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Robust Control and Scheduling Codesign for Networked Embedded Control Systems

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***Abstract*— Robust control and scheduling for networked embedded control systems (NECS) with uncertain but interval-bounded time-varying computation and transmission delay is addressed in this paper. The NECS is described by a set of continuous-time plant models and associated quadratic cost functions. Since the uncertainty of the computation and transmission delay affects the discretized plant models and cost functions in a nonlinear manner, a polytopic overapproximation of the uncertainty utilizing a Taylor series expansion is considered. For the resulting discrete-time switched system model with polytopic uncertainty, a periodic control and online scheduling (PCS_{on}) strategy is proposed to guarantee stability and performance of the resulting controlled system. The design is based on a periodic parameter-dependent Lyapunov function and exhaustive search. Furthermore, a method for reducing the online complexity of the PCS_{on} strategy is presented. The effectiveness of modeling and design is evaluated for networked embedded control of a set of inverted pendulums.**

I. INTRODUCTION

Networked embedded control systems (NECSs) are control systems where controllers are realized on embedded processors and connected with both sensors and actuators via a shared communication network. NECS occur in a wide range of domains such as transportation systems, power systems and industrial automation and bring many benefits, e.g. reduced installation and maintenance costs and increased flexibility, reusability and reconfigurability. However, also new challenges stemming from computation and transmission delays, packet loss, access constraints and quantization constraints have to be addressed, refer to [1] for a survey. Specifically for NECSs with access constraints, not only a control strategy but also a scheduling strategy is required.

Many approaches have been proposed for designing a controller and schedule jointly. These approaches can be roughly classified by offline and online scheduling. Under offline scheduling, the schedule is determined before runtime, where usually periodic schedules are considered [2], [3]. Under online scheduling, the schedule is determined at runtime based on the current plant states [4], [5]. Online scheduling is thus reactive to disturbances and consequently usually superior to offline scheduling. All approaches assume constant computation and transmission delays. This assumption is valid for many real-time computation and communication systems but not for general-purpose computation and communication systems. Control and scheduling strategies which are robust with respect to uncertain time-varying computation and transmission delays are clearly required.

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In this paper a periodic control and online scheduling (PCS_{on}) strategy for NECSs with uncertain but interval-bounded time-varying computation and transmission delays is presented. The NECS is modeled as a discrete-time switched linear system regarding access constraints and delays. A quadratic cost function with infinite time horizon is considered as a performance criterion. Due to the nonlinear dependency of the discrete-time model and cost function on the uncertain time-varying delays, a polytopic overapproximation technique based on a Taylor series expansion [6], [7], [8] is used. Periodicity is imposed as a "trick" to allow solving the control and scheduling codesign problem for the infinite time horizon by decomposition into a periodic control subproblem and an online scheduling subproblem. The periodic control subproblem is solved based on Lyapunov theory, particularly based on a periodic parameter-dependent Lyapunov function [9]. The online scheduling subproblem is solved based on exhaustive search at every time instant. Hence, the PCS_{on} strategy relates to an explicit receding-horizon control and scheduling strategy [10]. It is shown that the online complexity of the PCS_{on} strategy may grow exponentially. A method for reducing this complexity is presented. Finally, the methods are evaluated for networked embedded control of three inverted pendulums.

II. MODELING

A. Plant Model and Cost Function

Consider a fixed set of plants $\mathbb{P} = \{\text{P}_i,\ i = 1, \dots, N\}$ controlled over a network each described by a continuous-time state equation

$$\dot{\boldsymbol{x}}_{ci}(t) = \boldsymbol{A}_{ci}\boldsymbol{x}_{ci}(t) + \boldsymbol{B}_{ci}\boldsymbol{u}_i(t-\tau_{ik}), \quad \boldsymbol{x}_{ci}(0) = \boldsymbol{x}_{ci0} \quad (1)$$

where $\boldsymbol{A}_{ci} \in \mathbb{R}^{n\times n}$ is the system matrix, $\boldsymbol{B}_{ci} \in \mathbb{R}^{n\times m}$ is the input matrix, $\boldsymbol{x}_{ci}(t) \in \mathbb{R}^n$ is the state vector and $\boldsymbol{u}_i(t-\tau_{ik}) \in \mathbb{R}^m$ is the control vector with uncertain time-varying input delay $\tau_{ik} \in \mathcal{I}_i = [\underline{\tau}_i, \overline{\tau}_i]$ with known lower bound $\underline{\tau}_i$ and upper bound $\overline{\tau}_i$. The input delay τ_{ik} subsumes the computation and transmission delays. Associated with each plant $\text{P}_i \in \mathbb{P}$ is a continuous-time quadratic cost function

$$J_i = \int_0^\infty \begin{pmatrix} \boldsymbol{x}_{ci}(t) \\ \boldsymbol{u}_i(t-\tau_{ik}) \end{pmatrix}^T \begin{pmatrix} \boldsymbol{Q}_{ci} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{ci} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{ci}(t) \\ \boldsymbol{u}_i(t-\tau_{ik}) \end{pmatrix} dt \quad (2)$$

with the symmetric and positive semidefinite weighting matrix $\boldsymbol{Q}_{ci} \in \mathbb{R}^{n\times n}$ and the symmetric and positive definite weighting matrix $\boldsymbol{R}_{ci} \in \mathbb{R}^{m\times m}$.

The plants are controlled by a centralized discrete-time controller minimizing the overall cost $J = \sum_{i=1}^{N} J_i$ robustly. The controller samples the states of all plants at each time instant t_k where a time-varying sampling period $h_{j(k)}$ with $k = 0, 1, \ldots$ is utilized. The time-varying sampling period $h_{j(k)}$ results from the worst-case computation and transmission delay of the control task $\mathrm{T}_{j(k)}$ associated to plant $\mathrm{P}_{j(k)}$, i.e. $h_{j(k)} = \overline{\tau}_{j(k)}$. The task or switching index $j(k)$ indicates the plant $\mathrm{P}_{j(k)}$ for which the control signal will be updated and is determined based on the states. Then the control signal $\boldsymbol{u}_{j(k)}(k)$ is sent over the network to plant $\mathrm{P}_{j(k)}$ and delivered at $t_k + \tau_{jk}$. It is assumed that the control signal is held until a new one is delivered. For notational convenience, the time dependency indication of the switching index $j(k)$ is omitted if no ambiguity arises.

