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Model Reduction for Metzlerian Systems

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Abstract—**The problem of model reduction for the special class of continuous-time Metzlerian system is considered in this paper. The model reduction is performed under structural maintainability and stability preserving constraints. It is shown that model reduction based on aggregation method enables to maintain the structure of the original Metzlerian system while preserving its stability. Furthermore, we employ the frequency weighted balancing to Metzlerian systems and show that the reduced order models obtained by either truncation or singular perturbation approximation are guaranteed to be stable for the general case of double-sided weighting.**

I. Introduction

Model reduction problem has been an active area of research for several years. Algorithms have been developed to find reduced order systems both in time domain and frequency domain. The classical approaches to model reduction are summarized in [1]. After the publication of [2], and the development of balanced realization, researchers showed renewed interests in model and controller reduction techniques (see [3] and references therein). The existing methods are applicable to system without special structures. However, it is of particular interest to maintain the special structure of a system after model reduction. For discrete-time systems, a model reduction method is available with such properties [4].

In this paper, we consider the Metzlerian systems, which represent the continuous-time counterpart of discrete nonnegative systems [5]-[7]. Such systems appear in industrial processes involving chemical reactors, heat exchangers and distillation columns. A variety of nonnegative and Metzlerian systems can also be found in engineering, management, economics, biology and social sciences. Applications of discrete nonnegative systems can also be found in the input-output analysis model proposed by Leontief, Leslie population model, Markov Chains and Queueing systems. It is known that majority of such systems are of high order and model reduction techniques are required to be performed for obvious reasons. We apply model reduction techniques for these classes of systems such that the structure of the original high-order model and its stability are preserved after the reduction process.

Among the traditional techniques we show that the aggregation-based model reduction enables one to reduce a high order stable Metzlerian system with the properties that the reduced order model remains Metzlerian and stable.

We also apply the frequency weighted balanced model reduction technique to Metzlerian systems and investigate the stability behavior of the reduced system for the general case of double-sided weighting. Enns [12] has presented a scheme for reducing a stable high-order model with frequency weighting based on a modification of balanced truncation [13]. The method, known as frequency-weighted balanced truncation, may use input weighting, output weighting, or both. With only one weighting present, stability of the reduced-order model is guaranteed. With both weightings present, the method may yield unstable models. To overcome the potential drawback of instability, Lin and Chiu [14] proposed a new technique which yields stable models in case of double-sided weighting. Their technique was later generalized to include proper weights in [15]. However, this method had a drawback in controller reduction application, which was later rectified in [16]. Other modifications to Enns technique were proposed in [17] and [18] which had the shortcomings of being realization dependent.

A number of frequency weighted model reduction methods have been proposed based on partial-fraction-expansion idea (see [19]-[21]). However, the approximation error obtained using these methods are generally larger compared to Enns method with the exception of the method by Zhou [20] where optimization is used to improve the approximation error.

A parameterized method which combines the advantages of the unweighted balancing with the frequency weighted partial fraction expansion technique is also available [22]. This method, known as partial fraction expansion based frequency weighted model reduction, preserves the stability of the reduced order model in case of double-sided weighting.

Motivated by the fact that no result is available for model reduction of Metzlerain systems, we provide two techniques for model reduction of this class of systems. In this note, we show that a direct procedure for model reduction of Metzlerian system based on frequency weighted balancing maintains the stability property without the requirement of partial fraction or any other modifications as proposed by other researchers. We demonstrate that the stability of the reduced order model with double-sided weighting is guaranteed by applying either direct truncation or singular perturbation.

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II. Preliminary

In this Section, we present preliminary results and necessary background material needed for the development of the paper.

A. Metzlerian Systems

Definition 1: A matrix $A=[a_{ij}]\in\Re^{n\times n}$ is called a Metzlerian matrix if $a_{ii}\le 0$ for all *i* and $a_{ij}\ge 0$ $i\ne j$; $i=j=1,2,\ldots,n$.

Note that a necessary condition for stability of Metzlerian matrices is $a_{ii}<0$, which will be assumed in this paper.

