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Energy-Aware Opportunistic Channel Access with Decentralized Channel State Information

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Abstract— **Decentralized channel state information is utilized to design a multichannel access algorithm for uplink transmission with power adaptation. The base station does not coordinate the transmissions of mobile users and hence users employ random access transmission. The situation is modeled as a non-cooperative game, where mobile users attempt to maximize its individual throughput gain with the least transmission power consumption. It is shown that each user should access a channel with probability 1 if the channel gain exceeds some threshold and the energy consumption with threshold policy does not exceed randomized policy. The game is reformulated as a channel gain threshold adaptation game, whose Nash equilibrium is proven to exist and be unique with mild conditions. At last a distributed iterative algorithm is proposed for each user to find the optimal threshold without knowing the channel state distribution and other users' strategies, and is proven to converge to the unique Nash equilibrium.**

I. INTRODUCTION

This paper considers decentralized multichannel access control with power adaptation for uplink transmission in a cellular system. The objective is to maximize the throughput reward with the least transmission power by exploiting favorable channel conditions. The challenges in achieving this objective is that the implementation of a practical algorithm should only rely on each mobile user's local information without knowing the distribution of time-varying channel conditions.

The channel-aware channel access control has been extensively studied in the past several years. To adapt the channel access and transmission power to the time-varying channel state information (CSI), the system throughput maximization problem is usually formulated as a mixed integer programming or convex optimization problem by allowing time shared co-channel utilization [1]. Even if dual decomposition method is employed to reduce the exponential computational complexity in the related mixed integer programming, a substantial amount of message passing between the base station (BS) and mobile users is still needed (see *e.g.* [2]). Recently, [3] proposed a threshold-based schedule scheme, where mobile terminals transmit only when their channel gains exceed pre-defined thresholds. However, their solution method requires mobile users have the same channel gain distribution. Similar threshold-based scheduler was proposed by [4] to maximize the total logarithmic throughput in the case of heterogenous CSI distribution. However, the logarithmic transformation and variable substitution techniques in [4] can not decompose the variable coupling in the sum of product form in multichannel model in this paper. Non-cooperative game theory is a powerful tool to tackle the variable coupling, since a player or a user in a game always makes decision in reaction to others without considering its effects of decision making to others. The most related works include [5] and [6]. The authors in [5] consider the channel access game with the objective of minimizing the access probabilities of mobile users while satisfying rate constraint. We have different problem formulation and algorithm design in this paper. The tradeoff between throughput gain and energy consumption is also considered in [6]. There are several features distinguish this paper from [6]. Time-varying power consumption for channel contention and its analytical evaluation are considered and sufficient conditions for unique Nash equilibrium is established in this paper. A distributed algorithm is proposed and its convergence is proven. Thus, more smooth convergence process can be obtained compared with the numerical method in [6] and best response update in [5].

The rest of the paper is organized as follows. In section II the problem of multichannel throughput maximization with the least power consumption is formulated as a non-cooperative game. The threshold-based policy is proven to be optimal in solving this game in section III. Furthermore, its energy consumption is compared with the randomized channel access strategies. In section IV, a distributed algorithm is proposed to approach the optimal channel gain threshold without knowing channel gain distribution. In section V, the performance of proposed algorithm is evaluated and compared with other results. Conclusion remarks are given in section VI.

II. MULTICHANNEL ACCESS MODEL AND PROBLEM FORMULATION

In this section, we define the multichannel access model considered in this paper. We formulate the channel access problem as a non-cooperative game considering energy efficient transmission.

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A. Network Model

We considered a model of uplink transmission in a multichannel wireless network with a set of users $\mathscr{U} = \{1,\cdots,U\}$. The whole frequency band is divided into a set of channels $\mathscr{C} = \{1,\cdots,C\}$. Time is divided into frames identified by an index t. We assume that for each user, the channel gain h is constant in each channel and each frame, but varies independently between frames, users and channels. The channel gain probability density function (PDF) for user u on channel c is $f_u^c(h_u^c)$. It is assumed that each user can transmit on all channels simultaneously by using orthogonal frequency-division multiplexing (OFDM) techniques. We consider saturated time division duplex (TDD) systems, *i.e.*, all users always have data to transmit to the BS. Thus, the channel gains between users and the BS can be estimated at each user through a periodically-transmitted beacon signal from the BS. The BS can successfully receive the information bits on the channels given that there is no collision. Data is transmitted in frames and each frame consists of three periods: contention period, acknowledgment (ACK) period, and data transmission period. After the contention period, each user receives an instantaneous feedback from the BS for each contended channel during the ACK period. The address of the successful user is broadcasted through the ACK message. In the data transmission period, the successful user transmits its data packets.

