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Broadcast Control of Multi-Agent Systems

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***Abstract*— This paper addresses a broadcast control problem for multi-agent coordination. The "broadcast control" means governing multi-agent systems by sending the same signal to all the agents indiscriminatingly, under the assumption that there is no agent-to-agent communication. The problem studied here is to find both the information to be broadcasted and the local actions of the agents, to achieve a given motion-coordination task. We derive a solution which asymptotically achieves the task almost surely. The solution is demonstrated by a numerical simulation and an experiment.**

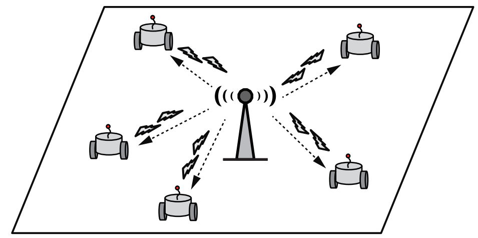

Fig. 1. Broadcast control of multi-agent systems.

I. INTRODUCTION

In recent years, there has been considerable interest in problems of multi-agent systems. This includes various subjects such as *rendezvous*, *coverage*, *formation*, and *synchronization* [1]–[4]. The motivation for the study comes from the great potential for engineering applications, e.g., cooperative robotics, mobile sensor networks, and grid computing.

So far, the focus of the topic is the *agent-to-agent communication* for achieving some global behavior. A number of results have been obtained from this point of view (see, e.g., [1]–[4] and references therein). On the other hand, our interest in this paper is somewhat different. Instead of using the agent-to-agent communication, we want to solve multi-agent problems with the *one-to-all broadcast*. More precisely, our goal is to complete given motion-coordination tasks just by sending the *same* signal to all the agents "indiscriminatingly", as shown in Fig. 1. This broadcast control has a different kind of theoretical difficulty from that of the communication based control. For instance, when a signal is broadcasted for moving an agent to a point, the signal also affects the other agents and may result in an undesirable configuration, which poses a new challenge for multi-agent control.

In this paper, we establish a broadcast control framework for multi-agent coordination. The control system considered here is composed of agents, local controllers, and a global controller as shown in Fig. 2, where the local controllers determine the local actions of each agent and the global controller broadcasts a signal to govern the global behavior. Our problem is to find local and global controllers achieving given motion-coordination tasks subject to the indiscriminating treatment of the agents. For the issue, this paper makes two contributions from theoretical and experimental sides, which are summarized as follows.

First, we present a solution to the broadcast control problem. For giving the solution, we clarify that the local controllers have to have *randomness* to attain given motion-coordination tasks. Based on this, the solution is given as the combination of

- the local controllers which let the agents alternately perform random walk and deterministic walk,
- the global controller which broadcasts the achievement degree of the task.

It is then proven that the solution asymptotically achieves the task with probability 1.

Second, in addition to a numerical simulation, an experiment has been conducted with seven mobile robots in order to evaluate the practical performance. This result demonstrates that the proposed controllers can be implemented even on a limited hardware device and the gap between theory and practice is small.

As a final remark of this section, the difference from the existing results on the broadcast control should be noted. The concept of the broadcast control has been originally proposed in [5] and has been applied to the control of biosystems in [6]. To our best knowledge, there are only the two results on the broadcast control (except for some variations). The aim in [5], [6] is to control the group of homogeneous two-state Markov chains, and so our idea of using the broadcast control for the multi-agent coordination is new. Moreover, from a technical point of view, a kind of the role assignment is often essential for multi-agent coordination, while it is not for the control of the group of Markov chains. Also in this respect, the contribution of this paper is distinguished.

