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Resource Pooling for Optimal Evacuation of a Large Building

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Abstract— **This paper is concerned with modeling, analysis and optimization/control of occupancy evolution in a large building. The main concern is efficient evacuation of a building in the event of threat or emergency. Complexity arises from the curse of dimensionality in a large building, as well as the uncertain and nonlinear dynamics of individuals. In this paper we propose relaxation techniques borrowed from queueing theory to address complexity. Then we provide tools to model occupancy evolution during egress, obtain lower bounds on evacuation time, and construct control solutions to instruct occupants in order to efficiently evacuate the building. The control solutions are based on recent generalizations of the MaxWeight policy for decentralized routing. These results are illustrated with the aid of simulations carried out using realistic building models.**

I. Introduction

The purpose of this paper is to introduce tools for the modeling, analysis, and optimization/control of occupancy evolution in a large building. There is currently great interest in these questions for various applications, including energy conservation and to aid first-responders in a burning building.[1]

The focus of this paper is *egress*: We restrict to a transient regime in which the occupants are exiting the building. Although, this is a daily routine for millions of people who egress from a building at the end of work day, we are especially motivated by situations arising due to an emergency, such as a fire or security threat. In such cases, the evacuation time is a critical factor whose reduction can help save lives by both evacuating people faster and providing earlier access to first responders.

Evacuation will be trivial if there is no congestion. In the simplest case in which there is a single occupant in the building, this agent will leave the building using the preferred/closest exit. However, congestion makes the evacuation problem much more interesting: If there are a large number of agents, and a large number of possible routes and exits, then the agent's *individual* best route to the closest exit may not be optimal for the overall evacuation problem.

Markovian models or more generally *Markov Decision Process (MDP) models* have been used to model the occupancy evolution in a single building floor example illustrated in Fig. 1. This particular example was also treated in [1] where each of 255 rectangular nodes can allow at most one agent. With a single agent, this leads to a Markov chain with 255 states. If the building is initially filled to capacity, that is the initial condition includes one agent at each node, then this grows to 2^{255} possible states. The complexity grows explosively with the introduction of control to obtain a MDP model. Moreover, the state space becomes infinite dimensional with the introduction of hidden Markov model to address the distributed sensing and control requirements. For the purposes of optimization, such models have no value in the applications of interest here.

In this paper we adapt workload relaxation techniques from queueing theory to formulate dynamic building models, and to address performance bounds and optimization problems in the context of building evacuation. The proposed model consists of two main components: 1) a family of Markov transition matrices that capture the typical behavior of a single agent who wishes to move towards one of the exits, and 2) a queue at each of the building nodes that indicates the number of agents at that node. There exist two mechanisms to model agent interactions: 1) the maximal service rate of queueing models is used to model local congestion, and 2) upper bounds on node occupancy are used to model physical constraints.

Two models are considered in this paper: a stochastic queueing model based on the Controlled Random Walk (CRW) model [2], and a fluid model describing the average behavior of the stochastic model. As in queueing theory, the fluid model is used to define workload and to obtain the bounds on evacuation time [3]. The CRW model is used for more fine-grained analysis, and also for simulation.

The policies introduced here are based on the h-MaxWeight policy of [4], which is a generalization of the MaxWeight policy of Tassiulas and Ephremides [5]. The MaxWeight policy can be expressed as the myopic policy for the fluid model with respect to a quadratic function

$$h(x) = \frac{1}{2}x^T Dx \tag{1}$$

in which the matrix D is positive and diagonal. Stability theory for this and similar classes of policies has been extended in multiple directions over the past fifteen years. In particular, these policies are known to be approximately optimal in heavy traffic under certain conditions on the network (see [6]).

It is also known that a MaxWeight policy may perform very poorly, in part because it makes use of so little information [7]. This has led to refinements such as the h-MaxWeight policy which is shown to be stabilizing and even approximately average-cost optimal under appropriate

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[1]For resources see the Illinois Fire Service Institute website http://www.fsi.uiuc.edu/content/research/agenda.cfm.

