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Output Control for Nonlinear System with Time-Varying Delay and Stability Analysis

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***Abstract*— This paper deals with the output stabilization of systems with sector-bounded nonlinearity and time-varying delay. In this paper we will consider the problem of absolute stability for a class of time-delay systems which can be represented as a feedback connection of a linear dynamical system with unknown parameters and a uncertain nonlinearity satisfying a sector constraint. For a class of output control algorithms a controller providing output exponential stability of nonlinear system is proposed. Certain results of application of theoretical robust and adaptive control algorithms for various Lego Mindstorms NXT mobile robots (track, wheel and walking ones) are presented in the article.**

I. INTRODUCTION

The absolute stability problem, formulated by Lurie and coworkers in the 40's, has been a well-studied and fruitful area of research, as presented in [19]. Since works of Lurie [19], interest of researches of control systems has been attracted by structures including linear block and nonlinear feedback static block. It is possible to allocate big enough number of works devoted to solution of problems of nonlinear systems stabilization for a case when output of the nonlinear block is given as control input of the linear block (see the review [13]). It is also possible to allocate a block of works [4], [12], [18] in which nonlinear part and static nonlinearity adjust with an input on control. However, approaches [4], [12], [18] are focused on stabilization of systems with nonlinearity, resulted to an input of system, and do not allow one to solve more general problems. Today problems of control of nonlinear systems in which nonlinearity is not coordinated with control are of interest. Among works devoted to these subjects one can allocate papers [2], [3], [14], [23] in which similar problem is considered. However, these results mentioned above are delay-independent. During the last two decades, the problem of stability of linear time- delay systems has been the subject of considerable research efforts. Many significant results have been presented in the literature (see for example [10]). However little attention has been focused on nonlinear time delayed systems.

The problem of stability for nonlinear systems with delay has been studied in [1], [7]–[9], [11], [20]–[22]. In paper [7] adaptive neural control was presented for a class of strict-feedback nonlinear systems with unknown time delays. For a case of measurability of the state vector [7], using appropriate Lyapunov-Krasovskii functionals, the uncertainties of unknown time delays are compensated for such that iterative backstepping design can be carried out. In [8] the relationships between the internal state and input dynamics of a controlled nonlinear delay system are studied. An interesting result is that a suitable stability assumption on the internal state dynamics ensures that, when the output is asymptotically driven to zero, both the state and control variables asymptotically decay to zero. In [22] the input-output linearization problem for retarded non-linear systems is considered, which have time-delays in the state. By using an extension of the Lie derivative for functional differential equations, authors derive coordinates transformation and a static state feedback to obtain linear input-output behaviour for a class of retarded non-linear systems. The obtained coordinates transformation is allowed to contain not only the current value of the state variables but also the past values of ones. In [11], a robust control problem of a class of nonlinear time delay systems is considered under the case when nonlinear uncertainties are bounded by first-order function. Geometrical method is employed to investigate the control problem of time delay systems in [20], [21]. Corresponding state feedback controller and output feedback controller are designed, but the strict conditions should be imposed on the investigated systems.

In this paper a robust version of the results on high-gain stabilization of nonlinear systems is extended to a class of nonlinear systems with time-varying delay without matching conditions. The given work, developing approaches obtained in [1], [7]–[9], [11], [20]–[22], offers new solution of stabilization problem of nonlinear system consisting of structures including a linear block and nonlinear feedback static block. Assuming, that parameters of the linear part and delay are unknown, the output is measured (but not its derivatives), and the characteristic of the nonlinear feedback block is unknown, a controller providing exponential stability of equilibrium position is designed. In this paper an interesting approach is offered, that does not use the procedure of linearization of nonlinear system, design of observer and iterative backstepping design.

II. PROBLEM FORMULATION

Consider the following nonlinear system

$$y(t) = \frac{b(p)}{a(p)}u(t) + \frac{c(p)}{a(p)}\omega(t), \qquad (1)$$

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where $p = d/dt$ denotes differential operator; output $y(t)$ is measured, but its derivatives are not measured; $b(p) = b_m p^m + \cdots + b_1 p + b_0$, $c(p) = c_r p^r + \cdots + c_1 p + c_0$, and $a(p) = p^n + \cdots + a_1 p + a_0$ are monic coprime polynomials with unknown coefficients; number $r \leq n-1$; transfer function $\frac{b(p)}{a(p)}$ has relative degree $\rho = n - m$; polynomial $b(p)$ is Hurwitz and parameter $b_m > 0$; unknown function $\omega(t) = \varphi(y(t-\tau))$ is such that:

$$|\varphi(y)| \leq C_0 \, |y| \quad \text{for all } y, \tag{2}$$

where $0 \leq \tau(t) \leq \tau_m$ is the unknown time-varying delay, τ_m is the maximum delay, $y(\vartheta) = \phi(\vartheta)$ for $\forall \vartheta \in [-\tau(0), 0]$ and number C_0 is unknown.