In this paper, we consider model reduction problems associated with the following continuous-time systems:

$$\dot{x}=Ax+Bu,\quad x(0)=x_0$$
$$y=Cx \tag{1}$$

where $x\in\Re^n$, $u\in\Re^m$ and $y\in\Re^p$ are state, control and output vectors, respectively; and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$ and $C\in\Re^{p\times n}$ are state space matrices.

Definition 2: A system (1) is called Metzlerian system if *A* is a Metzlerian matrix, and $B\ge 0, C\ge 0$ are non-negative matrices.

Metzlerian systems are closely related to the nonnegative discrete-time systems. The system matrix associated with this class is elementwise positive. The main characteristic of nonnegative matrices stems from the so-called Frobenius-Perron theorem, which states that the dominant eigenvalue of a nonnegative matrix is real and nonnegative. A number of nice properties can be derived from this theorem, specially in connection to the stability.

An equivalent result exists for the Metzlerian matrices associated with the continuous-time case. The magnitude of the largest eigenvalue of the Metzlerian matrix plays the same role as the Frobenius-Perron eigenvalue of a nonnegative matrix.

Definition 3: The system (1) is called internally positive if and only if for any $x_0\in\Re^n_+$, and every $u\in\Re^m_+$, we have $x\in\Re^n_+$ and $y\in\Re^p_+$ for all $t\ge 0$.

Theorem 1: The continuous-time system in (1) is internally positive if and only if the matrix *A* is a Metzlerian matrix and $B\in\Re^{n\times m}_+$, $C\in\Re^{p\times n}_+$ are non-negative matrices (i.e. a Metzlerian system).

The response of Metzlerian systems belongs to the positive cone. This is evident from the fact that $e^{At}\in\Re^{n\times n}_+$ if and only if *A* is the Metzlerian matrix. Since *B* and *C* are nonnegative, $x_0\in\Re^n_+$, $u(t)\in\Re^m_+$, then $x(t)\in\Re^n_+$ and $y(t)\in\Re^n_+$ for all $t\ge 0$.

Theorem 2: A Metzlerian system is asymptotically stable if and only if any one of the following equivalent conditions is satisfied:

1. All eigenvalues of the Metzlerian Matrix A have negative real parts.
2. All coefficient a_i $(i=0,1,\ldots,n-1)$ of the characteristic polynomial $\Delta(\lambda)=\det(\lambda I-A)=\lambda^n+a_{n-1}\lambda^{n-1}+\ldots+a_0\lambda+a_0$ are positive.
3. All leading principal minors of the matrix –*A* are positive.
4. The matrix A is nonsingular and $-A^{-1}>0$.

The proof of the above theorem can be deduced from the properties associated with the Metzlerian matrix (see [5]-[7]).

The discrepancies between the original high order system and the reduced order system can be characterized by the error analysis. Another important characteristic in the problem of model reduction is robustness issue. One way to compare the robustness of the reduced order model with the original high order system is to compute the robust stability radius. Robust stability and robust stability radius of nonnegative and Metzlerian system have originally been introduced in [8] - [10], whereby direct formulas for real and complex stability radius under unstructured and structured uncertainties have been derived in [10] (see also [11]). Here we only consider the stability radius of *A* under unstructured uncertainty Δ, which is given by $r=\dfrac{1}{\|A^{-1}\|}$.

B. Structural Maintainability

The main idea behind the concept of maintainability is to consider special properties of a certain class of systems and perform tasks such as realization, model reduction, control design, etc., and guarantee that those properties are preserved after the completion of those tasks.

Definition 4: A system (A,B,C) is said to be structurally maintainable if its corresponding reduced order system (F,G,H) preserves the same structure.

Structurally maintainable systems play an important role in system realization. Balanced truncation is a widely used model reduction method. It is well-known that the truncated system preserves the balanced structure and its properties.

Another example is the canonical form. A number of model reduction techniques both in time and frequency domains have been reported, whereby the reduced order model preserves the original canonical structures.

Finally, the class of Hessenberg forms has also been investigated and it is possible to apply model reduction procedures to obtain the reduced-order model with the same structure. As an example, consider the class of discrete-time Mansour form which has the structure of Hessenberg form. Mansour form can easily be constructed by Schur-Cohn coefficients from Schur-Cohn table. Mansour form has several interesting properties in relation to stability, filter realization, model reduction and control applications (see [4] for more details). In this reference, a model reduction technique for Mansour form is outlined, which maintains the original structure. In what follows we investigate the model reduction for the special structure of Metzlerian system.

