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Adaptive Spline Function Based Compensation of Synthetic Jet Actuators for Aircraft Flight Control

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Abstract— **In this paper, an adaptive scheme for controlling the aerodynamic flow using synthetic jet actuators over a commercial aircraft at a wide range of angles of attack is developed. Approximation of a nonlinearly parametrized model of the synthetic jet actuator characteristic by a linearly parametrized function is accomplished using a spline function approximator. An adaptive inverse is used to cancel the effect of the actuator nonlinearities and is implemented by another spline function approximator. An adaptive control design is applied to an aircraft model with synthetic jet actuators, which employs an adaptive actuator nonlinearity compensation scheme combined with a state feedback control law. Parameter projection based adaptive laws are employed to ensure desired closed-loop stability and tracking properties.**

Keywords: Actuator nonlinearity, adaptive inverse, spline function approximation, stability and tracking, synthetic jets.

I. Introduction

Future aircraft designs, such as micro air vehicles and stealth aircraft, may have wing shapes that are susceptible to flow separation and early stall. In these cases, active flow control can be applied to attempt to overcome these design deficiencies and result in acceptable aerodynamic performance. In the literature [1] and [11], synthetic jets have been proven to be an efficient method in controlling separated flow, resulting in an increase in lift and a delay in stall. Some of the main advantages of synthetic jet actuators are their zero-net-mass nature, low cost, compact structure, ease of operation and the fact that they produce a jet without the need for an external fluid source. Typically, synthetic jet actuators are made up of a piezoelectrically driven membrane that produces a synthetic jet flow.

As presented in [2] and [5], the control effect of a synthetic jet actuator is a nonlinear function of both the applied input $v(t)$ to drive the piezoelectric diaphragm to generate the air flow and the aircraft's angle of attack α. This feature is important to consider when designing an adaptive compensation scheme to cancel the effect of actuator nonlinearities inherent in the control of an aircraft system with synthetic jet actuators.

Recent developments in [4] and [5], present a nonlinear profile for the synthetic jet actuator modeling for a wide range of angles of attack and the adaptive compensation control of such signal-dependent actuator nonlinearities. In this paper, we will extend these results to apply such modeling and adaptive control techniques to a commercial transport aircraft model with nonlinear synthetic jet actuator characteristics. The dynamics include the aircraft's angle of attack for the study of adaptive approximation based compensation of the signal-dependent nonlinearities. An important issue of an approximation based adaptive compensation control scheme [10] is determining a suitable parameterization for the synthetic jet actuator nonlinearity and its inverse. Given that the actuator nonlinearity and its inverse are highly nonlinear, they are best approximately parametrized.

In this paper, we address this issue by approximating the synthetic jet actuator nonlinearity and its inverse with multivariable spline functions. Spline functions are a good fit for adaptive control applications because they are linear in their parameters, which are the spline coefficients [9]. A further advantage will become apparent in the development that follows: the synthetic jet actuator nonlinearity model is derived from data points and therefore interpolation of spline functions is suited to creating an accurate model.

The contributions of this paper are two-fold: spline function development for adaptive inverse compensation of synthetic jet actuator nonlinearities, and performance analysis for such an adaptive approximation scheme. This paper is organized as follows. In Section II, we describe the basic physical structure and mathematical representation of the synthetic jet actuator and we present our problem statement. In Section III, we propose and analyze our spline function approximation based adaptive inverse compensation model. We also present a modified inverse and the control error caused by parametric uncertainty. In Section IV, we design and analyze a state feedback parameter projection based adaptive control law. In Section V, we present the actuator nonlinear characteristic and flight dynamics for the aircraft model. In addition, we present a simulation for a feedback control law and the approximation structures.

II. Synthetic Jets for Aircraft

A single synthetic jet consists of an actuator cavity, an oscillating membrane and an orifice. A piezoelectric actuator driven at its resonant frequency functions as the oscillating membrane, and when it oscillates, fluid is alternately expelled and ingested through the orifice. A jet is synthesized by a train of vortices formed at the edge of the orifice.

