2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Anti-windup Synthesis for Optimizing Internal Model Control

Ambrose A. Adegbege* and William P. Heath**

Abstract— **We develop new design procedures for optimizing anti-windup control applicable to open-loop stable multivariable plants subject to input saturations. The optimizing anti-windup control falls into a class of compensator commonly termed directionality compensation. The computation of the control involves the on-line solution of a low-order quadratic program in place of simple saturation. We exploit the equivalence of the quadratic program to a feedthrough term in parallel with a deadzone-like nonlinearity that satisfies a sector bound condition. This allows for LMI-based anti-windup synthesis using a decoupled structure similar to that proposed in the literature for anti-windup schemes with simple saturation. We demonstrate the effectiveness of the design compared to several schemes using a highly ill-conditioned benchmark example.**

I. INTRODUCTION

Most practical control problems must deal with constraints imposed by equipment limitations such as actuator nonlinearities. While it may be of economic benefits to operate on or close to the constraints, the violation of actuator constraints during normal operation can result in serious performance degradation or even instability. One approach, that has recently received much attention, in dealing with such problems is the anti-windup technique [1], [2], [3], [4], [5], [6], [7], [8]. It has however, become a common practice to incorporate an additional artificial nonlinearity (directionality compensator) in multivariable anti-windup designs to address the problem of directionality change in control action [9], [10], [11], [12], [13], [14], [15], [16], [17]. Such directionality compensators often take the form of dynamic optimization problems that are solved either implicitly ([18], [19]) or explicitly ([13], [14], [15]) during control computation. When the control policy is obtained by an explicit solution of on-line optimization problem at each time step, the resulting scheme is termed optimizing anti-windup (for example [20], [21], [17]).

While the synthesis of non-optimizing anti-windup with both stability and performance guarantees has been studied extensively (see [3], [22], [4], [23], [24], [25], [7]), there have been few studies on the synthesis of optimizing anti-windup schemes with closed-loop stability guarantee. Most optimizing anti-windup schemes (e.g.[13], [15]) have focused on nonlinear performance optimization in the presence of input constraints without consideration of closed-loop stability. The design of directionality compensators is usually carried out independently of the control design and with the assumption that the resulting optimizing structures inherit the stability of the unsaturated loop. A notable exception is [21] where a sector-bound result (multivariable circle criterion) is extended to demonstrate the stability of optimizing anti-windup. In [16], [17], we employed the theory of Integral Quadratic Constraints (IQCs) to develop a sufficient robust stability condition for optimizing anti-windup subject to any infinity-norm-bounded uncertainty.

It is now standard in the saturating anti-windup synthesis literature to express the saturated loop in terms of a feedback interconnection involving a deadzone nonlinearity and a feedthrough term ([1], [3], [24], [22]). We note that the quadratic program in optimizing anti-windup can be similarly expressed as a corresponding nonlinearity satisfying a sector-bound condition. The information from the plant's structural characteristics and the quadratic program can then be incorporated into the optimizing anti-windup synthesis to guarantee closed-loop stability as well as improved nonlinear performance. The resulting synthesis problem can be cast as a convex optimization problem over linear matrix inequalities.

Notation: Given a square matrix X, we define $\mathrm{He}(X) := X + X^T$.

II. PROBLEM SETUP

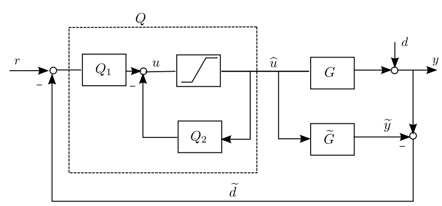

Fig. 1. The modified IMC anti-windup Structure

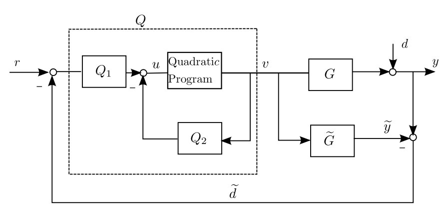

Fig. 2. IMC anti-windup with a quadratic program as directionality compensator

We consider the modified internal model control anti-windup structure of Fig. 1 where $G(s), \tilde{G}(s) : \mathcal{L}^m \rightarrow \mathcal{L}^p$ represent the plant and the nominal plant dynamics respectively. For compactness of expressions, we will henceforth