III. Model Reduction of Metzlerian Systems

In this section we consider two model reduction techniques for Metzlerian system with the properties of the structural maintainability and stability.

A. Aggregation Technique

It is evident that all model reduction techniques can be applied to Metzlerian systems. However, it is not guaranteed that the reduced order models maintain the original structure. In this section we employ the aggregation method for Metzlerian systems and investigate its structural maintainability.

There have been two schemes for model reduction in time domain: aggregation and perturbation. Aggregation is coarsing state variables while perturbation is ignoring certain interaction of the dynamic or structural nature in a system.

Here we provide a brief discussion on aggregation of large-scale linear time-invariant continuous system.

Consider a linear time-invariant controllable system (1). It is desired to describe the time behavior of

$$z = Tx, \quad z(0) = z_0 = Tx_0$$

where $T \in \Re^{l \times n}, (l < n)$ is a constant aggregation matrix and $z \in \Re^l$ is aggregation of x. Without loss of generality, it is assumed that $rank(T) = l$. Then the aggregated system is described by

$$\begin{aligned} \dot{z} &= Fz + Gu, \quad z(0) = z_0 \\ \hat{y} &= Hz \end{aligned} \tag{2}$$

where the pair (F, G) satisfy the following dynamic exactness (perfect aggregation) conditions:

$$\begin{aligned} FT &= TA \\ G &= TB \\ HT &\cong C \end{aligned} \tag{3}$$

If an error vector is defined as $e(t) = z(t) - Tx(t)$, then the dynamic behavior is given by $\dot{e}(t) = Fe(t)$. To asymptotically satisfy dynamic exactness condition, F should be a stable matrix to make $\lim_{t \to \infty} e(t) = 0$ for $e(0) \neq 0$. Thus, once matrix T is known, the aggregated matrix F is obtained by:

$$F = TAT^T (TT^T)^{-1} \tag{4}$$

and *G, H* are also determined from (3). The essential step of aggregation is to find *T*. According to different requirements and emphasis, *T* can be constructed by different algorithms, i.e,. modal, exact, aggregation by continuous fraction, and so on.

We begin by finding *T* to retain the dominant modes. When applying to Metzlerian systems, it is desirable that the resulting reduced order systems are also Metzlerian. To maintain the Metzlerian structure, we need to find a proper *T*.

Definition5: A subset M of $\Re^n$ is called an affine set if $(1-\lambda)x + \lambda y \in M$ for every $x \in M$, $y \in M$ and $\lambda \in \Re$.

Lemma 1: The subspaces of $\Re^n$ are the affine sets which contain the origin.

Theorem 3: Given a Metzlerian system (A, B, C), and let the aggregation matrix *T* be selected such that:

$$T = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{l1} & \alpha_{l2} & \cdots & \alpha_{\ln} \end{bmatrix} > 0 \tag{5}$$

$$\sum_{j=1}^{n} \alpha_{ij} = 1, \quad (i = 1,2,\ldots,n)$$

then the resulting reduced order model is also Metzlerian.

Proof: According to lemma 1, $\Re_+$ is an affine set. From Theorem 1, Metzlerian system (A, B, C) is internally positive, that is, $x \in \Re_+^n$, $x_i \in \Re_+^1$. Moreover, from Definition 5, it can easily be shown that for an affine set *M*, it also holds that, for every $x_i \in M$, $i = 1,2,\ldots,n$ we have $\sum_{i=1}^{n} \alpha_i x_i \in M$, in which $\sum_{i=1}^{n} \alpha_i = 1$. Since $z = Tx$, $z_j = \sum_{i=1}^{n} \alpha_{ij} x_i$ for $j = 1,2,\ldots,l$; we have $z \in \Re_+^1$. Similarly, we also have $\hat{y} \in \Re_+$. Thus (F, G, H) is an internally positive system, and by Theorem 1, it is Metzlerian.

Remark: It is interesting to point out that model reduction based on aggregation for the stable interval Metzlerian systems leads to the similar properties; namely, the reduced order models remain interval Metzlerian and stable.