A. Mathematical Model

Consider the actuator nonlinearity denoted by $N(\cdot)$:

$$u(t) = N(v(t)) = N(A_{pp}^2), \qquad (1)$$

where $v(t) = A_{pp}^2$, with A_{pp} being the input peak-to-peak amplitude voltage applied to the synthetic jet actuator's piezoelectric diaphragm which generates the air flow, and $u(t)$ is the equivalent virtual deflection on the airfoil. It has been observed through wind tunnel testing [5] that the synthetic jet actuator nonlinearity characteristic $N(\cdot)$ changes significantly with the varying values of the aircraft's angle of attack, denoted by α.

As shown in [3], at low angles of attack ($\alpha < 10$), a parametric model with parameters $\theta_l^* = [\theta_{1l}^*, \theta_{2l}^*]^T \in \Re^2$ for the actuator nonlinearity characteristic is

$$u(t) = N(v(t); \theta_l^*) = f_l(v(t)) = \theta_{2l}^* - \frac{\theta_{1l}^*}{v(t)}, \qquad (2)$$

where $v(t)$ is such that $u(t) \geq 0$

At high angles of attack ($22 < \alpha < 24$), the synthetic jet characteristic changes and is now nonlinearly parameterized and is represented by

$$u(t) = N(v(t); \theta_h^*) = f_h(v(t)) = \theta_{2h}^* + \theta_{1h}^* \sin^2(\theta_{3h}^* v), \quad (3)$$

for some parameters θ_{1h}^*, θ_{2h}^* and θ_{3h}^* [5].

An adaptive inverse compensation scheme can be easily obtained and used to cancel the effect of the unknown actuator nonlinearity at low angles of attack, that is,

$$v(t) = \widehat{NI}(u_d(t); \theta_l(t);) = \frac{\theta_{1l}(t)}{\theta_{2l}(t) - u_d(t)} \qquad (4)$$

where $\theta_l(t) = [\theta_{1l}(t), \theta_{2l}(t)]^T$, being the adaptive estimate of θ_l^*, and $u_d(t)$ being the desired feedback control to be designed based on the aircraft flight dynamics. An adaptive inverse compensation scheme for the actuator nonlinearity at high angles of attack was given in [5].

Combining the two models, we can see that a synthetic jet characteristic for a wide range of angles of attack is highly nonlinear in nature. Such a nonlinearity may be complicated to describe by an analytical function and it is denoted as $f(v, \alpha)$ whose characteristic depends on α.

B. Problem Statement

Our objective is to design a spline function approximation based adaptive inverse feedback control scheme for an aircraft flight control system using synthetic jet actuators. The developed scheme must cancel the nonlinearity $N(\cdot)$, in order to meet the control objective.

As a preliminary study, we consider a linear time-invariant plant to represent a linear aircraft model

$$\dot{x}(t) = Ax(t) + Bu(t), \ y(t) = Cx(t) \qquad (5)$$

where (A, B) is controllable, the angle of attack α is one of the state variables, and $A \in \Re^{n \times n}$, $B \in \Re^{n \times 1}$ and $C \in \Re^{1 \times n}$ are constant matrices. Recall that the control input $u(t)$ is implemented with the synthetic jet actuators that have a nonlinearity profile given by,
$u(t) = N(v(t)) = f(v, \alpha)$, where α is the angle of attack of the aircraft, and $v(t)$ is the applied input to the synthetic jet actuators.

To complete our framework for the implementation of spline function approximation applied to adaptive inverse compensation of synthetic jet actuator nonlinearities, in this paper we will construct two spline function approximators: one for the approximation of the synthetic jet actuator nonlinearity profile for a wide range of angles of attack and the other for the adaptive inverse to compensate for the synthetic jet actuator nonlinearity. Such an adaptive inverse compensator [10] is to adaptively and approximately cancel the effect of the uncertain actuator nonlinearity $N(\cdot)$ so that a feedback control law can be applied for aircraft control . The inverse scheme has the form $v(t) = \widehat{NI}(\theta; u_d, \alpha)$ which is also a nonlinear function of the angle of attack α , and a desired feedback control signal u_d, designed based on the aircraft flight dynamics. In other words, u_d is designed as if $u = u_d$ is true, so it is critical to determine and observe the control error $u - u_d$.

III. Spline Function Based Compensation Scheme

As evident from the variation of the nonlinearity models for varying angles of attack, synthetic jet actuators are dependent on the input signal v and the angle of attack α. This dependence motivates us to approximate the nonlinearity and its inverse to be used in the adaptive inverse control scheme. Although there are a number of different approximation methods (e.g., neural networks, fuzzy logic) that can be applied, we have chosen to perform the approximation with spline functions as a preliminary study and future research will combine the different approximation methods into a toolbox to help determine which would be best suited for practical applications.

