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Funnel control in mechatronics: An overview

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Abstract— **This overview presents a simple high-gain adaptive controller — the funnel controller — and its possible applications in mechatronics. The funnel controller neither identifies nor estimates the system under control and is applicable for (nonlinear) systems being minimum-phase (or having stable zero-dynamics in the nonlinear case), having relative degree one or two and known sign of the high-frequency gain. So only "structural system properties" must be satisfied to allow for controller implementation. Moreover, control performance is robust to parameter uncertainties or variations not affecting the system structure. The proportional funnel controller assures tracking of time-varying reference signals with *prescribed transient accuracy*, i.e. the tracking error evolves within a "funnel" with prescribed boundary (i.e. a continuous function of time chosen by the control designer). To illustrate applicability of funnel control in "real world" measurement results are presented for speed and position control of an unknown rotatory system subject to (varying) friction and load disturbances. The results are compared with classical PI/PID control.**

I. Introduction

Many industrial applications are only "structurally" known, i.e. the plant model is available in the form of an differential equation (a lumped parameter model) whereas the model parameters are uncertain. Moreover, mechatronic systems are subject to dynamic nonlinearities (such as friction) and unknown load disturbances deteriorating control performance. These effects and uncertainties (or variations) of the system parameters imply a robust and so often a conservative control design. In the majority of cases "standard" PI/PID controllers are implemented. To improve control performance, disturbance observer and/or time-consuming (cost-intensive) system identification strategies are necessary. Both is involved and not desirable [1]. This overview re-examines a robust, high-gain adaptive (time-varying) control concept—funnel control (FC)—as feasible alternative to standard PI/PID controllers in mechatronics. Funnel control, in general, is applicable to a wide class of (nonlinear) systems with relative degree *one* or *two*, stable zero-dynamics (minimum-phase in the LTI case) and known sign of the high-frequency gain[1] [3], [4]. Moreover, measurement noise and parameter uncertainties are tolerated. Identification or estimation of system parameters is not necessary. This non-identifier based adaptive controller guarantees global stability and good tracking performance, i.e. prescribed transient accuracy of the closed-loop system is achieved by adequate boundary design such that e.g. minimum rise time, maximum overshoot and minimum settling time (customer specifications) are assured (if the control input is sufficiently dimensioned [5], [3], [6], [7], [8], [9]). If constrained control actions impose severe performance issues (neglected in this paper), e.g. *Saturated Input Compensation* may be used [7], [9][2]. Funnel control is a 'proportional' (or memoryless) approach (i.e. no dynamics in the controller) and so asymptotic tracking and/or disturbance rejection are not obtained in general, therefore in addition a PI-like extension should be implemented attaining both goals (at least) in steady-state [10], [11]. All presented results are mathematically proved (see e.g. [3], [10], [11], [12]).

The goal of this overview is twofold: (i) to introduce funnel control as alternative to classical PI/PID control in mechatronics and (ii) to give an overview on the state-of-the-art and a comprehensive list on the literature of funnel control.

We conclude this section with some remarks on notation:

$\mathbb{N}, \mathbb{R}, \mathbb{C}$	natural, real and complex numbers
$[a, b)$	interval from a to b (excluded)
$\mathbb{R}_{\geq 0} := [0, \infty)$	set of positive real numbers with zero
$\Re(s), \Im(s)$	real and imaginary part of $s \in \mathbb{C}$
$\boldsymbol{v} \in \mathbb{R}^n$	column vector with n-entries, $n \in \mathbb{N}$
$\boldsymbol{M} \in \mathbb{R}^{n \times m}$	matrix with n-rows & m-columns
$\det(\boldsymbol{M})$	determinant of matrix $\boldsymbol{M} \in \mathbb{R}^{n \times n}$
$\boldsymbol{I}_n \in \mathbb{R}^{n \times n}$	identity matrix with rank n
$f(\cdot)$	a function, e.g. $f \colon X \to Y$
$f(x)$	the value of the function $f(\cdot)$ at x
$f^{(i)}(t) := \frac{\mathrm{d}^i}{\mathrm{d}t^i} f(t)$	the i-th (time) derivative of $f(\cdot)$
$\mathcal{C}^n(X; Y)$	space of n-times continuously differentiable functions mapping $X \to Y$
$\mathcal{L}^\infty_{(\mathrm{loc})}(X; Y)$	space of measurable, (locally) essentially bounded functions
$\mathcal{W}^{k,\infty}(X; Y)$	space of bounded locally absolutely continuous functions with essentially bounded derivatives $f^{(i)} \in \mathcal{L}^\infty(X; Y)$ for all $i \in \{1, \dots, k\}$ and norm:
$\|f\|_{k,\infty}$	$:= \sum_{i=1}^k \|f^{(i)}\|_\infty$

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[1]The "high-frequency gain" or also "instantaneous gain" describes the 'directional' effect of the control input on the first or second derivative of the system's output [2, p. 334]. For LTI SISO systems "high-frequency gain" denotes the "leading coefficient of the numerator of the transfer function.

[2]The strategy is not mathematically proven, hence not covered here. But it shows promising results in simulations and measurements.

II. High-gain (adaptive) control

For $n \in \mathbb{N}$ consider the linear time-invariant (LTI) single-input $u(t) \in \mathbb{R}$ single-output $y(t) \in \mathbb{R}$ (SISO) system given by the ordinary differential equation

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\,u(t) \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}^0 \in \mathbb{R}^n. \end{aligned} \tag{1}$$

where $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{b} \in \mathbb{R}^n$ and $\boldsymbol{c} \in \mathbb{R}^n$ represent system matrix, input and output coupling vector, resp. Now if system (1) has the following properties:

(i) relative degree[3] $r = 1$, i.e. $\boldsymbol{c}^\top \boldsymbol{b} \neq 0$;

(ii) known sign of high-frequency gain $\gamma := \boldsymbol{c}^\top \boldsymbol{b}$, i.e. $\operatorname{sign}(\gamma)$ known and

(iii) minimum phase[4], i.e. $\forall s \in \mathbb{C}$ with $\Re(s) \geq 0$: $\det\left(\begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix}\right) \neq 0$ (see e.g. [13, pp. 9-12]),

then there exists a minimal $k^* > 0$ (which depends on the system data!), such that simple output feedback of the form

$$u(t) = -\operatorname{sign}(\gamma) k\, y(t) = -\operatorname{sign}(\gamma) k\, \boldsymbol{c}^\top \boldsymbol{x}(t) \tag{2}$$

stabilizes system (1) for all $k > k^*$ [13]. If controller (2) is implemented, k^* must be known *a priori* and the output $y(t)$ must be available for feedback. This "high-gain property" of systems of form (1) with properties (i)-(iii) is the basis of high-gain adaptive control (for more details see the pioneering contributions [14], [15], [16], [17] or the self-contained surveys [18], [19]): in the adaptive case gain k in (2) becomes time-varying, i.e. $u(t) = -\operatorname{sign}(\gamma)k(t)y(t)$ where $\dot{k}(t) = y(t)^2$, $k(0) = k_0 > 0$. It is easy to see that the gain $k(\cdot)$ increases as long as $y(\cdot)$ is non-zero and hence there exists a time $t^* \geq 0$, where $k(t) \geq k^*$ for all $t \geq t^*$ yielding asymptotic stabilization of the closed-loop system. Due to dynamic adaption the controller gain is monotonically increasing. Moreover, if the output is perturbed by bounded measurement noise $n_m(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$ (i.e. $\dot{k}(t) = (y(t) + n_m(t))^2$) the gain might diverge. In contrast funnel control allows for gain increase *and* decrease (see Sec. IV) and therefore is more suitable for industrial application as for most mechatronic systems permanently large gains are not reasonable (e.g. noise amplification should be kept small).