Notation: Let $\mathbf{R}$, $\mathbf{R}_+$, $\mathbf{R}_{0+}$, and $\mathbf{N}$ be the real number field, the set of positive real numbers, the set of nonnegative real numbers, and the set of nonnegative integers, respectively. We denote by $0_{n\times m}$ (or 0) the $n \times m$ zero matrix. The Euclidian norm of the vector x is expressed by $\|x\|$. For the

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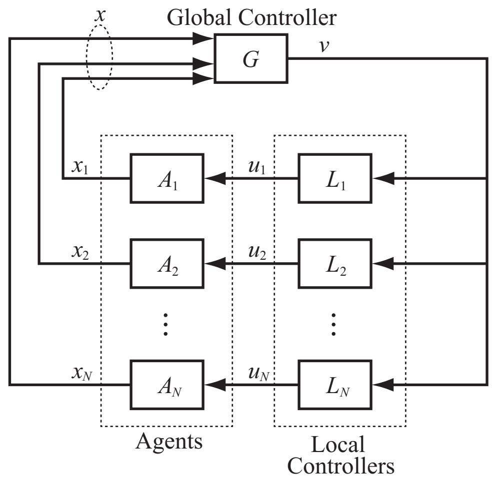


Fig. 2. Broadcast control system Σ.

vector $x = [x_1\ x_2\ \cdots\ x_n]^\top \in \mathbf{R}^n$ with nonzero elements, we use $x^{(-1)}$ to represent the vector composed of the elementwise inverse, i.e., $x^{(-1)} = [x_1^{-1}\ x_2^{-1}\ \cdots\ x_n^{-1}]^\top \in \mathbf{R}^n$. For example, $x^{(-1)} = [1/6\ \ 1/3\ \ 1/2]^\top$ for $x = [6\ \ 3\ \ 2]^\top$. The gradient of the differentiable function $J : \mathbf{R}^n \to \mathbf{R}$ (scalar-valued) is denoted by $\nabla J(x)$. Namely,

$$\nabla J(x) := \left[\frac{\partial J(x)}{\partial x_1}\ \ \frac{\partial J(x)}{\partial x_2}\ \cdots\ \frac{\partial J(x)}{\partial x_n}\right]^\top \in \mathbf{R}^n.$$

Finally, the function $f(x)$ is called the *stochastic function* if $f(x)$ contains randomness. In other words, the stochastic function $f(x)$ is expressed as $f(x) = g(x, \Delta)$ with a function g and a random variable Δ.

II. Problem Formulation

Consider the feedback system Σ in Fig. 2, composed of N *agents*, *local controllers*, and a *global controller*.

The agent A_i is given by

$$A_i:\ x_i(t+1) = x_i(t) + u_i(t) \tag{1}$$

where $x_i(t) \in \mathbf{R}^n$ is the position in the n-dimensional space, $u_i(t) \in \mathbf{R}^n$ is the control input, and $t \in \mathbf{N}$ is the time. The initial state is given as $x_i(0) = x_{i0} \in \mathbf{R}^n$. The group of the agents $(A_1, A_2, \ldots, A_N)$ is the control object in this paper, and the *group position* (the collective position of the agents) is denoted by $x \in \mathbf{R}^{nN}$, i.e., $x := [x_1^\top\ \ x_2^\top\ \ \cdots\ \ x_N^\top]^\top$. The *initial group position* is given as $x(0) = x_0 := [x_{10}^\top\ \ x_{20}^\top\ \ \cdots\ \ x_{N0}^\top]^\top \in \mathbf{R}^{nN}$.

The local controller L_i, which is an add-on to the agent A_i, is of the form

$$L_i: \begin{cases} \xi_i(t+1) = \alpha(\xi_i(t), v(t)), \\ \quad u_i(t) \quad = \beta(\xi_i(t), v(t)) \end{cases} \tag{2}$$

where $\xi_i(t) \in \mathbf{R}^\nu$, $v(t) \in \mathbf{R}$, and $u_i(t) \in \mathbf{R}^n$ are the state, the input (called the *broadcast signal*), and the output, and $\alpha : \mathbf{R}^\nu \times \mathbf{R} \to \mathbf{R}^\nu$ and $\beta : \mathbf{R}^\nu \times \mathbf{R} \to \mathbf{R}^n$ are functions. The functions α, β and the initial state $\xi_i(0)$ are assumed to be the *same* in the local controllers L_i $(i = 1, 2, \ldots, N)$, which implies that we deal with the agents indiscriminatingly, i.e., that we cannot move a single agent without affecting the other agents. For simplicity of discussion, we further assume

$$\xi_i(0) = 0_{\nu\times 1}. \tag{3}$$

The global controller G is given by

$$G:\ v(t) = \gamma(x(t)) \tag{4}$$

where $x(t) \in \mathbf{R}^{nN}$ and $v(t) \in \mathbf{R}$ are the input and output, and $\gamma : \mathbf{R}^{nN} \to \mathbf{R}$ is a function.