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Financial support from Petroleum Technology Development Fund (PTDF) is gratefully acknowledged.

drop the arguments (s or jw). Although we have restricted our discussions to the continuous-time systems, the discrete-time analysis follows naturally. The nominal IMC controller Q is assumed to have been designed to meet some nominal stability and performance specifications [26]. The anti-windup compensators Q_1 and Q_2 are related to the nominal IMC controller through

$$Q = (Q_2 + I)^{-1} Q_1. \tag{1}$$

Following the convention of anti-windup designs, we assume that the plant input is subject to saturation nonlinearities. The input signal u is constrained such that

$$u_i^{min} \le u_i(t) \le u_i^{max} \quad i = 1, \dots, m. \tag{2}$$

This can be represented by the saturation function sat(.) defined as

$$\text{sat}(u(t)) = \begin{bmatrix} \text{sat}(u_1(t)) \\ \vdots \\ \text{sat}(u_m(t)) \end{bmatrix}$$

where

$$\text{sat}(u_i(t)) = \begin{cases} u_i^{max} & u_i(t) > u_i^{max} \\ u_i(t) & u_i^{min} \le u_i(t) \le u_i^{max} \\ u_i^{min} & u_i(t) < u_i^{min} \end{cases}$$

denotes the saturation nonlinearity associated with each of the manipulated input $u_i(t)$.

However, for optimizing anti-windup, artificial nonlinearities (directionality compensators) are introduced such that the input saturation nonlinearities are never active and may be safely ignored as shown in Fig. 2. The artificial nonlinearities are assumed to take the form of a generalized positive hessian quadratic program (QP) as in (3).

$$\text{QP}_1 : \quad \begin{aligned} v^* = \arg\min \frac{1}{2} v^T H v - v^T H u \\ \text{subject to } Lv \preceq b \end{aligned} \tag{3}$$

where $H = H^T \in \mathbb{R}^{m \times m}$. The fixed terms $L \in \mathbb{R}^{2m \times m}$ and $b \in \mathbb{R}^{2m}$ in the inequality constraints are respectively obtained from (2) as

$$L = \begin{bmatrix} -I_m \\ I_m \end{bmatrix} \text{ and } b = \begin{bmatrix} -u^{min} \\ u^{max} \end{bmatrix} \tag{4}$$

with $u^{min} = \left[\begin{array}{ccc} u_1^{min}, & \cdots, & u_m^{min} \end{array} \right]^T$ and $u^{min} = \left[\begin{array}{ccc} u_1^{max}, & \cdots, & u_m^{max} \end{array} \right]^T$.

The quadratic program (QP_1) has some attractive properties. It can be solved efficiently on-line [27], [13], [28] and it has also been shown to satisfy the sector-bound condition [21]

$$\psi(u)^T H \left(\psi(u) - u\right) \le 0 \;\forall u. \tag{5}$$

Condition (5) is a generalized sector condition. A special case is when the nonlinearity is decoupled (i.e. $\psi_i(u) = \psi_i(u_i)$) with each component $\psi_i(u_i)$ inscribed in the sector $[0, 1]$. This corresponds to diagonal H. Besides containing information about the sector bound, the Hessian matrix H is also used to capture the directional characteristics of the plant in directionality compensation schemes [15], [17]. Typically, H takes the form $H = H_r^T H_r$ where H_r is the characteristic matrix of the plant [15]. Other choices of H_r are discussed in [17].

To obtain a decoupled representation of the optimizing IMC anti-windup, a related quadratic program to (3) is required in the forward path.

Lemma 1: Let the quadratic program (3) be set as $v = \psi(u)$ and let $w = \phi(u)$ be the quadratic program

$$\text{QP}_2 : \quad \begin{aligned} \phi(u) = \arg\min \frac{1}{2} w^T H w \\ \text{subject to } Lu - Lw \preceq b, \end{aligned} \tag{6}$$

the interconnection of $w = \phi(u)$ with $v = u - w$ is equivalent to $v = \psi(u)$.