Basis splines (B-splines) are important in that they can represent any spline function of the same degree by a linear combination. In particular, consider the Cardinal B-Spline for intervals of the form $[..., -2, -1, 0, 1, 2, ...]$. The most widely used in applications are the cubic splines due to their minimum curvature property [**?**]. They can be defined as symmetrical, bell shaped functions constructed from the functions $g_k : \Re^1 \rightarrow \Re^1$, defined recursively [6], by

$$\begin{aligned} g_k(x) &= \int_0^1 g_{k-1}(x - \lambda) d\lambda, \\ &= \int_{-\infty}^{\infty} g_{k-1}(x - \lambda) g_1(\lambda) d\lambda, \end{aligned} \qquad (6)$$

for $k > 1$. This defines the Cardinal B-spline of order k (degree $k - 1$) for the knot at 0, where

$$g_1(x) = \begin{cases} 1 & 0 \leq x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

The B-spline basis elements of order k for the knot at $x = j$ is $g_{kj}(x) = g_k(x - j)$, which is simply just a translation of the original B-spline $g_k(x)$. The basis element has support for $x \in (j, k+j)$, to form a partition of unity on $x \in (0,1)$ we need the B-splines g_{kj} for $j = 1-k, \ldots, 0$.

The spline function $s_k(x) = \sum_{j=1-k}^{n-1} \theta_j g_k(x-j)$ is a spline of order k, that has $(n+k-1)$ knots at the points $x = 1-k, 1, 2, \ldots, n-1$, with the coefficients θ_j. For $n > k$, the set of basis elements $\{g_k(x-j)\}_{j=1-k}^{n-1}$ form a partition of unity on $[0, n]$. Instead, if we write the basis elements as

$$\phi_j(x) = g_k\left(n\frac{x-a}{b-a} - j\right), \tag{7}$$

for $j = 1-k, \ldots, n-1$, then the new basis set is $\{\phi_j\}_{j=1-k}^{n-1}$, that is formed by translating and dilating the k-th order Cardinal B-spline, we form a partition of unity on $[a, b]$ [6]. Defining the approximator as

$$\hat{f}(x, \theta) = \theta^T \phi(x) = \sum_{j=1-k}^{n-1} \theta_j \phi_j(x), \tag{8}$$

with $\phi_j(x)$ as defined in (8), we are able to adjust the parameters of the approximator.

By representing the actuator nonlinearity profile and its inverse with spline functions, we enable the design of an adaptive inverse compensation technique and improve the accuracy in approximating the unknown nonlinearity functions required to meet the objectives.

A. Synthetic Jet Actuator Nonlinearity

Recall the signal-dependent synthetic jet actuator model

$$u = N(v; \theta^*, \alpha) \tag{9}$$

where u is the plant input generated by the synthetic jet actuators, v is the actuator input, α is the angle of attack, and θ^* is an unknown parameter vector. It is important to note that the actuator output u is not available for measurement, for a realistic study. In this case, an approximation is a good choice.

To overcome these challenges, we develop a linearly parametrized spline function approximation for $f(v, \alpha)$. That is, the nonlinear model can be expressed by multivariable B-splines defined as

$$u = N(v; \alpha) \triangleq f(v, \alpha) = \theta_N^{*T} B_N(v, \alpha) + \eta_N(v, \alpha), \tag{10}$$

where the expression is,

$$\theta_N^{*T} B_N(v, \alpha) = \sum_{j=1}^{M_1}\sum_{k=1}^{M_2} \theta_{jk}^* b_j(v) b_k(\alpha), \tag{11}$$