In the feedback system Σ, the group of the agents A_i $(i = 1, 2, \ldots, N)$ is governed by the combination of the global controller G and the local controllers L_i $(i = 1, 2, \ldots, N)$ so as to achieve pre-specified coordination tasks. Here, G evaluates the degree of the global achievement at each time and broadcasts the control signal v to L_i $(i = 1, 2, \ldots, N)$. On the other hand, L_i determines the local action of the agent A_i depending upon the broadcast signal v. This control scheme is called here the *broadcast control* and the $N+1$-tuple $(L_1, L_2, \ldots, L_N, G)$ is called the *broadcast controller*.

Then our problem is formulated as follows.

Problem 1: For the broadcast control system Σ in Fig. 2, suppose that the objective function $J : \mathbf{R}^{nN} \to \mathbf{R}_{0+}$, describing a motion-coordination task, is given. Then find a broadcast controller $(L_1, L_2, \ldots, L_N, G)$ (i.e., find functions α, β, and γ) satisfying

$$\lim_{t\to\infty} J(x(t)) = \min_{x\in\mathbf{R}^{nN}} J(x) \tag{5}$$

for every initial group position $x_0 \in \mathbf{R}^{nN}$. ■

Two remarks on Problem 1 are given.

First, as easily imagined, if $J(x)$ is nonconvex, it is in general hard to solve the minimization problem in the right-hand side of (5). This implies that, for such a nonconvex $J(x)$, it is hopeless to obtain a broadcast controller achieving (5) in the global sense. We thus address Problem 1 assuming that the right-hand side of (5) means a local minimum of $J(x)$.

Second, various motion-coordination tasks can be described by the objective function J. For example, the rendezvous is expressed by

$$J(x) = \sum_{i=1}^{N}\sum_{j=1}^{N} \|x_i - x_j\|, \tag{6}$$

and the coverage [7] is represented by

$$J(x) = \int_{\mathbf{Q}} \min_{i\in\{1,2,\ldots,N\}} \psi(\|q - x_i\|)\phi(q)dq \tag{7}$$

where $\mathbf{Q} \subset \mathbf{R}^n$ is the bounded region, $\psi : \mathbf{R}_{0+} \to \mathbf{R}_{0+}$ is the function describing the utility of placing an agent at a distance from a given location in $\mathbf{Q}$, and $\phi : \mathbf{R}^n \to \mathbf{R}_{0+}$ is the weighting function.

III. Broadcast Control

A. Characterization of Broadcast Controllers

To derive a solution to Problem 1, we first provide a necessary condition for the given broadcast controller $(L_1, L_2, \ldots, L_N, G)$ to satisfy (5). To this end, the notions of deterministic class and stochastic class are introduced to L_i $(i = 1, 2, \ldots, N)$.

Definition 1: Consider the local controller L_i in (2). If neither α nor β is a stochastic function, L_i is said to be *deterministic*; otherwise, it is said to be *stochastic* (see the end of Section I for the definition of the stochastic functions). ■

When the controller L_i is deterministic, the output $u_i(t)$ is uniquely determined from the past input sequence $(v(0), v(1), \ldots, v(t))$. While the output of the stochastic L_i is a random variable.

Using the notions, we obtain the following result.

Lemma 1: For the broadcast control system Σ, suppose that J and $(L_1, L_2, \ldots, L_N, G)$ are given. Let $\mathbf{X}^*(J) \subseteq \mathbf{R}^{nN}$ be the set of the local minimum points of $J(x)$ and assume that $\mathbf{X}^*(J)$ can be represented as the union of nonempty polyhedra $\mathbf{X}_j^*(J)$ of dimension $nN - m_j$ $(j = 1, 2, \ldots, \mu)$, i.e., $\mathbf{X}^*(J) = \bigcup_{j=1}^{\mu} \mathbf{X}_j^*(J)$, where $m_j \in \{0, 1, \ldots, nN\}$. If L_i $(i = 1, 2, \ldots, N)$ are deterministic and $n < \min_{j \in \{1,2,\ldots,\mu\}} m_j$, then (5) does not hold for almost all $x_0 \in \mathbf{R}^{nN}$.