To take the advantage of time-varying CSI, each user should adapt its channel access strategy to the decentralized CSI to save energy and improve throughput. For each user u, define the transmission vector as $\mathbf{s}_u(\mathbf{h}_u) \hat{=} \left[s_u^1(h_u^1),\cdots,s_u^C(h_u^C)\right]^T \in [0,1]^C$ [1], where the c^{th} entry corresponds to user u's channel access probability on channel c. The rate function of physical layer is denoted as $R_u^c(h_u^c)$, $\forall u \in \mathscr{U}$, $\forall c \in \mathscr{C}$, which is assumed to be continuously increasing over $[0,\infty)$.

B. Problem Formulation

Since multiple users contend for channel access, contention of user u in a frame t on channel c is successful if and only if user u is sending channel request. If the channel request from user u is successfully received by the BS, the average transmission rate for user u given current channel state $\mathbf{h}$ is

$$\sum_{c\in\mathscr{C}} r_u^c(\mathbf{h}^c),$$

where $r_u^c(\mathbf{h}^c) = s_u^c(h_u^c)\Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-s_j^c\left(h_j^c\right)\right)R_u^c(h_u^c)$ is user u's average rate on channel c with current channel state. Since the access strategy depends on the time-varying channel state, we are more interested in the expected throughput

$$\begin{aligned} & E_{\mathbf{h}}\left\{\sum_{c\in\mathscr{C}} r_u^c(\mathbf{h}^c)\right\} \\ = & \sum_{c\in\mathscr{C}}\int_0^\infty s_u^c(h_u^c)R_u^c(h_u^c)f_u^c(h_u^c)\,dh_u^c \\ & \times\Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-\int_0^\infty s_j^c\left(h_j^c\right)f_j^c\left(h_j^c\right)dh_j^c\right), \end{aligned}$$

where the above computation is due to the assumption that the channel gains of different users are independent over different channels and channel gains of the same user on the same channel are independent and identically distributed with PDF $f_u^c(\cdot)$. To map the time-varying CSI into a transmission opportunity is not only for throughput gain but also for energy saving consideration. Each user u should send the channel request with the smallest possible power $\frac{P_r}{h_u^c}$ to guarantee the received power P_r at the BS allows the channel request to be successfully detected when there are no collisions. Then the expected power consumption for user u to contend on channel c with strategy $s_u^c(h_u^c)$ is $E_{h_u^c}\{p_u^c(s_u^c(h_u^c))\} = \int_0^\infty s_u^c(h_u^c)\frac{P_r}{h_u^c}f_u^c(h_u^c)\,dh_u^c$ [2].

Under the above description, each user adapts its transmission opportunity to the time-varying channel gain in order to maximize its long-term throughput gain and minimize the energy consumption at the same time. This multi-objective optimization problem is formulated as a noncooperative game, namely energy-aware channel access game:

$$G_{EACA} = \left\{\mathscr{U},\{\mathscr{S}_u\}_{u\in\mathscr{U}},\{W_u\}_{u\in\mathscr{U}}\right\},$$

where $\mathscr{U}$ denotes the set of users contending a set of C channels, $\mathscr{S}_u = [0,1]^C$ and W_u denote the user u's strategy profile and payoff function, respectively. The payoff function in eq. (1) consists of both user u's throughput gain and energy cost:

$$W_u(\mathbf{s}) = \alpha_u^1 E_{\mathbf{h}}\left\{\sum_{c\in\mathscr{C}} r_u^c(\mathbf{h}^c)\right\} - \alpha_u^2\sum_{c\in\mathscr{C}} E_{h_u^c}\{p_u^c(h_u^c)\}. \quad (1)$$

In our context, $\left(\alpha_u^1,\alpha_u^2\right) \succ \mathbf{0}$ represents user u's reward for throughput gain and cost of dispensing one Joule energy for sending the channel request.

Given other users channel access strategies $\mathbf{s}_{-u}$, the opportunistic channel access problem faced by each user u is

$$\max_{s_u\in\mathscr{S}_u} W_u(\mathbf{s})$$

The equilibrium strategy profile $\mathbf{s}^* = [\mathbf{s}_1^*,\cdots,\mathbf{s}_U^*]$ is a Nash equilibrium (NE) if no user can benefit by unilaterally deviating from which while the other users keep their strategies fixed, *i.e.*,

$$\mathbf{s}_u^* = \arg\max_{\mathbf{s}_u\in\mathscr{S}_u} W_u\left(\mathbf{s}_u,\mathbf{s}_{-u}^*\right),\ \forall u\in\mathscr{U}, \quad (2)$$

where $\left\{\mathbf{s}_{-u}^*\right\}$ denotes the set of transmission policies of all nodes other than u.

[1] Hereafter, we use bold letter to denote a vector. We use $\mathbf{x}_{-u}$ to represent a vector except the u^{th} component.