Remark (Linear systems in the frequency domain)**.** *For systems of form* (1) *the transfer function is derived as follows*

$$\begin{aligned} F_S(s) &:= \frac{y(s)}{u(s)} = \boldsymbol{c}^\top (s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b} \\ &:= \gamma \frac{c_0 + c_1 s + \cdots + c_{m-1} s^{m-1} + s^m}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n}. \end{aligned} \tag{3}$$

It describes the linear system in the frequency domain, where $n \geq m$ for proper systems. If (1) *is a (minimal) realization of* (3)*, then the descriptions* (1) *and* (3) *are equivalent, and*

[3]The relative degree r of a system of form (1) indicates which output time derivative $y^{(r)}(t)$ is affected by control input $u(t)$, e.g. i) $r = 1$: $\dot{y}(t) = \boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{c}^\top \boldsymbol{b}\,u(t)$ or ii) $r = 2$: $\ddot{y}(t) = \boldsymbol{c}^\top \boldsymbol{A}^2\boldsymbol{x}(t) + \boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b}\,u(t)$

[4]In contrast to the classical definition of minimum-phase systems, in the present work the systems might be unstable, i.e. the poles might have positive real part. Only the zeros must have negative real part.

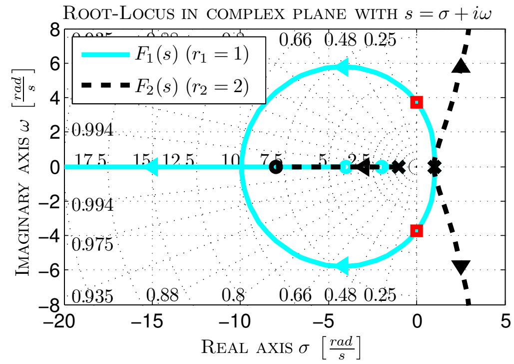


Fig. 1: Root-loci for exemplary systems $F_1(s)$ and $F_2(s)$ (the arrows indicate the pole shift for increasing gains).

analyzing transfer function (3) *yields relative degree $r = n - m$ and high-frequency gain $\gamma := \lim_{s\to\infty}\{s^r F_S(s)\}$. System* (3) *is minimum-phase if and only if the roots of the numerator polynomial have negative real parts. For illustration consider the two examples given by the transfer functions*

$$F_1(s) = \frac{s^2 + 6\,s + 8}{s^3 - s^2 - s + 1} \text{ and } F_2(s) = \frac{s + 8}{s^3 - s^2 - s + 1},$$

resp. Both systems are unstable but *minimum-phase*[5]*. Example one with transfer function $F_1(s)$ has relative degree $r_1 = 3 - 2 = 1$, whereas $F_2(s)$ has relative degree $r_2 = 3 - 1 = 2$. If output feedback* (2)[6] *is used to control the examples, the root-loci shown in Fig. 1 are obtained. The closed-loop $k \cdot F_1(s)/(1 + k \cdot F_1(s))$ is stable for all gains $k > k^* \approx 2.5$ (indicated by red squares), whereas the closed-loop $k \cdot F_2(s)/(1 + k \cdot F_2(s))$ is unstable for all $k \geq 0$. It exhibits two poles with positive real parts (see black dashed line in Fig. 1).*

III. System class

Funnel control is not limited to LTI SISO systems of form (1) it may be applied to a wider class of systems with relative degree one and two described by functional differential equations. To give a precise notion (sufficient for this overview) first introduce the following

Definition III.1 (Operator class $\mathcal{T}$)**.**
An operator $\mathfrak{T}$ is element of class $\mathcal{T}$ if and only if for some $h \geq 0$ and $n, m \in \mathbb{N}$, the following hold: (op$_1$) $\mathfrak{T} : \mathcal{C}([-h,\infty);\mathbb{R}^n) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_{\geq 0};\mathbb{R}^m)$, (op$_2$) bounded-input bounded-output: for every $\delta > 0$, there exists $\Delta > 0$, such that, for all $\boldsymbol{\zeta}(\cdot) \in \mathcal{C}([-h,\infty);\mathbb{R}^n)$: $[\![\, \sup_{t\in[-h,\infty)} \|\boldsymbol{\zeta}(t)\| < \delta \;\Rightarrow\; \|(\mathfrak{T}\boldsymbol{\zeta})(t)\| \leq \Delta$ for a.a. $t \geq 0 \,]\!]$, (op$_3$) for all $t \geq 0$, the following hold: (a) causality: for all $\boldsymbol{\zeta}(\cdot), \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty);\mathbb{R}^n)$: $[\![\, \boldsymbol{\zeta}(\cdot) \equiv \boldsymbol{\xi}(\cdot)$ on $[-h,t] \;\Rightarrow\; (\mathfrak{T}\boldsymbol{\zeta})(s) = (\mathfrak{T}\boldsymbol{\xi})(s)$ for a.a. $s \in [0,t] \,]\!]$ and (b) locally Lipschitz: for all $\boldsymbol{\beta}(\cdot) \in \mathcal{C}([-h,t];\mathbb{R}^n)$ there exist $\tau, \delta, c_0 > 0$, such that, for all $\boldsymbol{\zeta}(\cdot), \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty),\mathbb{R}^n)$ with

[5]The Routh-Hurwitz criterion [20, pp. 304-306] is violated for the denominator and is fulfilled for the numerator.

[6]The control law (2) becomes e.g. $u(s) = -k\,y(s) + v(s)$ in the frequency domain for some auxiliary input $v(s)$.

$\boldsymbol{\zeta}|_{[-h,t]} = \boldsymbol{\beta} = \boldsymbol{\xi}|_{[-h,t]}$ *and* $\boldsymbol{\zeta}(s), \boldsymbol{\xi}(s) \in \mathbb{B}^n_\delta(\boldsymbol{\beta}(t))$ *and for all* $s \in [t, t+\tau]$*:* $\text{ess-sup}_{s\in[t,t+\tau]} \|(\mathfrak{T}\boldsymbol{\zeta})(s) - (\mathfrak{T}\boldsymbol{\xi})(s)\| \leq c_0 \sup_{s\in[t,t+\tau]} \|\boldsymbol{\zeta}(s) - \boldsymbol{\xi}(s)\|$.

Here $h \geq 0$ quantifies the "memory" of the operator. As was shown in e.g. [21] nonlinear dynamic friction (common in mechatronics) is covered by the operator class $\mathcal{T}$. Moreover it subsumes e.g. relay, backlash, elasto-plastic and Preisach hysteresis or nonlinear delay systems (see [22], [23]). Now we are in the position to introduce the following

Definition III.2 (System class $\mathcal{S}_r$, $r \in \{1, 2\}$)**.**
Let $n, m \in \mathbb{N}$ *(unknown),* $h \geq 0$*,* $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}) \in \mathbb{R}^{n\times n} \times \mathbb{R}^n \times \mathbb{R}^n$ *and* $\boldsymbol{B}_\mathfrak{T} \in \mathbb{R}^{n\times m}$*. A system given by the functional differential equation*

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u(t) + u_d(t)\big) + \boldsymbol{B}_\mathfrak{T}\big((\mathfrak{T}\boldsymbol{x})(t) + \boldsymbol{d}(t)\big) \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \qquad \boldsymbol{x}|_{[-h,0]} = \boldsymbol{x}^0(\cdot) \in \mathcal{C}\big([-h, 0]; \mathbb{R}^n\big) \end{aligned}\right\} \tag{4}$$

with (measurable) disturbances $u_d(\cdot)$ *and* $\boldsymbol{d}(\cdot)$*, operator* $\mathfrak{T}\colon\ \mathcal{C}([-h,\infty);\mathbb{R}^n) \to \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R}^m)$ *and (measurable) control input* $u(\cdot)$*, is of Class* $\mathcal{S}_r$*, if the following hold:*