Proof: Due to the limited space, we omit the proof. ■

As can be imagined, the only thing the broadcast controllers including deterministic L_i $(i = 1, 2, \ldots, N)$ can do is to move the agent group translationally with keeping their relative positions. Therefore, such broadcast controllers cannot be a solution to Problem 1 with a general class of J (the class is specified by n and $\min_{j \in \{1,2,\ldots,\mu\}} m_j$). Lemma 1 captures this idea.

The above result suggests us to consider broadcast controllers including stochastic local controllers.

B. Proposed Broadcast Controllers

Now, we give a solution to Problem 1.

Assume the state $\xi_i(t)$ of L_i to be $(n+2)$-dimensional, and let $\xi_{i1}(t) \in \mathbf{R}^n$, $\xi_{i2}(t) \in \mathbf{R}$, and $\xi_{i3}(t) \in \mathbf{R}$ denote the components of $\xi_i(t)$, i.e.,

$$\xi_i(t) = \begin{bmatrix} \xi_{i1}(t) \\ \xi_{i2}(t) \\ \xi_{i3}(t) \end{bmatrix} \in \mathbf{R}^n \times \mathbf{R} \times \mathbf{R}. \tag{8}$$

By considering Lemma 1, our solution is given as follows:

$$\alpha(\xi_i(t), v(t)) := \begin{bmatrix} \Delta_i(t) \\ v(t) \\ \xi_{i3}(t) + 1 \end{bmatrix}, \tag{9}$$

$$\beta(\xi_i(t), v(t)) := \begin{cases} c(\xi_{i3}(t))\Delta_i(t) & \text{if } \xi_{i3}(t) \in \{0, 2, 4, \ldots\}, \\ -c(\xi_{i3}(t))\xi_{i1}(t) - a(\xi_{i3}(t))\dfrac{v(t) - \xi_{i2}(t)}{c(\xi_{i3}(t))}\xi_{i1}^{(-1)}(t) & \\ & \text{if } \xi_{i3}(t) \in \{1, 3, 5, \ldots\}, \end{cases} \tag{10}$$

$$\gamma(x(t)) := J(x(t)) \tag{11}$$

where $\Delta_i(t) \in (\mathbf{R} \setminus \{0\})^n$ is a "random" variable and $a(\xi_{i3}(t)) \in \mathbf{R}_+$ and $c(\xi_{i3}(t)) \in \mathbf{R}_+$ are the gains of this controller. Note here that

$$a(\xi_{i3}(t)) = a(t), \quad c(\xi_{i3}(t)) = c(t) \tag{12}$$

hold (which will be proven later around (13)), and so the gains are time-varying. Note also that the resulting $\xi_{i1}(t)$ has nonzero elements, under which $\xi_{i1}^{(-1)}(t)$ expresses the vector composed of the elementwise inverse of $\xi_{i1}(t)$ as defined in Section I.

In the proposed controller, the local controllers L_i $(i = 1, 2, \ldots, N)$ act letting each agent alternately perform two steps, (a) the random move and (b) the deterministic move to a specified point, as shown in Fig. 3, and the global controller G plays a role to broadcast the value of $J(x(t))$, i.e., broadcast the achievement degree of the desired motion-coordination task.

The role of G is clear from (4) and (11), while that of L_i may not be so clear. For this, the following explanation will be helpful. From (2), (8), and (9), the state ξ_{i3} evolves according to the dynamics $\xi_{i3}(t+1) = \xi_{i3}(t) + 1$. This and (3) mean that $\xi_{i3}(t)$ is equal to the time, i.e.,

$$\xi_{i3}(t) = t. \tag{13}$$

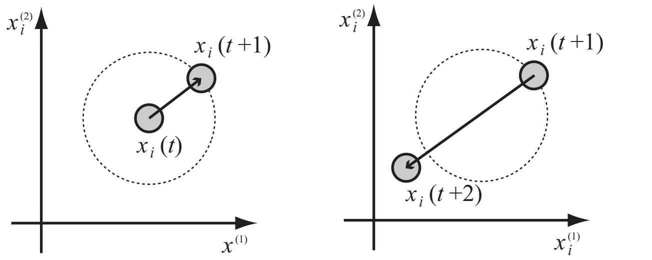
So the conditions in (10) respectively correspond to $t \in \{0, 2, 4, \ldots\}$ and $t \in \{1, 3, 5, \ldots\}$, which implies that the controller L_i periodically applies two kinds of inputs to the agent A_i. Moreover, by noting that $\Delta_i(t)$ is a random variable, (2) and (10) imply that the control input $u_i(t)$ is a random vector at $t \in \{0, 2, 4, \ldots\}$ and is a vector composed of pre-specified parameters at $t \in \{1, 3, 5, \ldots\}$. In this way, L_i steers A_i as shown in Fig. 3.