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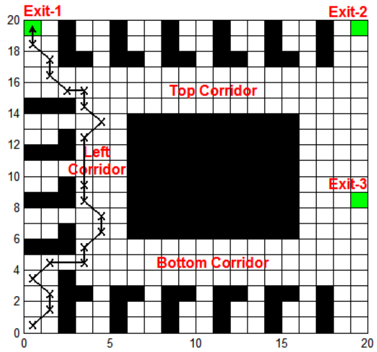

Fig. 1. Layout of Building where $\times$ corresponds to the agent, gray grid spaces are walls, and three green grids are the exits. The selected path shows that agent starts in the bottom-left office and moves towards the closest exit along the best route with the largest probability.

conditions on the function h and the network [4]. The proof of approximate optimality is based on a workload relaxation — a similar technique is used in this paper.

The remainder of the paper is organized as follows. In Section II we describe the queueing models and discuss the workload analysis. In Section III we use these models for the purposes of optimization using the h-MaxWeight policy based on workload relaxations of various dimensions. Section IV presents the simulation results for the building shown in Fig. 1. Conclusions and directions for future research are summarized in Section V.

II. Stochastic Agent Movement Model

Our starting point is a grid based model of the people (or agents) movement in a large building. We consider in this section only a highly stylized model in which all agents have identical behavior – this is captured by the specification of a single Markov transition matrix P that models local behavior as well as preference for exits. The model is constructed from two main parts:

1) The Markov transition matrix is used to model movement of a *single agent*, ignoring congestion effects.
2) Queues at every building nodes are used to describe the spatial distribution of the agents in the building. The queues are cleared according to a certain capacity constraint that serves to mimic the congestion effect due to agents blocking each other.

In section III we will extend this model for the purpose of control where an agent can choose from a family of Markov transition matrices. We begin by presenting details for the simplest model in the following:

A. Markov Transition Matrix

For the sake of exposition, we first assume that there is only a single agent. We denote the successive locations of the agent by $\{X(0), X(1), \ldots, X(t)\}$. Let the sequence $\{1, 2, \ldots, n\}$ denote the nodes in the building that can be occupied by the agent ($n = 255$ for the model shown in Fig. 1). The behavior of the agent is assumed to be Markovian: for each i, j, and each $t \geq 0$,

$$\begin{aligned} P(i,j) &:= \text{Prob}(X(t+1) = j \mid X(t) = i) \\ &= \text{Prob}(X(t+1) = j \mid X_0^t;\ X(t) = i) \end{aligned} \tag{2}$$

It is assumed that P is a sub-stochastic Markov transition matrix. If $p_i = \sum_j P(i,j) < 1$, then the agent leaves the building from node i with probability $(1 - p_i)$. In the evacuation applications considered in this paper the transition matrix P is defined by perturbing the baseline best route of an agent to the closest exit for each starting node in the building.

It is assumed that the substochastic matrix P has the following form: For certain special nodes, called *exit nodes*, we have $P(e, j) = 0$ for each $j = 1, \ldots, n$. The agent leaves the building via one of the exit nodes. For all other nodes i it is assumed that $P(i, \cdot)$ is an honest probability measure: An agent at node i will move according to this distribution. Finally, it is assumed that each node is transient in the sense that an agent eventually exits the building. This assumption is expressed by the existence of the inverse,

$$[I - P]^{-1} = \sum_{k=0}^{\infty} P^k. \tag{3}$$

In the following subsections, we construct two types of queueing models for the building: a stochastic model that simulates the agent movement in the building and a fluid model that describes the average behavior of occupancy evolution.

B. Stochastic Queueing Model

To describe the evolution of occupancy for a building with many agents we make following conventions:

1) The agents move according to the sub-stochastic Markov transition matrix introduced for the single agent model.
2) Associated with each node i in the building is a queue $Q_i(t)$ that indicates the number of agents at this node at time t. $Q(t) \in \mathbb{R}_+^n$ denotes the vector of n queues.
3) Each node with non-zero queue length serves at most c agents in one time step, i.e., at most c agents exit the node i at each time-step. We fix $c = 1$ in this paper.