(sp$_1$) known relative degree
- *(a)* $r = 1$*, i.e.* $\gamma := \boldsymbol{c}^\top \boldsymbol{b} \neq 0$*;*
- *(b)* $r = 2$*, i.e.* $\boldsymbol{c}^\top \boldsymbol{b} = 0$*,* $\boldsymbol{c}^\top \boldsymbol{B}_\mathfrak{T} = \boldsymbol{0}$*,* $\gamma := \boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b} \neq 0$*;*

(sp$_2$) known sign of high-frequency gain, i.e. $\text{sign}(\gamma)$ *known;*
(sp$_3$) minimum-phase (as in (ii));
(sp$_4$) globally bounded operator of class $\mathcal{T}$*, i.e.* $\mathfrak{T} \in \mathcal{T}$ *and* $M_\mathfrak{T} := \sup\{\|(\mathfrak{T}\boldsymbol{\xi})(t)\| \mid t \geq 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty),\mathbb{R}^n)\} < \infty$*;*
(sp$_5$) bounded disturbances, i.e. $u_d(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R})$ *and* $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R}^m)$*.*

For a generalization to a wide class of nonlinear systems with the high-gain property see e.g. [3], [4], [23].

IV. Funnel control

A. Funnel control for relative-degree-one systems

The idea of tracking with prescribed transient accuracy was initially presented in [26] utilizing a high-gain based switching controller which "provides an arbitrarily good transient and steady-state response". However, this controller has a non-decreasing gain and invokes switching, both is not desirable for industrial application. Whereas funnel control for systems with relative degree one (i.e. $r = 1$), developed by Ilchmann et al. in 2002 [3], is also high-gain based but has a continuous gain which might decrease again. Funnel control may be applied to systems of class $\mathcal{S}_1$. It employs an adjustable time-varying gain $k_0(t, e(t))$ to stabilize systems of form (4) with properties (sp_1)(a), (sp_2)–(sp_5). Furthermore, prescribed transient and asymptotic tracking of a (absolutely) continuous and bounded reference $y_{\text{ref}}(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};\mathbb{R})$ is assured. The tracking (or control) error

$$e(t) = y_{\text{ref}}(t) - y(t) - n_m(t) \tag{5}$$

perturbed by bounded measurement noise $n_m(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};\mathbb{R})$ evolves within the prescribed funnel (i.e. the set given by $\{(t,e) \in \mathbb{R}_{\geq 0}\times\mathbb{R} \mid |e| < \psi_0(t)\}$) with boundary $\psi_0(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};[\lambda_0,\infty))$ where $\lambda_0 > 0$ represents the prescribed asymptotic accuracy (see Fig. 2a). More formerly, the following holds $|e(t)| < \psi_0(t)$ for all $t \geq 0$, if the initial error $|e(0)| < \psi_0(0)$ 'starts' inside the funnel. The funnel controller generates the control action

$$u(t) = k_0\,(t, e(t)) \cdot e(t) \tag{FC$_1$}$$

$$\text{where } k_0\,(t, e(t)) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}. \tag{6}$$

The time-varying gain[7] $k_0\,(t, e(t))$ is instantaneously adjusted and is inversely proportional to the vertical[8] distance $\psi_0(t) - |e(t)|$ for all $t \geq 0$ (see Fig. 2a). For $m_0 > 0$, the bounded 'scaling function' $s_0(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};[m_0,\infty))$ e.g. allows to fix a minimal control gain $m_0/\|\psi_0\|_\infty > 0$.

Gain "adaption" (6) ensures that error $e(\cdot)$ evolves within the funnel limited by boundary $\psi_0(\cdot)$: gain $k_0(t, e(t))$ increases, if error $e(t)$ draws close to boundary $\psi_0(t)$ (more aggressive control) and decreases, if error $e(t)$ becomes small (more relaxed control). In [3] it is proved that both, gain $k_0(\cdot)$ and error $e(\cdot)$, stay bounded, if control input $u(\cdot)$ can adopt sufficiently large but finite values.

A proper choice of the funnel boundary is e.g. given by

$$\psi_0(t) = \psi_E(t) := (\Lambda_0 - \lambda_0)\exp(-t/T_E) + \lambda_0 \tag{7}$$

with initial value $\psi_E(0) = \Lambda_0 > |e(0)|$ enclosing the initial error, prescribed time constant $T_E > 0$ and asymptotic accuracy $\lambda_0 = \lim_{t\to\infty} \psi_E(t) > 0$. Note that the boundary may also increase temporarily (see Fig. 2). If e.g. sudden and drastic changes in reference and/or disturbance are to be expected or known a priori, then an increasing boundary is reasonable to avoid (too) large control actions.

To design the boundary properly the initial value of the error must be known *a priori*. For mechatronic applications this holds true for any given and bounded reference with $y_{\text{ref}}(0)$ and given output $y(0) + n_m(0)$. Theoretically this condition may be relaxed by choosing an "infinite" boundary 'starting at $\psi_0(0) = \infty$' [3], [4].

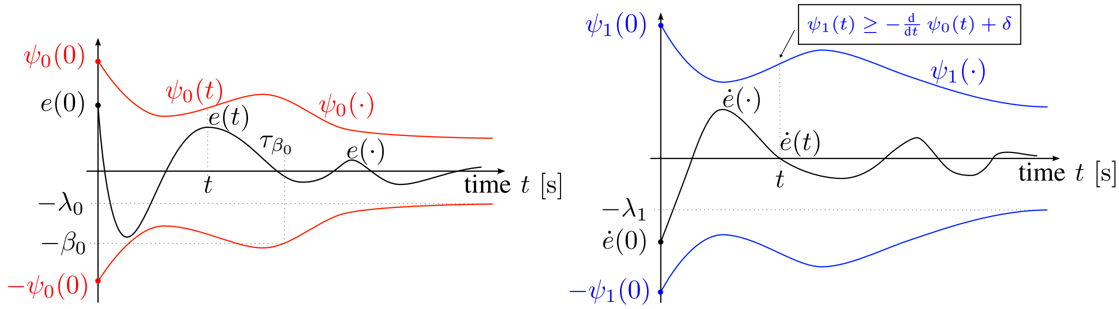
Clearly, costumer specifications $(\beta_0, \tau_{\beta_0}, \lambda_0)$ — where $\beta_0 > 0$ is the accuracy at time $\tau_{\beta_0} \geq 0$ and $\lambda_0 > 0$ is the desired asymptotic accuracy (see Fig. 2a) — can be easily met by adequate boundary design, i.e. $\psi_0(\tau_{\beta_0}) = \beta_0 > |e(t)|$ for all $t \geq \tau_{\beta_0}$ and $\lim_{t\to\infty} \psi_0(t) = \lambda_0 > \limsup_{t\to\infty} |e(t)|$.

B. Funnel control for relative-degree-two systems

It is well known (from e.g. the root locus method) that systems with relative degree two (i.e. $r = 2$), in general, might not be stabilizable by simple output feedback (2) or (FC$_1$) [18]. Over eight years it has been an unanswered question, if funnel control may be extended to the relative-degree-two case without the use of backstepping yielding an undesirable complex controller and gains occurring with

[7] The functions $\psi_0(\cdot)$ and $s_0(\cdot)$ are functions of time, hence the abbreviated definition of the gain $k(\cdot, e(\cdot))$ is correct.

[8] The minimal future distance $d_F(t_F, e(t)) := \min_{t_F \geq t} \sqrt{(\psi_0(t_F) - |e(t)|)^2 + (t_F - t)^2}$ at some future time $t_F \geq t$ is also admissible [4], [6], [8].