C. Convergence

Next, we show that the proposed broadcast controller in (2), (4), (9), (10), and (11) is a solution to Problem 1 under several conditions.

Theorem 1: For the broadcast control system Σ, suppose that J is given and assume that J is differentiable and there exists a vector $x^* \in \mathbf{R}^{nN}$ satisfying $\nabla J(x^*) = 0$. Let $(L_1, L_2, \ldots, L_N, G)$ be given by (2), (4), (9), (10), and (11). Let also Δ_{ij} denote the jth element of Δ_i and let $\Delta := [\Delta_1^\top \ \Delta_2^\top \ \cdots \ \Delta_N^\top]^\top$. If

(A1) J is twice differentiable (in addition to the differentiability assumed above),



(a) Step 1: Random move.

(b) Step 2: Deterministic move to a specified point.

Fig. 3. Motion of the agent A_i by the proposed controller (where $n := 2$, $x_i^{(j)}$ denotes the j-element of x_i, and the circles express A_i).

(A2) x^* is an asymptotically stable equilibrium of the gradient system $\dot{z}(\tau) = -\nabla J(z(\tau))$, where $\tau \in \mathbf{R}_{0+}$, $z(\tau) \in \mathbf{R}^{nN}$, and the stability is in the Lyapunov sense,

(A3) $a(t) = a(t+1)$ and $c(t) = c(t+1)$ for $t \in \{0, 2, 4, \ldots\}$, $\lim_{t\to\infty} a(t) = 0$, $\sum_{t=0}^{\infty} a(t) = \infty$, $\lim_{t\to\infty} c(t) = 0$, and $\sum_{t=0}^{\infty}(a(t)/c(t))^2 < \infty$ (note here that (12) holds),

(A4) $\Delta_{ij}(t)$ ($i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n$, $t = 0, 1, \ldots$) are independent identically distributed (independent also with $x(t)$ ($t = 0, 1, \ldots$)), and symmetrically distributed about zero with $|\Delta_{ij}(t)| < \infty$, $|\Delta_{ij}^{-1}(t)| < \infty$, and $|\Delta_{ij}^{-2}(t)| < \infty$ w.p.1,

(A5) $E[\,J(x(t) + c(t)\Delta(t))^2\,]$ is bounded for all $t \in \mathbf{N}$,

(A6) For a compact set $\mathbf{S} \subseteq \mathbf{R}^{nN}$ such that $\dot{z}(\tau) = -\nabla f(z(\tau))$ with $x(0) \in \mathbf{S}$ results in $x(\infty) = x^*$, $x(t) \in \mathbf{S}$ occurs infinitely often for almost all sample points of $\Delta_i(t)$ ($i = 1, 2, \ldots, N$ and $t = 0, 1, \ldots$),

(A7) $\sup_{t\in\mathbf{N}} \|x(t)\| < \infty$ w.p.1,

then

$$\lim_{t\to\infty} x(t) = x^* \quad \text{w.p.1.} \tag{14}$$

Proof: The following four facts prove the theorem.

(i) For $t \in \{0, 2, 4, \ldots\}$, the relation

$$x(t{+}2) = x(t) - a(t)\frac{v(t+1) - v(t)}{c(t)}\Delta^{(-1)}(t) \tag{15}$$

holds.

(ii) For the broadcast signal values at time $t \in \{0, 2, 4, \ldots\}$ and time $t+1$, the relations

$$v(t) = J(x(t)), \tag{16}$$
$$v(t+1) = J(x(t) + c(t)\Delta(t)) \tag{17}$$

hold.