η_N represents the spline function approximation error and is bounded by a positive constant $\|\eta_N\| \leq \eta_{NN}$, θ_{jk}^* is one of $(M_1 \cdot M_2)$ unknown B-spline coefficients, and $b_j(v)$, $b_k(\alpha)$ are the univariable B-spline basis elements, and the approximation of the synthetic jet actuator nonlinearity function as

$$\hat{u} = \hat{N}(v; \alpha) = \hat{\theta}_N^T B_N(v, \alpha). \tag{12}$$

B. Adaptive Inverse Design

To ensure that $u = N(NI(u_d, \alpha); \alpha) = u_d$ the input to the aircraft dynamics, we employ another spline function approximator for the modified inverse characteristic

$$u_{dS} = NI_o(u_d, \alpha) \triangleq f^{-1}(u_d, \alpha) - u_d, \tag{13}$$

$f^{-1}(u_d, \alpha)$ is the inverse function for the synthetic jet actuator nonlinearity, u_d, as mentioned before, is the desired input signal, and u_{dS} is the intermediate nonlinearity inverse function given by

$$u_{dS} = NI_o(u_d; \alpha) \triangleq \theta_s^{*T} B_s(u_d, \alpha) + \eta_s(u_d, \alpha), \tag{14}$$

where

$$\theta_s^{*T} B_s(u_d, \alpha) = \sum_{j=1}^{R_1}\sum_{k=1}^{R_2} \theta_{jk}^* b_j(u_d) b_k(\alpha), \tag{15}$$

η_s represents the spline function approximation error and is bounded by a positive constant $\|\eta_s\| \leq \eta_{NS}$, θ_{jk}^* is one of $(R_1 \cdot R_2)$ unknown B-spline coefficients, $b_j(u_d)$, $b_k(\alpha)$ are the univariable B-spline basis elements, and the approximation of the modified synthetic jet actuator nonlinearity inverse function as

$$\hat{u}_{dS} = \widehat{NI}_o(u_d; \alpha) = \hat{\theta}_s^T B_s(u_d, \alpha). \tag{16}$$

Using (14) and the estimation error $\tilde{u}_{dS} = u_{dS} - \hat{u}_{dS}$, we obtain

$$u_d = f(u_d + \hat{u}_{dS} + \tilde{u}_{dS}, \alpha). \tag{17}$$

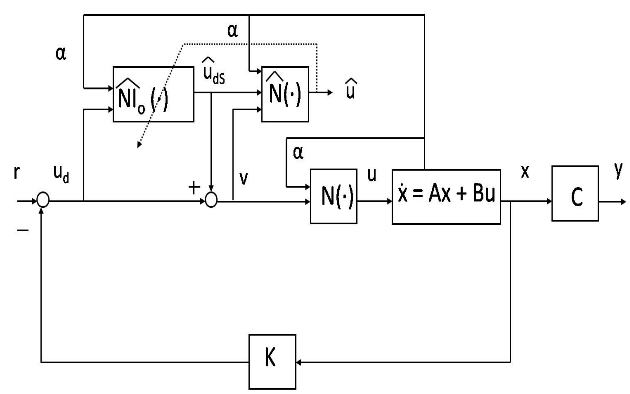

Fig. 1. Approximation based adaptive inverse compensation control.

With the applied control input defined as (see Figure 1)

$$v = u_d + \hat{u}_{dS}, \tag{18}$$

we take the Taylor series expansion about $u_{dS} = \hat{u}_{dS}$, to obtain the desired signal u_d as

$$u_d = f(v, \alpha) + \frac{\partial f(v, \alpha)}{\partial v}\tilde{u}_{dS} + O\Big(\frac{\partial f^i(v, \alpha)}{\partial v^i}, \tilde{u}_{dS}\Big), \tag{19}$$

where $i = 2, 3, \ldots$ and $O(\cdot)$ represents the remainder of the first Taylor polynomial. Figure 1 describes an adaptive state feedback inverse control system that contains two spline function approximations. The first approximation structure is used as an estimator for the synthetic jet actuator nonlinearity, while the second structure is a compensator. The state feedback control scheme and the aircraft flight dynamics described by $\dot{x} = Ax + Bu$ will be described in Section V. The actuator nonlinearity model $N(\cdot)$ and the spline function approximators $\hat{N}(\cdot)$ and $\widehat{NI}_o(\cdot)$ shown in Fig. 1, are defined in (11), (13), and (17) respectively. The above spline function based adaptive inverse design for compensation of synthetic jet actuator nonlinearities employs two approximators: one for the actuator nonlinearity and one for the modified nonlinearity inverse.

C. Control Error

Next, we derive an expression for the control error $u - u_d$ to describe the effectiveness of the nonlinearity functional approximations. This expression is critical in developing adaptive update laws for the parameter estimates.

