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Guidance Laws for Planar Motion Control

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Abstract— **This paper gives an overview of guidance laws that can be applied for planar motion control purposes. Considered scenarios include target tracking, where only instantaneous information about the target motion is available, as well as path scenarios, where spatial information is available apriori. For target-tracking purposes, classical guidance laws from the missile literature are reviewed. These laws encompass guidance principles such as line of sight, pure pursuit, and constant bearing. For the path scenarios, enclosure-based and lookahead-based guidance laws are presented. Considered paths include straight lines (zero curvature), circles (constant curvature), as well as general, regularly parameterized paths (variable curvature). Also, relations between the guidance laws are discussed, as well as interpretations toward saturated control.**

I. Introduction

Motion control is a fundamental enabling technology for any vehicle application, and all motion control systems require a guidance component. According to [1], guidance is defined as: *The process for guiding the path of an object towards a given point, which in general may be moving*. Furthermore, the father of inertial navigation, Charles Stark Draper, states in [2] that: *Guidance depends upon fundamental principles and involves devices that are similar for vehicles moving on land, on water, under water, in air, beyond the atmosphere within the gravitational field of earth and in space outside this field*. Thus, guidance is a basic methodology which is concerned with the transient motion behavior associated with achieving motion control objectives.

The most rich and mature literature on guidance is probably found within the guided missile community. In one of the earliest texts on the subject [3], a guided missile is defined as: *A space-traversing unmanned vehicle which carries within itself the means for controlling its flight path*. Today, most people probably think about unmanned aerial vehicles (UAVs) when hearing this definition. However, guided missiles have been operational since World War II [4], and thus organized research on guidance theory has been conducted almost as long as organized research on control theory. The continuous progress in missile hardware and software technology has made increasingly advanced guidance concepts feasible for implementation. Today, missile guidance theory encompass a broad spectrum of guidance laws, namely: classical guidance laws; optimal guidance laws; guidance laws based on fuzzy logic and neural network theory; differential-geometric guidance laws; and guidance laws based on differential game theory.

As already mentioned, a classical text on missile guidance concepts is [3], while more recent work include [5], [1], [6], and [7]. Relevant survey papers include [8], [9], [10], and [11]. Also, very interesting personal accounts of the guided missile development during and after World War II can be found in [12], [13], and [14], while [15] and [16] put the development of guided missile technology into a larger perspective.

The fundamental nature and diverse applicability of guidance principles can be further illustrated through a couple of examples. In nature, some predators are able to conceal their pursuit of prey by resorting to so-called motion camouflage techniques [17]. They adjust their movement according to their prey so that the prey perceive them as stationary objects in the environment. These predators take advantage of the fact that some creatures detect the lateral motion component relative to the predator-prey line of sight far better than the longitudinal component. Hence, approaching predators can appear stationary to such prey by minimizing the relative lateral motion, only changing in size when closing in for the kill. Interestingly, this behavior can be directly related to the classical guidance laws from the missile literature [18]. Also, such guidance laws have been successfully applied since the early 1990s to avoid computationally-demanding optimization methods associated with motion planning for robot manipulators operating in dynamic environments [19].

The main contribution of this paper is to give a convenient overview of available guidance laws applicable for planar motion control purposes. The exposition is deliberately kept at a basic level to make it accessible for a wide audience. Details and proofs can be found in the references.

II. Motion Control Fundamentals

This section reviews some basic motion control concepts, including operating spaces, vehicle actuation properties, motion control scenarios, as well as the motion control hierarchy. It concludes with some preliminaries.

A. Operating Spaces

It is useful to distinguish between different types of operating spaces when considering vehicle motion control, especially since such characterizations enable purposeful definitions of various motion control scenarios. The two most fundamental operating spaces to consider are the *work space* and the *configuration space*.

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The work space (also known as the operational space [20]) represents the physical space (environment) in which a vehicle moves. For a car, the work space is 2 dimensional (planar position), while it is 3 dimensional (spatial position) for an aircraft. Consequently, the work space is a position space which is common for all vehicles of the same type.

The configuration space (also known as the joint space [20]) is constituted by the set of variables sufficient to specify all points of a (rigid-body) vehicle in the work space [21]. Thus, the configuration of a car is given by its planar position and orientation, while the configuration of an aircraft is given by its spatial position and attitude.

B. Vehicle Actuation Properties

Each of the variables associated with the configuration of a vehicle is called a degree of freedom (DOF). Hence, a car has 3 DOFs, while an aircraft has 6 DOFs.

The type, amount, and distribution of vehicle thrust devices and control surfaces, hereafter commonly referred to as actuators, determine the actuation property of a vehicle. We mainly distinguish between two qualitatively different actuation properties, namely *full actuation* and *underactuation*. A fully actuated vehicle is able to independently control all its DOFs simultaneously, while an underactuated vehicle is not. Thus, an underactuated vehicle is generally unable to achieve arbitrary tasks in its configuration space. However, it will be able to achieve tasks in the work space as long as it can freely project its main thrust in this space, e.g., through a combination of thrust and attitude control. In fact, this principle is the mode by which most vehicles that move through a fluid operate, from missiles to ships. Even if these vehicles had the ability to roam the work space with an arbitrary attitude, this option would represent the least energy-efficient alternative.

