ki\bar{k}j}}{n} \left[\max\left(0, h(\Delta X_{ki\bar{k}j})\right)\right]^n \tag{13}$$

where $k_{ki\bar{k}j}$ are positive constants. The function $[max(0, h(.))]^n$ and potential function H is shown in figure 4. The inter-group interaction force can be defined by partial differentiation of (13) with respect to $\Delta X_{ki\bar{k}j}$,

$$\Delta\psi_{ki-\bar{k}j} = \sum_{\bar{k}j \in N_{ki}} \underbrace{k_{ki\bar{k}j} \left[\max\left(0, h(\Delta X_{ki\bar{k}j})\right)\right]^{n-1} \left(\frac{\partial h(\Delta X_{ki\bar{k}j})}{\partial \Delta X_{ki\bar{k}j}}\right)^T}_{\Delta\bar{\psi}_{ki-\bar{k}j}} \tag{14}$$

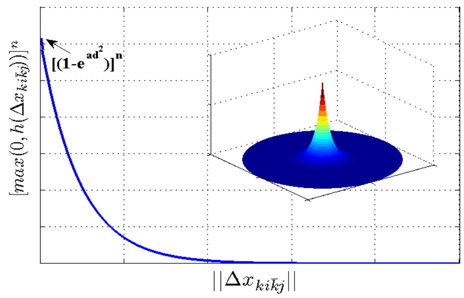

Fig. 4: Function $[max(0,h(.))]^n$ and potential function H

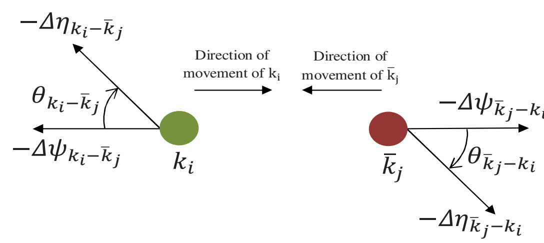

Fig. 5: Adaptive interaction force

In order to deal with the stability of multi-group system with inter-group interactions, we propose adaptive interaction force. In adaptive interaction force, the direction of interaction force is updating in such a way that robots from different groups will not get stuck in local minima (as illustrated in figure 5). Hence, adaptive interaction force, $\Delta\eta_{ki-\bar{k}j}$, for each ki and $\bar{k}j$, is considered as the interaction force which can update the direction of inter-robot force and can be mathematically expressed as follows

$$\Delta\eta_{ki-\bar{k}j} = \sum_{\bar{k}j\in N_{ki}} \mathcal{R}\left(\theta_{ki-\bar{k}j}\right)\Delta\bar{\psi}_{ki-\bar{k}j} \tag{15}$$

where $\Delta\bar{\psi}_{ki-\bar{k}j}$ can be computed based on (14) and $\mathcal{R}\left(\theta_{ki-\bar{k}j}\right)$ are rotation matrices and $\theta_{ki-\bar{k}j}$ are rotation angles that shall be defined in a way to guarantee the stability of system. The details will be given in later development.

IV. Asynchronous Dynamic Shape Control Methodology

In this section, we present the multi-group shape controller for formation of complex patterns. First, a reference vector $\dot{x}_{rki}$ is defined for member i of group k as follows:

$$\dot{x}_{rki} = \dot{x}_{ko} - (R_kS_k)\frac{d\left[(R_kS_k)^{-1}\right]}{dt}\Delta x_{ki} - (R_kS_k)\Delta\varepsilon_{ki} \tag{16}$$

where $\Delta x_{ki} = x_{ki} - x_{ko}$ and $\Delta\varepsilon_{ki} = \alpha_{ki}\Delta\xi_{ki} + \gamma\Delta\rho_{ki-kj}$, such that α_{ki} and γ are positive constants. By differentiating (16) with respect to time, we obtain

$$\begin{aligned}\ddot{x}_{rki} &= \ddot{x}_{ko} - \frac{d\left[(R_kS_k)\right]}{dt}\frac{d\left[(R_kS_k)^{-1}\right]}{dt}\Delta x_{ki} \\ &- (R_kS_k)\frac{d^2\left[(R_kS_k)^{-1}\right]}{dt^2}\Delta x_{ki} - (R_kS_k)\frac{d\left[(R_kS_k)^{-1}\right]}{dt}\Delta\dot{x}_{ki} \\ &- \frac{d\left[(R_kS_k)\right]}{dt}\Delta\varepsilon_{ki} - (R_kS_k)\Delta\dot{\varepsilon}_{ki}\end{aligned} \tag{17}$$

