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Exact Compensation of Unmatched Perturbation Via Quasi-Continuous HOSM

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Abstract— **This paper studies nonlinear systems control when unmatched perturbations are present. A new control scheme, based on block control and quasi-continuous HOSM techniques, is proposed. The proposed method assures exact finite time tracking, for the desired signal, regardless of unmatched perturbations. The proposed control design is tested through a simulation example.**

I. INTRODUCTION

The sliding mode control (SM) is using in many applications, applied to nonlinear plants enables high accuracy tracking and robustness to disturbances and plant parameters variations [20]. Nevertheless, classical SM are not able to compensate unmatched perturbations[23].

Combination of different robust techniques and SM has been applied to deal with systems with unmatched uncertainties [8]-[12]. A design method is developed in [11] the LMI-based switching surface is used. In order to reduce the effects of the unmatched uncertainties [7] propose a method that combines $\mathscr{H}_\infty$ and integral sliding mode control. The main idea is to choose such a projection matrix, ensuring that unmatched perturbations are not amplified and minimized. A similar projection minimization is proposed in [8].

For uncertain nonlinear systems in strict-feedback form [18],[17] develop the backstepping approach in a step by step design algorithm. The structure of the system enables that, in each step, some states can be considered as a virtual control input for other states. Thus a virtual control based on Lyapunov methods is constructed in each step. In a similar manner to backstepping, Multiple Surface Sliding control is proposed in [19] to simplify the controller design of systems where model differentiation is difficult. In [19] the control law has two parts, one is intended for cancel the nonlinear dynamic of the system, the other one is a high gain controller to overcome the virtually matched uncertainties.

The combination of the backstepping design and sliding mode control is studied in [4] for systems in strict-feedback form with parameter uncertainties and extended to the multi input case in [5]. The procedure proposed in [4],[5] reduces the computational load, as compared with the standard backstepping strategy, because only retains $n-2$ steps of the original backstepping technique, coupling them with with an auxiliary second order subsystem to which a second order sliding mode control is applied. In [6] the combination of dynamical adaptive backstepping and first and second order sliding mode control is applied to both triangular and nontriangular uncertain observable minimum phase nonlinear systems.

The problem of robust control for systems in Nonlinear Block Controllable form (NBC-form) is addressed in [1]. The sliding mode technique is applied to compensate the matched perturbations (i.e. matched disturbances and parameter variations). A high gain approach is used to achieve compensation of the unmatched uncertainty and stabilization of the sliding mode dynamics. In [13] a sliding mode controller is designed using the combination of: block control [14], a sigmoid approximation to the integral sliding mode control [15], and nested sliding mode control [16]. The approximation to the integral sliding mode control combined with nested control technique is used to suppress perturbations. Following the block control technique, some states are regarded as a virtual control input. A coordinate transformation is applied to design a nonlinear sliding manifold. This transformation requires smoothness of each virtual control. This leads to the lost of accuracy.

In this paper a new design algorithm for systems in strict feedback form, a special case of the BC-form, is proposed. This algorithm achieves finite-time **exact** tracking of the desired output in the presence of unmatched perturbations. Although the method uses the virtual control idea, it avoids coordinate transformation and high gain approach. These features are accomplished via the usage of quasi-continuous HOSM and a hierarchical design approach. In the first step the desired dynamic for the first state is defined by the desired tracking signal. After the first step the desired dynamic for each state is defined by the previous one. Each virtual control is divided in two parts, the first one is intended to compensate the nominal nonlinear part of the system and the second one is aimed to achieve the desired dynamics in spite of perturbations. In the second part the quasi-continuous HOSM is used for unmatched perturbations compensation.

The present paper proceeds as follows. In Section II the class of nonlinear systems to be treated and the problem formulation are described. Section III begins with an introduction to the quasi-continuous controller as proposed in [2]. The section continues with the presentation of hierarchical design algorithm proposed in this work. Section III ends with the convergence proof of the proposed algorithm. In Section IV the algorithm is applied and simulations results

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are presented. The note then concludes with a brief comment on the proposed algorithm.