C. Motion Control Scenarios

In the traditional control literature, motion control scenarios are typically divided into the following categories: point stabilization, trajectory tracking, and path following. More recently, the concept of maneuvering has been added to the fold as a means to bridge the gap between trajectory tracking and path following [22]. These scenarios are often defined by motion control objectives that are given as configuration-space tasks, which are best suited for fully actuated vehicles. Also, the scenarios typically involve desired motion that has been defined apriori in some sense. Little seems to be reported about tracking of target points for which only instantaneous motion information is available.

However, in what follows, both apriori and non-apriori scenarios are considered, and all motion control objectives are given as work-space tasks. Thus, the scenarios cover more broadly, and are also suited for underactuated vehicles.

The control objective of a *target-tracking scenario* is to track the motion of a target that is either stationary (analogous to point stabilization) or that moves such that only its instantaneous motion is known, i.e., such that no information about the future target motion is available. Thus, in this case it is impossible to separate the spatio-temporal constraint associated with the target into two separate constraints.

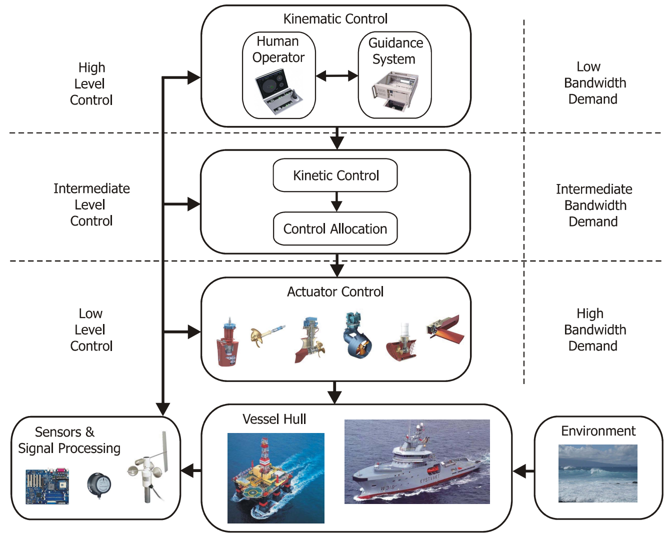

Fig. 1. The motion control hierarchy of a marine surface vessel.

In contrast, the control objective of a *path-following scenario* is to follow a predefined path, which only involves a spatial constraint. No restrictions are placed on the temporal propagation along the path.

However, the control objective of a *path-tracking scenario* is to track a target that moves along a predefined path (analogous to trajectory tracking), which means that it is possible to separate the related spatio-temporal constraint into two separate constraints. Often, the spatial constraint is considered more important than the temporal constraint, such that if both cannot be satisfied simultaneously, the spatial constraint takes precedence (i.e., to move along the path, albeit at a distance behind the target).

Finally, the control objective of a *path-maneuvering scenario* is to employ knowledge about vehicle maneuverability to feasibly negotiate (or somehow optimize the negotiation of) a predefined path. As such, path maneuvering represents a subset of path following, but is less constrained than path tracking since spatial constraints always take precedence over temporal constraints.

D. Motion Control Hierarchy

A vehicle motion control system can be conceptualized to involve at least three levels of control in a hierarchical structure, see Figure 1. This figure illustrates the typical components of a marine motion control system. All the involved building blocks represent autonomy-enabling technology, but more instrumentation and additional control levels are required to attain fully autonomous operation. An example involves collision avoidance functionality, which demands additional sense and avoid components.

This paper is mainly concerned with the highest control level of Figure 1. Termed the kinematic control level, it is responsible for prescribing vehicle velocity commands needed to achieve motion control objectives in the work space. Thus, in this paper, kinematic control is equivalent to work-space control, and *kinematic controllers* are referred to as *guidance laws*. This level purely considers the geometrical aspects of motion, without reference to the forces and moments that generate such motion.

The intermediate level encompass *kinetic controllers*, which do consider how forces and moments generate vehicle motion. These controllers are typically designed by model-based methods, and they must handle both parametric uncertainties as well as suppress environmental disturbances. For underactuated vehicles, they must actively employ the vehicle attitude as a means to adhere to the velocities ordered by the guidance module. The intermediate control level also contains a *control allocation* block which distributes the kinetic control commands among the various vehicle actuators.

Finally, the lowest level is constituted by the individual *actuator controllers*, which ensure that the actuators behave as requested by the intermediate control module.

E. Preliminaries

In what follows, a kinematic vehicle is represented by its planar position $\mathbf{p}(t) \triangleq [x(t), y(t)]^\top \in \mathbb{R}^2$ and velocity $\mathbf{v}(t) \triangleq \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}(t) \triangleq \dot{\mathbf{p}}(t) \in \mathbb{R}^2$, stated relative to some stationary reference frame. Note that even though the consideration is planar, the considered concepts can be extended to 3 dimensions and beyond.

In the missile literature, guidance laws are typically synonymous with steering laws, assuming that the speed is constant. Here, guidance laws are either directly prescribed for the velocity or partitioned into speed and steering laws.

Finally, all guidance-principle illustrations employ the marine convention of a right-handed coordinate system whose z-axis points down, into the plane. Thus, all angles are counted positive in the clockwise direction, as seen from above.

III. Guidance Laws for Target Tracking

In this section, guidance laws for target tracking are presented. The material is adapted from [23].