Lemma 1 : *With the functional compensator described by (17), (19) and the estimate represented by (13), the control error for the spline function based synthetic jet actuator nonlinearity compensation is given by*

$$u - u_d = \tilde{\theta}_N^T \frac{\partial B_N(v,\alpha)}{\partial v} \hat{\theta}_s^T B_s(u_d,\alpha) - \hat{\theta}_N^T \frac{\partial B_N(v,\alpha)}{\partial v} \tilde{\theta}_s^T B_s(u_d,\alpha) + \eta, \quad (20)$$

where η is the model mismatch error given by

$$\eta = -\tilde{\theta}_N^T \frac{\partial B_N(v,\alpha)}{\partial v} \theta_s^{*T} B_s(u_d,\alpha) - \frac{\partial \eta_N(v,\alpha)}{\partial v} \tilde{\theta}_s^T B_s(u_d,\alpha) - O\left(\frac{\partial f^i(v,\alpha)}{\partial v^i}, \tilde{u}_{dS}\right) - \theta_N^{*T} \frac{\partial B_N(v,\alpha)}{\partial v} \eta_s(u_d,\alpha). \quad (21)$$

This expresses the control error $u - u_d$ in terms of the parameter approximation errors $\tilde{\theta}_N$ and $\tilde{\theta}_s$. Note that the form of (21) is crucial in controller design and in deriving adaptive laws that guarantee closed-loop stability. The following result gives the upper bound of the norm $\eta(t)$, where $\|.\|$ as any suitable vector norm.

Lemma 2 : *The norm of the modelling mismatch term $\eta(t)$ in (21) is bounded by*

$$\|\eta(t)\| \leq \beta^T \Omega, \quad (22)$$

where $\beta = [\delta_1, \delta_2, \delta_3, \delta_4,]^T$ is an unknown constant vector, being composed of bounded constants and the known vector function is $\Omega = \left[1, \left\|\hat{\theta}_N\right\|, \left\|\hat{\theta}_s\right\|, \left\|\hat{\theta}_s\right\|^2\right]^T$.

Proof: From (25) and the fact that there should exist positive constants θ_M and θ_S satisfying $\|\theta_N^*\| \leq \theta_M$ and $\|\theta_s^*\| \leq \theta_S$, where θ_M and θ_S are not needed to be known. Based on the facts

$$\left\|\tilde{\theta}_N\right\| \leq \theta_M + \left\|\hat{\theta}_N\right\|, \left\|\tilde{\theta}_s\right\| \leq \theta_S + \left\|\hat{\theta}_s\right\|,$$

we have

$$\|\eta(t)\| \leq \theta_s\left(\theta_M + \left\|\hat{\theta}_N\right\|\right)\left\|\frac{\partial B_N(v,\alpha)}{\partial v}\right\| \|B_s(u_d,\alpha)\| + \theta_M \left\|\frac{\partial B_N(v,\alpha)}{\partial v}\right\| \eta_S + \|O(\cdot)\| + \left\|\frac{\partial \eta_N(v,\alpha)}{\partial v}\right\| \left(\theta_S + \left\|\hat{\theta}_s\right\|\right) \|B_s(u_d,\alpha)\| . (23)$$

With some algebraic simplifications, this becomes

$$\|\eta(t)\| \leq \delta_1 + \delta_2 \left\|\hat{\theta}_N\right\| + \delta_3 \left\|\hat{\theta}_s\right\| + \delta_4 \left\|\hat{\theta}_s\right\|^2 = \beta^T \Omega. \quad (24)$$

The vector β is concluded to be bounded because $\|B_s(u_d,\alpha)\|$ and $\left\|\frac{\partial B_N(v,\alpha)}{\partial v}\right\|$ are bounded for a bounded α. Note that the control error (21) reflects the mutual dependence of the two parametrized nonlinearity functions.

To proceed, we define an estimator for the bound of the model mismatch error $\beta^T \Omega$, as

$$\hat{\eta} = \hat{\beta}^T \Omega, \quad (25)$$

$\hat{\beta} = \left[\hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3, \hat{\delta}_4,\right]^T$, will be updated from a parameter projection based adaptive law. A similar development is given [13] with a neural network framework . ∇

As the estimates $\hat{\theta}_N^T$ and $\hat{\theta}_s^T$ approach the actual parameters θ_N^{*T} and θ_s^{*T}, the spline function approximator effectively provides an inverse for the synthetic jet actuator nonlinearity. Observing the form of the control error expression (21) is critical for adaptive controller design so as to guarantee closed-loop stability. Through the first-order Taylor expansion, the control error $u - u_d$ has been expressed conveniently in a linearly parameterizable form with respect to the parameter errors $\tilde{\theta}_N^T$ and $\tilde{\theta}_s^T$. This allows us to adaptively update the estimates $\hat{\theta}_N^T$ and $\hat{\theta}_s^T$. Moreover, the mismatch error term $\eta(t)$ is bounded by a constant vector multiplied by a known function vector. Indeed, adaptive control techniques can be employed to handle this residual term.