B. Discretized Model and Cost Function

In order to discretize the continuous-time state equation (1) using zero order hold (ZOH), it must be distinguished whether the control signal is updated or not. If the considered task T_i is the running task T_j ($\mathrm{T}_i = \mathrm{T}_j$), then due to $\tau_{jk} \le h_j$ the control signal is updated, i.e.

$$\boldsymbol{u}_i(t) = \begin{cases} \boldsymbol{u}_i(t_{k-1}) & \text{for} \quad t_k \le t < t_k + \tau_{ik} \\ \boldsymbol{u}_i(t_k) & \text{for} \quad t_k + \tau_{ik} \le t < t_{k+1} \end{cases} \tag{3}$$

where $h_j = t_{k+1} - t_k$. If the considered task T_i is not the running task T_j ($\mathrm{T}_i \neq \mathrm{T}_j$), then the control vector is not updated at all, i.e.

$$\boldsymbol{u}_i(t) = \boldsymbol{u}_i(t_{k-1}) \quad \text{for} \quad t_k \le t < t_{k+1}. \tag{4}$$

In the following the distinction between the two cases mentioned above is represented by the logical variable

$$\delta_{ij} = \begin{cases} 1 & \text{if} \quad \mathrm{T}_i = \mathrm{T}_j \\ 0 & \text{if} \quad \mathrm{T}_i \neq \mathrm{T}_j \,. \end{cases} \tag{5}$$

Considering this behavior, an augmented discrete-time state equation corresponding to (1) can be formulated as

$$\boldsymbol{x}_i(k+1) = \boldsymbol{A}_{ij}(k)\boldsymbol{x}_i(k) + \boldsymbol{B}_{ij}(k)\boldsymbol{u}_i(k) \tag{6}$$

with

$$\begin{aligned} \boldsymbol{x}_i(k) &= \begin{pmatrix} \boldsymbol{x}_{ci}^T(k) & \boldsymbol{u}_i^T(k-1) \end{pmatrix}^T \in \mathbb{R}^{n+m} \\ \boldsymbol{A}_{ij}(k) &= \begin{pmatrix} \boldsymbol{\Phi}_i(h_j) & \boldsymbol{\Gamma}_i(h_j) - \boldsymbol{\Gamma}_i(h_j - \grave{h}_{ij}) \\ \boldsymbol{0}_{m\times n} & (1-\delta_{ij})\boldsymbol{I}_{m\times m} \end{pmatrix} \\ \boldsymbol{B}_{ij}(k) &= \begin{pmatrix} \boldsymbol{\Gamma}_i(h_j - \grave{h}_{ij}) \\ \delta_{ij}\boldsymbol{I}_{m\times m} \end{pmatrix} \in \mathbb{R}^{(n+m)\times m} \\ \grave{h}_{ij} &= \delta_{ij}\tau_{ik} + (1-\delta_{ij})h_j \end{aligned}$$

where

$$\boldsymbol{\Phi}_i(t) = e^{\boldsymbol{A}_{ci}t}, \qquad \boldsymbol{\Gamma}_i(t) = \int_0^t \boldsymbol{\Phi}_i(s)ds\boldsymbol{B}_{ci}. \tag{7}$$

This representation is adopted from a representation of time-delay systems proposed in [11]. Using a block-diagonal structure, the overall system can be written as a discrete-time switched linear system

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}_j(k)\boldsymbol{x}(k) + \boldsymbol{B}_j(k)\boldsymbol{u}(k) \\ \boldsymbol{x}(0) &= \begin{pmatrix} \boldsymbol{x}_1^T(0) & \cdots & \boldsymbol{x}_N^T(0) \end{pmatrix}^T \end{aligned} \tag{8}$$

with

$$\begin{aligned} \boldsymbol{x}(k) &= \begin{pmatrix} \boldsymbol{x}_1^T(k) & \cdots & \boldsymbol{x}_N^T(k) \end{pmatrix}^T \in \mathbb{R}^{N(n+m)} \\ \boldsymbol{u}(k) &= \begin{pmatrix} \boldsymbol{u}_1^T(k) & \cdots & \boldsymbol{u}_N^T(k) \end{pmatrix}^T \in \mathbb{R}^{Nm} \\ \boldsymbol{A}_j(k) &= \mathrm{diag}\left(\boldsymbol{A}_{1j}(k), \ldots, \boldsymbol{A}_{Nj}(k)\right) \in \mathbb{R}^{[N(n+m)]\times[N(n+m)]} \\ \boldsymbol{B}_j(k) &= \mathrm{diag}\left(\boldsymbol{B}_{1j}(k), \ldots, \boldsymbol{B}_{Nj}(k)\right) \in \mathbb{R}^{[N(n+m)]\times(Nm)} \end{aligned}$$

where $\mathrm{diag}(\cdot)$ denotes a block diagonal matrix.

The discretized cost function associated with plant P_i for a time-varying sampling period h_j and an uncertain time-varying time delay $\tau_{ik} \in \mathcal{I}_i$ using ZOH is given by

$$J_i = \sum_{k=0}^{\infty} \begin{pmatrix} \boldsymbol{x}_{ci}(k) \\ \boldsymbol{u}_i(k-1) \\ \boldsymbol{u}_i(k) \end{pmatrix}^T \begin{pmatrix} \boldsymbol{Q}_{11ij} & \boldsymbol{Q}_{12ij} & \boldsymbol{Q}_{13ij} \\ * & \boldsymbol{Q}_{22ij} & \boldsymbol{0} \\ * & * & \boldsymbol{Q}_{33ij} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{ci}(k) \\ \boldsymbol{u}_i(k-1) \\ \boldsymbol{u}_i(k) \end{pmatrix} \tag{9}$$

with the discretized weighting matrices

$$\boldsymbol{Q}_{11ij} = \int_0^{h_j} \boldsymbol{\Phi}_i^T(t)\boldsymbol{Q}_{ci}\boldsymbol{\Phi}_i(t)dt \tag{10a}$$

$$\boldsymbol{Q}_{12ij} = \int_0^{\grave{h}_{ij}} \boldsymbol{\Phi}_i^T(t)\boldsymbol{Q}_{ci}\boldsymbol{\Gamma}_i(t)dt \tag{10b}$$

$$\boldsymbol{Q}_{13ij} = \int_{\grave{h}_{ij}}^{h_j} \boldsymbol{\Phi}_i^T(t)\boldsymbol{Q}_{ci}\boldsymbol{\Gamma}_i(t-\grave{h}_{ij})dt \tag{10c}$$