(a) Funnel for error (b) Funnel for error derivative

Fig. 2: Funnels for relative degree one and two. $r = 1$: Funnel for error $e(\cdot)$ with boundary $\psi_0(\cdot)$ (see (a)) and $r = 2$: Funnels for error $e(\cdot)$ and derivative $\dot{e}(\cdot)$ with boundaries $\psi_0(\cdot)$ and $\psi_1(\cdot)$, resp. (see (a) *and* (b)). Asymmetric funnels with upper $\psi_0^+(\cdot)$ and lower $\psi_0^-(\cdot)$ boundary, where $\psi_0^+(t) > \psi_0^-(t)$ for all $t \geq 0$, are admissible (but neglected here) [24], [25].

$k(t)$[7] [27]. The recent works [12] and [28] give an affirmative answer: if derivative feedback is admissible and two funnel boundaries $\psi_0(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}; [\lambda_0, \infty))$ and $\psi_1(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}; [\lambda_1, \infty))$ where $\lambda_0, \lambda_1 > 0$ (one for error e as in (5) and one for error derivative $\dot{e} = \dot{y}_{\mathrm{ref}} + \dot{y} + \dot{n}_m$, see Fig. 2a & b) are employed, then the extended funnel controller

$$u(t) = k_0(t, e(t))^2\, e(t) + k_0(t, e(t))\, k_1(t, \dot{e}(t))\, \dot{e}(t), \quad (\mathrm{FC}_2)$$

$$\text{where } k_i(t, e^{(i)}(t)) = \frac{s_i(t)}{\psi_i(t) - |e^{(i)}(t)|} \quad \text{for } i = 0, 1 \qquad (8)$$

assures prescribed transient accuracy for systems of class $\mathcal{S}_2$ with properties (sp_1)(b), (sp_2)–(sp_5). Again, for $m_0, m_1 > 0$, gains $k_0(\cdot)$ and $k_1(\cdot)$ may include bounded 'scaling functions' $s_0(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}, [m_0, \infty))$ and $s_1(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}, [m_1, \infty))$, resp., to allow for more degrees of freedom in the controller design. In contrast to the relative-degree-one funnel controller (FC_1), the relative-degree-two funnel controller (FC_2) ensures, that error $e(\cdot)$ *and* derivative $\dot{e}(\cdot)$ evolve within their respective funnels with boundaries $\psi_0(\cdot)$ and $\psi_1(\cdot)$, if $\psi_0(0) > |e(0)|$ and $\psi_1(0) > |\dot{e}(0)|$, resp. Again customer specifications $(\beta_0, \tau_{\beta_0}, \lambda_0)$ can be met by adequate boundary design. The funnel controller (FC_2) is a slight modified version of the originally introduced controller $u(t) = k_0(t, e(t))^2\, e(t) + k_1(t, e(t))\, \dot{e}(t)$ in [12]. The modification in (FC_2) allows for a better damped closed-loop system response (without overshoot, see [28]) and a more effective use of the control action (peaks in drive torque are reduced, see Fig. 10 in [29]).

Important to note that for the relative-degree-two case two more conditions are imposed on reference and boundary: (a) $y_{\mathrm{ref}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$ and (b) for some $\delta > 0$ the 'second' boundary must satisfy $\psi_1(t) \geq -\frac{\mathrm{d}}{\mathrm{d}t}\,\psi_0(t) + \delta$ for all $t \geq 0$ [12]. The second condition is obvious: to allow for errors $e(t)$ departing from the boundary $\psi_0(t)$ an error derivative with $\mathrm{sign}(e(t))\,\dot{e}(t) < \frac{\mathrm{d}}{\mathrm{d}t}\,\psi_0(t)$ must be admissible. For $\psi_0(t) = \psi_E(t)$ as in (7) and $\delta := \lambda_1 > 0$, following choice for the 'derivative' boundary

$$\psi_1(t) = (\Lambda_0 - \lambda_0)\,/T_E\,\exp\left(-t/T_E\right) + \lambda_1 \qquad (9)$$

is admissible, since $\psi_1(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\,\psi_E(t) + \lambda_1$.

C. Funnel control and steady-state accuracy

The condition '$\psi_0(t) \geq \lambda_0$ for all $t \geq 0$' possibly yields non-vanishing steady-state errors $\limsup_{t\to\infty} |e(t)| > 0$. This drawback is typically for proportional controllers without integral control action (see [30]) and hence is not a specific drawback of funnel control. For $k_P, k_I > 0$ and $u(t)$ as in (FC_1) or (FC_2), this drawback may be overcome by introducing a PI-like extension given by

$$v(t) = k_P u(t) + k_I \int_0^t u(\tau)\,\mathrm{d}\tau\,, \qquad (\mathrm{PI})$$

and connecting it in series to system (4) (see Fig. 3), which assures asymptotic accuracy and disturbance rejection for *constant* reference and disturbance signals if steady-state is reached [28].

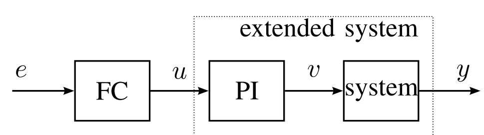


Fig. 3: Serial interconnection of funnel controller (FC_1) or (FC_2), PI-like extension (PI) and system (4).

The extension (PI) can be considered as simple internal model and does not deteriorate the affiliation to class $\mathcal{S}_r$, $r \in \{1, 2\}$ [10], [11], [28]. In the frequency domain this is easy to understand where (PI) becomes

$$F_{PI}(s) = k_P + k_I/s = k_P(s + k_I/k_P)/s. \qquad (10)$$

which has relative degree $r_{PI} = \deg(k_P\, s + k_I) - \deg(s) = 0$ and positive high-frequency gain $\gamma_{PI} := \lim_{s\to\infty} s^{r_{PI}} F_{PI} = k_P > 0$. It is minimum-phase since $k_I/k_P > 0$. For any LTI SISO system of form (1) with properties (i)–(iii) described by transfer function (3) with relative degree $r = n - m = 1$ or $r = n - m = 2$, positive high-frequency gain $\gamma > 0$, it is easy to see that the serial interconnection $F_{PI}(s) \cdot F_S(s)$ is still minimum-phase and has 'overall' positive high-frequency gain $\gamma \cdot k_P$ and relative degree $r_{PI} + r = r$. The same holds true for systems of the form (4) [10], [11], [28].

Remark (Remaining difficulties concerning implementation)**.** *Most modern controllers are implemented digitally* *on realtime platforms with limited computing power and sampling time: increasing gains accelerate the closed-loop system response, but to assure Shannon's theorem the sampling time might have to be decreased accordingly; yielding possibly non-realtime applicability of funnel control. Several experiments using funnel control in mechatronics did not reveal real-time issues if boundary design is not too demanding (see e.g. [11], [25], [12]). But still this is theoretically an open problem, first results (for λ-tracking) are presented in [31]. Nearly all actuator are saturated due to security reasons or power limitations. Funnel control with saturation is possible, see the theoretical results in [32], [33], [12], [24] for relative degree one and two: a (possibly very) conservative feasibility condition gives a sufficient lower bound on the required control input, however this bound is often not reasonable for application. More realistic and intuitive results for the relative-degree-one case — but only empirically examined — are proposed in [9], [7].*

V. Application: Speed and position control

A. Model of the plant

A simple rotatory model (translational is similar) for the standard speed or position control problem is presented. The model describes the laboratory setup consisting of two stiff coupled electrical machines with (overall) inertia $\Theta > 0\ \left[\mathrm{kg\,m^2}\right]$, see Figure 5. Motor drive and load drive induce accelerating (drive) torque $u(\cdot) = m_M(\cdot)$ [Nm] and load torque $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$ [Nm], resp. To allow for (ideal) gears the gear ratio $g_r \in \mathbb{R}\setminus\{0\}$ [1][9] is included in the model, whereas backlash in the gear is neglected.