Using the reference vector, a sliding vector can be defined for robot i of group k as follows:

$$s_{ki} = \dot{x}_{ki} - \dot{x}_{rki} = \Delta\dot{x}_{ki} + (R_kS_k)\frac{d\left[(R_kS_k)^{-1}\right]}{dt}\Delta x_{ki} + (R_kS_k)\Delta\varepsilon_{ki} \tag{18}$$

where $\Delta\dot{x}_{ki} = \dot{x}_{ki} - \dot{x}_{ko}$. The feedback law for group formation for robot i of group k can be expressed as

$$\begin{aligned}u_{ki} &= -K_{ski}s_{ki} - (R_kS_k)^{-T}k_{p_k}\left\{\Delta\varepsilon_{ki} + \lambda_k\Delta\eta_{ki-\bar{k}j}\right\} \\ &+ \mathrm{Y}_{ki}(x_{ki},\dot{x}_{ki},\dot{x}_{rki},\ddot{x}_{rki})\widehat{\theta}_{ki}\end{aligned} \tag{19}$$

where K_{ski} are positive matrices, k_{p_k} are positive constants and superscript $-T$ refers to inverse transpose; $\hat{\theta}_{ki}$ are estimated parameters vectors. The parameters update laws are expressed as follows

$$\dot{\hat{\theta}}_{ki} = -L_{ki}\mathrm{Y}_{ki}^T(x_{ki},\dot{x}_{ki},\dot{x}_{rki},\ddot{x}_{rki})s_{ki} \tag{20}$$

where L_{ki} are positive constants. Therefore the closed-loop equations can be written as

$$\begin{aligned}&M_{ki}(x_{ki})\dot{s}_{ki} + C_{ki}(x_{ki},\dot{x}_{ki})s_{ki} + D_{ki}(x_{ki})s_{ki} \\ &+ K_{ski}s_{ki} + \mathrm{Y}_{ki}(x_{ki},\dot{x}_{ki},\dot{x}_{rki},\ddot{x}_{rki})\Delta\theta_{ki} \\ &+ (R_kS_k)^{-T}k_{p_k}\left\{\Delta\varepsilon_{ki} + \lambda_k\Delta\eta_{ki-\bar{k}j}\right\} = 0\end{aligned} \tag{21}$$

where $\Delta\theta_{ki} = \theta_{ki} - \hat{\theta}_{ki}$ and λ_k are positive constants.

For stability analysis the following Lyapunov-like candidate is proposed

$$\begin{aligned}V &= \sum_{k=1}^{N}\left[\sum_{i=1}^{n_k}\frac{1}{2}s_{ki}^TM_{ki}(x_{ki})s_{ki} + \sum_{i=1}^{n_k}\frac{1}{2}\Delta\theta_{ki}^TL_{ki}^{-1}\Delta\theta_{ki}\right. \\ &+ \sum_{i=1}^{n_k}\frac{1}{n}\alpha_{ki}k_{p_k}\sum_{l=1}^{m_k}k_{kl}[max(0,f_{G_{kl}}(\Delta X_{kiol}))]^n \\ &+ \left.\frac{1}{2}\sum_{i=1}^{n_k}\frac{1}{n}\gamma k_{p_k}\sum_{kj\in N_{ki}}k_{kikj}[max(0,g(\Delta X_{kikj}))]^n\right]\end{aligned} \tag{22}$$