[2] If $h=0$ is detected, the user will refuse to access the considered channel in order to avoid infinite energy consumption.

III. Threshold Strategies and Properties of Nash Equilibria

A. Optimality of Threshold Strategies

Since the fading channel state takes on a continuum of variables, the problem (2) is an infinite-dimensional optimization problem. The following theorem gives a guideline to find the optimal solution to (2).

Theorem 1: There exists a threshold strategy (3) that maximizes each user's payoff (1).

$$s_u^{c*}\left(h_u^c\right)=\begin{cases}1, & h_u^c \geq H_u^c\\ 0, & h_u^c < H_u^c\end{cases},\forall u,\forall c, \tag{3}$$

where $H_u^c \in (0,\infty)$ is the channel gain threshold.

Proof: Recall that the payoff function in (1) can be rewritten into (4) since there is no correlation between different user's channel state.

$$W_u(\mathbf{s})=\alpha_u^1\sum_{c\in\mathscr{C}}\int_0^\infty s_u^c\left(h_u^c\right)\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)f_u^c\left(h_u^c\right)dh_u^c, \tag{4}$$

where $\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)\hat{=}-\frac{\alpha_u^2}{\alpha_u^1}\frac{P_r}{h_u^c}+R_u^c\left(h_u^c\right)\Pi_{j\in\mathscr{U}\setminus\{u\}}(1-\int_0^\infty s_j^c\left(h_j^c\right)f_j^c\left(h_j^c\right)dh_j^c)$. To maximize $W_u(\mathbf{s})$, we have two implications related to design of access strategy $\mathbf{s}_u^*=\arg\max_{\mathbf{s}_u\in\mathscr{S}_u}W_u\left(\mathbf{s}_u,\mathbf{s}_{-u}^*\right)$: (i) if $\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)>0$ then $s_u^{c*}\left(h_u^c\right)=1$; and (ii) if $s_u^{c*}\left(h_u^c\right)=1$, then $\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)\geq 0$. To prove that the optimal channel access is a threshold rule it is sufficient to prove that if $s_u^{c*}\left(h_u^c\right)=1$ for some given h_u^c then $s_u^{c*}\left(\hat{h}_u^c\right)=1$ for any $\hat{h}_u^c>h_u^c$. It is easy to check that $\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)$ is an increasing function of h_u^c given other users strategies. Thus, if $s_u^{c*}\left(h_u^c\right)=1$, we have $\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)\geq 0$ and $\Phi_u^c\left(\hat{h}_u^c,\mathbf{s}_{-u}^c\right)>\Phi_u^c\left(h_u^c,\mathbf{s}_{-u}^c\right)\geq 0$ for any $\hat{h}_u^c>h_u^c$. According to the implication (i) we have $s_u^{c*}\left(\hat{h}_u^c\right)=1$ for any $\hat{h}_u^c>h_u^c$. That the optimal channel access is a threshold strategy is proven. ■

The effectiveness of threshold strategy (3) can be strengthened by comparing its power consumption with randomized strategies. Before we state our results, the following definition is given.

Definition 2 ([7]): Let M and N be two cumulative distribution functions (CDFs). M is strongly stochastically smaller than N, written as $M<_{st}N$, if their complementary CDFs satisfy $\bar{M}(x)<\bar{N}(x)$ for all $x\in\mathbb{R}$. For random variables X, Y with distributions M and N respectively, we will write $X<_{st}Y$ as synonym for $M<_{st}N$. The following theorem characterizes the stochastic ordering.

Theorem 3 ([7]): $M\leq_{st}N$ if and only if

$$\int_{-\infty}^{\infty}\phi(x)\,dM(x)\leq\int_{-\infty}^{\infty}\phi(x)\,dN(x) \tag{5}$$

for all increasing functions ϕ, for which the integrals exist.

Corollary 4: $M\leq_{st}N$ if and only if

$$\int_{-\infty}^{\infty}\varphi(x)\,dM(x)\geq\int_{-\infty}^{\infty}\varphi(x)\,dN(x) \tag{6}$$

for all decreasing functions φ, for which the integrals exist.

Proof: By multiplying both sides of (5) by -1 and letting $\varphi(x)=-\phi(x)$, the proof follows from Theorem 3 directly. ■

Theorem 5: With the same expected access probability, *i.e.* $\int_H^\infty f(h)\,dh=\int_0^\infty s(h)f(h)\,dh=Q$, the power consumption with threshold strategy (3) is not greater than that with randomized strategy $\mathbf{s}\in[0,1]^{U\times C}$, *i.e.* $\int_{H_u^c}^\infty\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c\leq\int_0^\infty s_u^c\left(h_u^c\right)\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c$, $\forall u\in\mathscr{U},\forall c\in\mathscr{C}$.