The mechanical system is subject to friction on motor side with $\nu_1\omega(\cdot) + (\mathfrak{F}_1\omega)(\cdot)$ [Nm] and on the gear side with $\nu_2/g_r\,\omega(\cdot) + (\mathfrak{F}_2\omega/g_r)(\cdot)$ [Nm] (see Fig. 4). For viscous friction coefficients $\nu_1, \nu_2 \geq 0$ [Nms/rad], the friction characteristics are split into unbounded viscous friction (i.e. $\nu_1\omega(\cdot)$ and $\nu_2\omega(\cdot)/g_r$) and bounded friction (i.e. $\mathfrak{F}_1, \mathfrak{F}_2 \in \mathcal{T}$ with $M_{\mathfrak{F}_1}, M_{\mathfrak{F}_2} < \infty$, including e.g. Stribeck effect, stiction and Coulomb friction). The operators $\mathfrak{F}_1$ and $\mathfrak{F}_2$ allow for modeling of dynamic friction effects such as pre-sliding displacement and frictional lag (see e.g. [12], [34] or the survey [35]). For position control the mathematical model of the stiff coupled mechanical system with state variable $\boldsymbol{x}(t)^\top := \big(\phi(t),\ \ \omega(t)\big)$, representing angle $\phi(t)$ [rad] and speed $\omega(t)$ [rad/s] at time $t \geq 0$ [s], is given by

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(m_M(t) - (\mathfrak{F}_1\omega)(t) - \tfrac{1}{g_r}(m_L(t) + (\mathfrak{F}_2\tfrac{\omega}{g_r})(t)\big) \\
y(t) &= \boldsymbol{c}^\top\boldsymbol{x}(t) = \phi(t)/g_r, \qquad \boldsymbol{x}(0) = (\phi^0,\ \omega^0)^\top
\end{aligned}
\tag{11}
$$

where $\nu_1, \nu_2 \geq 0$, $g_r \in \mathbb{R}\setminus\{0\}$, $\Theta > 0$,

$$
\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\nu_1 + \nu_2/g_r^2}{\Theta} \end{bmatrix},\ \boldsymbol{b} = \begin{pmatrix} 0 \\ \frac{1}{\Theta} \end{pmatrix} \ \text{ and } \ \boldsymbol{c}^\top = \begin{pmatrix} \frac{1}{g_r} & 0 \end{pmatrix}. \tag{12}
$$

For speed control we obtain a simplified mathematical model

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + b\big(m_M(t) - (\mathfrak{F}_1\omega)(t) - \tfrac{1}{g_r}(m_L(t) + (\mathfrak{F}_2\tfrac{\omega}{g_r})(t)\big) \\
y(t) &= cx(t) = \omega(t)/g_r, \qquad x(0) = \omega^0
\end{aligned}
\tag{13}
$$

with state variable $x(t) := \omega(t)$ and the scalars

$$
A = -(\nu_1 + \nu_2/g_r^2)/\Theta,\ b = 1/\Theta \ \text{ and } \ c = 1/g_r. \tag{14}
$$

[9]In e.g. robotics exist gears with ratio $g_r < 0$.

The mechanical models (11) and (13) neglect the electrical drive (actuator) generating drive torque $m_M(\cdot)$. The mapping $m_{\mathrm{ref}} \mapsto m_M$, see Figure 4, is actually a saturated actuator comprising inverter and machine (with current/torque control-loop). That is a nonlinear dynamical system. Since its dynamics are very fast (compared to the mechanical model, see e.g. [36, pp. 775-779]), it may be modeled by $m_M(t) = \mathrm{sat}_{\overline{m}}(m_{\mathrm{ref}}(t) + u_d(t))$ where saturation $\mathrm{sat}_{\overline{m}}(\cdot)$ with $\overline{m} > 0$ [Nm] ensures that $|m_M(t)| \leq \overline{m}$ for all $t \geq 0$. $m_{\mathrm{ref}}(\cdot)$ and $u_d(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$ represent 'reference torque' and (actuator) disturbance, resp. In the following, we neglect the influence of constrained control inputs and assume that $|m_M(t)| \approx |m_{\mathrm{ref}}(t)| \leq \overline{m}$ for all $t \geq 0$. The control objectives for the controller in the closed-loop system (see Fig. 6) are speed or position reference tracking and disturbance rejection, where the references are given by $y_{\mathrm{ref}}(\cdot) = \omega_{\mathrm{ref}}(\cdot)/g_r \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};\mathbb{R})$ and $y_{\mathrm{ref}}(\cdot) = \phi_{\mathrm{ref}}(\cdot)/g_r \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0};\mathbb{R})$, resp.

Remark (Linear models for speed and position control in the frequency domain)**.** *Neglecting the bounded friction terms (i.e.* $\mathfrak{F}_1 = \mathfrak{F}_2 = 0$*) in* (11) *and* (13) *yields the transfer functions, resp., for speed control*

$$
F_\omega(s) = \frac{\omega(s)/g_r}{m_M(s)} = c(s-A)^{-1}b = \frac{1/(g_r\Theta)}{s + \frac{\nu_1 + \nu_2/g_r^2}{\Theta}} \tag{15}
$$

and for position control

$$
F_\phi(s) = \frac{\phi(s)/g_r}{m_M(s)} = \boldsymbol{c}^\top(s\boldsymbol{I}_2 - \boldsymbol{A})^{-1}\boldsymbol{b} = \frac{1/(g_r\Theta)}{s(s + \frac{\nu_1 + \nu_2/g_r^2}{\Theta})}. \tag{16}
$$

B. Speed control

Funnel control with PI-extension (FC_1)+(PI)*:* To show applicability of speed funnel control of (13), the system properties (sp_1)(a), (sp_2) –(sp_5) of class $\mathcal{S}_1$ must be verified. For the system data as in (14) and known sign of gear ratio g_r (normally the value can be read off on the gear box), we compute $c\,b = 1/(g_r\Theta) \neq 0$ where $\mathrm{sign}(1/(g_r\Theta))$ is known, which shows (sp_1)(a) and (sp_2), resp. Moreover, note that the following holds

$$
\det\begin{bmatrix} s - A & b \\ c & 0 \end{bmatrix} = \det\begin{bmatrix} s + \frac{\nu_1 + \nu_2/g_r^2}{\Theta} & \frac{1}{\Theta} \\ \frac{1}{g_r} & 0 \end{bmatrix} = \frac{-1}{g_r\Theta} \overset{(14)}{\neq} 0
$$

hence (sp_3) is satisfied. By assumption we have $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$, $\mathfrak{F}_i \in \mathcal{T}$ and $|\mathfrak{F}_i| \leq M_{\mathfrak{F}_i} < \infty$ for $i \in \{1,2\}$[10] and therefore (sp_4) and (sp_5) also hold. Concluding, funnel control with PI-extension is admissible for speed control of (13).

Classical PI control (PI): If we apply controller (PI) to the (linear) system (15), we obtain the closed-loop transfer function

$$
\begin{aligned}
F_{\omega,CL}(s) &:= \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{F_{PI}(s)F_\omega(s)}{1 + F_{PI}(s)F_\omega(s)} & (17)\\
&= \frac{\frac{k_P}{g_r\Theta}\,(s + k_I/k_P)}{s^2 + \left(\frac{\nu_1 + \nu_2/g_r^2}{\Theta} + \frac{k_P}{g_r\Theta}\right)s + \frac{k_I}{g_r\Theta}}. & (18)
\end{aligned}
$$

[10]considering actuator disturbance $u_d(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$ as bounded input disturbance in (4) is also possible.