By differentiating the Lyapunov-like candidate with respect to time, substituting the closed-loop equations and considering the skew-symmetricity of the matrix $\dot{M}_{ki}(x_{ki}) - 2C_{ki}(x_{ki},\dot{x}_{ki})$, $\dot{V}$ is obtained as follows

$$\begin{aligned}\dot{V} = \sum_{k=1}^{N}&\left[-\sum_{i=1}^{n_k}s_{ki}^TK_{ski}s_{ki} - \sum_{i=1}^{n_k}s_{ki}^TD_{ki}(x_{ki})s_{ki}\right. \\ &- \sum_{i=1}^{n_k}s_{ki}^T(R_kS_k)^{-T}k_{p_k}\left\{\Delta\varepsilon_{ki} + \lambda_k\Delta\eta_{ki-\bar{k}j}\right\} \\ &+ \left.\sum_{i=1}^{n_k}\alpha_{ki}k_{p_k}\Delta\dot{X}_{ki}^T\Delta\xi_{ki} + \frac{1}{2}\sum_{i=1}^{n_k}\gamma k_{p_k}\sum_{kj\in N_{ki}}\Delta\dot{X}_{kikj}^T\Delta\bar{\rho}_{ki-kj}\right]\end{aligned} \tag{23}$$

Using (9) and (10), it can be easily shown that

$$\sum_{i=1}^{n_k} \gamma k_{p_k} \sum_{kj \in N_{ki}} \Delta\dot{X}_{kikj}^T \Delta\bar{\rho}_{ki-kj} = 2\sum_{i=1}^{n_k} \gamma k_{p_k} \Delta\dot{X}_{ki}^T \Delta\rho_{ki-kj} \quad (24)$$

Therefore the derivative of Lyapunov-like candidate can be rewritten as

$$\dot{V} = \sum_{k=1}^{N} \left[-\sum_{i=1}^{n_k} s_{ki}^T K_{ski} s_{ki} - \sum_{i=1}^{n_k} s_{ki}^T D_{ki}(x_{ki}) s_{ki} - \sum_{i=1}^{n_k} k_{p_k} \Delta\varepsilon_{ki}{}^T \Delta\varepsilon_{ki} - \sum_{i=1}^{n_k} s_{ki}^T (R_k S_k)^{-T} k_{p_k} \lambda_k \Delta\eta_{ki-\bar{k}j} \right] \quad (25)$$

In order to meet the stability, we define inter-group interaction terms, $\Delta\eta_{ki-\bar{k}j}$, so that

$$\sum_{i=1}^{n_k} s_{ki}^T (R_k S_k)^{-T} k_{p_k} \lambda_k \Delta\eta_{ki-\bar{k}j} = 0 \quad (26)$$

In this regard, we update the adaptive interaction force (see (15)) between each member of group ki and its out-group neighbors $\bar{k}j$ as follows

$$\Delta\eta_{ki-\bar{k}j} = \sum_{\bar{k}j \in \mathbb{N}_{ki}} \mathcal{R}\left(\theta_{ki-\bar{k}j}\right) \Delta\bar{\psi}_{ki-\bar{k}j} \quad (27)$$

where $\mathcal{R}\left(\theta_{ki-\bar{k}j}\right)$ are rotation matrices and $\Delta\bar{\psi}_{ki-\bar{k}j}$ are introduced by equation (14). Substituting (27) into (26) and simplifying, (26) can be expressed as

$$y^T \mathcal{R}\left(\theta_{ki\bar{k}j}\right) \Delta\bar{\psi}_{ki-\bar{k}j} = 0 \quad (28)$$