Proof: The Karoush-Kuhn-Tucker (KKT) conditions [27] for ϕ are given by

$$\begin{aligned} Hw - L^T \lambda = 0 \\ Lu - Lw - b + s = 0 \\ s \succeq 0, \quad \lambda \succeq 0, \quad \lambda^T s = 0 \end{aligned} \tag{7}$$

If we substitute $w = u - v$ into (7), we obtain

$$\begin{aligned} Hv - Hu + L^T \lambda = 0 \\ Lv - b + s = 0 \\ s \succeq 0, \quad \lambda \succeq 0, \quad \lambda^T s = 0 \end{aligned} \tag{8}$$

The conditions in (8) are exactly the KKT conditions for ψ. ∎

It also follows from the KKT conditions (7) that $\phi(u)$ inhabits the same sector as the original nonlinearity $\psi(u)$ in (3). Using lemma 1, the optimizing anti-windup can be redrawn with the original quadratic program (3) replaced with a deadzone-like quadratic program (6) as shown in Fig. 3.

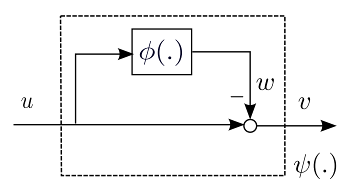


Fig. 3. Directionality compensator expressed a Deadzone-Like QP

With the assumption of perfect model, the closed loop equation of the equivalent structure can be expressed as

$$\begin{aligned} y &= y_{lin} - G(I + Q_2)^{-1} w \\ u &= u_{lin} + (I + Q_2)^{-1} Q_2 w \\ w &= \phi(u) \end{aligned} \tag{9}$$

with $y_{lin} = GQr + (I - GQ)d$ and $u_{lin} = Q(r - d)$ where y_{lin} and u_{lin} are the intended linear plant output and control input respectively. The disturbance filter $G(I + Q_2)^{-1}$ represents the difference between the intended linear performance and the degraded non-linear performance. The term $(I + Q_2)^{-1} Q_2$ plays an important role in determining closed-loop stability as it represents the loop transfer function around the nonlinearity. It is natural to choose $(I+Q_2)^{-1}$ as M from the coprime factorization of the plant $G = NM^{-1}$. With this choice, the slow modes of the plant are removed from the disturbance filter and thus recovery of linear performance after saturation is hastened. It also implies that there are no unstable pole-zero cancellations in the factorization of Q in (1). The IMC anti-windup can then be parameterized in terms of M as

$$Q_1 = M^{-1}Q, \quad Q_2 = M^{-1} - I. \tag{10}$$

The anti-windup design problem then reduces to finding an appropriate right coprime factorization of the nominal plant. This interpretation is similar to the coprime-factor parameterization in [29], [1], [5], [30], but here the optimizing framework offers an additional degree of freedom by incorporating the directional information of the plant through the parameter H.

The problem we seek to tackle in this paper is summarized as follows:

Problem: Given a stable plant G, a nominal internal model controller Q which meets certain linear performance specifications and a non-singular matrix H which contains the directional characteristics of the plant, synthesize the anti-windup compensator Q_1 and Q_2 such that the closed-loop system of Fig. 2 is stable, has a guaranteed level of non-linear performance and recovers the linear performance when there are no control saturations (i.e. $v = u$).

III. Stability and Performance Analysis

With the assumption that Q has been designed to meet some specified nominal stability and performance requirements, the stability of the optimizing anti-windup is then determined by the stability of the non-linear loop involving the nonlinearity $\phi(.)$ and the loop transfer function $(I + Q_2)^{-1}Q_2 = I - M$. The stability of such interconnections involving a class of nonlinearities has been widely studied using results from small gain, passivity, multiplier and IQC theories (see [29], [31] for saturating anti-windup). Property (5) allows the extension of such results to the optimizing anti-windup. Here, we exploit the structure of the quadratic program (6) to construct sufficient stability condition for the optimizing anti-windup in Fig. 2. We first show that after two linear transformations, the nonlinearity $\phi(.)$ belongs to the sector [0,I].