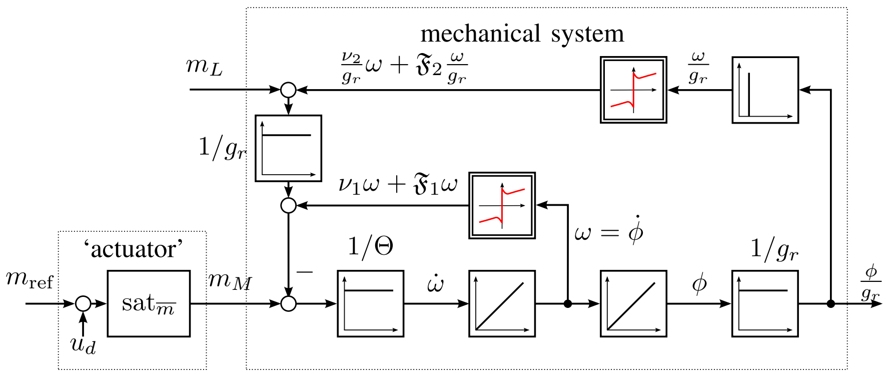


Fig. 4: Electrical drive system (actuator and mechanical system)

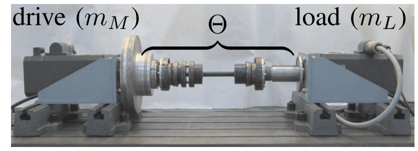


Fig. 5: Laboratory setup

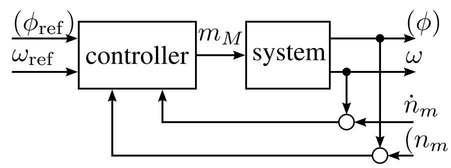


Fig. 6: Closed-loop system

which, using the Routh-Hurwitz criterion [20, pp. 304-306], is stable if $\mathrm{sign}(k_P) = \mathrm{sign}(k_I) = \mathrm{sign}(g_r)$.

C. Position control

Funnel control with PI-extension (FC_2)+(PI)*:* Now we need to verify properties (sp_1)(b), (sp_2)–(sp_5) of class $\mathcal{S}_2$. For system (11) with data (12) and known sign of g_r, we obtain $\boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b} = 1/(g_r\Theta) \neq 0$ and, for $\boldsymbol{B}_{\mathfrak{T}} := \boldsymbol{b}$, $\boldsymbol{c}^\top \boldsymbol{B}_{\mathfrak{T}}\boldsymbol{b} = 0$ which shows (sp_1)(b). Moreover, note that $\mathrm{sign}(1/(g_r\Theta))$ is known whence (sp_2). Property (sp_3) also holds since[11]

$$\det\begin{bmatrix} s\boldsymbol{I}_2 - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix} = \det\begin{bmatrix} s & -1 & 0 \\ 0 & s + \frac{\nu_1 + \nu_2/g_r^2}{\Theta} & \frac{1}{\Theta} \\ \frac{1}{g_r} & 0 & 0 \end{bmatrix} = \frac{-1}{g_r\Theta} \overset{(12)}{\neq} 0$$

Again since $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$, $\mathfrak{F}_i \in \mathcal{T}$ and $|\mathfrak{F}_i| \leq M_{\mathfrak{F}_i} < \infty$ for $i \in \{1,2\}$ also system properties (sp_4) and (sp_5) hold, resp. Concluding, funnel control with PI-extension is also admissible for position control of (12).

Classical PID control: For position error $e(t) = 1/g_r(\phi_{\mathrm{ref}}(t) - \phi(t)$ and its derivative $\dot{e}(t) = 1/g_r(\omega_{\mathrm{ref}}(t) - \omega(t))$ PID feedback with feedforward control $u_F(\cdot) \in \mathcal{L}^{\inf}(\mathbb{R}_{\geq 0};\mathbb{R})$ is given by

$$u(t) = k_P e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau + k_D\dot{e}(t) + u_F(t) \quad \text{(PID)}$$

with transfer function (neglecting u_F)

$$F_{PID}(s) = \frac{u(s)}{e(s)} = (k_P + k_I/s + k_D\,s)\,. \tag{19}$$

Applying the PID controller (19) to linear system (16) yields the closed-loop transfer function

$$F_{\phi,CL}(s) := \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{F_{PID}(s)F_\phi(s)}{1 + F_{PID}(s)F_\phi(s)} \tag{20}$$

$$= \frac{\frac{1}{g_r\Theta}\left(k_I + k_P\,s + k_D\,s^2\right)}{s^3 + \left(\frac{\nu_1+\nu_2/g_r^2}{\Theta} + \frac{k_D}{g_r\Theta}\right)s^2 + \frac{k_P}{g_r\Theta}\,s + \frac{k_I}{g_r\Theta}}. \tag{21}$$

which is Hurwitz stable for $\mathrm{sign}(k_P) = \mathrm{sign}(k_I) = \mathrm{sign}(k_D) = \mathrm{sign}(g_r)$ and $\mathrm{sign}(g_r)k_I \leq k_P \cdot k_D/(g_r\Theta) < k_P \cdot (k_D + \nu_1 + \nu_2/g_r^2)/(g_r\Theta)$. The numerator in (21) (differentiating action) causes an overshoot of the closed-loop system response, if e.g. a step reference is to be tracked. This overshoot may be avoided by adequate (constant) feed-forward control (see measurement results in Fig. 9 or [38]).

[11] using Laplace's Theorem to compute the determinant [37, pp. 36-37].

description	symbols & values (without dimensions)
inertia, gear	$\Theta = 0.3421$, $g_r = 1$
initial values	$\phi^0 = 0$, $\omega^0 = 0$
disturbances	$\|u_d\|_\infty \leq 0.56$, $\|M_L\|_\infty = 10$
speed control:	
(FC_1)+(PI)	$\Lambda_0 = 30$, $\lambda_0 = 1$, $T_E = 0.124$,
	$s_0(t) = 1.46 \cdot \psi_E(t)$, $k_P = 1$, $k_I = 4$
(PI)	$k_P = 2.2$, $k_I = 7.0$
position control:	
(FC_2)+(PI)	$\Lambda_0 = 2\pi$, $\lambda_0 = 0.09\pi$, $T_E = 0.35$,
	$\lambda_1 = 10$, $s_0(t) = 1.33 \cdot \psi_E(t)$
	$s_1(t) = 2 \cdot \psi_1(t)$, $k_P = 1$, $k_I = 5$
(PID)	$k_P = 11$, $k_I = 7$, $k_D = 4$, $u_F = -12.6$

TABLE I: System, implementation and controller data.

VI. Implementation and Measurement Results

Now, we apply the different controllers to the laboratory setup, i.e. (FC_1)+(PI) and (PI) for speed control and (FC_2)+(PI) and (PID) for position control (see Fig. 5 and 6). We implement the controllers using Matlab/Simulink and a xPC-Target PC with sampling time $h = 1$ [ms]. The laboratory setup consists of two permanent magnetic synchronous machines, each driven by its own inverter such that drive torque m_M and load torque m_L can be induced independently. HEIDENHAIN RON 3350 encoders measure angular position (with 2048 lines per revolution and 12-bit interpolation) and, by numeric differentiation, the capturing device (I/O board) provides speed information. For the comparison, we design all controllers such that the available torque $\overline{m} = 22$ [Nm] is not exceeded during the experiment. Moreover, *all* controllers are designed such that $u(0) = m_M(0) = 22$ [Nm] holds. With these constraints the (PI) and (PID) controllers are designed as fast as possible with minimal overshoot (tuning is performed empirically, then (linear) stability presuppositions are checked). The funnel controllers (FC_1)+(PI) and (FC_2)+(PI) are designed with the exponential boundaries as in (7) and (7),(9), resp. Implementation, system and controller data is collected in Tab. I (system data is not used for design of the funnel controllers!).