(iii) The dynamics (15) with (16) and (17) is equivalent to the stochastic approximation algorithm developed in [8], and so the sequence $x(0), x(2), x(4), \ldots$ converges to x^* w.p.1 under (A1)–(A7).

(iv) The sequence $\|x(1) - x(0)\|, \|x(3) - x(2)\|, \|x(5) - x(4)\|, \ldots$ converges to 0 w.p.1. ■

The intuitive interpretation of the proposed controller is as follows. From (15), (16), and (17), the resulting system Σ evolves according to

$$x(t{+}2) = x(t) - a(t)d(x(t), \Delta(t), c(t)) \tag{18}$$

for $t \in \{0, 2, 4, \ldots\}$, where

$$d(x(t), \Delta(t), c(t)) := \frac{J(x(t){+}c(t)\Delta(t)) - J(x(t))}{c(t)}\Delta^{(-1)}(t). \tag{19}$$

By applying Taylor's theorem to $J(x(t) + c(t)\Delta(t))$ and taking a similar way as in [8], [9], it can be shown that

$$E[d(x(t), \Delta(t), c(t))|x(t)] = \nabla J(x(t)) + O(c(t)) \quad (c(t) \to 0). \tag{20}$$

Namely, the expected value of $d(x(t), \Delta(t), c(t))$ is nearly equal to the gradient of $J(x(t))$. Thus (18) means $E[x(t+2)|x(t)] \simeq x(t) - a(t)\nabla J(x(t))$, i.e., a stochastic version of the so-called gradient-decent method.

The seven conditions in Theorem 1 are fairly standard in stochastic approximation [8], [10]. Condition (A1) means that the objective function J is sufficiently smooth and (A2) is a common requirement for decent-type algorithms. Condition (A3) is imposed for the gains a and c, and (A4) is for the random variables Δ_i ($i = 1, 2, \ldots, N$). The parameters a, c, and the probability distribution for Δ_i are usually designed by the users, so as to satisfy (A3)–(A4). The last (A5)–(A7) are technical conditions to guarantee the convergence. These may not be easy to check in an analytical way, but it is known that they are not restrictive conditions in practice, as addressed in [8], [10]. This fact has been demonstrated by a number of examples (a list of results are provided in [11]). In addition, as explained in [10], these can be ignored by replacing the method with its the projected version. Hence, (A1)–(A4) are essential in our problem.

A typical choice of the gains a and c (note (12)) will be

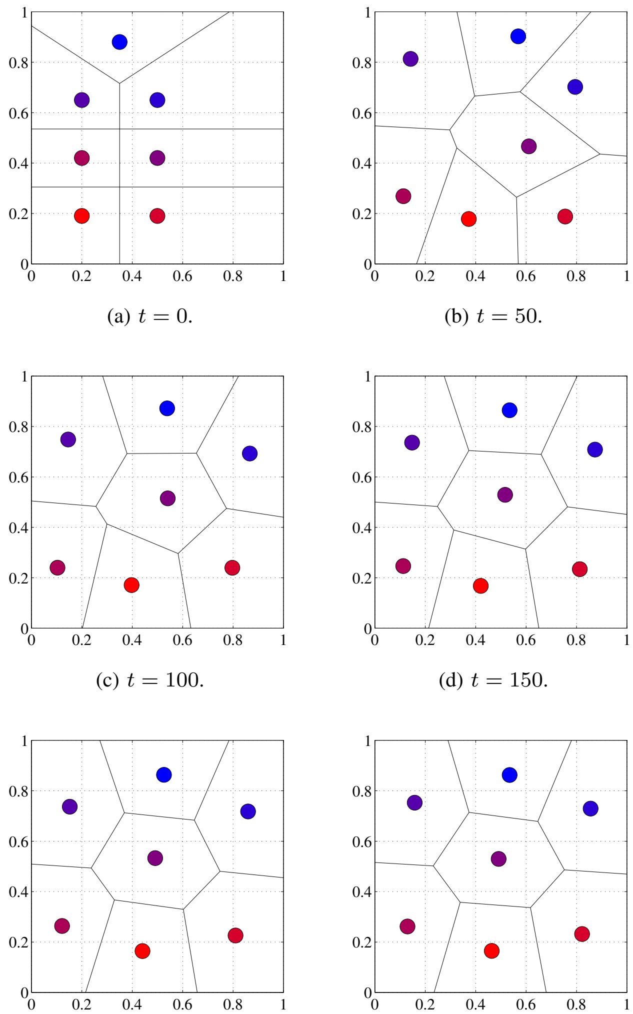
$$a(t) := \begin{cases} \dfrac{a_0}{((t/2) + 1 + a_v)^{a_p}} & \text{if } t \in \{0, 2, 4, \ldots\}, \\ a(t-1) & \text{if } t \in \{1, 3, 5, \ldots\}, \end{cases} \tag{21}$$