II. PROBLEM STATEMENT

Consider a class of nonlinear systems presented in the BC (block controllable) form [1]:

$$\left.\begin{array}{rcl} \dot{x}_1 &=& f_1(x_1,t)+B_1(x_1,t)x_2+g_1(x_1,t) \\ \dot{x}_i &=& f_i(\bar{x}_i,t)+B_i(\bar{x}_i,t)x_{i+1}+g_i(\bar{x}_i,t) \\ \dot{x}_n &=& f_n(\bar{x}_n,t)+B_n(\bar{x}_n,t)u+g_n(x,t) \\ i &=& 2,...,n-1 \end{array}\right\} \quad (1)$$

where $x\in R^n$ is the state vector, $x_i\in R$, $\bar{x}_i=[x_1\ldots x_i]^T$; $u\in R$ is the control vector. Moreover $f_i(\bar{x}_i,t)$ and $B_i(\bar{x}_i,t)$ are smooth vector fields, $g_i(\cdot)$ is a bounded unknown perturbation term due to parameter variations and external disturbances with at least $n-i$ bounded derivatives and $B_i(\cdot)\neq 0 \quad \forall x\in X\subset R^n, t\in[0,\infty)$.

The control problem is to design a controller such that the output $y=x_1$ in (1) tracks a desired reference with bounded derivatives, in spite of the presence of unknown bounded perturbations.

The whole state vector x is assumed to be known. In order to guarantee that y tracks a sufficiently smooth desired reference y_d, the first sliding surface is chosen as the difference between these signals, then a $(n-1)$-th order integral of the n-sliding homogeneous quasi-continuous controller is included in the first virtual control law. The error between this virtual control law and the next state is the next sliding surface.

III. HIERARCHICAL QUASI-CONTINUOUS CONTROLLER DESIGN

The first part of this section introduce the quasi-continuous homogeneous controller [2], after that the hierarchical quasi-continuous controller design algorithm is presented. Finally, the convergence proof is included.

A. Quasi-continuous controller [2]

Let a Single-Input-Single-Output system of the form

$$\begin{array}{rcl} \dot{\xi} &=& a(t,\xi)+b(t,\xi)u, \quad \xi\in R^n, u\in R \\ \sigma &:& (t,\xi)\mapsto\sigma(t,\xi)\in R \end{array} \quad (2)$$

where σ is the measured output of the system, u is the control. Smooth functions a,b,σ are assumed to be unknown, the dimension n can also be uncertain. The task is to make σ vanish in finite time by means of a possibly discontinuous feedback and to keep $\sigma\equiv 0$. Extend the system by means of a fictitious equation $\dot{t}=1$. Let $\tilde{\xi}=(\xi,t)^T$, $\tilde{a}(\tilde{\xi})=(a(t,\xi),1)^T$, $\tilde{b}(\tilde{\xi})=(b(t,\xi),0)^T$. Then system (2) takes on the form

$$\dot{\tilde{\xi}}=\tilde{a}(\tilde{\xi})+\tilde{b}(\tilde{\xi})u \quad \sigma=\sigma(\tilde{\xi}) \quad (3)$$

It is assumed that system (3) has relative degree r constant and known. As follow from [21] the equation

$$\sigma^{(r)}=h(t,\xi)+g(t,\xi)u, \quad g(t,\xi)\neq 0 \quad (4)$$

holds, where $h(t,\xi)=\sigma^{(r)}|_{u=0}$, $g(t,\xi)=\frac{\partial}{\partial u}\sigma^{(r)}$. The uncertainty prevents immediate reduction of (2) to (4). Suppose that the inequalities

$$0<K_m\leq\frac{\partial}{\partial u}\sigma^{(r)}\leq K_M, \quad |\sigma^{(r)}|_{u=0}|\leq C \quad (5)$$

holds for some $K_m,K_M,C>0$. Assume that (5) holds globally. Then (4),(5) imply the differential inclusion