Following the problem formulation presented in [17] consider an assumption.

Assumption 1: The function $\tau(t)$ is a continuously differentiable function that satisfies

$$\dot{\tau}(t) < 1\,, \tag{3}$$

and such that

$$\pi = 1 - \sup_{t \geq 0} \dot{\tau}(t) > 0\,. \tag{4}$$

The purpose of control is to provide the exponential stability of nonlinear system (1).

Remark 1: Let's imagine that the delay is a continuous tube on which substance passes. The value τ of delay is assigned with the size of the tube divided on speed of substance movement. What does $\dot{\tau}(t)$ mean? Increasing of delay corresponds to increasing of tube size. If tube enlarges faster than the substance is able to overcome the tube then the substance would never finish passage through the tube. From this point of view one can find that if inequality (3) isn't carried out the output of the time delay unit $y(t-\tau)$ is not defined, or may be accept as a zero, since signal $y(t)$ is not yet overcome the delay unit. Moreover, the signal can't achieve the end of the time delay unit while the inequality (3) holds. Establishment of the assumption (1) is not a restriction of considered class of systems, but it is a specification for system with time-varying delay.

III. CONTROL DESIGN

Let the system (1) be such that transfer function $\frac{b(p)}{a(p)}$ has relative degree $\rho = 1$ and polynomial $b(p)$ is Hurwitz. Choose control of the following form [3], [5], [6]:

$$u(t) = -\mu y(t) + \nu(t), \tag{5}$$

where $\nu(t)$ is an additional input and parameter $\mu > 0$.

Lemma 1 ([2], [3], [5], [6], [13]): There exists some positive number μ_0, such that for any $\mu \geq \mu_0$ the system $y(t) = \frac{b(p)}{a(p)+\mu b(p)} \nu(t) + \frac{c(p)}{a(p)+\mu b(p)} \omega(t)$ has the SPR (strictly positive real) transfer function

$$H(p) = \frac{b(p)}{a(p) + \mu b(p)}. \tag{6}$$

Let $\rho > 1$ and the derivatives of output $y(t)$ be measured. Choose control in the following form

$$u(t) = \alpha(p) \bar{u}(t), \tag{7}$$

where any polynomial $\alpha(p) = \alpha_\rho p^{\rho-1} + \cdots + \alpha_2 p + \alpha_1$ is Hurwitz and $\bar{u}(t)$ is a new variable.

Then we can rewrite the model (1) in the following form:

$$y(t) = \frac{b(p)\alpha(p)}{a(p)} \bar{u}(t) + \frac{c(p)}{a(p)} \omega(t), \tag{8}$$

where polynomial $b(p)\alpha(p)$ is Hurwitz and the relative degree of the transfer function $\frac{b(p)\alpha(p)}{a(p)}$ is equal 1.

Choose $\bar{u}(t)$ according to the equation (5)

$$\bar{u}(t) = -\mu y(t) + \nu(t). \tag{9}$$

Substituting (9) into the equation (8), we obtain closed-loop system

$$y(t) = \frac{b(p)\alpha(p)}{a(p) + \mu b(p)\alpha(p)} \nu(t) + \frac{c(p)}{a(p) + \mu b(p)\alpha(p)} \omega(t). \tag{10}$$

Then by using *lemma 1* for some $\mu \geq \mu_0 > 0$ it is easy to see that the following transfer function is SPR

$$W(p) = \frac{b(p)\alpha(p)}{a(p) + \mu b(p)\alpha(p)}. \tag{11}$$

However control of the form (7), (9) can not be applied because it is impossible to measure derivatives of function $y(t)$. Choose the control

$$u(t) = -\alpha(p)(\mu + \kappa)\hat{y}(t), \tag{12}$$

where number μ and polynomial $\alpha(p)$ are such that the transfer function (11) is SPR, the positive parameter κ is used for compensation of the uncertainty $\varphi(y(t-\tau))$ (see proof of the *theorem 2*, inequality (43)) and the function $\hat{y}(t)$ is the estimation of output $y(t)$. The function $\hat{y}(t)$ is calculated according to the following algorithm

$$\begin{cases} \dot{\xi}_1 = \sigma \xi_2, \\ \dot{\xi}_1 = \sigma \xi_2, \\ \dots \\ \dot{\xi}_{\rho-1} = \sigma\left(-k_1 \xi_1 - \ldots - k_{\rho-1}\xi_{\rho-1} + k_1 y\right), \end{cases} \tag{13}$$

$$\hat{y} = \xi_1, \tag{14}$$

where number $\sigma > \mu + \kappa$ (see proof of the *theorem 2*, inequality (41)) and parameters k_i are calculated for the system (13) to be exponentially stable. Such control law is known as "Consecutive compensator" approach [2], [3].