$$\boldsymbol{Q}_{22ij} = \int_0^{\grave{h}_{ij}} \boldsymbol{\Gamma}_i^T(t)\boldsymbol{Q}_{ci}\boldsymbol{\Gamma}_i(t) + \boldsymbol{R}_{ci}dt \tag{10d}$$

$$\boldsymbol{Q}_{33ij} = \int_{\grave{h}_{ij}}^{h_j} \boldsymbol{\Gamma}_i^T(t-\grave{h}_{ij})\boldsymbol{Q}_{ci}\boldsymbol{\Gamma}_i(t-\grave{h}_{ij}) + \boldsymbol{R}_{ci}dt. \tag{10e}$$

where $\boldsymbol{\Phi}_i(t)$ and $\boldsymbol{\Gamma}_i(t)$ are defined according to (7) and a matrix $\left(\begin{smallmatrix} \boldsymbol{A} & \boldsymbol{B} \\ * & \boldsymbol{C} \end{smallmatrix}\right)$ represents a symmetric matrix $\left(\begin{smallmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^T & \boldsymbol{C} \end{smallmatrix}\right)$. For a detailed discussion on the discretization of the cost function see [11] and [12]. Note that for $\delta_{ij} = 1$ the uncertain time-varying time delay $\tau_{jk} \in \mathcal{I}_j$ affects the matrices (7) and (10) in a nonlinear manner.

The overall cost function can be written as the sum of the individual cost functions

$$J = \sum_{i=1}^{N} J_i = \sum_{k=0}^{\infty} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{pmatrix}^T \underbrace{\begin{pmatrix} \boldsymbol{Q}_{1j} & \boldsymbol{Q}_{12j} \\ * & \boldsymbol{Q}_{2j} \end{pmatrix}}_{\boldsymbol{Q}_j(\tau_{jk})} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{pmatrix} \tag{11}$$

where the block-diagonal weighting matrices are given by

$$\begin{aligned} \boldsymbol{Q}_{1j} &= \mathrm{diag}\left(\left(\begin{smallmatrix} \boldsymbol{Q}_{111j} & \boldsymbol{Q}_{121j} \\ * & \boldsymbol{Q}_{221j} \end{smallmatrix}\right), \ldots, \left(\begin{smallmatrix} \boldsymbol{Q}_{11Nj} & \boldsymbol{Q}_{12Nj} \\ * & \boldsymbol{Q}_{22Nj} \end{smallmatrix}\right)\right) \\ \boldsymbol{Q}_{12j} &= \mathrm{diag}\left(\left(\begin{smallmatrix} \boldsymbol{Q}_{131j} \\ \boldsymbol{0} \end{smallmatrix}\right), \ldots, \left(\begin{smallmatrix} \boldsymbol{Q}_{13Nj} \\ \boldsymbol{0} \end{smallmatrix}\right)\right) \\ \boldsymbol{Q}_{2j} &= \mathrm{diag}\left(\boldsymbol{Q}_{331j}, \ldots, \boldsymbol{Q}_{33Nj}\right). \end{aligned}$$

Problem 1: For the switched system (8) find a control sequence $\boldsymbol{u}^*(0), \ldots, \boldsymbol{u}^*(\infty)$ and a switching sequence $j^*(0), \ldots, j^*(\infty)$ such that the cost function (11) is robustly minimized for all sequences $\tau_{jk} \in \mathcal{I}_j$, i.e.

$$\min_{\substack{\boldsymbol{u}(0),\ldots,\boldsymbol{u}(\infty) \\ j(0),\ldots,j(\infty)}} \max_{\tau_{jk}\in\mathcal{I}_j} J \quad \text{subject to (8).} \tag{12}$$

Remark 1: Problem 1 is essentially computationally intractable. Three modifications are introduced in the following sections to obtain a tractable problem: First, the nonlinear uncertainty is overapproximated by a polytopic uncertainty to enable the application of robust control design methods. Second, an upper bound on the cost function is derived to obtain a tractable problem, cf. Section III-A. Third, periodic switching sequences are considered to enable a solution for an infinite time horizon. These modifications allow utilizing periodic parameter-dependent Lyapunov functions for an LMI-based control design, but introduce some conservatism which can be adjusted e.g. by increasing the period and the approximation order, however, at the cost of complexity.

C. Polytopic Formulation

For the case that $\delta_{ij} = 1$ the matrix exponentials contained in the matrices (7) and (10) are expanded in Taylor series. The resulting Taylor series are then partitioned into a truncation part and a remainder part, yielding

$$\begin{aligned}\boldsymbol{\Phi}_i(t) &= \sum_{q=0}^{\infty} \frac{\boldsymbol{A}_{ci}^q}{q!} t^q = \sum_{q=0}^{M_\Phi} \frac{\boldsymbol{A}_{ci}^q}{q!} t^q + \Delta\boldsymbol{\Phi}_i \\ &= \hat{\boldsymbol{\Phi}}_i(t, M_\Phi) + \Delta\boldsymbol{\Phi}_i \end{aligned} \tag{13a}$$

$$\begin{aligned}\boldsymbol{\Gamma}_i(t) &= \int_0^t \sum_{n=0}^{\infty} \frac{\boldsymbol{A}_{ci}^n}{n!} s^n ds \boldsymbol{B}_{ci} = \sum_{q=1}^{M_\Gamma+1} \frac{\boldsymbol{A}_{ci}^{q-1}}{q!} t^q \boldsymbol{B}_{ci} + \Delta\boldsymbol{\Gamma}_i \\ &= \hat{\boldsymbol{\Gamma}}_i(t, M_\Gamma) + \Delta\boldsymbol{\Gamma}_i \end{aligned} \tag{13b}$$

with $\Delta\boldsymbol{\Phi}_i = \Delta\boldsymbol{\Phi}_i(\underline{t}, \overline{t}, M_\Phi), \Delta\boldsymbol{\Gamma}_i = \Delta\boldsymbol{\Gamma}_i(\underline{t}, \overline{t}, M_\Gamma)$ and the general uncertain parameter $t \in [\underline{t}, \overline{t}]$.