IV. Adaptive Feedback Control System

We present a state feedback adaptive inverse control scheme to cancel the nonlinearity $N(\theta^*; v, \alpha)$, in order to ensure the control objectives are met. Such a control system is that shown in Figure 1 (note that the output y is not used in this study, but the control scheme to be developed can be made for output tracking).

A. State Feedback Control

Consider the linear time-invariant aircraft model

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t), \quad (26)$$

where the control input $u(t)$ is implemented with the synthetic jet actuators that have a nonlinearity profile given by, $u(t) = N(v(t)) = f(v,\alpha)$, where α is the angle of attack of the aircraft and is one of the state variables, and $v(t)$ is the applied input to the synthetic jet actuators.

The desired state feedback control signal is

$$u_d(t) = -Kx(t) + r(t) + v_\eta(t), \quad (27)$$

where $r(t)$ is a bounded reference input signal and $K \in \Re^{1\times n}$ is a constant gain vector such that the eigenvalues of $A - BK$ are set to the desired closed-loop system poles. The $v_\eta(t)$ term, is a commonly applied term to compensate for the approximation error $\eta(t)$ uncertainty and is defined as

$$v_\eta(t) = -\frac{e^T PB}{\rho + |e^T \beta B|}, \quad 0 < \rho << 1.$$

The choice of K can be made from a linear-quadratic regulator (LQR) design [7] or a pole placement technique[8].

Applying (20), (26) and (27), we obtain

$$\begin{aligned}\dot{x}(t)=&(A-BK)x(t)+Br(t)+Bv_\eta(t)+B\eta(t)\\&+B\tilde{\theta}_N^T(t)\frac{\partial B_N(v(t),\alpha)}{\partial v(t)}\hat{\theta}_s^T(t)B_s(u_d(t),\alpha)\\&-B\hat{\theta}_N^T(t)\frac{\partial B_N(v(t),\alpha)}{\partial v(t)}\tilde{\theta}_s^T(t)B_s(u_d(t),\alpha).\end{aligned} \quad (28)$$

This representation motivates us to choose the reference model system as

$$\dot{x}_m(t)=(A-BK)x_m(t)+Br(t). \quad (29)$$

The control objective is to choose a feedback gain K and adaptive laws for the parameters $\hat{\theta}_N^T$ and $\hat{\theta}_s^T$, such that all closed-loop system signals are bounded, and the tracking error $x(t)-x_m(t)$ is as small as possible (due to the uncertainty of $\eta(t)$ and the related approximation, $\lim_{t\to\infty}(x(t)-x_m(t))=0$ may not be theoretically achievable).

B. Adaptive Laws

In this section, we formulate adaptive laws to update the estimates $\hat{\theta}_N^T$, $\hat{\theta}_s^T$ and $\hat{\beta}(t)$ so that the control objectives are achievable. Parameter projection is applied to ensure that the parameters remain bounded. Applying the parameter projection algorithm in [12], we determine our adaptive laws as

$$\dot{\hat{\theta}}_N(t)=g_N(t)+h_N(t), \quad (30)$$

$$\dot{\hat{\theta}}_s(t)=g_s(t)+h_s(t), \quad (31)$$

$$\dot{\hat{\beta}}(t)=\Gamma_3 PB\Omega e(t)+h_\beta(t), \quad (32)$$

where $g_N(t)$ and $g_s(t)$ are adaptation functions given by

$$g_N(t)=-\Gamma_1 e^T(t)PB\frac{\partial B_N(v,\alpha)}{\partial v}\hat{\theta}_s^T B_s(u_d,\alpha),$$

$$g_s(t)=\Gamma_2 e^T(t)PB\frac{\partial B_N(v,\alpha)}{\partial v}\hat{\theta}_N^T B_s(u_d,\alpha),$$