It is obvious, that the control (12)–(14) is technically possible as contains known or measurable signals.

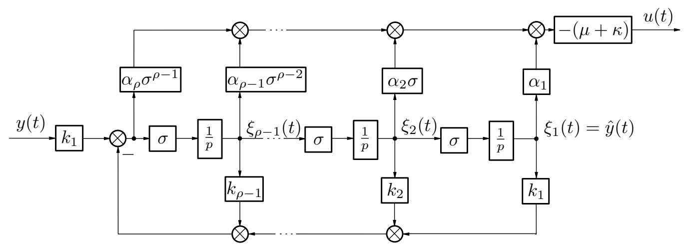


Fig. 1: The control law "Consecutive compensator".

Substituting (12) into equation (1), we obtain

$$\begin{aligned} y(t) &= \frac{b(p)}{a(p)}\left[-\alpha(p)(\mu+\kappa)\hat{y}(t)\right] + \frac{c(p)}{a(p)}\omega(t) \\ &= \frac{b(p)}{a(p)}\left[-\alpha(p)(\mu+\kappa)y(t) + \alpha(p)(\mu+\kappa)\varepsilon(t)\right] \\ &\quad + \frac{c(p)}{a(p)}\omega(t), \end{aligned} \tag{15}$$

where the error $\varepsilon(t) = y(t) - \hat{y}(t)$.

After simple transformations, for model (15) we have

$$\begin{aligned} (a(p) + \mu\alpha(p)b(p))\, y(t) = b(p)\alpha(p)\left[(\mu+\kappa)\varepsilon(t) - \kappa y(t)\right] \\ + c(p)\omega(t) \end{aligned} \tag{16}$$

and

$$\begin{aligned} y(t) &= \frac{b(p)\alpha(p)}{a(p) + \mu\alpha(p)b(p)}\left[-\kappa y(t) + (\mu+\kappa)\varepsilon(t)\right] \\ &\quad + \frac{c(p)}{a(p)+\mu\alpha(p)b(p)}\omega(t), \end{aligned} \tag{17}$$

where transfer function $W(p) = \frac{b(p)\alpha(p)}{a(p)+\mu\alpha(p)b(p)}$ is SPR (see equation (11)).

Let us present model (17) in the form

$$\dot{x}(t) = Ax(t) + b(-\kappa y(t) + (\mu+\kappa)\varepsilon(t)) + q\omega(t), \tag{18}$$

$$y(t) = c^T x(t), \tag{19}$$

where $x \in \mathbb{R}^n$ is a state vector of system (18); A, b, q and c are appropriate matrix and vectors of transition from model (17) to model (18), (19).

Since transfer function $W(p)$ is SPR then

$$A^T P + PA = -R, \qquad Pb = c, \tag{20}$$

where $R = R^T$ and parameters of matrix R depend on μ and do not depend on κ.

Let us rewrite model (13), (14) in the form

$$\dot{\xi}(t) = \sigma(\Gamma\xi(t) + dy(t)), \tag{21}$$

$$\hat{y}(t) = h^T\xi(t), \tag{22}$$

where $\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{\rho-1} \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ k_1 \end{bmatrix}$, and $h^T = \begin{bmatrix} 1 & 0 & 0 & \dots 0 \end{bmatrix}$.

Consider vector

$$\eta(t) = hy(t) - \xi(t), \tag{23}$$

then by force of vector h structure the error $\varepsilon(t)$ will become

$$\begin{aligned} \varepsilon(t) &= y(t) - \hat{y}(t) = h^T hy(t) - h^T\xi(t) \\ &= h^T(hy(t) - \xi(t)) = h^T\eta(t). \end{aligned} \tag{24}$$

For derivative of $\eta(t)$ we obtain

$$\begin{aligned} \dot{\eta}(t) &= h\dot{y}(t) - \sigma(\Gamma(hy(t) - \eta(t)) + dy(t)) \\ &= h\dot{y}(t) + \sigma\Gamma\eta(t) - \sigma(d + \Gamma h)y(t). \end{aligned} \tag{25}$$

Since $d = -\Gamma h$ (can be checked by substitution), then

$$\dot{\eta}(t) = h\dot{y}(t) + \sigma\Gamma\eta(t), \varepsilon(t) = h^T\eta(t), \tag{26}$$

where matrix Γ is Hurwitz by force of calculated parameters k_i of system (13) and

$$\Gamma^T N + N\Gamma = -M, \tag{27}$$

where $N = N^T > 0$, $M = M^T > 0$.

Theorem 2: Consider the nonlinear system (18), (19), (26). Let number $\rho = n - m \geq 1$ and unknown function $\omega(t) = \varphi(y(t-\tau))$ be such that:

$$|\varphi(y(t-\tau))| \leq C_0\,|y(t-\tau)| \;\text{ for all } y(t-\tau), \tag{28}$$

where $\tau = \tau(t) > 0$ is the time-varying delay, $y(\vartheta) = \phi(\vartheta)$ for $\forall\vartheta \in [-\tau(0), 0]$ and number C_0 is unknown.