The Markov transition matrix is used to model the preference of a typical agent. Service rate constraints mimic the effect of congestion due to agents blocking each other. In the presence of congestion, the queues build up and the evacuation time increases. The modeling framework also allows upper bound constraints on the number of agents at a node.

To describe the evolution of Q in a recursive form we introduce an i.i.d. process Δ that models potential transitions. For each t, the matrix $\Delta(t)$ has entries that are zero or one. It is assumed that the mean of $\Delta(t)$ is equal to P. If $\Delta_{ij}(t+1) = 1$ this means that an agent at node i will move to node j at time t, provided that there is an agent present at that node.

Letting $U_i(t) = \mathbb{1}\{Q_i(t) \geq 1\}$, the stochastic queueing model is defined by the recursion,

$$Q_i(t+1) = Q_i(t) + \sum_k \Delta_{ki}(t+1) U_k(t) - U_i(t) \tag{4}$$

for $i = 1, 2, \ldots, n$.

The model (4) given here is just the Controlled Random Walk (CRW) model defined in [2] (without arrivals).

C. Fluid Model

The fluid model is motivated by considering the average behavior of the CRW model. On denoting $\bar{q}_i(t) = \mathsf{E}[Q_i(t)]$ and $\bar{U}_i(t) = \mathsf{E}[U_i(t)]$ we obtain under the i.i.d. assumption on Δ,

$$\bar{q}_i(t+1) = \bar{q}_i(t) + \sum_k P(k,i)\bar{U}_k(t) - \bar{U}_i(t). \tag{5}$$

Letting $\bar{z}_i(t) = \sum_{s=0}^{t-1} \bar{U}_i(s)$ be the mean *cumulative activity process*, and denoting

$$B := -(I - P^T), \tag{6}$$

the recursion (5) can be expressed as

$$\bar{q}(t) = \bar{q}(0) + B\bar{z}(t). \tag{7}$$

The fluid model obeys analogous equations in continuous time. Its state process q evolves on $\mathbb{R}^n_+$ along with a cumulative activity process z that also evolves on $\mathbb{R}^n_+$. Following (7), the fluid model equations are given by,

$$q(t) = q(0) + Bz(t). \tag{8}$$

A related model is used in [8] for the purposes of occupancy estimation.

On denoting $C = I$, the *constituency matrix*, we impose the following constraints for this model: For each $0 \le t_0 \le t_1 < \infty$,

$$z(t_1) - z(t_0) \ge \mathbf{0}, \quad C(z(t_1) - z(t_0)) \le (t_1 - t_0)\mathbf{1}, \tag{9}$$

where $\mathbf{1}$ and $\mathbf{0}$ are column vectors of ones and zeros, respectively.

We can also write this as a *controlled differential equation*,

$$\tfrac{d}{dt} q(t) = B\zeta(t) \tag{10}$$

in which $\zeta(t)$ is the time derivative of $z(t)$. It is subject to the constraints $0 \le \zeta_i(t) \le 1$ for each i and t.

D. Workload Vectors and Evacuation Time

In this section we construct workload vectors based on the fluid model.

The minimal evacuation time T^* for the fluid model from an initial condition $q(0) = x$ can be obtained by solving the following linear program,

$$\begin{aligned} T^*(x) = \min\ & T \\ \text{s.t.}\ & x + Bz = \mathbf{0} \\ & Cz \le T\mathbf{1},\ z \ge \mathbf{0}. \end{aligned} \tag{11}$$

In this simple model the matrix B is square and invertible, so that $z = -B^{-1}x$. The existence of the inverse is due to the assumption that every node is transient (see (3)).