Proof: The expected power consumption with randomized strategies for user u on channel c is

$$E_{M_u^c(\cdot)}\left\{p_u^c\left(h_u^c\right)\right\}\hat{=}Q_u^c\int_0^\infty\frac{1}{Q_u^c}s_u^c\left(h_u^c\right)\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c,$$

where $M_u^c\left(h_u^c\right)\hat{=}\frac{1}{Q_u^c}\int_0^{h_u^c}s_u^c(x)f_u^c(x)\,dx$ is the CDF for energy consumption. Since $s_u^c\left(h_u^c\right)\in[0,1]$ we have

$$\begin{aligned}\bar{M}_u^c\left(h_u^c\right)&\hat{=}\frac{1}{Q_u^c}\int_{h_u^c}^\infty s_u^c(x)f_u^c(x)\,dx\\&\leq\frac{1}{Q_u^c}\int_{h_u^c}^\infty f_u^c(x)\,dx\hat{=}\frac{1}{Q_u^c}F_u^c\left(h_u^c\right),\end{aligned}$$

where $\bar{M}_u^c\left(h_u^c\right)$ is the complementary CDF of expected power consumption. Thus

$$\bar{M}_u^c\left(h_u^c\right)\leq\min\left\{Q_u^{c-1}F_u^c\left(h_u^c\right),1\right\}. \tag{7}$$

Consider the threshold strategy

$$s_u^c\left(h_u^c\right)=\begin{cases}1 & h_u^c\geq H_u^c\\ 0 & h_u^c<H_u^c\end{cases},$$

where H_u^c is determined by $\int_{H_u^c}^\infty f\left(h_u^c\right)dh_u^c=Q_u^c=F_u^c\left(H_u^c\right)$. The corresponding complementary CDF for energy consumption with threshold strategy is

$$\bar{N}_u^c\left(h_u^c\right)=\begin{cases}Q_u^{c-1}F_u^c\left(H_u^c\right) & h_u^c\geq H_u^c\\ 1 & h_u^c<H_u^c\end{cases} \tag{8}$$

Comparing (7) and (8), we deduce that $\bar{M}_u^c\left(h_u^c\right)\leq\bar{N}_u^c\left(h_u^c\right)$, and $M_u^c\leq_{st}N_u^c$ $\forall u\in\mathscr{U},\forall c\in\mathscr{C}$. Therefore, it follows from Corollary 4 that $\int_{H_u^c}^\infty\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c\leq\int_0^\infty s_u^c(x)\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c$ since the instantaneous transmission power $\frac{P_r}{h}$ is a decreasing function of h. ■

B. Existence and Uniqueness of Equilibrium Point

Due to the optimality of threshold based channel access strategy we reformulate the G_{EACA} game allowing only threshold-based channel access. Then the reformulated problem is how to adapt thresholds $\{H_u^c\}$ for each user over all channels to maximize its own payoff. In light of $q_u^c\left(H_u^c\right)\hat{=}\int_{H_u^c}^\infty f_u^c\left(h_u^c\right)dh_u^c\hat{=}F_u^c\left(H_u^c\right)$ (or $H_u^c=F_u^{c-1}\left(q_u^c\right)$), the reformulated game G_{T-EACA} is defined as:

$$\max_{\mathbf{q}_u\in\mathscr{Q}_u}\bar{W}_u\left(\mathbf{q}_u,\mathbf{q}_{-u}^*\right),\ \forall u\in\mathscr{U}$$

where the payoff $\bar{W}_u$ is defined as

$$\begin{aligned}&\bar{W}_u(\mathbf{q}) \qquad (9)\\ =\ &-\alpha_u^2\sum_{c\in\mathscr{C}}\int_{F_u^{c-1}\left(q_u^c\right)}^\infty\frac{P_r}{h_u^c}f_u^c\left(h_u^c\right)dh_u^c\\&+\alpha_u^1\sum_{c\in\mathscr{C}}\int_{F_u^{c-1}\left(q_u^c\right)}^\infty R_u^c\left(h_u^c\right)f_u^c\left(h_u^c\right)dh_u^c\prod_{j\in\mathscr{U}\setminus\{u\}}\left(1-q_j^c\right)\end{aligned}$$

with

$$\mathscr{Q}_u = \left\{ q_u \in \mathbb{R}^C \middle| 0 < q_u^{\min} \le q_u^c \le q_u^{\max} < 1, \forall c \in \mathscr{C} \right\}. \quad (10)$$

Here $q_u^{\min}$ is set to be non-zero to avoid time out in long time backoff.

The equilibrium strategy profile $\mathbf{q}^* = [\mathbf{q}_1^*, \cdots, \mathbf{q}_U^*]$ at the NE satisfies the following conditions simultaneously, *i.e.*,

$$\mathbf{q}_u^* = \arg\max_{\mathbf{q}_u \in \mathscr{Q}_u} \bar{W}_u \left(\mathbf{q}_u, \mathbf{q}_{-u}^*\right), \ \forall u \in \mathscr{U}. \quad (11)$$

Having reformulated the game, we provide results of existence and uniqueness of NE for the game G_{T-EACA}.