For all $\kappa \geq \kappa_0 > 0$ and $\sigma \geq \sigma_0 > 0$, where κ_0 and σ_0 are some constants that depend on plant parameters, the nonlinear system (18), (19), (26) is exponentially stable at the origin in the sense of the norm

$$\left(\|x(t)\|^2 + \|\eta(t)\|^2 + \int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta\right)^{1/2}. \tag{29}$$

Proof: Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \eta^T(t)N\eta(t) \\ &\quad + \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta. \end{aligned} \tag{30}$$

Differentiating (30) yields

$$\begin{aligned} \dot{V}(t) &= x^T(t)\left(A^T P + PA\right)x(t) - 2\kappa x^T(t)Pby(t) \\ &\quad + 2(\mu+\kappa)x^T(t)Pbh^T\eta(t) + 2x^T(t)Pq\omega(t) \\ &\quad + \eta^T(t)\sigma(\Gamma^T N + N\Gamma)\eta(t) + 2\eta^T(t)Nhc^T Ax(t) \\ &\quad + 2(\mu+\kappa)\eta^T(t)Nhc^T bh^T\eta(t) \\ &\quad + 2\eta^T(t)Nhc^T q\omega(t) - 2\kappa\eta^T(t)Nhc^T by(t) \\ &\quad + \kappa y^2(t) - \kappa e^{-\tau}y^2(t-\tau)\left(1 - \dot{\tau}\right) \\ &\quad - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta. \end{aligned} \tag{31}$$

Substituting in (31) equations (20), (27) and taking into account inequalities

$$\begin{aligned} 2x^T(t)Pbh^T\eta(t) &\leq \delta x^T(t)Pbb^T Px(t) \\ &\quad + \delta^{-1}\eta^T(t)hh^T\eta(t), \end{aligned} \tag{32}$$

$$\begin{aligned} 2x^T(t)Pq\omega(t) &\leq \delta x^T(t)Pqq^T Px(t) \\ &\quad + \delta^{-1}[\omega(t)]^2, \end{aligned} \tag{33}$$

$$\begin{aligned} 2\eta^T(t)Nhc^T bh^T\eta(t) &\leq \eta^T(t)Nhc^T bb^T ch^T N\eta(t) \\ &\quad + \eta^T(t)hh^T\eta(t), \end{aligned} \tag{34}$$

$$\begin{aligned} 2\eta^T(t)Nhc^T Ax(t) &\leq \delta^{-1}\eta(t)Nhc^T AA^T ch^T N\eta^T(t) \\ &\quad + \delta x^T(t)x(t), \end{aligned} \tag{35}$$

$$\begin{aligned} 2\eta^T(t)Nhc^T q\omega(t) &\leq \kappa\eta^T(t)Nhc^T qq^T ch^T N\eta(t) \\ &\quad + \kappa^{-1}[\omega(t)]^2, \end{aligned} \tag{36}$$

$$\begin{aligned} -2\kappa\eta^T(t)Nhc^T by(t) &\leq \delta^{-1}\kappa\eta^T(t)Nhc^T bb^T ch^T N\eta(t) \\ &\quad + \delta\kappa x^T(t)Pbb^T Px(t), \end{aligned} \tag{37}$$

we obtain

$$\begin{aligned}\dot V(t) \le & -x^T(t)Rx(t) - \sigma\eta^T(t)M\eta(t)\\ & - \kappa x^T(t)Pbb^TPx(t) + \delta(\mu+\kappa)x^T(t)Pbb^TPx(t)\\ & + \delta^{-1}(\mu+\kappa)\eta^T(t)hh^T\eta(t) + \delta x^T(t)Pqq^TPx(t)\\ & + \delta^{-1}[\omega(t)]^2 + (\mu+\kappa)\eta^T(t)Nhc^Tbb^Tch^TN\eta(t)\\ & + (\mu+\kappa)\eta^T(t)hh^T\eta(t)\\ & + \delta^{-1}\eta(t)Nhc^TAA^Tch^TN\eta^T(t)\\ & + \delta x^T(t)x(t) + \kappa\eta^T(t)Nhc^Tqq^Tch^TN\eta(t)\\ & + \kappa^{-1}[\omega(t)]^2 + \delta^{-1}\kappa\eta^T(t)Nhc^Tbb^Tch^TN\eta(t)\\ & + \delta\kappa x^T(t)Pbb^TPx(t) - \kappa\pi e^{-\tau_m}y^2(t-\tau)\\ & - \int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta. \qquad (38)\end{aligned}$$

where the number $\delta > 0$, $e^{-\tau} \ge e^{-\tau_m}$, and $1-\dot\tau \ge \pi$ with respect to the assumption 1.