$$c(t) := \begin{cases} \dfrac{c_0}{((t/2) + 1)^{c_p}} & \text{if } t \in \{0, 2, 4, \ldots\}, \\ c(t-1) & \text{if } t \in \{1, 3, 5, \ldots\} \end{cases} \tag{22}$$

where $a_0, a_v, a_p, c_0, c_p \in \mathbf{R}_+$ are arbitrarily given so as to satisfy (A3), i.e., $a_p \le 1$ and $a_p - c_p > 0.5$ [12]. Note that $t/2$ is equal to the number of the iterations of the two steps in Fig. 3. On the other hand, a typical probability distribution for Δ_{ij} (i.e., for Δ_i) is the Bernoulli distribution with outcome ± 1 and equal probabilities.

IV. Numerical Simulation

Consider the broadcast control system Σ in Fig. 2, where $N := 7$ and $n := 2$. We consider the uniform coverage as a motion-coordination task, and thus the objective function J is given by (7) with $\mathbf{Q} := [0,1]^2$, $\psi(\|q - x_i\|) := \|q - x_i\|^2$, and $\phi(x) := 1$ ($\phi(x) \equiv 1$). This satisfies condition (A1)



(a) $t = 0$. (b) $t = 50$.

(c) $t = 100$. (d) $t = 150$.

(e) $t = 200$. (f) $t = 250$.

Fig. 4. Snapshots of the group position (in simulation).

in Theorem 1 [13]. We employ the broadcast controller $(L_1, L_2, \ldots, L_7, G)$ in (2), (4), (9), (10), and (11). The gains are given by (21) and (22) with $a_0 := 2.38$, $a_v := 15$, $a_p := 0.7$, $c_0 := 0.026$, and $c_p := 0.16$. The probability distribution for $\Delta_i(t)$ $(t = 0, 1, \ldots)$ is the Bernoulli type as described at the end of Section III-C. For these parameters, conditions (A2)–(A4) hold. The other conditions are checked numerically.

Fig. 4 illustrates the snapshots of the group position $x(t)$ at $t = 0, 50, \ldots, 250$, where the circles express the agents A_i $(i = 1, 2, \ldots, 7)$ and the solid line represents the boundary of the Voronoi cells for the agents. Fig. 5, on the other hand, depicts the time evolution of the objective function $J(x(t))$. These show that the coverage by seven agents is achieved by the proposed broadcast controller.

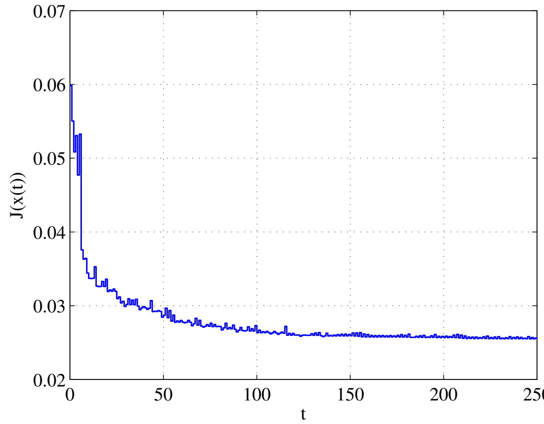


Fig. 5. Time evolution of the objective function (in simulation).