Denoting the position of the target by $\mathbf{p}_\mathrm{t}(t) \triangleq [x_\mathrm{t}(t), y_\mathrm{t}(t)]^\top \in \mathbb{R}^2$, the control objective of a target-tracking scenario can be stated as

$$\lim_{t\to\infty} (\mathbf{p}(t) - \mathbf{p}_\mathrm{t}(t)) = \mathbf{0}, \tag{1}$$

where $\mathbf{p}_\mathrm{t}(t)$ is either stationary or moving by a (non-zero and bounded) velocity $\mathbf{v}_\mathrm{t}(t) \triangleq \dot{\mathbf{p}}_\mathrm{t}(t) \in \mathbb{R}^2$.

Concerning tracking of moving targets, the missile guidance community has probably the most comprehensive experience. The object that is supposed to destroy another object is commonly referred to as either a missile, an interceptor, or a pursuer. Conversely, the threatened object is typically called a target or an evader. Here, the designations interceptor and target will be employed.

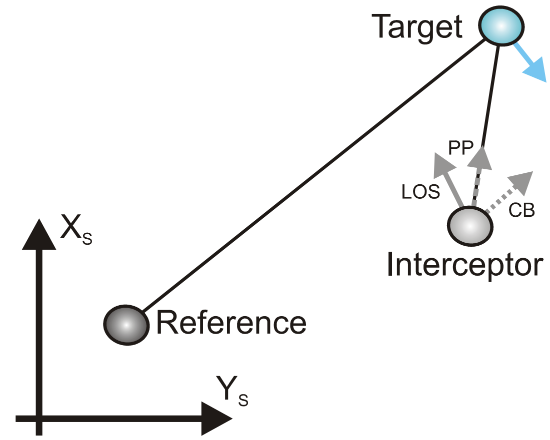

Fig. 2. The interceptor velocity commands that are associated with the classical guidance principles line of sight (LOS), pure pursuit (PP), and constant bearing (CB).

An interceptor typically undergoes 3 phases during its operation; a launch phase, a midcourse phase, and a terminal phase. The greatest accuracy demand is associated with the terminal phase, where the interceptor guidance system must compensate for the accumulated errors from the previous phases to achieve a smallest possible final miss distance to the target. Thus, 3 terminal guidance strategies will be presented in the following, namely line of sight, pure pursuit, and constant bearing. The associated geometric principles are illustrated in Figure 2.

Note that while the main objective of a guided missile is to hit (and destroy) a physical target in finite time, we recognize the analogy of hitting (converging to) a virtual target asymptotically, i.e., the concept of asymptotic interception, as stated by (1).

A. Line of Sight Guidance

Line of sight (LOS) guidance is classified as a so-called three-point guidance scheme since it involves a (typically stationary) reference point in addition to the interceptor and the target. The LOS denotation stems from the fact that the interceptor is supposed to achieve an intercept by constraining its motion along the line of sight between the reference point and the target. LOS guidance has typically been employed for surface-to-air missiles, often mechanized by a ground station which illuminates the target with a beam that the guided missile is supposed to ride, also known as beam-rider guidance. The LOS guidance principle is illustrated in Figure 2, where the associated velocity command is represented by a vector pointing to the left of the target.

B. Pure Pursuit Guidance

Pure pursuit (PP) guidance belongs to the so-called two-point guidance schemes, where only the interceptor and the target are considered in the engagement geometry. Simply put, the interceptor is supposed to align its velocity along the line of sight between the interceptor and the target. This strategy is equivalent to a predator chasing a prey in the animal world, and very often results in a tail chase. PP guidance has typically been employed for air-to-surface missiles. The PP guidance principle is represented in Figure 2 by a vector pointing directly at the target.

Deviated pursuit guidance is a variant of PP guidance where the velocity of the interceptor is supposed to lead the interceptor-target line of sight by a constant angle in the direction of the target movement. An equivalent term is fixed-lead navigation.

C. Constant Bearing Guidance

Constant bearing (CB) guidance is also a two-point guidance scheme, with the same engagement geometry as PP guidance. However, in a CB engagement the interceptor is supposed to align the *relative* interceptor-target velocity along the line of sight between the interceptor and the target. This goal is equivalent to reducing the LOS rotation rate to zero such that the interceptor perceives the target at a constant bearing, closing in on a direct collision course. CB guidance is often referred to as parallel navigation, and has typically been employed for air-to-air missiles. Also, the CB rule has been used for centuries by mariners to avoid collisions at sea; steering away from a situation where another vessel approaches at a constant bearing. Hence, guidance principles can just as well be applied to avoid collisions as to achieve them. The CB guidance principle is indicated in Figure 2 by a vector pointing to the right of the target.

The most common method of implementing CB guidance is to make the rotation rate of the interceptor velocity directly proportional to the rotation rate of the interceptor-target LOS, which is widely known as proportional navigation (PN).