B. Frequency Weighted Balanced Model Reduction Technique

This subsection is devoted to balanced model reduction with double-sided frequency weightings. Consider the transfer function of a linear time invariant system:

$$G(s) = C(sI - A)^{-1} B + D \tag{6}$$

where (A,B,C,D) is its nth-order minimal realization. Let transfer functions of the stable input and output weights be given by (7) and (8), respectively

$$V(s)=C_V(sI-A_V)^{-1}B_V+D_V \tag{7}$$

$$W(s)=C_W(sI-A_W)^{-1}B_W+D_W \tag{8}$$

where (A_V,B_V,C_V,D_V) and (A_W,B_W,C_W,D_W) are their minimal realizations of orders n_V and n_W, respectively. The augmented systems given by

$$G(s)V(s)=\overline{C}_i(sI-\overline{A}_i)^{-1}\overline{B}_i+\overline{D}_i$$

$$W(s)G(s)=\overline{C}_o(sI-\overline{A}_o)^{-1}\overline{B}_o+\overline{D}_o$$

have the following realizations:

$$\overline{A}_i=\begin{bmatrix}A & BC_V\\ 0 & A_V\end{bmatrix} \quad , \quad \overline{B}_i=\begin{bmatrix}BD_V\\ B_V\end{bmatrix}$$

$$\overline{C}_i=\begin{bmatrix}C & DC_V\end{bmatrix} \quad , \quad \overline{D}_i=DD_V$$

and

$$\overline{A}_o=\begin{bmatrix}A & 0\\ B_WC & A_W\end{bmatrix} \quad , \quad \overline{B}_o=\begin{bmatrix}B\\ B_WD\end{bmatrix}$$

$$\overline{C}_o=\begin{bmatrix}D_WC & C_W\end{bmatrix} \quad , \quad \overline{D}_o=D_WD$$

And the frequency weighted Gramians

$$\overline{P}_i=\begin{bmatrix}P_{11} & P_{12}\\ P_{12}{}^T & P_V\end{bmatrix} \text{ and } \overline{Q}_o=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12}{}^T & Q_W\end{bmatrix}$$

satisfy the following Lyapunov equations:

$$\overline{A}_i\overline{P}_i+\overline{P}_i\overline{A}_i{}^T+\overline{B}_i\overline{B}_i{}^T=0 \tag{9}$$

$$\overline{A}_o{}^T\overline{Q}_o+\overline{Q}_o\overline{A}_o+\overline{C}_o{}^T\overline{C}_o=0 \tag{10}$$

Expanding (1, 1) blocks of the above Lyapunov equations yield the following:

$$AP_{11}+P_{11}A^T+P_E=0 \tag{11}$$

$$A^TQ_{11}+Q_{11}A+Q_E=0 \tag{12}$$

where

$$P_E=BC_VP_{12}^T+P_{12}C_V^TB^T+BD_VD_V^TB^T \tag{13}$$

$$Q_E=C^TB_W^TQ_{12}^T+Q_{12}B_WC+C^TD_W^TD_WC \tag{14}$$

Simultaneously diagonalizing the weighted Gramians, we get

$$T^TQ_{11}T=T^{-1}P_{11}T^{-T}=\begin{bmatrix}\Sigma_{E1} & 0\\ 0 & \Sigma_{E2}\end{bmatrix}$$

in which

$$\Sigma_{E1}=diag\{\sigma_1,\sigma_2,\ldots,\sigma_r\}$$

$$\Sigma_{E2}=diag\{\sigma_{r+1},\ldots,\sigma_r\}$$

where $\sigma_i\le\sigma_{i+1}$, $i=1,2,\ldots,n-1$ and $\sigma_r>\sigma_{r+1}$. Now transform and partition the original system as shown

$$T^{-1}AT=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} \quad , \quad B=\begin{bmatrix}B_1\\ B_2\end{bmatrix}$$

$$C=\begin{bmatrix}C_1 & C_2\end{bmatrix}, \quad D=D$$

Where $A_{11}\in\Re^{r\times r}$, $B_1\in\Re^{r\times p}$, $C_1\in\Re^{q\times r}$ and $r<n$. Then the reduced order model can be obtained by

(i) Direct Truncation where (A_{11},B_1,C_1,D) is the reduced order model.