Lemma 2: Let $z = \varphi(x)$ be the quadratic program

$$\text{QP}_3: \quad \begin{aligned} \varphi(x) &= \arg\min \frac{1}{2} z^T z \\ &\text{subject to } Rx - Rz \preceq b, \end{aligned} \tag{11}$$

$\phi(u)$ and $\varphi(x)$ are equivalent after two linear transformations $\phi(u) = H_r^{-1}\varphi(x)$ and $u = H_r^{-1}x$. Furthermore, $\varphi(x)$ belongs to the sector $[0, I]$.

Proof: The KKT conditions for $\varphi(x)$ are given by

$$\begin{aligned} z - R^T\lambda &= 0 \\ Rx - Rz - b + s &= 0 \\ s \succeq 0, \quad \lambda \succeq 0, \quad \lambda^T s &= 0 \end{aligned} \tag{12}$$

Equivalence follows by substituting $w = H_r^{-1}z$, $u = H_r^{-1}x$ and finally $L = RH_r$ in (12) to obtain (7). Pre-multiplying the first KKT condition in (12) by z^T and substituting gives

$$z^T z - z^T x = -b^T\lambda \leq 0. \tag{13}$$

Hence, we may say $\varphi(x)^T(\varphi(x) - x) \leq 0$ or analogously $\varphi(x) \in$ sector$[0, I]$. ■

Using the linear transformations of lemma 2 followed by loop transformations, the nonlinear loop can be redrawn as shown in Figures 4 and 5 respectively.

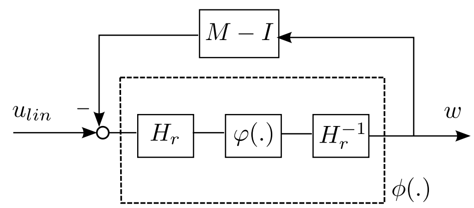


Fig. 4. Nonlinear loop with quadratic program $\varphi \in$ sector $[0, I]$

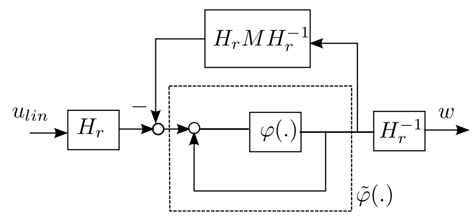


Fig. 5. Nonlinear loop with quadratic program $\varphi \in$ sector$[0, I]$ transformed to $\tilde{\varphi} \in$ sector $[0, \infty]$ via loop transformation

It then follows from passivity result or the multivariable circle criterion [21] that a sufficient condition for asymptotic stability of the nonlinear-loop is that HM is strongly positive real (SPR). This implies that

$$HM^* + MH > 0 \tag{14}$$

for all frequency and where the superscript $(*)$ denotes the complex conjugate transpose. Stronger stability result may be obtained by introducing multipliers such as those discussed in [17], [32], [33] into the nonlinear loop of Fig. 5. We would like to incorporate the above stability result into the choice of the compensators Q_1 and Q_2 for optimizing anti-windup of Fig. 2.

Apart from ensuring closed-loop stability, the main goal of anti-windup designs is to ensure graceful performance degradation during control inputs saturations and swift recovery of linear performance after a period of saturation. This objective can be achieved if the disturbance filter is small in some sense. Typically, the anti-windup performance criterion is specified in terms of minimization of the $\mathcal{L}_2$ gain from the unconstrained control input u_{lin} to the difference between the constrained output and the unconstrained output (i.e. $y - y_{lin}$) [24], [5], [22]. From the closed-loop equation (9), the map from u_{lin} to $(y - y_{lin})$ can be expressed in state-space form using the coprime-factor parameterization of (10).

Let the right coprime factors of the plant $G = NM^{-1}$ admit the following state space realizations

$$\begin{bmatrix} M \\ N \end{bmatrix} = \left[\begin{array}{c|c} A+BF & B \\ \hline F & I \\ C+DF & D \end{array}\right] \tag{15}$$

where F must be chosen such that $A+BF$ is Hurwitz, the map $u_{lin} \to (y - y_{lin})$ is obtained as

$$\begin{bmatrix} \dot{x} \\ u - u_{lin} \\ y_d \end{bmatrix} = \left[\begin{array}{c|c} A+BF & B \\ \hline -F & 0 \\ C+DF & D \end{array}\right] \begin{bmatrix} x \\ w \end{bmatrix} \quad \text{with} \quad w = \phi(u). \tag{16}$$

The optimizing anti-windup synthesis problem is thus that of finding a free parameter F such that the closed-loop system of Fig 2 has both stability and performance guarantees.