CB guidance can also be implemented through the direct velocity assignment

$$\mathbf{v}(t) = \mathbf{v}_{\mathrm{t}}(t) - \kappa(t)\frac{\tilde{\mathbf{p}}(t)}{|\tilde{\mathbf{p}}(t)|}, \tag{2}$$

where

$$\tilde{\mathbf{p}}(t) \triangleq \mathbf{p}(t) - \mathbf{p}_{\mathrm{t}}(t) \tag{3}$$

is the line of sight vector between the interceptor and the target, $|\tilde{\mathbf{p}}(t)| \triangleq \sqrt{\tilde{\mathbf{p}}(t)^\top \tilde{\mathbf{p}}(t)} \geq 0$ is the Euclidean length of this vector, and where $\kappa(t) > 0$ can be chosen as

$$\kappa(t) = U_{\mathrm{a,max}} \frac{|\tilde{\mathbf{p}}(t)|}{\sqrt{\tilde{\mathbf{p}}(t)^\top \tilde{\mathbf{p}}(t) + \triangle_{\tilde{\mathrm{p}}}^2}}, \tag{4}$$

where $U_{\mathrm{a,max}} > 0$ specifies the maximum approach speed toward the target, and $\triangle_{\tilde{\mathrm{p}}} > 0$ specifies the interceptor-target rendezvous behavior.

Note that CB guidance becomes equal to PP guidance for a stationary target, i.e., the basic difference between the two guidance schemes is whether the target velocity is used as a kinematic feedforward or not.

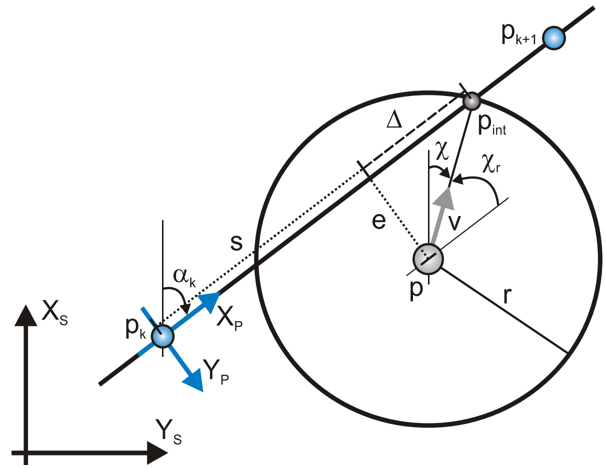

Fig. 3. The main variables associated with steering laws for straight-line paths.

Returning to the example on motion camouflage, it seems that two main strategies are in use; camouflage against an object close by and camouflage against an object at infinity. The first strategy clearly corresponds to LOS guidance, while the second strategy equals CB guidance since it entails a non-rotating predator-prey line of sight.

IV. Guidance Laws for Path Scenarios

In this section, guidance laws for different path scenarios are considered, including path following, path tracking, and path maneuvering. Specifically, the guidance laws are composed of speed and steering laws, which can be combined in different ways to achieve different motion control objectives. The speed is denoted $U(t) \triangleq |\mathbf{v}(t)| = \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2} \geq 0$, while the steering is denoted $\chi(t) \triangleq \mathrm{atan2}\left(\dot{y}(t), \dot{x}(t)\right) \in \mathbb{S} \triangleq [-\pi, \pi]$, where $\mathrm{atan2}\,(y, x)$ is the four-quadrant version of $\arctan\left(\frac{y}{x}\right) \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle$. Path following is ensured by proper assignments to $\chi(t)$ as long as $U(t) > 0$ since the scenario only involves a spatial constraint, while the spatio-temporal path-tracking and path-maneuvering scenarios both require explicit speed laws in addition to the steering laws. The following material is adapted from [24], [25], and [26].

A. Steering Laws for Straight Lines

Consider a straight-line path implicitly defined by two waypoints through which it passes. Denote these waypoints as $\mathbf{p}_{\mathrm{k}} \triangleq [x_{\mathrm{k}}, y_{\mathrm{k}}]^\top \in \mathbb{R}^2$ and $\mathbf{p}_{\mathrm{k+1}} \triangleq [x_{\mathrm{k+1}}, y_{\mathrm{k+1}}]^\top \in \mathbb{R}^2$, respectively. Also, consider a path-fixed reference frame with origin in $\mathbf{p}_{\mathrm{k}}$, whose x-axis has been rotated a positive angle $\alpha_{\mathrm{k}} \triangleq \mathrm{atan2}\left(y_{\mathrm{k+1}} - y_{\mathrm{k}}, x_{\mathrm{k+1}} - x_{\mathrm{k}}\right) \in \mathbb{S}$ relative to the x-axis of the stationary reference frame. Hence, the coordinates of the kinematic vehicle in the path-fixed reference frame can be computed by

$$\boldsymbol{\varepsilon}(t) = \mathbf{R}(\alpha_{\mathrm{k}})^\top (\mathbf{p}(t) - \mathbf{p}_{\mathrm{k}}), \tag{5}$$

where

$$\mathbf{R}(\alpha_k) \triangleq \begin{bmatrix} \cos\alpha_k & -\sin\alpha_k \\ \sin\alpha_k & \cos\alpha_k \end{bmatrix}, \tag{6}$$

and $\boldsymbol{\varepsilon}(t) \triangleq [s(t), e(t)]^\top \in \mathbb{R}^2$ consists of the along-track distance $s(t)$ and the cross-track error $e(t)$, see Figure 3. For path-following purposes, only the cross-track error is relevant since $e(t) = 0$ means that the vehicle has converged to the straight line. Expanding (5), the cross-track error can be explicitly stated by

$$e(t) = -(x(t) - x_k)\sin\alpha_k + (y(t) - y_k)\cos\alpha_k, \tag{7}$$

and the associated control objective for straight-line path-following purposes become

$$\lim_{t\to\infty} e(t) = 0. \tag{8}$$

In the following, two steering laws that ensure stabilization of $e(t)$ to the origin will be presented. The first method is used in ship motion control systems [27], and will be referred to as enclosure-based steering. The second method is called lookahead-based steering, and has links to the classical guidance principles from the missile literature. The two steering methods essentially operate by the same principle, but as will be made clear, the lookahead-based scheme has several advantages over the enclosure-based approach.