Theorem 6: A NE exists in the game G_{T-EACA}.

Proof: The proof is based on Theorem 4.3 in [8]. The payoff function $\bar{W}_u(\mathbf{q})$ in (9) is continuous on $\times_{u\in\mathscr{U}}\mathscr{Q}_u$ and strictly concave in $\mathbf{q}_u$ by showing that

$$\nabla^2_{\mathbf{q}_u\mathbf{q}_u} \bar{W}_u = \mathrm{diag}\left\{ w_{uu}^c(\mathbf{q}^c), \forall c \in \mathscr{C} \right\}$$

is negative definite, since $w_{uu}^c(\mathbf{q}^c) = -\alpha_u^1 \frac{dR_u^c(h_u^c)}{dh_u^c}\Big|_{h_u^c = F_u^{c-1}(q_u^c)} \times \frac{1}{f_u^c\left(F_u^{c-1}(q_u^c)\right)} \times \Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-q_j^c\right) - \alpha_u^2 \frac{P_r}{\left(F_u^{c-1}(q_u^c)\right)^2 f_u^c\left(F_u^{c-1}(q_u^c)\right)} < 0$. In addition, $\mathscr{Q}_u$ in (10) is a non-empty compact and convex subset of a finite-dimensional Euclidean space. According to Theorem 4.3 in [8], the game G_{T-EACA} has a NE. ■

To study the uniqueness of NE, let's differentiate the payoff function $\bar{W}_u(\mathbf{q})$ with respect to q_u^c, $\forall u, \forall c$,

$$\begin{aligned} & \frac{\partial \bar{W}_u(\mathbf{q})}{\partial q_u^c} \\ = \ & \alpha_u^1 R_u^c\left(F_u^{c-1}(q_u^c)\right) \Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-q_j^c\right) - \alpha_u^2 \frac{P_r}{F_u^{c-1}(q_u^c)} \\ \hat{=} \ & G_u^c(\mathbf{q}^c). \end{aligned}$$

Define the Jacobian matrix $\mathbf{J}^c(\mathbf{q}^c) = \left[J_{uj}^c(\mathbf{q}^c)\right]$, $\forall c$ where $J_{uj}^c(\mathbf{q}^c) = \begin{cases} \frac{\partial G_u^c(\mathbf{q}^c)}{\partial q_u^c}, & \text{if } u = j \\ \frac{\partial G_u^c(\mathbf{q}^c)}{\partial q_j^c} & \text{if } u \neq j \end{cases}$. It has been proven in [9] that the NE is unique regardless of whether it is an inner solution or a boundary solution, if the symmetric matrix $\mathbf{J}^c(\mathbf{q}^c) + \mathbf{J}^c(\mathbf{q}^c)^T$ is negative definite, where $\mathbf{J}^c(\mathbf{q}^c) \hat{=} \left[J_{uj}^c(\mathbf{q}^c)\right]$ and $\mathbf{J}^c(\mathbf{q}^c)^T$ is its transposed matrix.

Theorem 7: If

$$\left|J_{uu}^c(\mathbf{q}^c)\right| > \max\left(\sum_{j\in\mathscr{U}\backslash\{u\}} \left|J_{uj}^c(\mathbf{q}^c)\right|, \sum_{j\in\mathscr{U}\backslash\{u\}} \left|J_{ju}^c(\mathbf{q}^c)\right| \right) \quad (12)$$

the game G_{T-EACA} has a unique NE.

Proof: (12) indicates that the matrices $\mathbf{J}^c(\mathbf{q}^c)$ and $\mathbf{J}^c(\mathbf{q}^c)^T$ are both strictly diagonally dominant. Following Gershgorin's theorem [10], all the eigenvalues of $\mathbf{J}^c(\mathbf{q}^c) + \mathbf{J}^c(\mathbf{q}^c)^T$ are negative due to $J_{uu}^c(\mathbf{q}^c) < 0$, for all u. Therefore the matrices $\mathbf{J}^c(\mathbf{q}^c) + \mathbf{J}^c(\mathbf{q}^c)^T$ are negative definite and the game G_{T-EACA} has a unique NE. ■

Remark 8: For each user u, if it imposes a larger ration α_u^2/α_u^1, *i.e.* users are more sensitive to energy consumption than throughput gain, the condition (12) is more easily to meet.

In the following, we assume the game G_{T-EACA} has a unique NE $\mathbf{q}^*$ (or equivalently $H^* = F^{-1}(q^*)$).