$$\sigma^{(r)}\in[-C,C]+[K_m,K_M]u \quad (6)$$

The bounded feedback control

$$u=-\alpha\Psi(\sigma,\dot{\sigma},\ldots,\sigma^{(r-1)}) \quad (7)$$

is constructed such that $\sigma=\dot{\sigma}=\ldots=\sigma^{(r-1)}=0$ is established in finite time. In order to reduce the chattering, a controller is designed which is continuous everywhere except this set. Such a controller is naturally called quasi-continuous. In practice, in the presence of switching delays, measurement noises and singular perturbations, the motion will take place in some vicinity of the r sliding set $\sigma=\dot{\sigma}=\ldots=\sigma^{(r-1)}=0$ never hitting it with $r>1$. Denote

$$\begin{array}{rcl} \varphi_{0,r} &=& \sigma,\ N_{0,r}=|\sigma|,\ \Psi_{0,r}=\varphi_{0,r}/N_{0,r}=sign\sigma \\ \varphi_{i,r} &=& \sigma^{(i)}+\beta_i N_{i-1,r}^{(r-i)/(r-i+1)}\Psi_{i-1,r} \\ N_{i,r} &=& |\sigma^{(i)}|+\beta_i N_{i-1,r}^{(r-i)/(r-i+1)} \\ \Psi_{r-1,r}(\cdot) &=& \varphi_{r-1,r}/N_{r-1,r}; \qquad i=0,\ldots,r-1 \end{array} \quad (8)$$

where $\beta_1,\ldots,\beta_{r-1}$, are positive numbers. The following proposition is easily proved by induction.

Proposition 1. [2] *Let $i=0,...,r-1$. $N_{i,r}=0$ is positive definite, i.e. $N_{i,r}=0$ iff $\sigma=\dot{\sigma}=\ldots=\sigma^{(i)}=0$.The inequality $|\Psi_{r-1,r}|\leq 1$ holds whenever $N_{i,r}>0$. The function $\Psi_{i,r}(\sigma,\dot{\sigma},\ldots,\sigma^{(i)})$ is continuous everywhere (i.e. it can be redefined by continuity) except the point $\sigma=\dot{\sigma}=\ldots=\sigma^{(i)}=0$.*

Theorem 1. [2] *Provided $\beta_1,\ldots,\beta_{r-1},\alpha>0$ are chosen sufficiently large in the list order, the above design result in* *the r-sliding homogeneous controller*

$$u = -\alpha\Psi(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}) \quad (9)$$

providing for the finite-time stability of (6),9). *The finite-time stable r-sliding mode $\sigma \equiv 0$ is established in the system (2),(9).*

It follows from Proposition 1 that control (*9*) is continuous everywhere except the r-sliding mode $\sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0$.

B. Design algorithm

Consider the state x_2 in (1) as a virtual control, then uncertainty $g_1(x_1,t)$ is seen as matched, the objective is achieve tracking of y_d. The next algorithm is proposed:

Step 1: $x_2 = \phi_1(x_1)$, taking $\sigma_1 = x_1 - y_d$ as sliding surface, the *n*-sliding homogeneous quasi-continuous controller is included in $\phi_1(x_1)$, where $\phi_1(x_1)$ is an $n-1$ times differentiable function defined as:

$$\begin{aligned} \phi_1(x_1) &= B_1(\cdot)^{-1}\{-f_1(\cdot) + u_{1,1}\} \\ \dot{u}_{1,1} &= u_{1,2} \\ &\vdots \\ \dot{u}_{1,n-1} &= -\alpha_1\Psi_{n-1,n}(\sigma_1, \dot{\sigma}_1, \ldots, \sigma_1^{(n-1)}) \end{aligned} \quad (10)$$

The first part of control $\phi_1(x_1)$ is aimed to compensate the nominal part of the system, $\Psi_{n-1,n}(\cdot)$ as defined in (8) with the subtitutions $r = n$, $\sigma = \sigma_1$. The controller $-\alpha\Psi_{n-1,n}(\cdot)$ is *n*-sliding homogeneous. Derivatives $\sigma_1, \dot{\sigma}_1, \ldots, \sigma_1^{(n-1)}$ are calculated by means of the robust differentiators with finite-time convergence [3]:

$$\begin{aligned} \dot{u}_{1,n-1} &= -\alpha_1\Psi_{n-1,n}(z_0, z_1, \ldots, z_{n-1}) \\ \dot{z}_0 &= \upsilon_0 \\ \upsilon_0 &= -\lambda_n L^{\frac{1}{n}}|z_0 - \sigma|^{\frac{(n-1)}{n}} sign(z_0 - \sigma) + z_1 \\ \dot{z}_k &= \upsilon_k \\ \upsilon_k &= -\lambda_{n-k} L^{\frac{1}{(n-k)}}|z_k - \upsilon_{k-1}|^{\frac{(n-k-1)}{(n-k)}} \ldots \\ & \quad sign(z_k - \upsilon_{k-1}) + z_{k+1} \\ k &= 1, \ldots, n-2 \\ \dot{z}_{n-1} &= -\lambda_1 L sign(z_{n-1} - \upsilon_{n-2}) \end{aligned} \quad (11)$$

where $\left|\sigma^{(n)}\right| \le L$ and the parameters $\lambda_n > \lambda_{n-1} > \ldots > \lambda_1 > 0$ are chosen such that the estimates $z_0, \ldots, z_{n-1}$ converge to $\sigma_1, \ldots, \sigma_1^{(n-1)}$ in finite time.

Step i: Since the desired dynamic for x_i is $\phi_i(x_i)$, the *i*-th sliding surface is chosen as $\sigma_i = x_i - \phi_{i-1}(x_{i-1})$. The control proposed is analogous to (10), but with some changes in the order:

$$\begin{aligned} \phi_i(x_i) &= B_i(\cdot)^{-1}\{-f_i(\cdot) + u_{i,1}\} \\ \dot{u}_{i,1} &= u_{i,2} \\ &\vdots \\ \dot{u}_{i,n-i} &= -\alpha_i\Psi_{n-i,n-i+1}(\sigma_i, \dot{\sigma}_i, \ldots, \sigma_i^{(n-i)}) \end{aligned} \quad (12)$$

$\Psi_{n-i,n-i+1}(\cdot)$ defined as in (8),(11) obviously using $\sigma = \sigma_i$, in those equations.

Step n: In this step, the use of quasi-continuous controller is no longer needed, therefore the next controller is chosen with $\sigma_n = x_n - \phi_{n-1}(x_{n-1})$:

$$\begin{aligned} \phi_n(x_n) &= B_n(\cdot)^{-1}\{-f_n(\cdot) + u_{n,1}\} \\ \text{where } u_{n,1} &= -\alpha_n sign(\sigma_n) \end{aligned} \quad (13)$$

The discontinuous control in (13) has been chosen for simplicity but it is possible to choose an algorithm that achieves a smother control signal, e.g. super twisting.

Theorem 2. Provided that $g_i(\cdot)$ in system (1) and y_d are smooth functions with $n-i$ and n bounded derivatives respectively the above hierarchic design results in an ultimate controller $u = \phi_n(x_n)$ providing for the finite time stability of $\sigma_1 = x_1 - y_d = \dot{\sigma}_1 = \ldots = \sigma_1^{(n-1)} = 0$ in system (1).

C. Convergence proof

- For the state n

$$\begin{aligned} \dot{x}_n &= f_n(\cdot) + B_n(\cdot)u + g_n(x,t) \\ \text{with} \quad u &= B_n(\cdot)^{-1}\{-f_n(\cdot) + \alpha_n sign(\sigma_n)\} \\ \sigma_n &= x_n - \phi_{n-1}; \quad \phi_{n-1} \text{ sufficiently smooth} \end{aligned}$$

Thus $\dot{\sigma}_n = -\alpha_n sign(\sigma_n) + g_n(x,t) + \dot{\phi}_{n-1}$, taking $\alpha_n \ge |g_n(\cdot)| + |\dot{\phi}_{n-1}|$ provides for the appearance of a 1-sliding mode for the constraint σ_n.