(ii) singular Perturbation approximation where reduced order model $(A_{SP},B_{SP},C_{SP},D_{SP})$ is defined by:

$$\begin{aligned}A_{SP}&=A_{11}-A_{12}A_{22}^{-1}A_{21}\\ B_{SP}&=B_1-A_{12}A_{22}^{-1}B_2\\ C_{SP}&=C_1-C_2A_{22}^{-1}A_{21}\\ D_{SP}&=D-C_2A_{22}^{-1}B_2\end{aligned} \tag{15}$$

The well-known frequency weighted balanced model reduction with the Enns method is based on simultaneously diagonalizing the solution of Lyapunov equation (11) and (12). Since the matrices P_E and Q_E in these equations may not be positive semi-definite, the reduced order model obtained by this technique may not be stable.

However, it is known that with only one weighting present the stability of the reduced order model is guaranteed. The main contribution of this paper is the fact that for Metzlerian system and double-sided weights the reduced order models remain stable.

In the stability of the reduced order Metzlerian system, it is of interest to find conditions under which the solution of the Lyapunov matrix equation

$$A^TP+PA=-Q \tag{17}$$

is positive matrix, that is, its element are all positive in addition to its positive definiteness. The following theorem is useful for our main result.

Theorem 4: If a matrix A is Metzlerian and stable, then for any positive and positive definite symmetric Q there is a positive and positive definite symmetric matrix P as a solution of the Lyapunov equation (17).

Proof: The Lyapunov matrix equation (17) can be rewritten as a linear matrix equation

$$Mp=-q \tag{18}$$

where

$$M=A^T\otimes I+I\otimes A \tag{19}$$

is $n^2\times n^2$ an matrix with $\otimes$ denoting the Kronecker product. The matrix M is stable and by construction it is also Metzlerian. Since for any Metzlerian matrix $-M^{-1}>0$ (see Theorem 2) we conclude that for any $q>0$ we have $p>0$. The positive definiteness of P follows directly from stability result of the Lyapunov matrix equation.

Theorem 5: Let the system $\Sigma_S=(A,B,C,D)$ be Metzlerian stable and let $\Sigma_V=(A_V,B_V,C_V,D_V)$ and $\Sigma_W=(A_W,B_W,C_W,D_W)$ be minimal Metzlerian realization of input and output weights. Then the reduced order models obtained by direct truncation or singular perturbation approximation are stable for double-sided weightings.

Proof: It is easy to see that the augmented Metzlerian systems consists of cascade connections of Σ_S and Σ_V as well as Σ_W and Σ_S remain Metzlerian and stable. Consequently, the Lyapunov equations (9) and (10) have positive definite solutions $\overline{P}_i$ and $\overline{Q}_o$. By theorem 4 both of these matrices are elementwise positive as well since $\overline{A}_i$ and $\overline{A}_o$ are Metzlerian stable. Therefore, submatrices P_{11} and Q_{11} are also positive and positive definite. Consequently, the reduced order model $G_r(s) = C_1(sI - A_{11})^{-1}B_1 + D$ defined by truncation is stable. Similarly one can show that the reduced order model by singular perturbation approximation is also stable.

Theorem 6: Let $G(s)$ be stable transfer function of order n, $V(s)$ and $W(s)$ be the weighting functions. If $G_r(s)$ is stable reduced order model obtained by the above procedure, then the following error bound holds

$$\|W(s)(G(s) - G_r(s))V(s)\|_\infty \le 2\|W(s)\|_\infty \|V(s)\|_\infty \sum_{i=r+1}^{n} \sigma_i \quad (20)$$

Proof: The proof of the above theorem can directly be deduced from [14].