Partitioning the Taylor series in an approximation of order M_Φ/M_Γ and a remainder allows the polytopic formulation of the approximation part. Furthermore, substituting (13) in the weighting matrices (10) leads to

$$\begin{aligned}\boldsymbol{Q}_{12ij} &= \int_0^{\grave{h}_{ij}} \hat{\boldsymbol{\Phi}}_i^T(t, M_{a12}) \boldsymbol{Q}_{ci} \hat{\boldsymbol{\Gamma}}_i(t, M_{b12}) dt + \Delta\boldsymbol{Q}_{12ij} \\ &= \hat{\boldsymbol{Q}}_{12ij}(\grave{h}_{ij}, M_{a12}, M_{b12}) + \Delta\boldsymbol{Q}_{12ij} \end{aligned} \tag{14a}$$

$$\begin{aligned}\boldsymbol{Q}_{13ij} &= \int_{\grave{h}_{ij}}^{h_j} \hat{\boldsymbol{\Phi}}_i^T(t, M_{a13}) \boldsymbol{Q}_{ci} \hat{\boldsymbol{\Gamma}}_i(t - \grave{h}_{ij}, M_{b13}) dt + \Delta\boldsymbol{Q}_{13ij} \\ &= \hat{\boldsymbol{Q}}_{13ij}(\grave{h}_{ij}, M_{a13}, M_{b13}) + \Delta\boldsymbol{Q}_{13ij} \end{aligned} \tag{14b}$$

$$\begin{aligned}\boldsymbol{Q}_{22ij} &= \int_0^{\grave{h}_{ij}} \hat{\boldsymbol{\Gamma}}_i^T(t, M_{a22}) \boldsymbol{Q}_{ci} \hat{\boldsymbol{\Gamma}}_i(t, M_{b22}) + \boldsymbol{R}_{ci} dt + \Delta\boldsymbol{Q}_{22ij} \\ &= \hat{\boldsymbol{Q}}_{22ij}(\grave{h}_{ij}, M_{a22}, M_{b22}) + \Delta\boldsymbol{Q}_{22ij} \end{aligned} \tag{14c}$$

$$\begin{aligned}\boldsymbol{Q}_{33ij} &= \int_{\grave{h}_{ij}}^{h_j} \hat{\boldsymbol{\Gamma}}_i^T(t - \grave{h}_{ij}, M_{a33}) \boldsymbol{Q}_{ci} \hat{\boldsymbol{\Gamma}}_i(t - \grave{h}_{ij}, M_{b33}) + \\ &\quad + \boldsymbol{R}_{ci} dt + \Delta\boldsymbol{Q}_{33ij} \\ &= \hat{\boldsymbol{Q}}_{33ij}(\grave{h}_{ij}, M_{a33}, M_{b33}) + \Delta\boldsymbol{Q}_{33ij} \end{aligned} \tag{14d}$$

with the remainder $\Delta\boldsymbol{Q}_{pqij} = \Delta\boldsymbol{Q}_{pqij}(\underline{\tau}_i, \overline{\tau}_i, M_{apq}, M_{bpq})$ and $pq \in \{12, 13, 22, 33\}$. Note that $\boldsymbol{\Phi}_i(h_j), \boldsymbol{\Gamma}_i(h_j)$ and $\boldsymbol{Q}_{11ij}$ depend only on the certain time-varying sampling period h_j and can therefore be determined by evaluating the matrix exponentials numerically. Hence, a Taylor series expansion is not required. The truncation parts of the above matrices correspond to matrix polynomials. The orders of the truncation parts for the matrices (13), (14) are design parameters and can be chosen differently. However, the resulting polynomial orders must be identical to allow for a polytopic formulation. A reasonable choice of the truncation orders ensuring that each matrix in (13), (14) has at least a truncation order equal to M is shown in [8, Table 1]. This leads to the minimum polynomial order $\tilde{M} = 2M + 3$. The matrix polynomials can now be enveloped by polytopes as illustrated in [13, Lemma 1]. Hence, the matrix polynomials contained in (13) and (14) can be expressed as

$$\hat{\boldsymbol{\Gamma}}_i(\tau_{ik}, M_\Gamma) = \sum_{l=0}^{\tilde{M}} \mu_l(\tau_{ik}) \hat{\boldsymbol{\Gamma}}_{il} \tag{15a}$$

$$\hat{\boldsymbol{Q}}_{pqij}(\tau_{ik}, M_{apq}, M_{bpq}) = \sum_{l=0}^{\tilde{M}} \mu_l(\tau_{ik}) \hat{\boldsymbol{Q}}_{pqijl} \tag{15b}$$

with $pq \in \{12, 13, 22, 33\}$, $\mu_l(\tau_{ik}) \geq 0$, $\sum_{i=0}^{\tilde{M}} \mu_l(\tau_{ik}) = 1$. $\hat{\boldsymbol{\Gamma}}_{il}$ and $\hat{\boldsymbol{Q}}_{pqijl}$ represent the vertices of the resulting polytopes. Substituting (15a) into (13b) and further (13b) into (7) results after factorizing $\mu_l(\tau_{ik})$ in a discrete-time plant model with polytopic and additive norm-bounded uncertainty

$$\begin{aligned}\boldsymbol{x}_i(k+1) = &\left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{ik}) \boldsymbol{A}_{ijl} + \Delta\boldsymbol{A}_{ij} \right) \boldsymbol{x}_i(k) + \\ &\left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{ik}) \boldsymbol{B}_{ijl} + \Delta\boldsymbol{B}_{ij} \right) \boldsymbol{u}_i(k) \end{aligned} \tag{16}$$

where

$$\boldsymbol{A}_{ijl} = \begin{pmatrix} \boldsymbol{\Phi}_i(h_j) & \boldsymbol{\Gamma}_i(h_j) - \hat{\boldsymbol{\Gamma}}_{il} \\ \boldsymbol{0}_{m\times n} & (1-\delta_{ij})\boldsymbol{I}_{m\times m} \end{pmatrix}, \ \boldsymbol{B}_{ijl} = \begin{pmatrix} \hat{\boldsymbol{\Gamma}}_{il} \\ \delta_{ij}\boldsymbol{I}_{m\times m} \end{pmatrix}$$

$$\Delta\boldsymbol{A}_{ij} = \begin{pmatrix} \boldsymbol{0}_{n\times n} & -\Delta\boldsymbol{\Gamma}_i \\ \boldsymbol{0}_{m\times n} & \boldsymbol{0}_{m\times m} \end{pmatrix}, \qquad \Delta\boldsymbol{B}_{ij} = \begin{pmatrix} \Delta\boldsymbol{\Gamma}_i \\ \boldsymbol{0}_{m\times m} \end{pmatrix}.$$