and Γ_i, $i=1,2,3$ are the adaptation gain matrices that satisfy $\Gamma_i=\Gamma_i^T>0$ and tracking errors

$$e(t)=x(t)-x_m(t). \quad (33)$$

$P\in\Re^{5\times5}$, $P=P^T>0$ is determined by the solution to the Lyapunov equation for continuous time systems

$$PA_m+A_m^TP=-Q, \quad (34)$$

for a constant matrix $Q\in\Re^{n\times n}$, $Q=Q^T>0$ (recall $A_m=A-BK$). The functions $h_N(t), h_s(t)$ and $h_\beta(t)$ are parameter projection functions defined such that the parameter estimates stay in a convex region for certain desired physical properties, and are represented as

$$h_{ij}(t)=\begin{cases}0 & \text{if } \hat{\theta}_{ij}>\theta_{ij}^b,\text{or}\\ & \text{if } \hat{\theta}_{ij}=\theta_{ij}^b, g_{ij}(t)\geq 0,\\ -g_{ij} & \text{otherwise,}\end{cases} \quad (35)$$

where $i=1,\ldots,(M_1\cdot M_2)$ for $j=N$ and $i=1,\ldots,(R_1\cdot R_2)$ for $j=s$ and

$$h_{\beta k}(t)=\begin{cases}0 & \text{if } \hat{\delta}_k\in(\delta_k^b,\delta_k^d),\text{or}\\ & \text{if } \hat{\delta}_k=\delta_k^b, \Gamma_3\Omega e(t)\leq 0,\text{or}\\ & \text{if } \hat{\delta}_k=\delta_k^d, \Gamma_3\Omega e(t)\geq 0,\\ -\Gamma_3\Omega e(t) & \text{otherwise,}\end{cases} \quad (36)$$

where $k=1,\ldots,4$ and $h_\beta=[h_{\beta1},\ldots,h_{\beta4}]^T$. Note the coupled nature of the adaptive laws $\hat{\theta}_N(t)$ and $\hat{\theta}_s(t)$ clearly showing the mutual dependence of the two nonlinearity spline function approximators.

This adaptive control scheme ensures desired closed-loop stability and tracking properties which, like those with other approximation based designs, are in a local and average sense due to approximation errors, that is, for approximation errors with some non-zero bounds and initial conditions within some regions, all closed-loop system signals remain bounded and the tracking error $e(t)$ is bounded by the approximation errors in a mean square sense.

V. Aircraft Flight Control Performance Evaluation

In this section, we demonstrate our adaptive inverse compensation control design applied to a commercial transport linear aircraft flight dynamic model with actuator nonlinearities, to evaluate the adaptive control system performance. We present the details of the aircraft dynamic model, actuator nonlinearities, nonlinearity inverse design, feedback control law, and simulation steps.

A. Actuator Nonlinearity Characteristic

From what we have analyzed in Section II.A, the synthetic jet actuator characteristic is a general nonlinear function of v and α. Therefore, we need to build a profile for this actuator nonlinear feedback control. One possible realistic choice of the nonlinear profile $f(v,\alpha)$, based on interpolation of the low and high angle of attack models, is

$$f(v,\alpha)=a(\alpha)f_l(v)+b(\alpha)f_h(v). \quad (37)$$

This candidate nonlinearity profile depends on the applied input to the synthetic jets v and the angle of attack α, where $f_l(v)$ is the actuator nonlinearity function at a specific low angle ($\alpha_l=3^o$) of attack, $f_h(v)$ is the nonlinearity function at a high angle ($\alpha_h=24^o$) of attack, and, $a(\alpha)$ and $b(\alpha)$ are functions that determine the dependency of the actuator nonlinearity on α. In order for this function to be meaningful it must satisfy

$$a(\alpha_l)=1, a(\alpha_h)=0, b(\alpha_l)=0, b(\alpha_h)=1. \quad (38)$$

That is the nonlinear profile is equal to the actuator nonlinearity functions at the two different angles of attack mentioned above that were determined by experimental data points taken from wind tunnel tests. There are many functions that can be used to represent $a(\alpha)$ and $b(\alpha)$. Given that we have data points for two different angles of attack, the simplest of these are linear functions. With the constraint (43), we find

$$a(\alpha)=-0.0479\alpha+1.1429,\ b(\alpha)=0.0479\alpha-0.1429. \quad (39)$$

As determined in [4], the actuator nonlinearity functions at $\alpha_l = 3^o$ and $\alpha_h = 24^o$ are given by $f_l(v) = 15 - \frac{33.335}{v}$ and $f_h(v) = 20 + 5\sin^2(\pi\frac{v}{32})$, respectively. As our further wind tunnel tests are performed (as a part of our research project). Therefore, this model will be employed in our study for the approximation based adaptive inverse compensation control designs, as the actual model $f(v, \alpha)$ is not currently available.