Speed control: Fig. 7 (step-response) and Fig. 8 (long-run) show the measurement results for speed control. During the initial phase $0 - 7s$ set-point tracking of 10 $\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$ is the control task. Load torques are induced at 3, 10 and 30 [s] (see

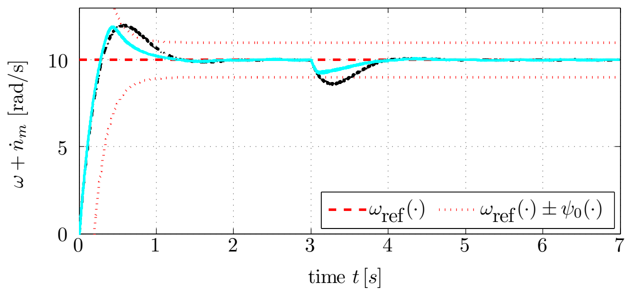


Fig. 7: Measurement results for speed control (step-response): measured speed $\omega(\cdot) + \dot{n}_m$ ——(FC$_1$)+(PI) and -·-·-(PI).

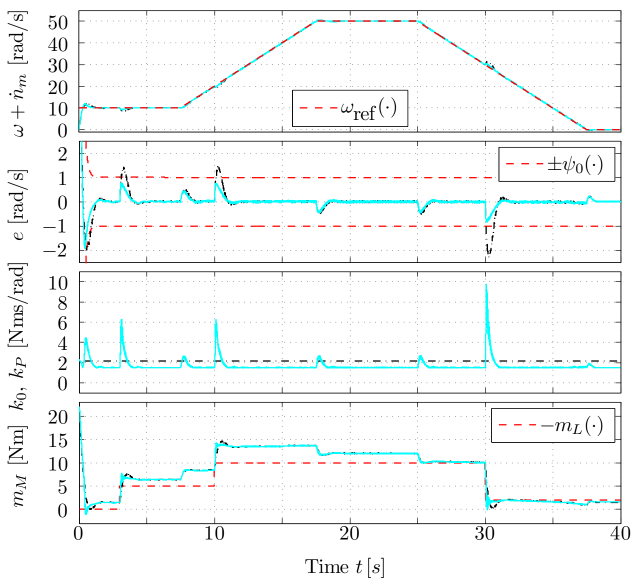


Fig. 8: Measurement results for speed control (long-run): ——(FC$_1$)+(PI) and -·-·-(PI) (from top to bottom: measured speed $\omega(\cdot) + \dot{n}_m$, error $e(\cdot)$, proportional gain $k_0(\cdot), k_P$ and torque $u(\cdot) = m_M(\cdot)$).

Fig. 8). Both controllers show good tracking performance and disturbance rejection. The (FC$_1$)+(PI) controller reacts slightly faster on reference changes and disturbances (see Fig. 7 and 8). For (FC$_1$)+(PI), the control error evolves within the funnel whereas, for (PI), the error leaves the prescribed region (during load action).

Position control: Fig. 9 (step-response) and Fig. 10 (long-run) show the measurement results for position control. During the first $10\,[\mathrm{s}]$ the control-loops have to track a set-point of π [rad] then smoothed ramps[12]. Load steps at $5, 15$ and 35 [s] disturb the closed-loop systems (see Fig. 10). (FC$_2$)+(PI) and (PID) controller attain a comparable performance without overshoot (see Fig. 10). For (FC$_2$)+(PI), the error and its derivative evolve within their corresponding funnels whereas, for (PID), it leaves the prescribed regions (during load action and reference changes).

Remark. *Obviously, the PI/PID controllers could be designed more properly, using e.g. anti-wind up strategies, such that a better tracking and disturbance performance is to be expected. But for the presented comparison, we intended to show the limits of classical PI/PID control with constant gains, if the generated control action* $u(\cdot)$ *should not exceed the available torque* $\overline{m} = 22$ [Nm].

[12]The position reference is low-pass filtered to obtain a signal element of $\mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$.

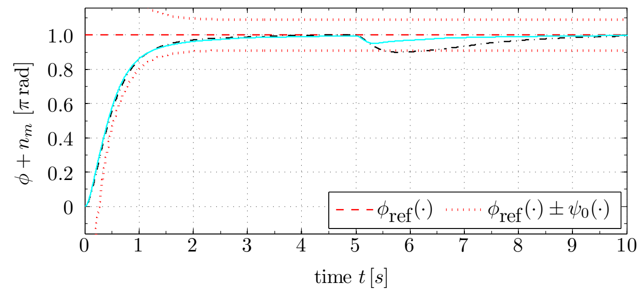


Fig. 9: Measurement results for position control (step-response): measured position $\phi(\cdot) + n_m(\cdot)$ ——(FC$_2$)+(PI) and -·-·-(PID).

VII. Conclusion

This overview introduced and showed possible applications of a time-varying (adaptive) proportional funnel controller for speed and position control of a (stiff) mechatronic servo-system. The presented funnel controllers instantaneously adjust their time-varying gains inversely proportional to the distances between measured output and prescribed funnel boundary. This "adaption" guarantees that error (and its derivative) evolves within the prescribed funnel(s). Funnel control is applicable to systems with relative degree one or two, stable zero-dynamics (minimum-phase) and known sign of the high-frequency gain. Since only structural assumptions on the system must hold for controller implementation, funnel control is inherently robust to parameter uncertainties. Measurement results illustrate that the funnel controllers can keep up with the control performance and the disturbance rejection of classical PI/PID controllers. Moreover, when unknown load disturbances deteriorate control performance, then the funnel controllers guarantee tracking with prescribed transient behavior in contrast to the PI/PID controllers.

References

[1] K. Ohnishi, M. Shibata, and T. Murakami, "Motion control for advanced mechatronics," *IEEE/ASME Transactions on Mechatronics*, vol. 1, no. 1, pp. 56–67, 1996.

[2] K. J. Åström and B. Wittenmark, *Adaptive Control*. Boston: Addison-Wesley Publishing Company, Inc., 2nd ed., 1995.

[3] A. Ilchmann, E. P. Ryan, and C. J. Sangwin, "Tracking with prescribed transient behaviour," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 7, pp. 471–493, July 2002.

[4] A. Ilchmann, E. P. Ryan, and S. Trenn, "Tracking control: Performance funnels and prescribed transient behaviour," *Systems & Control Letters*, vol. 54, pp. 655–670, 2005.

[5] A. Ilchmann, M. Thuto, and S. Townley, "Input constrained adaptive tracking with applications to exothermic checmical reaction models," *SIAM Journal of Control and Optimization*, vol. 43, no. 1, pp. 154–173, 2004.

[6] C. M. Hackl, Y. Ji, and D. Schröder, "Enhanced funnel-control with improved performance," in *Proceedings of the 15th Mediterranean Conference on Control and Automation, Athens, Greek*, pp. Paper T01–016, June 27-29 2007.

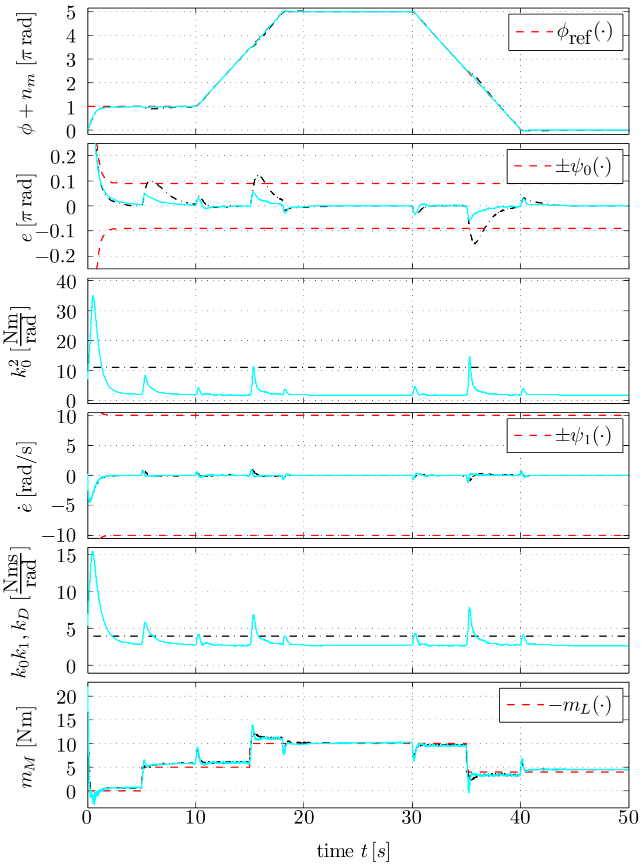


Fig. 10: Measurement results for position control (long-run): —(FC$_2$)+(PI) and -·-·(PID) (from top to bottom: measured position $\phi(\cdot) + n_m(\cdot)$, error $e(\cdot)$, error derivative $\dot{e}(\cdot)$, proportional gain $k_0(\cdot)^2, k_P$, derivative gain $k_0(\cdot)k_1(\cdot), k_D$ and torque $u(\cdot) = m_M(\cdot)$).