Notice that in case $\delta_{ij} = 0$, there is no need to create a polytopic and additive norm-bounded uncertainty for the plant model since there is no dependency on the uncertain parameter τ_{ik}. Based on the block-diagonal structure, the overall discrete-time switched system with polytopic and additive norm-bounded uncertainty is given by

$$\begin{aligned}\boldsymbol{x}(k+1) = &\left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \boldsymbol{A}_{jl} + \Delta\boldsymbol{A}_j \right) \boldsymbol{x}(k) + \\ &\left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \boldsymbol{B}_{jl} + \Delta\boldsymbol{B}_j \right) \boldsymbol{u}(k). \end{aligned} \tag{17}$$

Furthermore, substituting (15b) into (14) and further (14) into (10) leads to a discrete-time cost function with polytopic and additive norm-bounded uncertainty

$$J_i = \sum_{k=0}^{\infty} \begin{pmatrix} \boldsymbol{x}_i(k) \\ \boldsymbol{u}_i(k) \end{pmatrix}^T \left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{ik}) \boldsymbol{Q}_{ijl} + \Delta\boldsymbol{Q}_{ij} \right) \begin{pmatrix} \boldsymbol{x}_i(k) \\ \boldsymbol{u}_i(k) \end{pmatrix} \tag{18}$$

where

$$\boldsymbol{Q}_{ijl} = \begin{pmatrix} \boldsymbol{Q}_{11ij} & \hat{\boldsymbol{Q}}_{12ijl} & \hat{\boldsymbol{Q}}_{13ijl} \\ * & \hat{\boldsymbol{Q}}_{22ijl} & \hat{\boldsymbol{0}}_{m\times m} \\ * & * & \hat{\boldsymbol{Q}}_{33ijl} \end{pmatrix}$$

$$\Delta\boldsymbol{Q}_{ij} = \begin{pmatrix} \boldsymbol{0}_{n\times n} & \Delta\boldsymbol{Q}_{12ij} & \Delta\boldsymbol{Q}_{13ij} \\ * & \Delta\boldsymbol{Q}_{22ij} & \Delta\boldsymbol{0}_{m\times m} \\ * & * & \Delta\boldsymbol{Q}_{33ij} \end{pmatrix}.$$

Based on the block-diagonal structure, the overall discrete-time switched cost function with polytopic and additive norm-bounded uncertainty is given by

$$J = \sum_{k=0}^{\infty} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{pmatrix}^T \left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \boldsymbol{Q}_{jl} + \Delta\boldsymbol{Q}_j \right) \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{pmatrix}. \tag{19}$$

The matrices $\boldsymbol{A}_{jl}$, $\boldsymbol{B}_{jl}$ and $\boldsymbol{Q}_{jl}$ represent vertices of a polytopic uncertainty, the matrices $\Delta\boldsymbol{A}_j$, $\Delta\boldsymbol{B}_j$ and $\Delta\boldsymbol{Q}_j$ describe an additive norm-bounded uncertainty since the time varying time delay τ_{jk} is bounded on the interval $\mathcal{I}_j$.

Remark 2: A polytopic overapproximation can also be determined based on the Cayley-Hamilton Theorem or the Jordan normal form, see [6], [7]. These approaches are to our best knowledge not applicable for the cost function since $\boldsymbol{Q}_j(\tau_{jk})$ can not be expressed as potentials of a single matrix.

III. Periodic Control and Scheduling

Problem 1 can by imposing periodicity be decomposed into a periodic control subproblem and an online scheduling subproblem. The periodic control subproblem can be solved by using periodic parameter-dependent Lyapunov function; the online scheduling subproblem can be solved by using the receding-horizon control and scheduling concept [10].

A. Periodic Control Design

For formulating both problem and solution of the periodic control subproblem, some definitions are in order.

Definition 1: A switching sequence $j(k)$ is called p-periodic if

$$j(k) = j(k+p) \quad \forall k \in \mathbb{N}_0. \tag{20}$$

It is furthermore called admissible if all tasks under consideration are contained.

Definition 2: The set of admissible p-periodic switching sequences $\mathbb{J}_{p,\text{adm}}$ is defined by

$$\mathbb{J}_{p,\text{adm}} = \{ j(k) | j(k) = j(k+p) \; \forall k \in \mathbb{N}_0, j(k) \text{ admissible} \}. \tag{21}$$

Consider the full state feedback control law

$$\boldsymbol{u}(k) = \boldsymbol{K}_j(k)\boldsymbol{x}(k) \tag{22}$$

with the constant p-periodic feedback matrices $\boldsymbol{K}_j(k) \in \mathbb{R}^{(Nm)\times[N(n+m)]}$. The subindex j indicates the affiliation to the p-periodic switching sequence $j(k)$. Substituting (22) into (17) leads to the discrete-time periodic closed-loop system

$$\boldsymbol{x}(k+1) = \left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \tilde{\boldsymbol{A}}_{jl} + \Delta\tilde{\boldsymbol{A}}_j \right) \boldsymbol{x}(k) \tag{23}$$

with $\tilde{\boldsymbol{A}}_{jl} = \boldsymbol{A}_{jl} + \boldsymbol{B}_{jl}\boldsymbol{K}_j$ and $\Delta\tilde{\boldsymbol{A}}_j = \Delta\boldsymbol{A}_j + \Delta\boldsymbol{B}_j\boldsymbol{K}_j$. Substituting further (22) into (19) yields the discrete-time cost function

$$J = \sum_{k=0}^{\infty} \boldsymbol{x}^T(k) \left(\sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \tilde{\boldsymbol{Q}}_{jl} + \Delta\tilde{\boldsymbol{Q}}_j \right) \boldsymbol{x}(k) \tag{24}$$

with $\tilde{\boldsymbol{Q}}_{jl} = \left(\begin{smallmatrix} \boldsymbol{I} \\ \boldsymbol{K}_j \end{smallmatrix}\right)^T \boldsymbol{Q}_{jl} \left(\begin{smallmatrix} \boldsymbol{I} \\ \boldsymbol{K}_j \end{smallmatrix}\right)$ and $\Delta\tilde{\boldsymbol{Q}}_j = \left(\begin{smallmatrix} \boldsymbol{I} \\ \boldsymbol{K}_j \end{smallmatrix}\right)^T \Delta\boldsymbol{Q}_j \left(\begin{smallmatrix} \boldsymbol{I} \\ \boldsymbol{K}_j \end{smallmatrix}\right)$. The matrices $\tilde{\boldsymbol{A}}_{jl}$, $\tilde{\boldsymbol{Q}}_{jl}$ again represent vertices of a switched polytopic uncertainty while the matrices $\Delta\tilde{\boldsymbol{A}}_j$, $\Delta\tilde{\boldsymbol{Q}}_j$ describe a switched additive norm-bounded uncertainty. The norm bounds of these additive uncertainties are specified by