- Now for the state $(n-1)$, ϕ_{n-1} as defined in (12), the constraint function is $\sigma_{n-1} = x_{n-1} - \phi_{n-2}$ then:

$$\begin{aligned} \dot{\sigma}_{n-1} &= \dot{x}_{n-1} - \dot{\phi}_{n-2} \\ &= f_{n-1}(\cdot) + B_{n-1}(\cdot)\phi_{n-1} + g_{n-1}(\bar{x}_{n-1},t) - \dot{\phi}_{n-2} \\ &= u_{n-1,1} + g_{n-1}(\bar{x}_{n-1},t) - \dot{\phi}_{n-2} \\ \ddot{\sigma}_{n-1} &= \dot{u}_{n-1,1} + \dot{g}_{n-1}(\bar{x}_{n-1},t) - \ddot{\phi}_{n-2} \end{aligned} \quad (14)$$

and according to (12):

$$\dot{u}_{n-1,1} = -\alpha_{(n-1)}\Psi_{1,2}(\sigma_{n-1}, \dot{\sigma}_{n-1})$$

That is (14) takes the form:

$$\begin{aligned} \ddot{\sigma}_{n-1} &= h_{n-1}(t,x) + g_{n-1}(t,x)u_{n-1} \quad (15)\\ \text{with} \quad h_{n-1}(t,x) &= \ddot{\sigma}_{n-1}|_{u_{n-1}=0} = \dot{g}_{n-1}(\cdot) - \ddot{\phi}_{n-2}\\ g_{n-1}(t,x) &= \frac{\partial}{\partial u_{n-1}}\ddot{\sigma}_{n-1}\\ u_{n-1} &= -\alpha_{n-1}\Psi_{1,2}(\sigma_{n-1},\dot{\sigma}_{n-1}) \quad (16) \end{aligned}$$

If for some $K_{m_{n-1}}, K_{M_{n-1}}, C_{n-1} > 0$

$$0 < K_{m_{n-1}} \leq \frac{\partial}{\partial u_{n-1}}\ddot{\sigma}_{n-1} \leq K_{M_{n-1}}, \quad |\ddot{\sigma}_{n-1}|_{u_{n-1}=0}| \leq C_{n-1} \quad (17)$$

holds then (16),(17) imply the differential inclusion

$$\ddot{\sigma}_{n-1} \in [-C_{n-1}, C_{n-1}] + [K_{m_{n-1}}, K_{M_{n-1}}]u_{n-1} \quad (18)$$

and controller (16) provides for the finite time stability of (18),(16). The finite-time stable 2-sliding mode is established for the constraint σ_{n-1}.

• For the first state $\sigma_1 = x_1 - y_d$, ϕ_1 as defined in (10) then:

$$\begin{aligned} \dot{\sigma}_1 &= \dot{x}_1 - \dot{y}_d\\ &= f_1(\cdot) + B_1(\cdot)\phi_1 + g_1(x_1,t) - \dot{y}_d\\ &= u_{1,1} + g_1(x_1,t) - \dot{y}_d\\ \sigma_1^{(n)} &= \dot{u}_{1,n-1} + g_1^{(n-1)}(x_1,t) - y_d^{(n)} \quad (19) \end{aligned}$$

where $\dot{u}_{1,n-1} = -\alpha_1\Psi_{n-1,n}(\sigma_1,\dot{\sigma}_1,\ldots,\sigma_1^{(n-1)})$. Note that equation (19) is analogous to (14) then

$$\begin{aligned} \sigma_1^{(n)} &= h_1(t,x) + g_1(t,x)u_1 \quad (20)\\ h_1(t,x) &= \sigma_1^{(n)}|_{u_1=0} = g_1^{(n-1)}(\cdot) - y_d^{(n)}\\ g_1(t,x) &= \frac{\partial}{\partial u_1}\sigma_1^{(n)}\\ u_1 &= -\alpha_1\Psi_{n-1,n}(\sigma_1,\dot{\sigma}_1,\ldots,\sigma_1^{(n-1)}) \quad (21) \end{aligned}$$

If the inequalities

$$0 < K_{m_1} \leq \frac{\partial}{\partial u_1}\sigma_1^n \leq K_{M_1}, \quad |\sigma_1^n|_{u_1=0}| \leq C_1 \quad (22)$$

holds for some $K_{m_1}, K_{M_1}, C_1 > 0$ then (21),(22) imply the differential inclusion

$$\sigma_1^{(n)} \in [-C_1, C_1] + [K_{m_1}, K_{M_1}]u_1 \quad (23)$$

and controller (21) provides for the finite time stability of (23),(21). The finite time stable r-sliding mode is established for the constraint σ_1.