Let the number $0 < \delta < 0.5$ be such that

$$\begin{aligned}-R + \delta I + (\delta\mu + 2\delta\kappa - \kappa)Pbb^TP\\ +\delta Pqq^TP \le -Q_1 < 0. \qquad (39)\end{aligned}$$

Substituting (39) into the inequality (38), we obtain

$$\begin{aligned}\dot V(t) \le & -x^T(t)Q_1x(t) - \sigma\eta^T(t)M\eta(t)\\ & + \delta^{-1}(\mu+\kappa)\eta^T(t)hh^T\eta(t)\\ & + \delta^{-1}[\omega(t)]^2 + (\mu+\kappa)\eta^T(t)Nhc^Tbb^Tch^TN\eta(t)\\ & + (\mu+\kappa)\eta^T(t)hh^T\eta(t)\\ & + \delta^{-1}\eta(t)Nhc^TAA^Tch^TN\eta^T(t)\\ & + \kappa\eta^T(t)Nhc^Tqq^Tch^TN\eta(t)\\ & + \kappa^{-1}[\omega(t)]^2 + \delta^{-1}\kappa\eta^T(t)Nhc^Tbb^Tch^TN\eta(t)\\ & -\kappa\pi e^{-\tau_m}y^2(t-\tau) - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta. \quad (40)\end{aligned}$$

Let number σ be such that the following ratio is executed

$$\begin{aligned}-\sigma M + \delta^{-1}(\mu+\kappa)hh^T + (\mu+\kappa)Nhc^TbbTch^TN\\ +\kappa Nhc^Tqq^Tch^TN + \delta^{-1}\kappa Nhc^Tbb^Tch^TN \le -Q_2 < 0,\end{aligned} \qquad (41)$$

or $\sigma \ge \sigma_0$ where $\sigma_0 > 0$ corresponds to an equality in (41).

Substituting (41) into the inequality (40), we have

$$\begin{aligned}\dot V(t) \le & -x^T(t)Q_1x(t) - \eta^T(t)Q_2\eta(t) + \left(\delta^{-1}+\kappa^{-1}\right)[\omega(t)]^2\\ & -\kappa\pi e^{-\tau_m}y^2(t-\tau) - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta\\ \le & -x^T(t)Q_1x(t) - \eta^T(t)Q_2\eta(t)\\ & + \left(\left(\delta^{-1}+\kappa^{-1}\right)C_0^2 - \kappa\pi e^{-\tau_m}\right)y^2(t-\tau)\\ & - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta \qquad (42)\end{aligned}$$

where from (2) $[\omega(t)]^2 \le C_0^2y^2(t-\tau)$.

Let number κ be such that

$$\kappa \ge C_0^2\pi e^{\tau_m}\left(\kappa^{-1}+\delta^{-1}\right). \qquad (43)$$

Remark 2: It is easy to show that the number $\kappa_0 > 0$ exists such that for $\forall\kappa \ge \kappa_0$ the inequality (43) holds.

Then we have

$$\begin{aligned}\dot V(t) \le & -x^T(t)Q_1x(t) - \eta^T(t)Q_2\eta(t)\\ & - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta. \qquad (44)\end{aligned}$$

From the expression (44) follows asymptotic stability of system (18), (19), (26). Now we are ready to show the exponential stability of the closed-loop system.

$$\begin{aligned}\dot V(t) \le & -\lambda_{\min}\{Q_1\}\|x(t)\|^2 - \lambda_{\min}\{Q_2\}\|\eta(t)\|^2\\ & - \kappa\int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta\\ \le & -\gamma_1\left(\|x(t)\|^2 + \|\eta(t)\|^2 + \int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta\right), \quad (45)\end{aligned}$$

where $\gamma_1 = \min\{\lambda_{\min}\{Q_1\}; \lambda_{\min}\{Q_2\}; \kappa\} > 0$. $\lambda_{\min}\{Q_1\}$ and $\lambda_{\min}\{Q_2\}$ are the minimum eigen values of the matrices Q_1 and Q_2.

From (30) we have

$$V(t) \le \gamma_2\left(\|x(t)\|^2 + \|\eta(t)\|^2 + \int_{t-\tau}^{t} e^{-t+\theta}y^2(\theta)d\theta\right), \quad (46)$$

where $\gamma_2 = \max\{\lambda_{\max}\{P\}; \lambda_{\max}\{N\}; \kappa\} > 0$, $\lambda_{\max}\{P\}$ and $\lambda_{\max}\{N\}$ are the maximum eigen values of the matrices P and N correspondingly.