Feasibility requires that $CB^{-1}x \ge -\mathbf{1}T$ and $B^{-1}x \le \mathbf{0}$. The second condition is automatically satisfied for the models described using (6). The first condition motivates the definition of the *workload matrix*

$$\Xi = -CB^{-1}. \tag{12}$$

The rows of the workload matrix are called *workload vectors*, and are denoted,

$$\Xi = [\xi^1 \mid \xi^2 \mid \cdots \mid \xi^n]^T. \tag{13}$$

The feasibility constraint $CB^{-1}x \ge -\mathbf{1}T$ is equivalent to the constraint that $\langle \xi^j, x\rangle \le T$ for each j. This bound can be achieved, from which it follows that the solution to the linear program (11) is obtained as the maximum,

$$T^*(x) = \max_{1 \le j \le n} \langle \xi^j, x\rangle. \tag{14}$$

The quantity $\xi^j_i x_i = \xi^j_i q_i(0)$ is the time that node j serves the traffic originated from node i. $T^*(x)$ is determined by the node j that needs to work the longest. This node has a natural interpretation as the most congested node in the building.

E. Bounds on Evacuation Time

$T^*(x)$ gives a lower bound on the evacuation time for the fluid model. As the following proposition shows, it also provides a lower bound for the CRW model (4).

Proposition 1 *The following bound on the time τ to egress (empty the building) holds for any policy applied to the CRW model:*

$$\mathsf{E}_x[\tau] \ge T^*(x). \tag{15}$$

Proof: The proof relies on the dynamic programming equation for the fluid model,

$$\min_{\zeta} \tfrac{d}{dt} T^*(q(t)) = -1. \tag{16}$$

This implies that for any time t, and any feasible control for the fluid model,

$$T^*(q(t+1)) \ge T^*(q(t)) - 1. \tag{17}$$

We now turn to the CRW model. Since the function T^* is convex, then by Jensen's inequality,

$$\begin{aligned} &\mathsf{E}[T^*(Q(t+1)) \mid Q(t) = x,\ U(t) = u] \\ &\qquad \ge T^*\big(\mathsf{E}[Q(t+1) \mid Q(t) = x,\ U(t) = u]\big) \\ &\qquad = T^*(x + Bu) \end{aligned} \tag{18}$$

Returning to the fluid model, if $q(t) = x$, then $q(t+s) = x + (Bu)s$, $0 \le s \le 1$ is feasible for the fluid model. The bound (17) then gives,

$$\mathsf{E}[T^*(Q(t+1)) \mid Q(t) = x,\ U(t) = u] \ge T^*(x) - 1. \tag{19}$$

We conclude that the process defined below is a submartingale,

$$M(t) := T^*(Q(t)) + t, \qquad t = 0, 1, 2, \ldots \tag{20}$$

The submartingale property gives, for any initial condition $Q(0) = x$,

$$\mathsf{E}_x[M(t \wedge \tau)] \ge M(0) = T^*(x). \tag{21}$$

It can be shown that $\{M(t\wedge\tau) : t\ge 0\}$ is uniformly integrable if the mean of τ is finite. Consequently, we can let $t\to\infty$ to obtain,

$$\mathsf{E}_x[T^*(Q(\tau))+\tau] = \mathsf{E}_x[M(\tau)] \ge M(0) = T^*(x). \qquad (22)$$

The proof is completed on observing that $T^*(Q(\tau)) = 0$. ■

III. Optimization and Control

We now extend the homogeneous fluid model introduced in the last section to allow a wider range of decisions by the agents in the building.

A. Control Model

Suppose that an agent has a choice of behaviors, or that the population is heterogeneous. We model this more general situation using a family of transition matrices $\{P_a : 1\le a\le r\}$ parametrized according to r routes out of the building. In the example considered here we take $r=3$. The a^{th} matrix captures an agent's preference for the a^{th} exit. P_0 is used to denote the transition matrix where an agent always prefers the closest exit.

We maintain the linear fluid model (10) in which the matrix B and the constituency matrix C are redefined as follows,

$$B = [B_1 \mid B_2 \mid \cdots \mid B_r] \qquad (23)$$

where $B_a = -(I-P_a)^T$ for $a=1,\ldots,r$. The constraints (9) are maintained, where the constituency matrix is redefined as,

$$C = [I \mid I \mid \cdots \mid I]. \qquad (24)$$

The constraint $C\zeta \le \mathbf{1}$ reflects the assumption that an agent can choose at most one behavior at each time step. The control problem will involve choosing, in real-time, from one of these matrices thereby deciding or modifying the route.