$$\|\Delta\tilde{\boldsymbol{A}}_j\|_2^2 \leq \alpha_j, \qquad \alpha_j = \sup_{\tau_{jk}\in\mathcal{I}_j} \overline{\sigma}^2(\Delta\tilde{\boldsymbol{A}}_j) \tag{25a}$$

$$\|\Delta\tilde{\boldsymbol{Q}}_j\|_2^2 \leq \beta_j, \qquad \beta_j = \sup_{\tau_{jk}\in\mathcal{I}_j} \overline{\sigma}^2(\Delta\tilde{\boldsymbol{Q}}_j) \tag{25b}$$

where $\overline{\sigma}(\cdot)$ represents the maximum singular value. The upper bounds α_j and β_j depend on the feedback matrix $\boldsymbol{K}_j$.

Many approaches to handle the additive norm-bounded uncertainty have been proposed in literature, refer to [8, Remark 5]. For brevity of presentation, the additive uncertainty will not be considered in the following. The periodic control subproblem can now be defined as

Problem 2: For the admissible schedule $j(k) \in \mathbb{J}_{p,\text{adm}}$ find a p-periodic feedback matrix $\boldsymbol{K}_j$ for the closed-loop system (23) with $\Delta\tilde{\boldsymbol{A}}_j = \boldsymbol{0}$ such that the cost function (24) with $\Delta\tilde{\boldsymbol{Q}}_j = \boldsymbol{0}$ is robustly minimized for all $\tau_{jk} \in \mathcal{I}_j$, i.e.

$$\min_{\boldsymbol{K}_j} \max_{\tau_{jk}\in\mathcal{I}_j} J \qquad \text{subject to (23).} \tag{26}$$

Problem 2 is computationally intractable as pointed out in [14, Sec. 3.1]. Therefore, an upper bound on the objective function is derived in the following to obtain a computationally tractable minimization problem.

Consider the p-periodic parameter-dependent quadratic Lyapunov function

$$V(\boldsymbol{x}(k), k) = \boldsymbol{x}^T(k)\boldsymbol{\mathcal{P}}_j(k)\boldsymbol{x}(k) \tag{27}$$

where

$$\boldsymbol{\mathcal{P}}_j(k) = \sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \boldsymbol{P}_{jl} \tag{28}$$

with $\boldsymbol{P}_{jl}$ symmetric and positive definite. The subindex j again indicates the affiliation to the p-periodic switching sequence $j(k)$. Suppose that the difference $\Delta V(\boldsymbol{x}(k), k) = V(\boldsymbol{x}(k+1), k+1) - V(\boldsymbol{x}(k), k)$ along trajectories of the closed-loop system (23) satisfies

$$\Delta V(\boldsymbol{x}(k), k) < -\boldsymbol{x}^T(k) \sum_{l=0}^{\tilde{M}} \mu_l(\tau_{jk}) \tilde{\boldsymbol{Q}}_{jl} \boldsymbol{x}(k) \tag{29}$$

for all $\tau_{jk} \in \mathcal{I}_j$ and $\boldsymbol{x}(k) \neq \boldsymbol{0}$. Furthermore, suppose that the cost function (24) is finite, then $\lim_{k\to\infty} \boldsymbol{x}(k) = \boldsymbol{0}$ and therefore $\lim_{k\to\infty} V(\boldsymbol{x}(k), k) = 0$ holds. Summing (29) over $k = 0, \ldots, \infty$ yields

$$\max_{\tau_{jk}\in\mathcal{I}_j} J < \boldsymbol{x}^T(0)\boldsymbol{\mathcal{P}}_j(0)\boldsymbol{x}(0), \tag{30}$$

giving an upper bound on the cost function (24).

The upper bound depends on the initial state $\boldsymbol{x}(0)$ which is often unknown. Therefore, the worst-case cost value

$$J_{\text{w.c.}} = \max_{\|\boldsymbol{x}(0)\|=1} \boldsymbol{x}^T(0)\boldsymbol{\mathcal{P}}_j(0)\boldsymbol{x}(0) \leq \lambda_{\max}(\boldsymbol{\mathcal{P}}_j(0)) \quad (31)$$

can be considered instead where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue. Alternatively, the expected cost value or the maximum cost degradation can be utilized as proposed in [8]. In the following the worst-case cost value will be considered. Problem 2 can then be redefined to the following computationally tractable minimization problem

Problem 3: For the admissible schedule $j(k) \in \mathbb{J}_{p,\text{adm}}$ find a p-periodic feedback matrix $\boldsymbol{K}_j$ for the closed-loop system (23) with $\Delta\tilde{\boldsymbol{A}}_j = \boldsymbol{0}$ such that the cost function (24) with $\Delta\tilde{\boldsymbol{Q}}_j = \boldsymbol{0}$ is robustly minimized for all $\tau_{jk} \in \mathcal{I}_j$, i.e.

$$\min_{\boldsymbol{K}_j} \lambda_{\max}(\boldsymbol{\mathcal{P}}_j(0)) \quad \text{subject to (29).} \quad (32)$$

Theorem 1: Problem 3 is solved for a given $j(k) \in \mathbb{J}_{p,\text{adm}}$ by the LMI optimization problem

$$\min_{\boldsymbol{K}_j(k)} -\lambda \quad \text{subject to}$$

$$\boldsymbol{Z}_{jl}(0) - \lambda\boldsymbol{I} \geq \boldsymbol{0}$$

$$\begin{pmatrix} \boldsymbol{G}_j(k)^T + \boldsymbol{G}_j(k) - \boldsymbol{Z}_{jl}(k) & * & * \\ \boldsymbol{Q}_{j(k)l}^{1/2}\begin{pmatrix}\boldsymbol{G}_j(k)\\ \boldsymbol{W}_j(k)\end{pmatrix} & \boldsymbol{I} & * \\ \boldsymbol{A}_{j(k)l}\boldsymbol{G}_j(k) + \boldsymbol{B}_{j(k)l}\boldsymbol{W}_j(k) & \boldsymbol{0} & \boldsymbol{Z}_{jm}(k+1)\end{pmatrix} > \boldsymbol{0}$$

$\forall l, m = 0, \ldots, \tilde{M}$ and $k = 0, \ldots, p-1$ with $\boldsymbol{G}_j(k) \in \mathbb{R}^{[N(n+m)]\times[N(n+m)]}$ and $\boldsymbol{W}_j(k) \in \mathbb{R}^{(Nm)\times[N(n+m)]}$ regular, $\boldsymbol{Z}_{jl}(k) = \boldsymbol{P}_{jl}^{-1}(k) \in \mathbb{R}^{[N(n+m)]\times[N(n+m)]}$ symmetric and positive definite and $\lambda \in \mathbb{R}$. The optimal p-periodic feedback matrix results from $\boldsymbol{K}_j(k) = \boldsymbol{W}_j(k)\boldsymbol{G}_j^{-1}(k)$.