[7] C. M. Hackl, Y. Ji, and D. Schröder, "Funnel-control with constrained control input compensation," in *Proceedings of the 9th IASTED International Conference CONTROL AND APPLICATIONS, Montreal, Canada*, pp. 147–152, May 30 - June 1 2007.

[8] C. M. Hackl and D. Schröder, "Funnel-control with online foresight," in *Proceedings of the 26th IASTED International Conference MODELLING, IDENTIFICATION AND CONTROL, Innsbruck, Austria*, pp. 171–176, February 12-14 2007.

[9] C. M. Hackl, Y. Ji, and D. Schröder, "Nonidentifier-based adaptive control with saturated control input compensation," in *Proceedings of the 15th Mediterranean Conference on Control and Automation, Athens, Greek*, June 27–29 2007.

[10] C. M. Hackl and D. Schröder, "Extension of high-gain controllable systems for improved accuracy," in *Proceedings of the IEEE International Conference on Control Applications, Munich, Germany*, pp. 2231–2236, October 4-6 2006.

[11] A. Ilchmann and H. Schuster, "PI-funnel control for two mass systems," *IEEE Transactions on Automatic Control*, vol. 54, pp. 918–923, April 2009.

[12] C. M. Hackl, N. Hopfe, A. Ilchmann, M. Mueller, and S. Trenn, "Funnel control for systems with relative degree two," *submitted to SIAM Journal on Control and Optimization (preprint available at the author)*, 2010.

[13] A. Ilchmann, *Non-Identifier-Based High-Gain Adaptive Control*, vol. 189 of *Lecture notes in control and information sciences*. London: Springer-Verlag, 1993.

[14] C. I. Byrnes and J. C. Willems, "Adaptive stabilization of multivariable linear systems," in *Proceedings of the 23rd Conference on Decision and Control*, (Las Vegas, USA), pp. 1574–1577, December 1984.

[15] I. Mareels, "A simple selftuning controller for stably invertible systems," *Systems & Control Letters*, vol. 4, pp. 5–16, February 1984.

[16] B. Mårtensson, "The oder of any stabilizing regulator is sufficient a priori information for adaptive stabilization," *Systems & Control Letters*, vol. 6, pp. 87–91, July 1985.

[17] B. Mårtensson and J. Polderman, "Correction and simplification to "The order of a stabilizing regulator is sufficient a priori information for adaptive stabilization"," *Systems & Control Letters*, vol. 20, pp. 465–470, June 1993.

[18] A. Ilchmann, "Non-identifier-based adaptive control of dynamical systems: A survey," *IMA Journal of Mathematical Control & Information*, vol. 8, pp. 321–366, 1991.

[19] A. Ilchmann and E. Ryan, "High-gain control without identification: A survey," *GAMM-Mitteilungen*, vol. 31, no. 1, pp. 115–125, 2008.

[20] D. Hinrichsen and A. Pritchard, *Mathematical Systems Theory I — Modelling, State Space Analysis, Stability and Robustness*. No. 48 in Texts in Applied Mathematics, Berlin: Springer-Verlag, 2005.

[21] A. Ilchmann and A. Isidori, "Adaptive dynamic output feedback stabilization of nonlinear systems," *Asian Journal of Control*, vol. 4, pp. 246–254, September 2002.

[22] R. P. Ryan and C. J. Sangwin, "Controlled functional differential equations and adaptive stabilization," *International Journal of Control*, vol. 74, pp. 77–90, 2001.

[23] A. Ilchmann, E. Ryan, and C. Sangwin, "Systems of controlled functional differential equations and adaptive tracking," *SIAM Journal on Control and Optimization*, vol. 40, no. 6, pp. 1746–1764, 2002.

[24] D. Liberzon and S. Trenn, "The bang-bang funnel controller," in *Proceedings of the 49th IEEE Conference on Decision and Control*, (Atlanta, USA), pp. 690–695, December 15-17 2010.

[25] C. M. Hackl, C. Endisch, and D. Schröder, "Contributions to non-identifier based adaptive control in mechatronics," *Robotics and Autonomous Systems, Elsevier*, vol. 57, pp. 996–1005, October 2009.

[26] D. E. Miller and E. J. Davison, "An adaptive controller which provides an arbitrarily good transient and steady-state response," *IEEE Transactions on Automatic Control*, vol. 36, pp. 68–81, January 1991.

[27] A. Ilchmann, E. Ryan, and P. Townsend, "Tracking with prescribed transient behaviour for nonlinear systems of known relative degree," *SIAM Journal on Control and Optimization*, vol. 46, no. 1, pp. 210–230, 2007.

[28] C. M. Hackl, "High-gain adaptive position control," *accepted for publication in "International Journal of Control" (preprint available at the author)*, 2011.

[29] C. M. Hackl, A. G. Hofmann, R. W. De Doncker, and R. M. Kennel, "Funnel control for speed & position control of electrical drives: A survey," in *Proceedings of the 19th Mediterranean Conference on Control and Automation*, (Corfu, Greece), pp. xxxx–xxxx, June 20-23 2011.

[30] H. Khalil, "Universal integral controllers for minimum–phase nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 45, pp. 490–494, March 2000.

[31] A. Ilchmann and S. Townley, "Adaptive sampling control of high-gain stabilizable systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 10, pp. 1961–1965, 1999.

[32] N. Hopfe, A. Ilchmann, and E. P. Ryan, "Funnel control with saturation: Linear MIMO systems," *IEEE Transactions on Automatic Control*, vol. 55, pp. 532–538, February 2010.

[33] N. Hopfe, A. Ilchmann, and E. P. Ryan, "Funnel control with saturation: Nonlinear SISO systems," *to appear in IEEE Transactions on Automatic Control (Paper submission no. TN-09-583, Paper-Id 09-1092)*, 2010.

[34] C. Canudas de Wit, H. Olsson, K. Åström, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Transactions on Automtic Control*, vol. 40, pp. 419–425, March 1995.

[35] B. Armstrong-Hélouvry, P. Dupont, and C. C. De Wit, "A survey of models, analysis tools and compensation methods for the control of machines with friction," *Automatica*, vol. 30, no. 7, pp. 1083–1138, 1994.

[36] D. Schröder, *Elektrische Antriebe - Regelung von Antriebssystemen (3., bearb. Auflage)*. Berlin: Springer-Verlag, 2009.

[37] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*. Computer Science and Applied Mathematics, London New York Paris: Academic Press, Inc., 2nd., with applications ed., 1985.

[38] G. Ellis and R. D. Lorenz, "Comparison of motion control loops for industrial applications," in *Conference Record of the IEEE Industry Applications Conference and 34th Industrial Application Society Annual Meeting*, vol. 4, pp. 2599–2605, 1999.