where $y = k_{p_k}(R_k S_k)^{-1} s_{ki} - k_{p_{\bar{k}}}(R_{\bar{k}} S_{\bar{k}})^{-1} s_{\bar{k}j}$. We define rotation matrices such that equation (26) is satisfied; that is rotating the inter-group interaction force such that becomes perpendicular to y. In fact, asynchronous transformation of shapes for different groups causes different desired speed during transient response. In this regard, $\mathcal{R}\left(\theta_{ki-\bar{k}j}\right)$ is responsible to rotate inter-group interaction force in such a way that robots will not get stuck into local minima. For steady state response which groups move within the desired region, $y = 0$ therefore no updates in the rotation matrices are needed since equation (28) is satisfied for any $\mathcal{R}\left(\theta_{ki-\bar{k}j}\right)$, hence previous value of $\theta_{ki-\bar{k}j}$ can be used. Similar idea is also used in force control of robots [23]. An example of a rotation matrix in 2D is given as follows

$$\mathcal{R}(\theta_{ki\bar{k}j}) = \begin{bmatrix} \cos(\theta_{ki\bar{k}j}) & -\sin(\theta_{ki\bar{k}j}) \\ \sin(\theta_{ki\bar{k}j}) & \cos(\theta_{ki\bar{k}j}) \end{bmatrix} \quad (29)$$

where the rotation angle $\theta_{ki\bar{k}j}$ is defined as

$$\theta_{ki\bar{k}j} = \tan^{-1}\left\{ -\frac{y^T \Delta\bar{\psi}_{ki-\bar{k}j}}{y^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Delta\bar{\psi}_{ki-\bar{k}j}} \right\} \quad (30)$$

As it was mentioned, for the case that $y = 0$ no update in rotation angles are needed since equation (28) is satisfied for any $\mathcal{R}(\theta_{ki\bar{k}j})$; therefore, previous value of $\theta_{ki\bar{k}j}$ can be used. Hence, $\mathcal{R}(\theta_{ki\bar{k}j})$ is always defined.

Now we are able to state the following theorem:

Theorem: The multi-group system, represented by equation (1) with the adaptive control scheme (19) and the parameter update law (20), result in the convergence of $s_{ki} \to 0$ and $\Delta\varepsilon_{ki} \to 0$ for all $k = 1,2,...,N$, and $i = 1,2,...,n_k$, as $t \to \infty$.

Proof: Since $M_{ki}(x_{ki})$ and L_{ki} are positive definite, so V is positive definite in s_{ki} and $\Delta\theta_{ki}$. Therefore, s_{ki}, $f_{G_{kl}}(\Delta x_{kiol})$, $g(\Delta x_{kikj})$ and $h(\Delta x_{ki\bar{k}j})$ are bounded. Hence, $\Delta\xi_{ki}$, $\Delta\rho_{ki-kj}$ and $\Delta\psi_{ki-\bar{k}j}$ are bounded and from (16) we reach to the boundedness of $\dot{x}_{kri}$ and consequently from (18) we can conclude the boundedness of $\dot{x}_{ki}$. Since $\dot{x}_{ki}$ are bounded, we can conclude that $\Delta\dot{x}_{kikj}$ and $\Delta\dot{x}_{ki\bar{k}j}$ are bounded. Therefore $\Delta\dot{\xi}_{ki}$, $\Delta\dot{\rho}_{ki-kj}$ and $\Delta\dot{\psi}_{ki\bar{k}j}$ are bounded and the boundedness of $\ddot{x}_{ko}$ ensure the boundedness of $\ddot{x}_{rki}$. Hence, from closed loop equation (21) we can conclude the boundedness of $\dot{s}_{ki}$ and this lead to boundedness of $\ddot{V}$, which means that $\dot{V}$ is uniformly continuous. By Barbalat's Lemma, it then follows $\dot{V}$ goes to zero as $t \to \infty$, so it can be concluded that s_{ki} and $\Delta\varepsilon_{ki}$ goes to zero.

V. SIMULATIONS

In this section, we consider a scenario to demonstrate the performance of the proposed method in dealing with dynamic formation problem. In this scenario, swarm of 150 robots is decomposed to 6 groups to form a plane-shaped pattern. The building blocks of regions used to achieve the specified shape are consisted of circle, triangle rectangle and parallelogram. To examine the capabilities of the proposed method in dynamic situation, the swarm goes through a narrow passageway. Initial positions were generated randomly for all robots as shown in figure 6(a). Figures 6(b)-6(d) depict snapshots of the swarm movement. The solid lines represent the desired shape of each group. The controller parameters are set as $K_{ski} = diag\{50,50\}$, $k_{p_k} = 15$, $k_{kl} = 5$, $k_{kikj} = 1$ and $k_{ki\bar{k}j} = 1$.