IV. ILLUSTRATIVE EXAMPLE

Example 1: Consider the fourth-order Metzlerian system (*A,B,C*) with the following parameter matrices

$$A = \begin{bmatrix} -0.9 & 0.2 & 0.1 & 0 \\ 1.5 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0.1 \\ 0 & 0 & 1 & -0.8 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 2 & 8 & 4.5 \end{bmatrix}$$

Applying the aggregation procedure with

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 \end{bmatrix}$$

the resulting third-order system (*F,G,H*) is obtained as:

$$F = \begin{bmatrix} -0.9 & 0.2 & 0.12 \\ 1.5 & -1 & 0 \\ 0 & 0.7 & -0.57 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 2 & 11.98 \end{bmatrix}$$

which is Metzlerian and stable. To check how dynamic exactness is achieved, the step response of the original and reduced system is shown for the purpose of comparison.

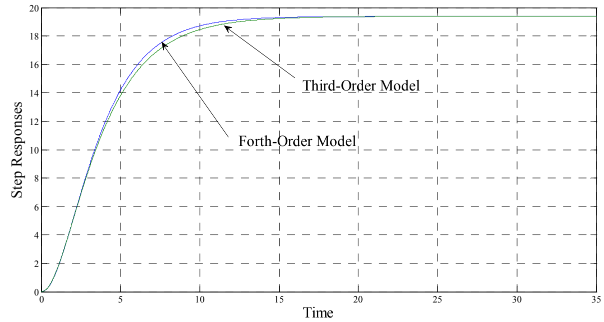

Fig. 1. Step response of of 4th and 3rd order systems

Since both the original model and the reduced order model are Metzlerian, one can use the direct formula of stability radius [10] (see also section III-A) as a measure of robustness. In this example, $r(A) = 0.3655$ and $r(F) = 0.3762$, which shows that full and reduced order model have comparable stability robustness properties.

Example 2: Now Consider an MIMO stable forth-order Metzlerian system (*A,B,C,D*) with the following parameter matrices

$$A = \begin{bmatrix} -2 & 1 & 2 & 0.5 \\ 1 & -3 & 0.25 & 0.6 \\ 2 & 0.25 & -5 & 0.75 \\ 0.7 & 0.6 & 0.75 & -6 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 0.5 & 1 \\ 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 & 2 & 1.5 \\ 3 & 1 & 0.5 & 2 \end{bmatrix}, \ D = \underline{0}$$

Moreover, let us use the following input and output weights

$$A_V = A_W = \begin{bmatrix} -4.5 & 0 \\ 0 & -4.5 \end{bmatrix}, \ B_V = B_W = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$C_V = C_W = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \ D_V = D_W = \underline{0}$$

Then, using the frequency weighted balanced technique described is section III, the matrices P_{11} and Q_{11} are obtained as

$$P_{11} = \begin{bmatrix} 44.6030 & 20.6446 & 21.5866 & 13.9881 \\ 20.6446 & 9.6418 & 9.9780 & 6.1340 \\ 21.5866 & 9.9780 & 10.6851 & 6.9514 \\ 13.9881 & 6.1340 & 6.9514 & 5.8521 \end{bmatrix}$$

$$Q_{11} = \begin{bmatrix} 63.0961 & 33.3649 & 31.1594 & 16.1370 \\ 33.3649 & 19.8226 & 17.5035 & 9.2333 \\ 31.1594 & 17.5035 & 15.9662 & 8.1553 \\ 16.1370 & 9.2333 & 8.1553 & 4.7149 \end{bmatrix}$$

and the system matrices of 2nd and 3rd order reduced models obtained by direct truncation are given by

$$A_{11,3}=\begin{bmatrix} -0.4011 & -0.2824 & -0.0691 \\ -0.2519 & -6.4360 & 0.1046 \\ 0.0959 & -0.3753 & -6.5588 \end{bmatrix}$$

$$A_{11,2}=\begin{bmatrix} -0.4011 & -0.2824 \\ -0.2519 & -6.4360 \end{bmatrix}$$

remain stable in the presence of double-sided weighting as it is proved.

V. CONCLUSION

In this paper, the problem of model reduction for the special class of continuous-time Metzlerian system was investigated. Two techniques were proposed under structural maintainability and stability preserving constraints. The first technique was based on aggregation, which led to a reduced order stable Metzlerian system. The second technique was the frequency weighted balancing to Metzlerian systems and it was shown that the reduced order model obtained by either truncation or singular perturbation approximation are guaranteed to be stable for the general case of double-sided weightings.

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