IV. Anti-windup Compensator Synthesis

The results in this section extend the synthesis approach of [24], [30] to the optimizing anti-windup. Here, we consider a multi-objective synthesis approach which addresses *a*) closed-loop stability through a Lyapunov-based stability criterion; *b*) nonlinear performance by minimizing the $\mathcal{L}_2$-norm of the difference between the constrained output y and the unconstrained (nominal) output y_{lin} via the quadratic program (3); and *c*) recovery of linear performance through the minimization of the $\mathcal{L}_2$ gain of the map $u_{lin} \to (y - y_{lin})$.

Theorem 1 (Synthesis with Lyapunov stability criterion): Given a stable plant G with coprime factorization (15) and a stable Q. Suppose there exists positive quadratic function V and $\tau > 0$ such that for all t,

$$\dot{V}(x) + 2\tau w^T H(u - w) < 0 \tag{17}$$

for all x, u, u_{lin} and w satisfying (16). Then the optimizing anti-windup in Fig. 2 is stable. Moreover, condition in (17) is equivalent to the existence of $P = P^T > 0$ such that the following LMI in P, L and $\alpha > 0$

$$\begin{bmatrix} AP + PA^T + BL + L^T B^T & \alpha B - L^T H \\ \alpha B^T - HL & -2\alpha H \end{bmatrix} < 0. \tag{18}$$

is satisfied. A suitable choice of F is given as $F = LP^{-1}$ where L and P are feasible solutions of LMI (18).

Proof: Choosing V as $V = x^T X x$ with $X = X^T > 0$, the expression in (17) is a direct application of S-procedure to $\dot{V}(x) < 0$ and the sector condition $w^T H(u - w) \geq 0 \ \forall \ \tau > 0$. Condition (17) is guaranteed $\forall \ [x^T \ w^T]^T \neq 0$ with $u_{lin} = 0$ if (19) is satisfied.

$$\begin{bmatrix} XA + A^T X + XBF + F^T B^T X & XB - \tau F^T H \\ B^T X - \tau FH & -2\tau H \end{bmatrix} < 0. \tag{19}$$

By a simple congruence transformation $\text{diag}(X^{-1}, \tau^{-1} I)$ and defining $P = X^{-1}$, $\alpha = \tau^{-1}$, $L = FP$ in equation (19), we obtain the LMI in (18). ■

Remark 1: The main result of theorem 1 is that existing optimizing anti-windup schemes such as those in [13], [15], [19] can now be equiped with stability guarantees for all nonlinearities of the form of (3) and satisfying the generalized sector condition (5). By construction, the Hessian matrix H is always positive definite. The key feature of this theorem is the freedom in choosing H which may now assume a more general non-diagonal structure as compared to existing saturating anti-windup schemes [3], [24], [5]. The appropriate choice of H is made based on the plant characteristics [17], giving the designer more control on the anti-windup design as well as offering insights into the anti-windup computation.

The result (18) can also be obtained by applying the positive real lemma (e.g.[34]) to the stability condition (14). We note that similar sufficient stability result based on the KKT conditions of the associated input nonlinearities have earlier been suggested in the literature [20], [21] but only for posteriori stability checks. Here, the information from the QP (3) is incorporated into the synthesizing LMI such that closed loop stability is assured.

Theorem 2 (Synthesis with $\mathcal{L}_2$-gain performance): Given a stable plant G with coprime factorization (15) and a stable Q. Suppose there exists positive quadratic function V, $\tau > 0$ and $\gamma > 0$ such that for all t,

$$\dot{V}(x) + y_d^T y_d - \gamma^2 u_{lin}^T u_{lin} + 2\tau w^T H(u - w) < 0 \tag{20}$$

for all x, u, u_{lin} and w satisfying (16). Then the $\mathcal{L}_2$ gain of the map from u_{lin} to $(y - y_d)$ is less than γ. Moreover, condition (20) is equivalent to the existence of $P = P^T > 0$ such that the following LMI in P, L, $\alpha > 0$ and $\gamma > 0$

$$\text{He} \begin{bmatrix} AP + BL & 0 & 0 & 0 \\ \alpha B^T - HL & -\alpha H & 0 & 0 \\ 0 & H & -\gamma I/2 & 0 \\ CP + DL & \alpha D & 0 & -\gamma I/2 \end{bmatrix} < 0. \tag{21}$$

is satisfied. A suitable choice of F is given as $F = LP^{-1}$ where L and P are feasible solutions of LMI (21).