1) Enclosure-Based Steering: Imagine a circle with radius $r > 0$ enclosing $\mathbf{p}(t)$. If the circle radius is chosen sufficiently large, the circle will intersect the straight line at two points. The enclosure-based strategy for driving $e(t)$ to zero is then to direct the velocity toward the intersection point that corresponds to the desired direction of travel, which is implicitly defined by the sequence in which the waypoints are ordered. Such a solution involves directly assigning

$$\chi(t) = \text{atan2}(y_{\text{int}}(t) - y(t), x_{\text{int}}(t) - x(t)), \tag{9}$$

where $\mathbf{p}_{\text{int}}(t) \triangleq [x_{\text{int}}(t), y_{\text{int}}(t)]^\top \in \mathbb{R}^2$ represents the intersection point of interest. In order to calculate $\mathbf{p}_{\text{int}}(t)$ (two unknowns), the following two equations must be solved

$$(x_{\text{int}}(t) - x(t))^2 + (y_{\text{int}}(t) - y(t))^2 = r^2 \tag{10}$$

$$\begin{aligned} \tan(\alpha_k) &= \frac{y_{k+1} - y_k}{x_{k+1} - x_k} \\ &= \frac{y_{\text{int}}(t) - y_k}{x_{\text{int}}(t) - x_k}, \end{aligned} \tag{11}$$

where (10) represents the theorem of Pythagoras, while (11) states that the slope of the line between the two waypoints is constant.

2) Lookahead-Based Steering: Here, the steering assignment is separated into two parts

$$\chi(e) = \chi_p + \chi_r(e), \tag{12}$$

where

$$\chi_p = \alpha_k \tag{13}$$

is the path-tangential angle, while

$$\chi_r(e) \triangleq \arctan\left(-\frac{e(t)}{\triangle}\right) \tag{14}$$

is a velocity-path relative angle which ensures that the velocity is directed toward a point on the path that is located a *lookahead distance* $\triangle > 0$ ahead of the direct projection of $\mathbf{p}(t)$ onto the path [28], see Figure 3.

As can be immediately noticed, this lookahead-based steering scheme is less computationally intensive than the enclosure-based approach. It is also valid for all cross-track errors, whereas the enclosure-based strategy requires $r \geq |e(t)|$. Furthermore, Figure 3 shows that

$$e^2 + \triangle^2 = r^2, \tag{15}$$

which means that the enclosure-based approach corresponds to a lookahead-based scheme with a time-varying $\triangle(t) = \sqrt{r^2 - e(t)^2}$, varying between 0 (when $|e(t)| = r$) and r (when $|e(t)| = 0$). Only lookahead-based steering will be considered in the following.

B. Piecewise Linear Paths

If a path is made up of n straight-line segments connected by $n+1$ waypoints, a strategy must be employed to purposefully switch between these segments as they are traversed. In [27], it is suggested to associate a so-called circle of acceptance with each waypoint, with radius $R_{k+1} > 0$ for waypoint $k+1$, such that the corresponding switching criterion becomes

$$(x_{k+1} - x(t))^2 + (y_{k+1} - y(t))^2 \leq R_{k+1}^2, \tag{16}$$

i.e., to switch when $\mathbf{p}(t)$ has entered the waypoint-enclosing circle. Note that for the enclosure-based approach, such a switching criterion entails the additional (conservative) requirement $r \geq R_{k+1}$.

A perhaps more suitable switching criterion solely involves the along-track distance $s(t)$, such that if the total along-track distance between waypoints $\mathbf{p}_k$ and $\mathbf{p}_{k+1}$ is denoted s_{k+1}, a switch is made when $(s_{k+1} - s(t)) \leq R_{k+1}$. This approach is similar to (16), but has the advantage that $\mathbf{p}(t)$ does not need to enter the waypoint-enclosing circle for a switch to occur, i.e., no restrictions are put on the cross-track error.

C. Steering for Circles

Denote the centre of a circle with radius $r_c > 0$ as $\mathbf{p}_c \triangleq [x_c, y_c]^\top \in \mathbb{R}^2$. Subsequently, consider a path-fixed reference frame with origin at the direct projection of $\mathbf{p}(t)$ onto the circular path, see Figure 4. The x-axis of this reference frame has been rotated a positive angle (relative to the x-axis of the stationary reference frame)

$$\chi_p(t) = \chi_c(t) + \lambda\frac{\pi}{2}, \tag{17}$$

where

$$\chi_c(t) \triangleq \text{atan2}\left(y(t) - y_c, x(t) - x_c\right), \tag{18}$$

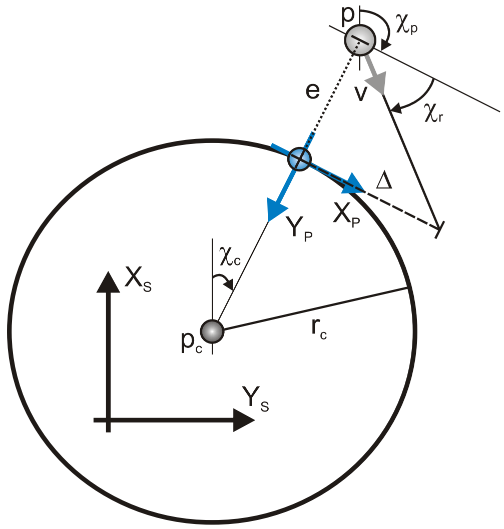

Fig. 4. The main variables associated with steering for circles.