Proof: The proof follows from the proof of Theorem 8 in [8] by reformulation for the worst-case cost value as an objective function and using a p-periodic parameter-dependent Lyapunov function instead of only a parameter-dependent Lyapunov function. ■

Solving problem 3 for each admissible p-periodic switching sequence $j(k) \in \mathbb{J}_{p,\text{adm}}$ results in a set of p-periodic stabilizing feedback matrices. These admissible feedback matrices are then stored in a lookup table with their first step Lyapunov matrix

$$\boldsymbol{P}_j(0) = \arg \max_{\boldsymbol{P}_{jl}(0), l\in\{0,\ldots,\tilde{M}\}} \lambda_{\max}(\boldsymbol{P}_{jl}(0)). \quad (33)$$

So far we have solved the periodic control subproblem. The online scheduling subproblem is addressed in the following subsection.

B. Online Scheduling

The main idea is to consider at every time instant t_k the measured state $\boldsymbol{x}(k)$ as the initial state of the system and to solve Problem 1 based on the receding horizon concept. The imposed periodicity is thus suspended and an additional degree of freedom for the optimization is obtained. The procedure is summarized in the following

Theorem 2: The solution to Problem 1 is given by the state feedback control law

$$\boldsymbol{u}^*(k) = \boldsymbol{K}_{j^*}(k)\boldsymbol{x}(k) \quad (34)$$

where $\boldsymbol{x}(k)$ represents the current state of the switched system (8) at the time instant t_k and

$$j^*(k) = \arg \min_{j(k)\in\mathbb{J}_{p,\text{adm}}} \boldsymbol{x}^T(k)\boldsymbol{P}_j(0)\boldsymbol{x}(k). \quad (35)$$

Proof: The upper bound on the cost function is given in (30). Hence, the minimum cost J^* among all admissible schedules can also be upper bounded as follows

$$J^* < \min_{j(k)\in\mathbb{J}_{p,\text{adm}}} \boldsymbol{x}(0)^T\boldsymbol{P}_j(0)\boldsymbol{x}(0). \quad (36)$$

Hence, the optimal p-periodic switching sequence is given by

$$j^*(k) = \arg \min_{j(k)\in\mathbb{J}_{p,\text{adm}}} \boldsymbol{x}(0)^T\boldsymbol{P}_j(0)\boldsymbol{x}(0)\ . \quad (37)$$

and $\boldsymbol{P}_j(0)$ is defined according to (33). Under online scheduling, the current state $\boldsymbol{x}(k)$ at time instant t_k represents a shifted initial time instant. Hence, the upper bound on the cost at time instant t_k follows immediately from (30) as

$$\max_{\tau_{jk}\in\mathcal{I}_j} J < \boldsymbol{x}^T(k)\boldsymbol{\mathcal{P}}_j(0)\boldsymbol{x}(k). \quad (38)$$

Therefore, the optimal switching index for the current state $\boldsymbol{x}(k)$ is given by (35). ■

Remark 3: Theorem 2 actually provides the solution of Problem 1 w.r.t. the modifications introduced in Remark 1.

An algorithm for solving Problem 1 divides into an offline part given in Algorithm 1 and an online part given in Algorithm 2.

Algorithm 1 Periodic Control Design (Offline Part)

Input: $\boldsymbol{A}_{jl}$, $\boldsymbol{B}_{jl}$, $\boldsymbol{Q}_{jl}$, p
Output: $\boldsymbol{P}_j(0)$, $\boldsymbol{K}_j(0)$ for each $j(k) \in \mathbb{J}_{p,\text{adm}}$

1. Determine set of all admissible periodic schedules $\mathbb{J}_{p,\text{adm}}$
2. Determine $\boldsymbol{K}_j(0)$, $\boldsymbol{P}_j(0)$ from Theorem (1) for each $j(k)\in\mathbb{J}_{p,\text{adm}}$
3. Store $\boldsymbol{P}_j(0)$ and $\boldsymbol{K}_j(0)$ for each $j(k) \in \mathbb{J}_{p,\text{adm}}$

Algorithm 2 Online Scheduling (Online Part)

Input: $\boldsymbol{P}_j(0)$, $\boldsymbol{K}_j(0)$, $\boldsymbol{x}(k)$ for each $j(k) \in \mathbb{J}_{p,\text{adm}}$
Output: Switching index $j^*(k)$ and control vector $\boldsymbol{u}^*(k)$

```
for each time instant t_k do
    Determine the optimal switching index j*(k) from (35)
    Determine the control vector u*(k) = K_{j*}(k)x(k)
    Apply j*(k) and u*(k) to the switched system (8)
end for
```

The period $p \geq N$ can be considered as a design parameter. With increasing period, the overall cost decreases but the computational complexity of Algortihms (1) and (2) which is characterized by $|\mathbb{J}_{p,\text{adm}}| \leq N^p$ with $|\cdot|$ denoting the cardinality increases. Global asymptotic stability of the switched system (8) under the PCS_{on} strategy (34) is guaranteed inherently due to the infinite horizon cost monotonicity. The computational complexity of Algorithm (2) may, however, be critical.

IV. Complexity Reduction

An approach for reducing the online complexity is proposed in [4]. The idea is to determine the optimal admissible p-periodic switching sequence $j^*(k)$ from (37). Then, the first step Lyapunov matrices associated to $j^*(k)$ and its cyclic shifts $\mathrm{S}^{k_s}(j^*)$ for all $k_s \in \{0, \dots, p-1\}$ are determined. Till now every thing is completely done offline. The online part is to reoptimize for the current state $\boldsymbol{x}(k)$ over all cyclic shifts, i.e.

$$j^*(k) = \arg \min_{k_s \in \{0,\dots,p-1\}} \boldsymbol{x}^T(k) \boldsymbol{P}_{\mathrm{S}^{k_s}(j^*)}(0) \boldsymbol{x}(k). \qquad (39)$$

The online complexity is consequently determined by the period p. This procedure is called Optimal Pointer Placement (OPP) where k_s is denoted as pointer. The switched system (8) under the OPP strategy is globally asymptotically stable. A proof of this property is given in [4, Theorem 2].