IV. EXAMPLE

Consider the perturbed third order system:

$$\begin{aligned} \dot{x}_1 &= 2sin(x_1) + 1.5x_2 + g_1(x_1,t)\\ \dot{x}_2 &= 0.8x_1x_2 + x_3 + g_2(\bar{x}_2,t) \quad (24)\\ \dot{x}_3 &= -\frac{1.5}{x_3^2+1} + 2u + g_3(x,t) \end{aligned}$$

the functions g_1, g_2 are the unmatched bounded perturbations and the function g_3 is the matched perturbation, these functions were defined as follows

$$\begin{aligned} g_1(x_1,t) &= 0.2sin(t) + 0.1x_1 + 0.12\\ g_2(\bar{x}_2,t) &= 0.3sin(2t) + 0.2x_1 + 0.2x_2 - 0.4\\ g_3(x,t) &= 0.2sin(2t) + 0.2x_1 + 0.3x_2 + 0.2x_3 + 0.3 \end{aligned}$$

a controller that achieves tracking of $y_d = 2sin(0.15t) + 4cos(0.1t) - 4$ by x_1 is desired. Thus the first sliding surface is $\sigma_1 = x_1 - y_d$ and the virtual control for x_1:

$$\begin{aligned} \phi_1(x_1,t) &= \frac{1}{1.5}\{-2sen(x_1) + u_{11}\}\\ \dot{u}_{11} &= u_{12}\\ \dot{u}_{12} &= -\alpha_1\Psi_{2,3}(\sigma_1,\dot{\sigma}_1,\ddot{\sigma}_1) \end{aligned}$$

where

$$\Psi_{2,3}(\cdot) = \frac{\ddot{\sigma}_1 + 2(|\dot{\sigma}_1| + |\sigma_1|^{2/3})^{-1/2}(\dot{\sigma}_1 + |\sigma_1|^{2/3}sign(\sigma_1))}{|\ddot{\sigma}_1| + 2(|\dot{\sigma}_1| + |\sigma_1|^{2/3})^{1/2}}$$

for next state $\sigma_2 = x_2 - \phi_1(x_1,t)$ then

$$\begin{aligned} \phi_2(x_2,t) &= -0.8x_1x_2 + u_{21}\\ \dot{u}_{21} &= -\alpha_2\Psi_{1,2}(\sigma_2,\dot{\sigma}_2)\\ \Psi_{1,2}(\sigma_2,\dot{\sigma}_2) &= \frac{\dot{\sigma}_2 + |\sigma_2|^{1/2}sign(\sigma_2)}{|\dot{\sigma}_2| + |\sigma_2|^{1/2}} \end{aligned}$$

Finally for state x_3, $\sigma_3 = x_3 - \phi_2(x_2,t)$

$$u = \frac{1}{2}\{\frac{1.5}{x_3^2+1} + u_{31}\}$$
$$u_{31} = -\alpha_3 sign(\sigma_3)$$

Results obtained in simulation are shown in figures (1)-(4), taking $\alpha_1 = 0.92, \alpha_2 = 1, \alpha_3 = 3.2$. Since $\sigma_1 = x_1 - y_d$, straightforward algebra reveals that $B_1u_{11} = \dot{y}_d - g_1(\cdot)$ has to be accomplished in order to achieve that x_1 tracks y_d, these signals are shown in figure (4).

V. Conclusion

The problem of control design for nonlinear systems with unmatched perturbations is treated in this paper. A design algorithm providing for finite time exact tracking of the desired signal is given. The proposed algorithm uses nested quasi-continuous HOSM in a hierarchic manner. Results obtained in simulation are presented in figures (1)-(4).