Recall that the constraints defined in (9) are used to model the effect of agents blocking each other. The control problem for the fluid model is to choose $\zeta(t)$ with the objective of minimizing the total evacuation time T. The solution to this problem is again obtained by the construction of workload vectors.

B. Workload Vectors

Let $x=q(0)$ denote the initial distribution of agents inside the building. One can then pose a linear program whose solution gives the minimal evacuation time. It has the same form as (11), but in the more general model considered here the matrix B is not square, so the solution must be obtained by considering the linear program in a greater detail.

We obtain a representation of $T^*(x)$ in terms of workload vectors by considering the dual of (11). Letting $\xi\in\mathbb{R}^n$ denote the dual variable corresponding to the equality constraint $x+Bz=\mathbf{0}$, and $\nu\in\mathbb{R}^n$ the dual variable corresponding to the inequality constraint $Cz\le T\mathbf{1}$, the dual of (11) is expressed,

$$\begin{aligned} T^*(x) = \max\ & \langle \xi, x\rangle \\ \text{s.t.}\ & -B^T\xi - C^T\nu \le \mathbf{0} \\ & \mathbf{1}^T\nu \le 1,\ \nu\ge \mathbf{0}. \end{aligned} \qquad (25)$$

Letting $\{\xi^i,\nu^i\}$ denote the extreme points of this linear program, the minimal evacuation time can be expressed as the finite maximum,

$$T^*(x) = \max\ \langle \xi^i, x\rangle. \qquad (26)$$

Proposition 6.1.5 of [2] states that each $\nu^i\in\mathbb{R}^n_+$ satisfies $\langle \nu^i,\mathbf{1}\rangle = 1$. Since this is the dual variable corresponding to the resource constraints $Cz\le T\mathbf{1}$, the vector ν^i can be interpreted as a probability distribution on these resources.

The complementary slackness condition holds: If z^* is an optimizer of (11), and i^* a maximizer in (26), then

$$\langle \nu^{i^*}, Cz^*\rangle = T^*(x)\langle \nu^{i^*},\mathbf{1}\rangle = T^*(x). \qquad (27)$$

This and the fact that ν^{i^*} is a probability distribution implies that whenever the index j satisfies $\nu^{i^*}_j > 0$, the corresponding resource j must work at capacity for all $t\in[0,T^*)$. Written in terms of the rate $\zeta^* = z^*/T^*$, this conclusion is expressed as,

$$(C\zeta^*)_j = 1, \qquad \text{whenever } \nu^{i^*}_j > 0. \qquad (28)$$

Following the terminology in the stochastic networks literature, we say that there exists *resource pooling* among the resources $\{j : \nu^{i^*}_j > 0\}$ [9].

In the example considered in this paper we find that typically $\nu^{i^*}_j$ is zero for all values of j except those corresponding to the exit nodes (see discussion surrounding Fig. 3).

The value ξ_j is the sensitivity of the minimal evacuation time with respect to queue x_j. It follows that $\xi^i\in\mathbb{R}^n_+$ for each i that is a unique maximizer of (26) for some x. In the numerical examples considered we find that there may be many entries of ξ^i that are zero (see Fig. 3). We say that each workload vector defines a partition of the building graph based on the region $A=\{i : \xi^i_j > 0\}$ and its complement $A^c = \{j : \xi^i_j = 0\}$. The boundary between A and A^c is analogous to a *cut* as defined in classical graph theory [10]. The region in the building defined by the set A is congested, in the sense that the evacuation time is sensitive to the total occupancy in this region, while sensitivity to the occupancy in A^c is zero. This intuition will be used to guide the construction of policies that concentrate on pushing agents from the congested region towards the exits or the un-congested region. Congestion may vary with time, so the policies are necessarily dynamic and based on feedback.