Substitution (46) in (45) yields the condition

$$\dot V(t) \le -\frac{\gamma_1}{\gamma_2}V(t), \qquad (47)$$

which completes the proof of exponential stability. ■

IV. ADAPTIVE CONTROL LAW

In some cases, the problem of choosing the parameters κ, μ, and σ of regulator (12)–(14) which satisfy *theorem 2* can arise (see (39), (41), (43)). This choice presents no appreciable difficulties for known polynomials $a(p)$, $b(p)$, and $c(p)$ of the plant (1) and also for a definite number C_0. However, if the parameters of the plant (1) are unknown, the problem of calculating κ, μ, and σ may prove to be problematic. As was demonstrated by theorem, if condition (41) is met, then there exists a number such that $\sigma > \mu + \kappa$. A possible variant of adjustment of κ, μ, and σ lies in increasing them until the following goal condition is met

$$|y(t)| < \delta_0, \quad \forall t \ge t_1, \qquad (48)$$

where the number δ_0 is set by the designer of the control system.

This concept may be realized by using the following adjustment algorithm

$$\tilde k(t) = \int_{t_0}^{t}\lambda(\theta)d\theta, \qquad (49)$$

where $\tilde k(t) = \kappa + \mu$ and the function $\lambda(t)$ is as follows

$$\lambda(t) = \begin{cases}\lambda_0, & \text{for } |y(t)| > \delta_0,\\ 0, & \text{for } |y(t)| \le \delta_0,\end{cases}$$

where the number $\lambda_0 > 0$.

We take σ as follows

$$\sigma = \varsigma_0 \tilde{k}^2, \tag{50}$$

where $\varsigma_0 > 0$. Obviously, for this calculation of σ there will be a time instant $t_1 > t_0$ such that the following conditions (39), (41), (43) of *theorem 2* are satisfied.

V. SIMULATION RESULTS

Consider the following nonlinear system

$$y(t) = \frac{b(p)}{a(p)}u(t) + \frac{c(p)}{a(p)}\omega(t), \tag{51}$$

where polynomials $b(p) = b_0$, $a(p) = p(p - a_1)(p - a_2)$ $c(p) = c_0 p - c_1$ with unknown parameters b_0, a_1, a_2, c_0, c_1, and the nonlinear function $\omega(t) = \varphi(y(t - \tau)) = y(t - \tau)\sin y(t - \tau)$.

Choose the control according to equation (12)

$$u(t) = -\alpha(p)(\mu + \kappa)\hat{y}(t), \tag{52}$$

where for relative degree $\rho = 3$ the polynomial $\alpha(p)$ we choose as

$$\alpha(p) = (p + 1)^2, \tag{53}$$

Then we can rewrite the control in the following form

$$\begin{aligned} u(t) &= -(p + 1)^2(\mu + \kappa)\hat{y}(t), \\ &= -(\mu + \kappa)\left(\ddot{\hat{y}}(t) + 2\dot{\hat{y}}(t) + \hat{y}(t)\right), \end{aligned} \tag{54}$$

where the positive numbers $\mu > 0$, $\kappa > 0$ and the function $\hat{y}(t)$ is calculated by the following algorithm

$$\begin{cases} \dot{\xi}_1(t) = \sigma\xi_2(t), \\ \dot{\xi}_2(t) = \sigma\left(-k_1\xi_1(t) - k_2\xi_2(t) + k_1 y(t)\right), \end{cases} \tag{55}$$

$$\hat{y} = \xi_1, \tag{56}$$

where the number $\sigma > \mu + \kappa$ and parameters k_i are $k_1 = 1$, $k_2 = 2$.

Choose the parameters $\tilde{k} = \mu + \kappa = 7$ and $\sigma = 15$. The results of a computer simulation for variable $y(t)$ for the case $b_0 = 1$, $a_1 = 1$, $a_2 = 0.5$, $c_0 = 1$, $c_1 = 3$, $\tau(t) = 3 + 0.5\sin(0.7t + 1) + \sin(0.6t)$ are presented in Fig. 2. For adaptive case we use parameters $\tilde{k}(0) = 5$, $\lambda = 2$ and $\varsigma_0 = 20$.

We can see that proposed controller provides stability of equilibrium $y = 0$ as for static as for adaptive adjusted parameters $\tilde{k}$ and σ .

VI. EXPERIMENTAL APPROVAL

To approve the effectiveness of proposed controller "consecutive compensator" we have assembled a number of Lego NXT based mobile robots. These robots are the part of laboratory setups built by students and for students who would like to start research and develop activity during the education period.

The first robot (fig. 3a) with full-track drive is used as plant. Any object, for example, a book, can be a tracking object . Ultrasonic sensor installed in the model measures the distance to the book. Control algorithm compares the current distance with the given distance and forms a control signal to the servodrives in accordance with the tracking error.