VI. CONCLUSIONS

In this paper, we have proposed the multi-group shape transformation to deal with asynchronous dynamic formation of robot swarms. Asynchronous dynamic formation control increases the ability of whole system in adaptation to the environment. In multi-group robotic system, independent control of each group gives capability in achieving group separation, multiple group reconfiguration and asynchronous shape transformation. Unlike synchronous shape transformations which lead to unnecessary relocation of robots, asynchronous shape transformation is more efficient in the sense that redundant deformations are eliminated. A Lyapunov-like function has been presented for the stability analysis of the system. Finally, simulation results have been presented to illustrate the performance of the proposed method in achieving asynchronous dynamic formation.

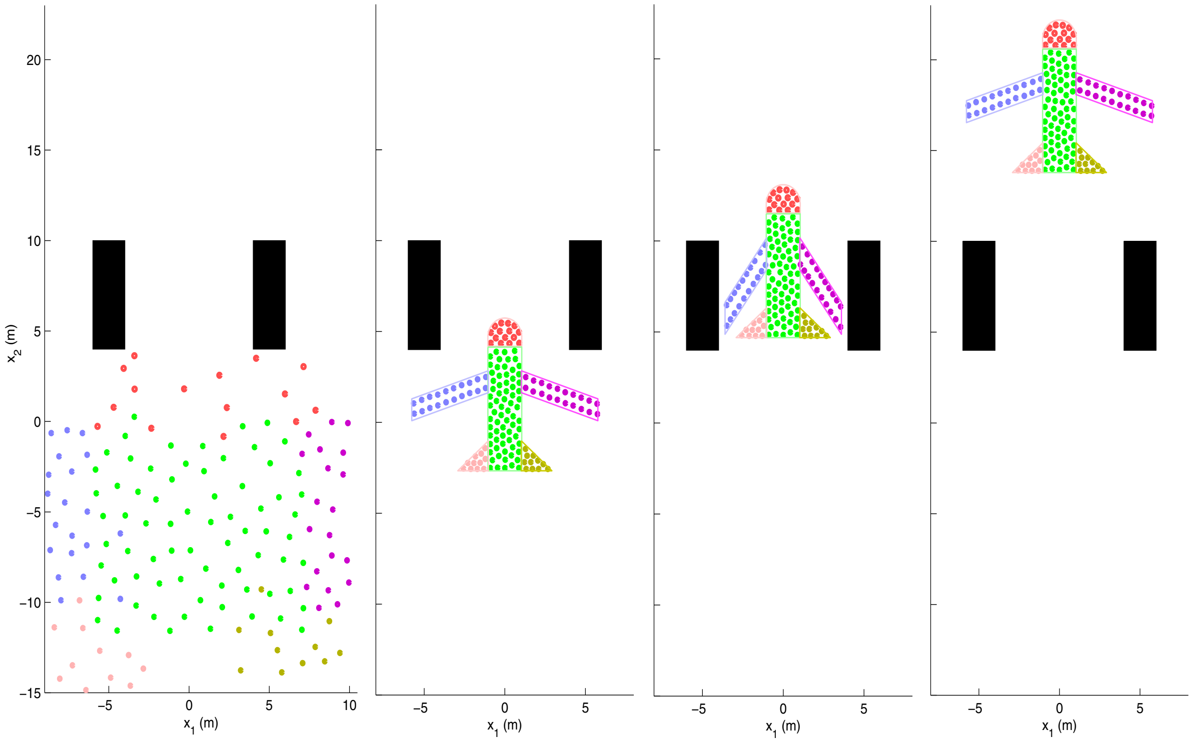

Fig. 6: Asynchronous dynamic plane-shaped formation by 6-group of robots.

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