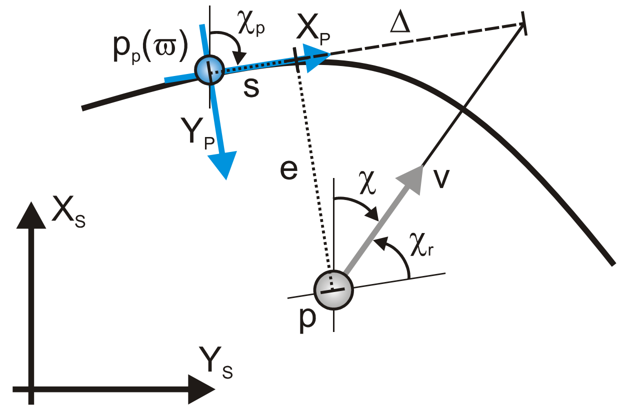

Fig. 5. The main variables associated with steering for regularly parameterized paths.

and where $\lambda = 1$ corresponds to clockwise motion and $\lambda = -1$ to anti-clockwise motion. Hence, χ_{p} becomes time-varying for circular (curved) motion, as opposed to the constant χ_{p} associated with straight lines (13). Also, note that (18) is undefined for $\mathbf{p}(t) = \mathbf{p}_{\mathrm{c}}$, i.e., when the kinematic vehicle is located at the centre of the circle. In this case, any projection of $\mathbf{p}(t)$ onto the circular path is valid, but in practice this problem can be alleviated by, e.g., purposefully choosing $\chi_{\mathrm{c}}(t)$ based on the motion of $\mathbf{p}(t)$.

Since the path-following control objective for circles is identical to (8), lookahead-based steering can be employed, implemented by using (12) with (17) instead of (13), and

$$\begin{aligned} e(t) &= r_{\mathrm{c}} - |\mathbf{p}(t) - \mathbf{p}_{\mathrm{c}}| \\ &= r_{\mathrm{c}} - \sqrt{(x(t) - x_{\mathrm{c}})^2 - (y(t) - y_{\mathrm{c}})^2} \end{aligned}$$

in (14), see Figure 4. Note that the lookahead distance $\triangle$ is no longer defined along the path, but (in general) along the x-axis of the path-fixed frame (i.e., along the path-tangential associated with the origin of the path-fixed frame). An along-track distance $s(t)$ can also be computed relative to some fixed point on the circle perimeter if required.

D. Steering for Regularly Parameterized Paths

Consider a planar path continuously parameterized by a scalar variable $\varpi \in \mathbb{R}$, such that the position of a point belonging to the path is represented by $\mathbf{p}_{\mathrm{p}}(\varpi) \in \mathbb{R}^2$. Thus, the path is a one-dimensional manifold that can be expressed by the set

$$\mathcal{P} \triangleq \left\{ \mathbf{p} \in \mathbb{R}^2 \mid \mathbf{p} = \mathbf{p}_{\mathrm{p}}(\varpi) \;\forall \varpi \in \mathbb{R} \right\}. \tag{19}$$

Regularly parameterized paths belong to the subset of $\mathcal{P}$ for which $\left|\mathbf{p}'_{\mathrm{p}}(\varpi)\right| \triangleq \left|\frac{\mathrm{d}\mathbf{p}_{\mathrm{p}}}{\mathrm{d}\varpi}(\varpi)\right|$ is positive and finite, which means that they never degenerate into a point nor have corners. Such paths include both straight lines (zero curvature) and circles (constant curvature). However, most are paths with varying curvature. For such paths, it is not trivial to calculate the cross-track error $e(t)$ required in (14).

Although it is possible to calculate the exact projection of $\mathbf{p}(t)$ onto the path by applying the so-called Serret-Frenet equations, such an approach suffers from a kinematic singularity associated with the osculating circle of the instantaneous projection point [29]. For every point along a curved path, there exists an associated tangent circle with radius $r(\varpi) = 1/c(\varpi)$, where $c(\varpi)$ is the curvature at the path point. This circle is known as the osculating circle, and if at any time $\mathbf{p}(t)$ is located at the origin of the osculating circle, the projected point on the path will have to move infinitely fast, which is not possible. This kinematic singularity effect necessitates a different approach to obtain the cross-track error required for steering purposes. The solution considered in this paper was originally suggested in [30], where it was employed for arc-length parameterized paths in a Serret-Frenet framework, and later extended to general, regularly parameterized paths in [31].