Control law is of the form $u_1(t) = u(t)$, $u_2(t) = u(t)$, where u_1 and u_2 are control signals input directly to servodrives. To calculate control signal $u(t)$ we will apply the algorithm (12)–(14) where the track error $y(t)$ is stabilized as the output system variable.

The second problem of mobile robot control is the motion along an unknown trajectory (fig. 3b). The sensor measures the distance to the wall along which the motion is occurring. The wall curvature is not defined in advance. Walking towards the wall mobile robot defines the distance to it, compares the value with the given value and forms control signals on the drives on the basis of tracking error. Control law is of the form $u_1(t) = u_v(t) + u(t)$, $u_2(t) = u_v(t) - u(t)$, where u_1 and u_2 are control signals input directly to servodrives, function $u_v(t)$ is chosen proportionally to the velocity of the robot on the direct site of the trajectory, function $u(t)$ brings in mismatch between the drives and it makes robot to turn left or right. To calculate control signal $u(t)$ we will use the algorithm (12)–(14). Let us make the task more difficult again, and let us suppose the robot has only two wheels (fig. 3c). Figure 3c shows the result of control: the robot stands in the vertical position and keeps the given distance equal to 40 cm to the object. Figure 3c shows that the surface, where robot is balancing, is tilted, and the tilting angle is slowly changed, at that the robot keeps vertical position and the given distance to the barrier. Let us make one more step together with the bipedal walking robot. Figure 3d shows the result of solving the classic problem of tracking for the object (folder in this case) at the given distance.

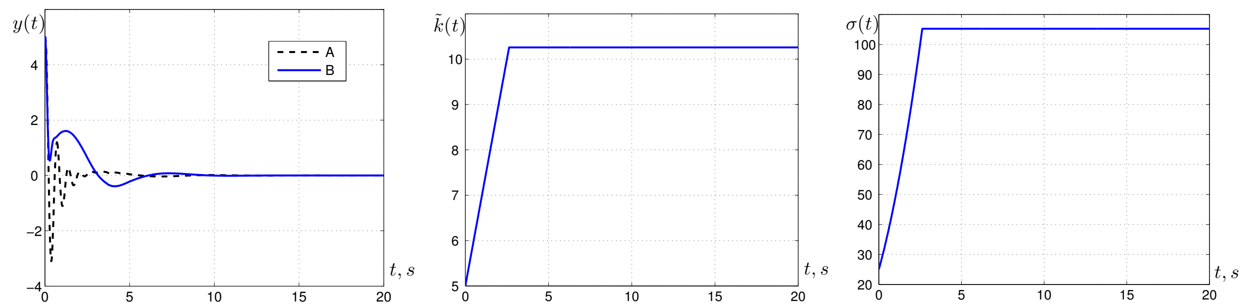
VII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper has extended the theory of output feedback control of time-delay nonlinear systems. The control law (12)–(14), providing output exponential stability of system (1) was designed. Only measurements of the output, but not its derivatives were used. Dimension of the robust controller (12)–(14) is $\rho - 1$ and dimension in the adaptive case (12)–(14), (49), (50) is ρ. Several application results of theoretical robust and adaptive control algorithms for various Lego Mindstorms NXT mobile robots are presented.

B. Future Works

The most interesting problem is the output control of the nonlinear system with parametric and functional uncertainties and the input delay. In [24], [25] the control problem is considered for the linear plants with the input delay. The feedback controller based on approaches presented in [15], [16] and [26] allows to reject an unknown biased sinusoidal disturbance for an internally unstable plant with the input delay. Also it is worth to study the adaptive case, when the adaptive law of controller's parameters adjusting provides exponential stability for the closed-loop system with unknown parameters.



(a) Transients for $y(t)$ with static $\tilde{k}$ and σ (line A) and adaptive adjusting (line B)

(b) Adjusting of parameter $\tilde{k}$

(c) Adjusting of parameter σ

Fig. 2: Transients for the closed-loop system.

(a)

(b)

(c)

(d)

Fig. 3: Motion of mobile robot along indeterminate trajectory.

REFERENCES

[1] P. - A. Bliman, "Lyapunov-Krasovskii functionals and frequency domain: delay-independent absolute stability criteria for delay systems", *International Journal of Robust and Nonlinear Control,* N. 11, pp. 771–788, 2001.

[2] A. Bobtsov, "A note to output feedback adaptive control for uncertain system with static nonlinearity", *Automatica*, vol. 41, N.12, pp. 1277–1280, 2005.

[3] A. Bobtsov and N. Nikolaev, "Fradkov theorem-based design of the control of nonlinear systems with functional and parametric uncertainties", *Automation and Remote Control,* vol. 66, N. 1, pp. 108–118, 2005.

[4] C. I. Byrnes and A. Isidori, "Asymptotic stabilization of minimum phase nonlinear systems", *IEEE Trans. Automat. Contr.,* vol. 36, N. 10, pp. 1122–1137, 1991.