Proof: With a Lyapunov function choice of $V = x^T Y x$ with $Y = Y^T > 0$, condition (20) reduces to

$$\begin{bmatrix} Y\tilde{A} + \tilde{A}^T Y + \tilde{C}^T \tilde{C} & YB + \tilde{C}^T D - \tau F^T H & 0 \\ B^T Y + D^T \tilde{C} - \tau HF & D^T D - 2\tau H & \tau H \\ 0 & \tau H & -\gamma^2 I \end{bmatrix} < 0 \tag{22}$$

for all $[x^T \ w^T \ u_{lin}^T]^T \neq 0$ where $\tilde{A} = A + BF_2$ and $\tilde{C} = C + DF_2$. By applying Schur complement, change of variables $Y = \gamma X, \tau = \gamma\beta$ and congruence transformation using $diag(X^{-1}, \ \beta^{-1} I, \ I, \ I)$, (22) reduces to

$$\text{He} \begin{bmatrix} \tilde{A}X^{-1} & 0 & 0 & 0 \\ \beta^{-1} B^T - HFX^{-1} & -\beta^{-1} H & 0 & 0 \\ 0 & H & -\gamma I/2 & 0 \\ \tilde{C}X^{-1} & \beta^{-1} D & 0 & -\gamma I/2 \end{bmatrix} < 0. \tag{23}$$

Defining $P = X^{-1}$, $\alpha = \beta^{-1}$, $L = FP$ in (23) as well as substituting for $\tilde{A}$ and $\tilde{C}$ gives the LMI result (21). ■

Remark 2: The feasibility of LMI (21) is sufficent for the feasibility of LMI (18) since the LMI (18) is a principal submatrix (upper left 2×2 block) of LMI (21). On the other hand, LMI (21) is feasible if and only if LMI (18) is feasible and γ is sufficiently large (as seen from the lower right 2×2 block of LMI(21)).

V. Simulation Example

In order to demonstrate the effectiveness of our anti-windup design method, we consider a case-study example involving an ill-conditioned distillation column [26], [35]. This is a well studied problem because of the strong directionality and interaction that exist in the plant dynamics as well as its high sensitivities to diagonal input nonlinearities and uncertainties. We compare three anti-windup approaches, namely the modified IMC anti-windup ([18]), the LMI-based anti-windup design without directionality compensation ([1], [30]) and the LMI-based anti-windup design with directionality compensation of theorem 2. The second example demonstrates the superiority of the proposed method compared to existing optimization-based anti-windup when applied to plants with lightly damped modes.

Example 1: The nominal plant dynamics is given by the transfer function matrix

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \qquad (24)$$

with both inputs constrained as $|u_i| \leq 100$, $i = 1, 2$. In the absence of control input saturations, the linear controller is designed to achieve a completely decoupled closed-loop response represented as follows

$$G_F(s) = \frac{1}{1.43s+1} I.$$

The classical IMC controller design for a step input is

$$Q(s) = \frac{75s+1}{(1.43s+1)}\begin{bmatrix} 39.94 & -31.49 \\ 39.43 & -32.00 \end{bmatrix} \qquad (25)$$

and the corresponding unity feedback controller is

$$K(s) = \frac{75s+1}{1.43s}\begin{bmatrix} 39.94 & -31.49 \\ 39.43 & -32.00 \end{bmatrix}. \qquad (26)$$

We chose $H = H_r^T H_r$ with H_r as the characteristic matrix of the plant (in the notion of [15]).