B. Linear Aircraft Dynamic Model

In this study, we will use a linearized model of a commercial transport aircraft that is,

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (40)$$

where the state vector is $x = [x_1, x_2, x_3, x_4, x_5]^T$ whose components are the angle of attack α, roll rate p, side-slip angle β, pitch rate q, and yaw rate r, the control input vector is $u = [u_1, u_2, u_3]^T$ whose components are the elevator angle δ_e, aileron angle δ_a and rudder angle δ_r, respectively. The elevator angle δ_e is to be equivalently implemented through synthetic jet actuators, and $A \in R^{5\times5}$ and $B \in R^{5\times3}$ which are given by

$$A = \begin{bmatrix} -0.5656 & 0.9730 & 0 & 0 & 0 \\ -0.8985 & -0.4755 & 0 & 0 & 0 \\ 0 & 0 & -0.1178 & 0.0501 & -0.9881 \\ 0 & 0 & -1.4828 & -1.0674 & 0.6121 \\ 0 & 0 & 0.5364 & -0.0644 & -0.3057 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.0009 & 0 & 0 \\ -0.0161 & 0 & 0 \\ 0 & 0 & 0.0007 \\ 0 & 0.0136 & 0.0063 \\ 0 & 0.0003 & -0.0079 \end{bmatrix}.$$

C. Linear Feedback Law

Recall the desired linear feedback control law for the case when $u(t) = u_d(t)$ (that is, when there is no actuator nonlinearity $N(\cdot)$ and there is no inverse $\widehat{NI}(\cdot)$) is

$$u_d(t) = -Kx(t) + r(t) \quad (41)$$

where $r(t)$ is a reference input signal, and $K \in R^{3\times5}$ is a feedback gain vector to be determined. Using the LQR design, with simple $Q = I_5$ and $R = I_3$, we determine the optimal gain matrix K as

$$K = \begin{bmatrix} -0.0005 & -0.0163 & 0 & 0 & 0 \\ 0 & 0 & -0.0046 & 0.0061 & 0.0033 \\ 0 & 0 & 0.0090 & 0.0016 & -0.0434 \end{bmatrix}.$$

The reference model matrix $A_m = A - BK$ has eigenvalues: $-0.5207 + 0.9339i$, $-0.5207 - 0.9339i$, $-0.2179 + 0.7904i$, $-0.2179 - 0.7904i$ and -1.0555.

D. Simulation

To simulate the design, we consider an approximation structure for the design and performance evaluation of the adaptive controller. We chose to use cubic Cardinal B-splines for the approximation structure. Next, we need to generate the signal $u_d(t) = -Kx(t) + r(t)$, for the reference signal

$$r(t) = \begin{cases} 2.3sin(t), & 0 \le t \le 70, \\ 2.3sin(t) + 3.7sin(3t) & t \ge 70. \end{cases}$$

The initial states are: $x_0 = [5 \;\; 0.01 \;\; 0.01 \;\; 0.01 \;\; 0.001]^T$. The parameter estimates are initialized as $\hat{\theta}^N_{j,k} = 6.1$, $\hat{\theta}^s_{j,k} = -0.04$, these values are based on offline learning of the nonlinear profile to obtain a good estimate of the nominal parameters $f(v_0, \alpha_0) = 7.35$ and $NI_o(u_{d0}; \alpha_0) = -0.0027$.

VI. Concluding Remarks

In this paper, we have proposed to use multivariable B-splines to construct two approximation structures for an adaptive inverse compensation controller for aircraft flight control. The first approximator estimates the unknown synthetic jet actuator nonlinearity, while the other estimates the inverse function for compensation. The use of spline functions for implementing these functions leads to a linear parametrization of the control error. A state feedback control law, combined with the adaptive inverse compensation scheme, has been designed and analyzed for desired stability and tracking performance, and applied to an aircraft control example. Simulation is currently being carried out and its results will be reported.

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