Thus, consider an arbitrary path point $\mathbf{p}_{\mathrm{p}}(\varpi)$. Subsequently, consider a path-fixed reference frame with origin at $\mathbf{p}_{\mathrm{p}}(\varpi)$, whose x-axis has been rotated a positive angle (relative to the x-axis of the stationary reference frame)

$$\chi_{\mathrm{p}}(\varpi) = \operatorname{atan2}\left(y'_{\mathrm{p}}(\varpi), x'_{\mathrm{p}}(\varpi)\right), \tag{20}$$

such that

$$\boldsymbol{\varepsilon}(t) = \mathbf{R}(\chi_{\mathrm{p}})^\top (\mathbf{p}(t) - \mathbf{p}_{\mathrm{p}}(\varpi)), \tag{21}$$

where $\boldsymbol{\varepsilon}\ (t) = [s(t), e(t)]^\top \in \mathbb{R}^2$ represents the along-track and cross-track errors relative to $\mathbf{p}_{\mathrm{p}}(\varpi)$, decomposed in the path-fixed reference frame by

$$\mathbf{R}(\chi_{\mathrm{p}}) = \begin{bmatrix} \cos\chi_{\mathrm{p}} & -\sin\chi_{\mathrm{p}} \\ \sin\chi_{\mathrm{p}} & \cos\chi_{\mathrm{p}} \end{bmatrix}. \tag{22}$$

In contrast to (8), the path-following control objective now becomes

$$\lim_{t\to\infty} \boldsymbol{\varepsilon}(t) = \mathbf{0}, \tag{23}$$

and in order to reduce $\boldsymbol{\varepsilon}(t)$ to zero, $\mathbf{p}(t)$ and $\mathbf{p}_{\mathrm{p}}(\varpi)$ can collaborate with each other. Specifically, $\mathbf{p}_{\mathrm{p}}(\varpi)$ can contribute by moving toward the direct projection of $\mathbf{p}(t)$ onto the x-axis of the path-fixed reference frame by assigning

$$\dot{\varpi} = \frac{U(t)\cos\chi_{\mathrm{r}}(e) + \gamma s(t)}{\left|\mathbf{p}_{\mathrm{p}}'(\varpi)\right|}, \tag{24}$$

where $\chi_{\mathrm{r}}(e)$ is given by (14), $\gamma > 0$, and $\left|\mathbf{p}_{\mathrm{p}}'(\varpi)\right| = \sqrt{x_{\mathrm{p}}'(\varpi)^2 + y_{\mathrm{p}}'(\varpi)^2}$. Hence, $\mathbf{p}_{\mathrm{p}}(\varpi)$ basically tracks the motion of $\mathbf{p}(t)$, which maneuvers by the cross-track error of (21) through employing (12) with (20) and (14) for $U(t) > 0$. Such an approach suffers from no kinematic singularities, and ensures that $\boldsymbol{\varepsilon}(t)$ asymptotically converges to zero for regularly parameterized paths.

Drawing a line to the classical guidance principles of the missile literature, lookahead-based steering can be seen as pure pursuit of the lookahead point. Thus, convergence to $\mathbf{p}_{\mathrm{p}}(\varpi)$ is achieved as $\mathbf{p}(t)$ in vain chases the carrot located a distance $\triangle$ further ahead along the path-tangential.

Furthermore, the concept of *off-path traversing of curved paths*, which requires the use of two virtual points to avoid kinematic singularities, was originally suggested in [32]. The scheme was used for formation control of ships in [26].

Although the recently-presented guidance method can also be applied for both straight lines and circles, the analytic, path-specific approaches presented previously are often preferable since they do not require numerical integrations such as (24). However, for completeness, applicable (arc-length) parameterizations of straight lines and circles are stated in the following.

1) Parameterization of Straight Lines: A planar straight line can be parameterized by ϖ as

$$x_{\mathrm{p}}(\varpi) = x_{\mathrm{f}} + \varpi\cos\alpha \tag{25}$$

$$y_{\mathrm{p}}(\varpi) = y_{\mathrm{f}} + \varpi\sin\alpha, \tag{26}$$

where $\mathbf{p}_{\mathrm{f}} \triangleq [x_{\mathrm{f}}, y_{\mathrm{f}}]^\top \in \mathbb{R}^2$ represents a fixed point on the path (for which ϖ is defined relative to), and α represents the orientation of the path relative to the x-axis of the stationary reference frame (defined in the direction of increasing ϖ).

2) Parameterization of Circles: A planar circle can be parameterized by ϖ as

$$x_{\mathrm{p}}(\varpi) = x_{\mathrm{c}} - \lambda r_{\mathrm{c}}\sin\left(\frac{\varpi}{r_{\mathrm{c}}}\right) \tag{27}$$

$$y_{\mathrm{p}}(\varpi) = y_{\mathrm{c}} + r_{\mathrm{c}}\cos\left(\frac{\varpi}{r_{\mathrm{c}}}\right), \tag{28}$$

where $\mathbf{p}_{\mathrm{c}} = [x_{\mathrm{c}}, y_{\mathrm{c}}]^\top \in \mathbb{R}^2$ represents the circle centre, r_{c} represents the circle radius, and λ decides in which direction $\mathbf{p}_{\mathrm{p}}(\varpi)$ traces the circumference; $\lambda = 1$ for clockwise motion and $\lambda = -1$ for anti-clockwise motion.