IV. Threshold Adaptation

Since each user's payoff function (9) is concave with respect to q_u^c, the necessary condition for $\mathbf{q}^*$ being a NE:

$$\begin{aligned} & \frac{\partial \bar{W}_u\left(\mathbf{q}_u, \mathbf{q}_{-u}^*\right)}{\partial q_u^c}\Big|_{q_u^c = q_u^{c*}} \\ = \ & \alpha_u^1 R_u^c\left(F_u^{c-1}(q_u^{c*})\right) \Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-q_j^{c*}\right) \\ & -\alpha_u^2 \frac{P_r}{F_u^{c-1}(q_u^{c*})} = 0 \end{aligned} \quad (13)$$

is also a sufficient condition. Eq. (13) means that at equilibrium the marginal throughput reward is equal to the instantaneous power cost. Since it is difficult to compute the NE explicitly without knowing the channel state distribution, we propose an iterative algorithm for each user to compute the equilibrium. Recalling the one-to-one mapping $H_u^c = F_u^{c-1}(q_u^c)$ and the first order necessary condition (13), we propose the following iteration to update the channel gain threshold for each user on each channel:

$$\begin{aligned} & H_u^c(t+1) \\ = \ & [H_u^c(t) + \delta(\frac{\alpha_u P_r}{R_u^c(H_u^c(t))H_u^c(t)} \\ & - \prod_{j\in\mathscr{U}\backslash\{u\}} \left(1 - q_j^c\left(H_j^c(t)\right)\right))]_{H_u^{\min}}^{H_u^{\max}} \end{aligned} \quad (14)$$

where $[x]_a^b \hat{=} \max\{\min\{x,b\},a\}$, $\delta \in (0,1)$ is the stepsize and $\alpha_u = \alpha_u^2/\alpha_u^1$. Here the boundary $0 < H_u^{\min} \le H_u^c \le H_u^{\max}$ corresponds to $q_u^{\min} \le q_u^c \le q_u^{\max}$ equivalently for each u and c. The engineering implication behind (14) is that each user raises its channel threshold if the energy cost of unique rate cannot be compensated by the offered transmission opportunity $\Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-q_j^c\left(H_j^c(t)\right)\right)$, and the threshold is decreased vice versa. Note that the iteration (14) is used to find $\mathbf{q}_u^*$ solving (11) equivalently in light of $H_u^{c*} = F_u^{c-1}(q_u^{c*})$.

Each user will contend multiple channels based on the comparison of measured channel gain and threshold updated at each time in (14). Iteration (14) shows that the update of threshold only depends on each user's local information on each channel. The transmission opportunity $\Pi_{j\in\mathscr{U}\backslash\{u\}}\left(1-q_j^c\left(H_j^c(t)\right)\right)$ for each user u can be estimated via the channel idle probability $\Pi_{j\in\mathscr{U}}\left(1-q_j^c\left(H_j^c(t)\right)\right)$, whose local computation method has been given in [11]. Each user's backoff probability $(1-q_u^c(H_u^c(t)))$ can be obtained by counting the number of times N_u^c that the actual channel gain $h_u^c(t)$ is smaller than the threshold $H_u^c(t)$ within the last several transmission attempts $N_u^{\max}$, *i.e.* $(1-q_u^c(H_u^c(t))) = N_u^c/N_u^{\max}$. Note that this computation can be further elaborated by an exponential weighted average between the current backoff probability and history value to smooth the estimated value.

We now show the convergence of (14) to the unique NE $\mathbf{H}^*$. Our analysis follows the technique in [12].

Theorem 9: Suppose the game G_{T-EACA} has a unique inner NE point $\mathbf{H}^*$, (14) converges to the unique NE, if (15) holds $\forall u \in \mathscr{U}, \forall c \in \mathscr{C}$

$$\begin{aligned} & \frac{\alpha_u P_r \left(H_u^c \frac{dR_u^c(H_u^c)}{dH_u^c} + R_u^c\left(H_u^c\right)\right)}{H_u^{c2}\left(R_u^c\left(H_u^c\right)\right)^2} \\ > & \sum_{k \in \mathscr{U} \backslash \{u\}} \Pi_{j \in \mathscr{U} \backslash \{u,k\}} \left(1 - q_j^c\left(H_j^c\right) f_k^c\left(H_k^c\right)\right) \end{aligned} \quad (15)$$

and δ is sufficiently small.