$$H_r = \begin{bmatrix} 0.012 & -0.012 \\ 0.014 & -0.015 \end{bmatrix}$$

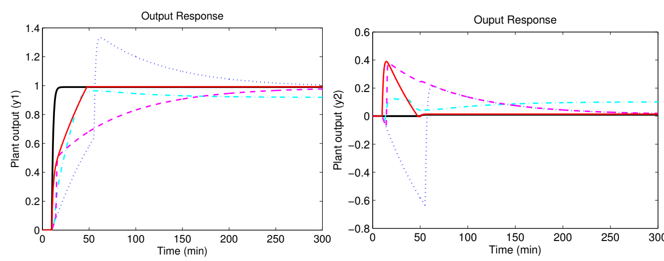
For the modified IMC anti-windup design [18], the plant model is slightly modified as

$$\widetilde{G}(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & \frac{-0.864}{0.1s+1} \\ \frac{1.082}{0.1s+1} & -1.096 \end{bmatrix} \qquad (27)$$

and the compensator Q_1 is designed as $Q_1 = f_A \widetilde{G} Q$ where

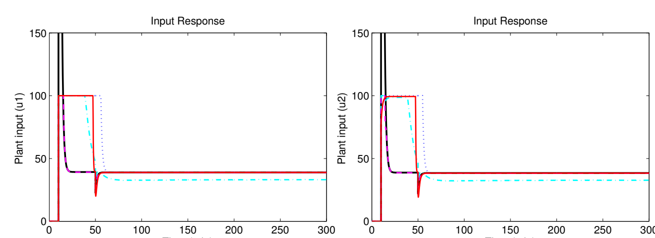
$$f_A = \begin{bmatrix} 85.42(s+1) & 0 \\ 0 & -68.43(s+1) \end{bmatrix}. \qquad (28)$$

Figs 6 and 7 show the input and output responses of the nominal plant to a set-point change from $[0 \quad 0]^T$ to $[0.99 \quad 0]^T$ at time $t = 10$ and from $[0.99 \quad 0]^T$ to $[0.99 \quad 0.01]^T$ at time $t = 50$ respectively for the different control configurations listed in Table I. In the saturated case (without anti-windup and directionality compensations), the control inputs are kept saturated longer than required resulting in serious performance degradation on both channels as compared to the nice decoupled response of the unconstrained case. Note that the unconstrained case requires a very aggressive control action during transient condition to achieve the decoupled response. The modified IMC anti-windup [18] results in improved transient performance but degraded steady-state behaviour. This is not unexpected as the anti-windup compensator Q_1 is designed to instantaneously minimize the 1-norm of a filtered difference between the unconstrained and the constrained outputs which is based on a related plant model. In addition, there are no design guidelines for the filter which is chosen purely based on intuition and has nothing to do with the plant's characteristics. The optimal LMI-based anti-windup design (without directionality compensation) [1], [30] shows an improved steady-state performance as compared with the modified IMC but with a sluggish transient response. Since the LMI synthesis results in a compensator with a very fast pole (requiring a very high sampling frequency), we have constrained the poles to a region comparable to those of Theorem 2 for ease of implementation. The solid lines in Figs. 6 through 7 show the improved performance achievable when an anti-windup scheme is augmented with a directionality compensator especially for an ill-conditioned plant. The response is closest to the unconstrained response for both transient and steady-state behaviours.



(a) Channel 1 (b) Channel 2

Fig. 6. Output Response



(a) Channel 1 (b) Channel 2

Fig. 7. Input Response

TABLE I

Legend for the responses in Fig. 6a through 7b

System	Line Type
Unconstrained	Bold
Saturated (without anti-windup)	Dotted
Modified IMC [18]	Dashdot
Optimal anti-windup (without QP)[1]	Dashed
Optimal anti-windup (with QP)	Solid

VI. CONCLUSIONS

We have presented a multivariable optimizing anti-windup design which guarantees closed-loop stability while compensating for both windup and directionality change in the control input vector. The simulated examples demonstrate the benefits that ensue: both from introducing directionality compensation into an anti-windup structure and from applying our proposed design procedures. The results are especially beneficial when the plant is ill-conditioned or has lightly damped modes. The method allows an explicit trade-off between stability and performance. We are currently investigating how significantly the balance is shifted when we replace input saturations with a quadratic program. We have also restricted our discussions to the nominal case where there are no model uncertainties. An area of further work is to adapt the synthesis approach of [5] or [36] for incorporating robustness into the optimizing anti-windup design.

VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the Financial support from Petroleum Technology Development Fund (PTDF).

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