E. Speed Law for Path Tracking

As previously stated, the control objective of a path-tracking scenario is to track a target that is constrained to move along a path. Denoting the path-parameterization variable associated with the path-traversing target by $\varpi_{\mathrm{t}}(t) \in \mathbb{R}$, the control objective is identical to (1) with $\mathbf{p}_{\mathrm{t}}(t) = \mathbf{p}_{\mathrm{p}}(\varpi_{\mathrm{t}}(t))$. Here, $\varpi_{\mathrm{t}}(t)$ can be updated by

$$\dot{\varpi}_{\mathrm{t}} = \frac{U_{\mathrm{t}}(t)}{\left|\mathbf{p}_{\mathrm{p}}'(\varpi_{\mathrm{t}})\right|}, \tag{29}$$

which means that the target point traverses the path with the speed profile $U_{\mathrm{t}}(t) > 0$, which can also vary with ϖ_{t}.

Due to apriori knowledge about the path, the path-tracking problem can be divided into two tasks, i.e., a spatial task and a temporal task [22]. The spatial task was solved in the previous section, while the temporal task can be solved by employing the speed law

$$U(t) = \left|\mathbf{p}_{\mathrm{p}}'(\varpi)\right|\left(\frac{U_{\mathrm{t}}(t)}{\left|\mathbf{p}_{\mathrm{p}}'(\varpi_{\mathrm{t}})\right|} - \mu\frac{\tilde{\varpi}(t)}{\sqrt{\tilde{\varpi}(t)^2 + \triangle_{\tilde{\varpi}}^2}}\right), \tag{30}$$

where

$$\tilde{\varpi}(t) \triangleq \varpi(t) - \varpi_{\mathrm{t}}(t), \tag{31}$$

μ can be chosen as

$$\mu = \rho\frac{U_{\mathrm{t}}(t)}{\left|\mathbf{p}_{\mathrm{p}}'(\varpi_{\mathrm{t}})\right|},\ \rho \in \langle 0, 1], \tag{32}$$

and where $\triangle_{\tilde{\varpi}} > 0$ specifies the rendezvous approach toward the target, such that

$$U(t) = U_{\mathrm{t}}(t)\left(1 - \rho\frac{\tilde{\varpi}(t)}{\sqrt{\tilde{\varpi}(t)^2 + \triangle_{\tilde{\varpi}}^2}}\right)\frac{\left|\mathbf{p}_{\mathrm{p}}'(\varpi(t))\right|}{\left|\mathbf{p}_{\mathrm{p}}'(\varpi_{\mathrm{t}}(t))\right|}, \tag{33}$$

which means that the kinematic vehicle speeds up to catch the target when located behind it, and speeds down to wait when located in front of it. Hence, this solution just entails a synchronization-law extension of the path-following scenario.

F. Path Maneuvering Aspects

The path-maneuvering scenario involves the use of knowledge about vehicle maneuverability constraints to design purposeful speed and steering laws that allow for feasible path negotiation. Since this paper only deals with kinematic considerations, such deliberations are outside of its scope. However, relevant marine applications can be found in [33] and [34]. Much work still remains to be done on this topic, which represents a rich source of interesting problems.

G. Steering Laws as Saturated Control Laws

Rewriting (14) as

$$\chi_{\mathrm{r}}(e) = \arctan\left(-k_{\mathrm{p}}e(t)\right), \tag{34}$$

where $k_{\mathrm{p}} = \frac{1}{\triangle} > 0$, it can be seen that the lookahead-based steering law is equivalent to a saturated proportional control law, effectively mapping $e \in \mathbb{R}$ into $\chi_{\mathrm{r}}(e) \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$.

As can be inferred from the geometry of Figure 3, a small lookahead distance implies aggressive steering, which intuitively is confirmed by a correspondingly large proportional gain in the saturated control interpretation. This interpretation also suggests the possibility of introducing, e.g., integral action into the steering law, such that

$$\chi_{\mathrm{r}}(e) = \arctan\left(-k_{\mathrm{p}}e(t) - k_{\mathrm{i}}\int_0^t e(\tau)\mathrm{d}\tau\right), \tag{35}$$

where $k_{\mathrm{i}} > 0$ represents the integral gain. However, to avoid overshoot and windup issues, a better solution is to let the integral term involve spatial rather than temporal integration [35]. For a straight-line path, spatial integration results in

$$\begin{aligned}\int_0^s e(\sigma)\mathrm{d}\sigma &= \int_0^t e(\tau)\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\mathrm{d}\tau & (36)\\ &= \int_0^t e(\tau)U(\tau)\cos\chi_{\mathrm{r}}(e(\tau))\mathrm{d}\tau & (37)\\ &= \int_0^t e(\tau)U(\tau)\frac{\triangle}{\sqrt{e(\tau)^2+\triangle^2}}\mathrm{d}\tau, & (38)\end{aligned}$$

which means that the integration only occurs when the velocity has a component along the path. The inclusion of spatial integration in the steering law can be particularly useful for underactuated ships that can only steer by heading information, enabling them to follow straight-line paths while under the influence of constant environmental disturbances even without having access to directional velocity information.

V. CONCLUSIONS

This paper has given an overview of guidance laws applicable to planar motion control scenarios. Specifically, target tracking and path scenarios were considered. For target-tracking purposes, classical guidance laws from the missile literature were reviewed. For the path scenarios, enclosure-based and lookahead-based guidance laws were presented. Considered paths included straight lines, circles, as well as regularly parameterized paths. Finally, relations between the guidance laws has been discussed, as well as interpretations toward saturated control.

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