Proof: Define a function $x_u^c(\tau) : [0,1] \to \mathbb{R}$ for user u on channel c as

$$\begin{aligned} & x_u^c(\tau) \quad (16) \\ = \ & \tau H_u^c(t) + (1-\tau) H_u^{c*} + \delta\Big(\frac{\alpha_u P_r}{R_u^c\left(H_u^c\right) H_u^c} \\ & -\Pi_{j \in \mathscr{U} \backslash \{u\}} \left(1 - q_j^c\left(H_j^c\right)\right)\Big)\Big|_{\mathbf{H}^c = \tau \mathbf{H}^c(t) + (1-\tau)\mathbf{H}^{c*}}, \end{aligned}$$

where the unique NE H_u^{c*}, which corresponds to $H_u^{c*} = F_u^{c-1}\left(q_u^{c*}\right)$ in (11) is the fixed point of the mapping $H_u^c(t) \to H_u^c(t+1)$ in (14). By (14) and (16), we have

$$\begin{aligned} & \left|H_u^c(t+1) - H_u^{c*}\right| \\ \leq \ & \left|x_u^c(1) - x_u^c(0)\right| \\ = \ & \left|\int_0^1 \frac{dx_u^c(\tau)}{d\tau} d\tau\right| \leq \int_0^1 \left|\frac{dx_u^c(\tau)}{d\tau}\right| d\tau \\ \leq \ & \max_{\tau \in [0,1]} \left|\frac{dx_u^c(\tau)}{d\tau}\right| \end{aligned}$$

Noticing that δ is usually sufficiently small, we have

$$\begin{aligned} & \left|\frac{dx_u^c(\tau)}{d\tau}\right| \\ \leq \ & |1 - \delta \frac{\alpha_u P_r \left(H_u^c \frac{dR_u^c(H_u^c)}{dH_u^c} + R_u^c\left(H_u^c\right)\right)}{H_u^{c2}\left(R_u^c\left(H_u^c\right)\right)^2} \\ & + \delta \sum_{k \neq u} \Pi_{j \neq u,k} \left(1 - q_j^c\left(H_j^c\right) f_k^c\left(H_k^c\right)\right)| \, \|\mathbf{H}^c(t) - \mathbf{H}^{c*}\|_\infty \end{aligned}$$

where $\|\mathbf{H}^c(t) - \mathbf{H}^{c*}\|_\infty \hat{=} \max_u \left|H_u^c(t) - H_u^{c*}\right|$. The sufficiently small δ is used to guarantee that

$$\delta \frac{\alpha_u P_r \left(H_u^c \frac{dR_u^c(H_u^c)}{dH_u^c} + R_u^c\left(H_u^c\right)\right)}{H_u^{c2}\left(R_u^c\left(H_u^c\right)\right)^2} < 1$$

for all $H_u^c \in \left[H_u^{\min}, H_u^{\max}\right]$. Furthermore, if condition (15) is satisfied, we have

$$\max_{\tau \in [0,1]} \left|\frac{dx_u^c(\tau)}{d\tau}\right| \leq \Psi_u^c \|\mathbf{H}^c(t) - \mathbf{H}^{c*}\|_\infty, \quad (17)$$

where $0 < \Psi_u^c = 1 - \delta\Big(\frac{\alpha_u P_r \left(H_u^c + R_u^c(H_u^c) \frac{dR_u^c(H_u^c)}{dH_u^c}\right)}{H_u^{c2}(R_u^c(H_u^c))^2 \frac{dR_u^c(H_u^c)}{dH_u^c}} - \sum_{k \in \mathscr{U} \backslash \{u\}} \Pi_{j \in \mathscr{U} \backslash \{u,k\}} \left(1 - q_j^c\left(H_j^c\right) f_k^c\left(H_k^c\right)\right)\Big) < 1$. Given (15) and sufficiently small δ it can easily be proved that the synchronous update (14) converges to the unique inner NE as $t \to \infty$. ■

In the iteration (14), each user updates its channel gain threshold at the same time instance. A practical generalization is the asynchronous update scheme where only a random subset of users perform update at a given time. The proof for asynchronous convergence of a nonlinear iterative mapping consists of two well known sufficient conditions and is omitted here. Detailed proof can be found in [12].

V. Numerical Examples

The considered simulation scenario is the same as in [13]. The channels are modeled as independent 3-tap Rayleigh fading channels with an exponential power delay profile. Each frame contains 48 OFDM symbols. There are total 256 subcarriers, and among which 64 subcarriers are grouped into one sub-channel. Only 32 subcarriers are used to transmit contention packet. Within one frame, the first phase is dedicated to contention, which consists of K mini-slots. At the end of contention, the BS takes one mini-slot to feed back ACK. Since one mini-slot is set to be $1/2$ of one OFDM symbol duration, the length of the data transmission period is $L = 2 \times 48 - K - 1$ mini-slots, where "1" denotes one mini-slot used by BS to feed back ACK. In our algorithm we set $K = 1$. The received power required by the BS to detect the channel request successfully is set to $P_r = 1$.

We first show the convergence property of our proposed algorithms. There are 10 users deployed in the above network scenario. The convergence of the adaptation of channel gain threshold in the selected channel is shown in Fig. 1. The convergence of channel gain adaptation on the other three channles has the similar properties as in Fig. 1.