V. Example

Consider simultaneous stabilization of three inverted pendulums as shown in [3]. The linearized dynamic model of each undisturbed inverted pendulum under networked embedded control is given by

$$\begin{pmatrix} \dot{\phi}_i(t) \\ \ddot{\phi}_i(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{(m_i+M_i)g}{M_i l_i} & 0 \end{pmatrix} \begin{pmatrix} \phi_i(t) \\ \dot{\phi}_i(t) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{M_i l_i} \end{pmatrix} F_i(t - \tau_i)$$

where ϕ_i is the pendulum angle, F_i is the force acting on the cart and $i = 1, 2, 3$. The inverted pendulums have the same pendulum mass $m_i = 0.3\,\mathrm{kg}$ and cart mass $M_i = 0.1\,\mathrm{kg}$, but different pendulum length $l_{1/2/3} = 0.136/0.242/0.545\,\mathrm{m}$, yielding different natural frequencies $w_{1/2/3} = 12/9/6\,\mathrm{s}^{-1}$. The input delay τ_i is bounded on the interval $\mathcal{I}_i = [1\,, 3\,]\,\mathrm{ms}$ $\forall i$. Notation g represents the gravitational acceleration. Further, consider the quadratic cost function (2) with the weighting matrices $\boldsymbol{Q}_{ci} = \left(\begin{smallmatrix} 1000 & 0 \\ 0 & 10 \end{smallmatrix}\right)$, $R_{ci} = 1$. Applying more than 3000 different initial values to the system and computing the mean cost $\overline{J}_{\text{strategy}}$ for each strategy with $p = 4$ leads to the results summarized in Table I. From Table I we can notice that the $\mathrm{PCS_{off}}$ strategy, a special case of $\mathrm{PCS_{on}}$ strategy where the optimal sequence is determined once from (37) and followed till the end, leads to the largest mean cost while the online complexity is negligible since a predefined p-periodic switching sequence must be followed forever. The $\mathrm{PCS_{on}}$ strategy yields the smallest mean cost. However, the online complexity is considerable. The OPP strategy enables a compromise between complexity and performance and guarantees that the resulting mean cost is smaller or equal to the resulting cost under $\mathrm{PCS_{off}}$ strategy, cf. [4, Thm. 2].

VI. Conclusions and Future Work

In this paper robust control and scheduling codesign of NECSs based on the $\mathrm{PCS_{on}}$ strategy is addressed. Using a polytopic overapproximation, the discrete-time NECS model is expressed as a switched system with polytopic and additive norm-bounded uncertainty. The control and scheduling codesign problem is decomposed into a periodic control subproblem and an online scheduling subproblem. The online complexity can be reduced considerably by the OPP strategy. The effectiveness of the proposed strategies is shown by a practical example. Future work will focus on alternative methods for reducing the online complexity.

TABLE I
Comparison of the $\mathrm{PCS_{on}}$, OPP and $\mathrm{PCS_{off}}$ Strategy

Strategy	Online Complexity	$\overline{J}_{\text{strategy}}$
$\mathrm{PCS_{on}}$	$\lvert\hat{\mathbb{J}}_{p,\text{adm}}\rvert = 36$	340.1648
OPP	$p = 4$	340.4934
$\mathrm{PCS_{off}}$	negligible	350.9192

References

[1] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, “A survey of recent results in networked control systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[2] H. Rehbinder and M. Sanfridson, “Scheduling of a limited communication channel for optimal control,” *Automatica*, vol. 40, no. 3, pp. 491–500, 2004.

[3] D. Görges, M. Izák, and S. Liu, “Optimal control of systems with resource constraints,” in *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007, pp. 1070–1075.

[4] M. E. M. Ben Gaid, A. Çela, and Y. Hamam, “Optimal integrated control and scheduling of networked control systems with communication constraints: Application to a car suspension system,” *IEEE Transactions on Control Systems Technology*, vol. 14, no. 4, pp. 776–787, 2006.

[5] D. Görges, M. Izák, and S. Liu, “Optimal control and scheduling of networked control systems,” in *Proceedings of the 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009, pp. 5839–5844.

[6] R. H. Gielen, S. Olaru, M. Lazar, W. P. M. H. Heemels, N. van de Wouw, and S.-I. Niculescu, “On polytopic inclusions as a modeling framework for systems with time-varying delays,” *Automatica*, vol. 46, no. 3, pp. 615–619, 2010.

[7] W. P. M. H. Heemels, N. van de Wouw, R. H. Gielen, M. C. F. Donkers, L. Hetel, S. Olaru, M. Lazar, J. Daafouz, and S.-I. Niculescu, “A comparison of overapproximation methods for stability analysis of networked control systems,” in *Proceedings of the 13th International Conference on Hybrid Systems: Computation and Control*, 2010, pp. 181–190.

[8] M. Izák, D. Görges, and S. Liu, “Optimal control of networked control systems with uncertain time-varying transmission delay,” in *Proceedings of the 2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems*, 2010, pp. 13–18.

[9] C. Farges, D. Peaucelle, D. Arzelier, and J. Daafouz, “Robust $\mathcal{H}_2$ performance analysis and synthesis of linear polytopic discrete-time periodic systems via LMIs,” *Systems & Control Letters*, vol. 56, no. 2, pp. 159–166, 2007.

[10] D. Görges, M. Izák, and S. Liu, “Optimal control and scheduling of switched systems,” *IEEE Transactions on Automatic Control*, vol. 56, no. 1, pp. 135–140, 2011.

[11] K. J. Åström and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1990.

[12] M. Sanfridson, “Quality of control and real-time scheduling,” Ph.D. dissertation, Royal Institute of Technology, Department of Machine Design, Stockholm, Sweden, 2004.

[13] L. Hetel, J. Daafouz, and C. Iung, “LMI control design for a class of exponential uncertain systems with application to network controlled switched systems,” in *Proceedings of the 2007 American Control Conference*, 2007, pp. 1401–1406.

[14] M. V. Kothare, V. Balakrishnan, and M. Morari, “Robust constrained model predictive control using linear matrix inequalities,” *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.