[5] A. L. Fradkov, "Synthesis of adaptive system of stabilization of linear dynamic plants", *Automation and Remote Control,* vol. 35, N. 12, pp. 1960–1966, 1974.

[6] A. L. Fradkov, "Passification of nonsquare linear systems and Yakubovich-Kalman-Popov Lemma", *European Journal of Control*, N. 6, pp. 573–582, 2003.

[7] S. S. Ge, F. Hong, T. H. Lee, "Adaptive neural network control of nonlinear systems with unknown time delays", *IEEE Trans. Automat. Contr.*, vol. 48, N. 11, pp. 2004–2010, 2003.

[8] A. Germani, C. Manes and P. Pepe, "Input-output linearization with delay cancellation for nonlinear delay systems: the problem of the internal stability", *International Journal of Robust and Nonlinear Control,* vol. 13, N. 9, pp. 909–937, 2003.

[9] F. Gouaisbaut, Y. Blanco, J. P. Richard, "Robust control of nonlinear time delay system: a sliding mode control design", in *Proñ. 5th IFAC Symposium Nonlinear Control Systems,* Russia, Saint-Petersburg, 2001.

[10] K. Gu, V. L. Kharitonov, J. Chen, *Stability of time-delay systems*, Boston: Birkhuser, 2003.

[11] C. Hua, C. Long and X. Guan, "Robust stabilization of uncertain dynamic time delay systems with unknown bounds of uncertainties", in *Proc. Amer. Control Conf.*, pp. 3365–3370, 2002.

[12] H.K. Khalil, F. Esfandiari, "Semiglobal stabilization of a class of nonlinear systems using output feedback", *IEEE Trans. Automat. Contr.*, vol. 38, N. 9, pp. 1412–1415, 1993.

[13] P. V. Kokotovic, M. Arcak, "Constructive nonlinear control: a historical perspective", *Automatica,* vol. 37, N. 5, pp. 637–662, 2001.

[14] M. Krstic, P. Kokotovic, "Adaptive nonlinear output-feedback schemes with Marino-Tomei controller", *IEEE Trans. Automat. Contr.*, vol. 41, N. 2, pp. 274–280, 1996.

[15] M. Krstic, and A. Smyshlyaev, "Backstepping boundary bontrol for first-order hyperbolic PDEs and application to systems with actuator and sensor delays," *Systems & Control Letters*, vol. 57, pp. 750–758, 2008.

[16] M. Krstic, *Delay Compensation for Nonlinear, Adaptive and PDE Systems*, Birkhauser, 2009.

[17] M. Krstic, "Lyapunov Stability of Linear Predictor Feedback for Time-Varying Input Delay", *IEEE Trans. Automat. Contr.*, vol. 55, N. 2, pp. 554–559, 2010.

[18] Z. Lin, A. Saberi, "Robust semiglobal stabilization of minimum-phase input-output linearizable systems via partial state and output feedback", *IEEE Trans. Automat. Contr.,* vol. 40, N. 6, pp. 1029–1041, 1995.

[19] A. I. Lurie, V. N. Postnikov, "On the stability theory of control systems", *Prikl. Mat. i Mekh.,* pp. 246–248, 1944.

[20] L. A. Marquez-Martines, C.H. Moog, "Input-output feedback linearization of time-delay systems". IEEE Trans. Automat. Contr., 49(5), 781-786.

[21] C. H. Moog, R. Castro-Linares, M. Velasco-Villa, L. A. Marquez-Martines, "The disturbance decoupling problem for time delay nonlinear systems", *IEEE Trans. Automat. Contr.,* vol. 45, N. 2, pp. 305–309, 2000.

[22] T. Oguchi, A. Watanabe, T. Nakamizo, "Input-output linearization of retarded non-linear systems by using an extension of Lie derivative", *International Journal of Control,* vol. 75, N. 8, pp. 582–590, 2002.

[23] L. Praly, "Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate", *IEEE Trans. Automat. Contr.,* vol. 48, N. 6, pp. 1103–1108, 2003.

[24] A. Pyrkin, A. Smyshlyaev, N. Bekiaris-Liberis, M. Krstic, "Rejection of Sinusoidal Disturbance of Unknown Frequency for Linear System with Input Delay", in *Proc. American Control Conference*, Baltimore, 2010.

[25] A. Pyrkin, A. Smyshlyaev, N. Bekiaris-Liberis, M. Krstic, "Output control algorithm for unstable plant with input delay and cancellation of unknown biased harmonic disturbance", in *Proc. 9th IFAC Workshop on Time Delay System*, Prague, Czech Republic, 2010.

[26] A. Pyrkin, The adaptive compensation algorithm of an uncertain biased harmonic disturbance for the linear plant with the input delay, *Automation and Remote Control*, 2010, vol. 71, N. 8, pp. 1562–1577.