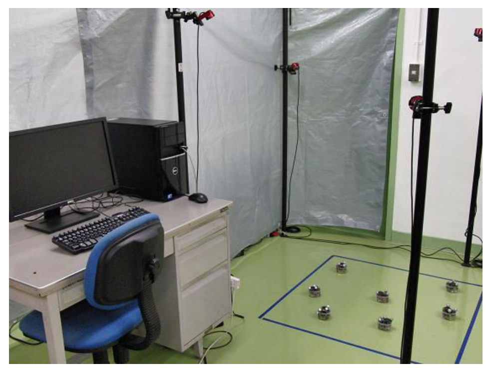


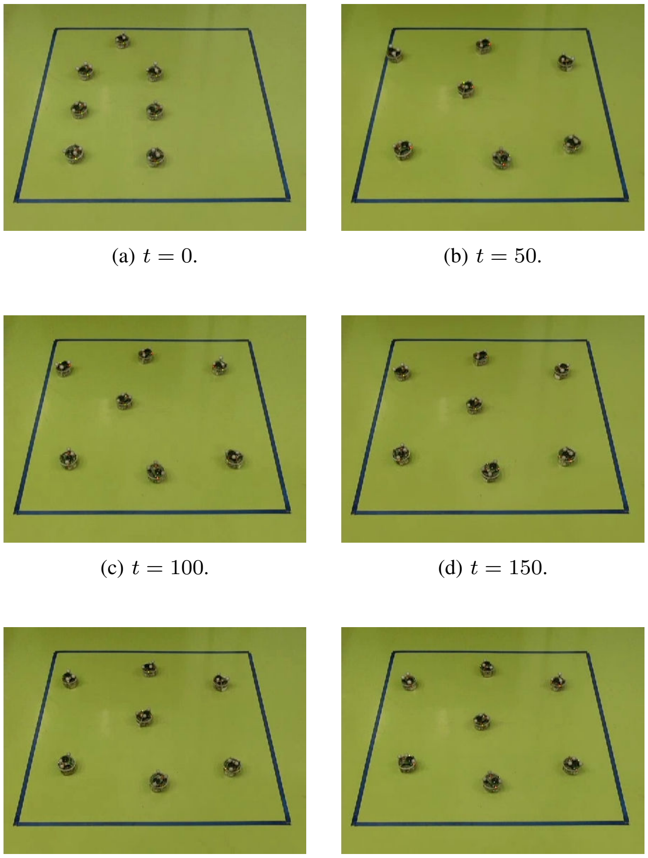
Fig. 6. Experimental setup.

V. Experiment

For testing our broadcast control framework, we have set up the experimental system in Fig. 6, composed of an environment, seven agents, a motion capture system, and a desktop computer.

The dimension of the environment is 1×1 [m]. As the agents A_i $(i = 1, 2, \ldots, 7)$, we have employed the two-wheeled mobile robot *e-puck* [14] of a diameter of 75 [mm] and a height of 60 [mm]. This is actuated by two stepper motors. Though the mobility of the agent model in (1) is different from that of the two-wheeled robot, we substitute the two-wheeled robot for the agent model in (1) by converting the control input in the Cartesian coordinates to that in the polar coordinates. The local controllers L_i $(i = 1, 2, \ldots, 7)$ are implemented in the micro computers embedded in the *e-puck* robots. The translational and rotational positions of the agents are measured by the motion capture system *OptiTrack* [15]. This integrates six cameras with a frame rate of 100 [Hz] and a resolution of 640×480 [pixel]. The global controller G is implemented in the desktop computer and the broadcast signal is sent to the agents via Bluetooth [16].

For the above experimental system, we consider again the uniform coverage as a motion-coordination task. The objective function J and the broadcast controller $(L_1, L_2,$



(a) $t = 0$. (b) $t = 50$.

(c) $t = 100$. (d) $t = 150$.

(e) $t = 200$. (f) $t = 250$.

Fig. 7. Snapshots of the group position (in experiment).

$\ldots, L_7, G)$ are the same as considered in Section IV.

Fig. 7 illustrates the snapshots of the group position $x(t)$. We can see that the experimental result agrees with the simulation result in Figs. 4. Thus the gap between theory and practice is small and the proposed controller works well in practical situations.

VI. Conclusion

A broadcast control method for multi-agent coordination has been given. By focusing on the necessity of randomness in the broadcast control system, we have derived a broadcast controller, composed of local controllers inducing agents to take random actions and a global controller broadcasting the achievement degree of a given motion-coordination task. It has been proven that the proposed controller achieves the motion-coordination task with probability 1. Furthermore, a simulation and experimental evaluation have been performed to demonstrate that the proposed controller is useful in practice.

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