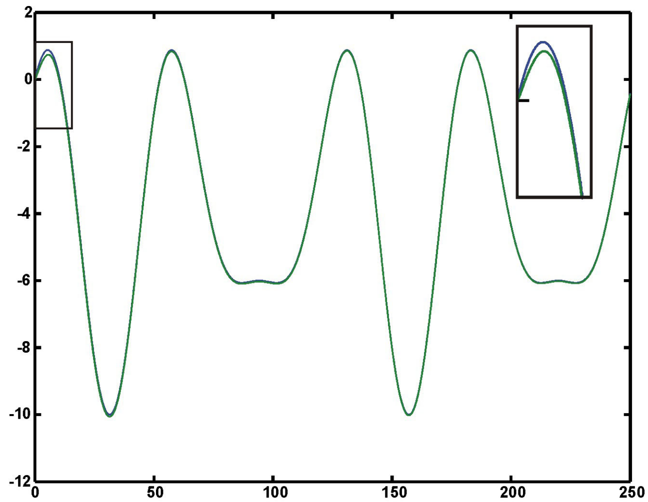

Fig. 1. Signals yd, x_1

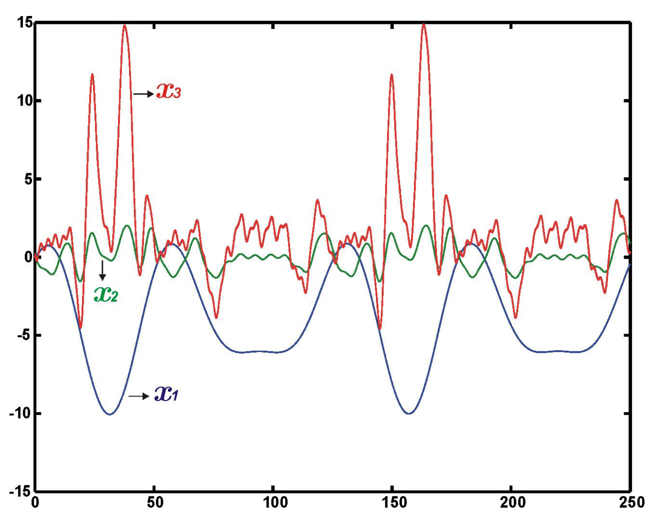

Fig. 2. States x_1, x_2, x_3

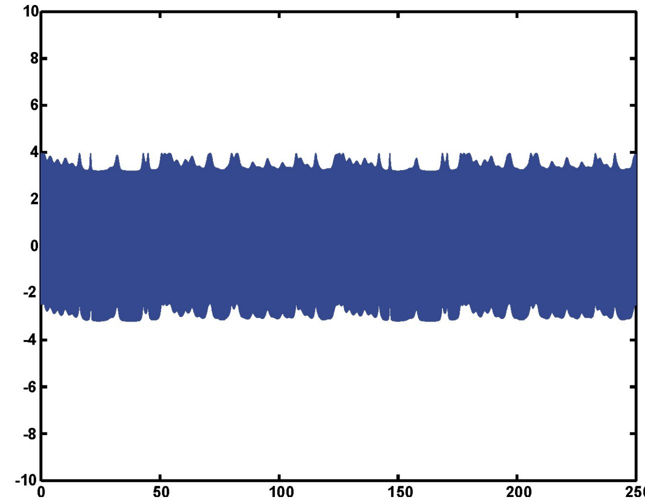

Fig. 3. Control signal u

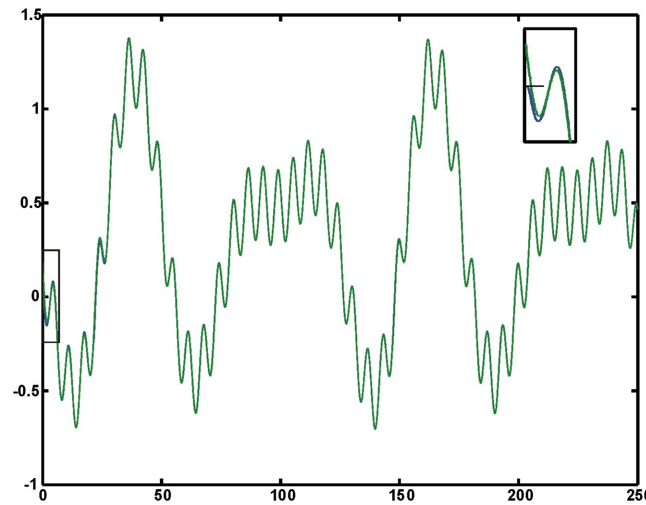

Fig. 4. $B_1 u_{11}, \dot{y}_d - g_1(\cdot)$

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