Then we compare the system throughput obtained by different algorithms in the same simulation scenario except for the number of users. The simulation was obtained by running 100,000 independent frames. It was shown in [13] that CAC algorithm can achieve the best results that approach the ideal centralized OFDMA scheme in this simulation scenario with $K = 7$. The basic principle of CAC algorithms in single channel is to distributively set channel gain threshold for each user on one mini-time slot by solving a system throughput maximization problem. For multichannel case, the principle in single channel will be repeated over the selected subchannels. It can be found in Fig. 2 that the system throughput of CAC with $K = 7$ can achieve the best results since multiple rounds of contention in a single transmission frame may increase the possibility of successful contention. Fig. 2 also shows that the system throughput of CAC algorithm highly depends on the number of contention before formal data transmission. The performance of proposed algorithm can approach CAC ($K = 7$) and outperforms random channel selection schemes and CAC ($K = 1$). Since we take into account the energy consumption explicitly in the payoff function, the effects of energy saving with the proposed algorithm is notable in Fig. 3. Although multiple rounds of contention before formal data transmission can increase system throughput, CAC algorithms ignore the energy consumption for channel contention, which consumes more energy as shown in Fig. 3. Fig. 3 also illustrate that the proposed algorithm can achieve a better tradeoff between energy consumption and system throughput.

To understand the effects of parameter α_u on the throughput-energy tradeoff, we plot Fig. 4. With the increment of α, each user is more sensitive to energy consumption and thus we have the monotonically decreasing energy consumption with respect to α. On the contrary, the equilibrium channel gain threshold is increasing with the increment of α, which results in less channel access opportunity and less system throughput.

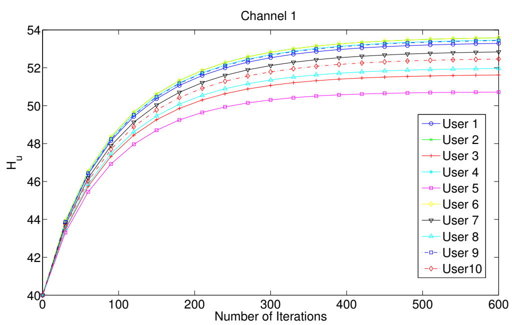

Fig. 1. Throughput comparison with $\alpha_u = 550$ for all u.

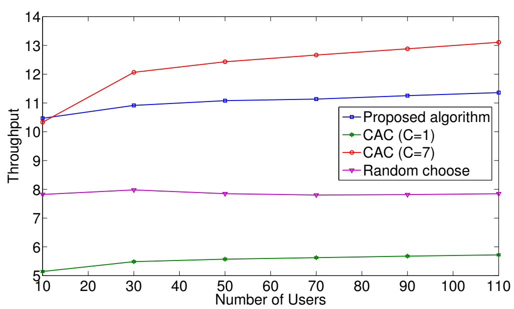

Fig. 2. Throughput comparison with $\alpha_u = 550$ for all u.

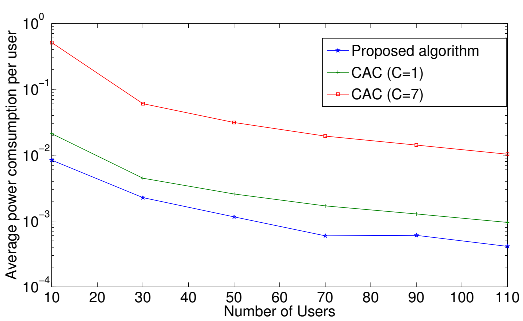

Fig. 3. Energy consumption comparison with $\alpha_u = 550$ for all u.

VI. Conclusions

Energy-aware multi-channel access control is considered in this paper. The design of decentralized channel access adapting to time-varying CSI is formulated to a non-cooperative game. The optimal strategy is shown to have threshold structure, with which each user contends the channel if its actual channel gain exceeds the threshold. The existence and uniqueness of Nash equilibrium of the game is

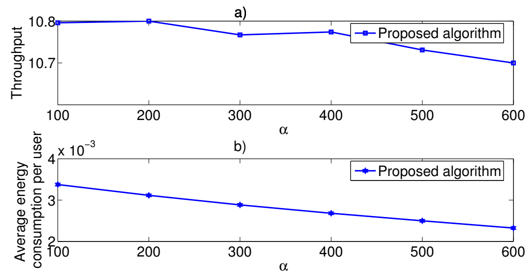

Fig. 4. Energy-throughput tradeoff with 20 users

proven under certain conditions. A distributed algorithm is proposed for each user to update the channel gain threshold without knowing the distribution of channel gain and its convergence is proven. Simulation results verify that the tradeoff between system throughput and energy consumption can be achieved by each user's local tuning parameter.

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