C. MaxWeight Evacuation

For the purposes of control we utilize a generalization of the MaxWeight policy [5]. The methodology and the notation here follows closely [2], [4]. Let $h_0\colon \mathbb{R}^n_+\to\mathbb{R}_+$ denote a convex, monotone function that vanishes only at the origin. This is interpreted as an approximate value function for an associated optimal control problem.

We apply a change of variables using the *perturbed state vector* with entries,

$$\tilde{x}_i = x_i \log(1+\theta^{-1}x_i), \qquad 1\le i\le n \qquad (29)$$

where θ is a parameter that is selected to obtain the best performance. We denote by h the perturbed function,

$$h(x) := h_0(\tilde{x}), \qquad x \in \mathbb{R}^n_+. \tag{30}$$

In one class of policies considered in the numerical results that follow we first fix a single workload vector ξ, and set h_0 to be a quadratic,

$$h_0(x) = \tfrac{1}{2}\left(w^2 + (c(x) - \bar{c}(w))^2\right) \tag{31}$$

where $w = \xi^T x$ is the *workload*, $c(x) = \sum_i x_i$ is the *cost function*, and $\bar{c}(w)$ is the *effective cost function* obtained from the following linear program (see [2, Chap. 6] for more details),

$$\begin{aligned} \bar{c}(w) = \min\ & c(x) \\ \text{s.t. } & \xi^T x = w,\ x \in \mathbb{R}^n_+. \end{aligned} \tag{32}$$

Its solution is $\bar{c}(w) = \bar{c}_0 w$ where $\bar{c}_0 = (\max_j \xi_j)^{-1}$.

The h-MaxWeight policy is simply the associated myopic policy for the fluid model:

$$\zeta(t) = \arg\min_{\zeta} \tfrac{d}{dt} h(q(t)) = \arg\min_{\zeta} \langle \nabla h(q(t)), B\zeta \rangle. \tag{33}$$

We express this as the feedback law $\zeta(t) = \phi^{\text{MW}}(q(t))$ where,

$$\phi^{\text{MW}}(x) := \arg\min_{\zeta} \langle \nabla h(x), B\zeta \rangle, \qquad x \in \mathbb{R}^n_+. \tag{34}$$

The gradient of h is given by,

$$\nabla h(x) = L_\theta(x) \nabla h_0(\tilde{x}) \tag{35}$$

where

$$L_\theta(x) = \text{diag}\left(\frac{x}{\theta + x} + \log\left(1 + \frac{x}{\theta}\right)\right), \tag{36}$$

and

$$\nabla h_0(x) = (\xi^T x)\xi + ((\mathbf{1} - \bar{c}_0 \xi)^T x)(\mathbf{1} - \bar{c}_0 \xi). \tag{37}$$

Note that h is convex and monotone since this is assumed for h_0.

The perturbed state vector is introduced to ensure that the h-myopic policy for the fluid model (the policy (34) in which the minimum is not subject to integral constraints) is feasible for the CRW model. The form of the gradient (35) implies that $\partial h(x)/\partial x_i = 0$ whenever $x_i = 0$. This property ensures that there is a disincentive to work on an empty queue in the fluid model.

D. Information Structure for the h-MaxWeight Policy

For the i^{th} node in the building, the h-MaxWeight policy arises as:

$$\begin{aligned} \phi_i^{\text{MW}}(x) \in & \arg\max_{a=1,\dots,r} \left(\sum_{j=1}^{n} -B_a^T(i,j) \nabla h_j(x) \right) \\ = & \arg\max_{a=1,\dots,r} \Big(\nabla h_i(x) - \sum_{P_a(i,k)>0} P_a(i,k) \nabla h_k(x) \Big), \end{aligned} \tag{38}$$

where $\nabla h_i(x) = \frac{\partial h}{\partial x_i}(x)$. Note that the resulting policy is not distributed. However, the evaluation of the gradient term $\nabla h(x)$ requires knowledge of only two pieces of global information, the values of $\xi^T \tilde{x}$ and $\mathbf{1}^T \tilde{x}$. The evaluation of the second-term on the right hand side in (38) also requires queue length information from the one-hop neighborhood of node i (nodes k such that $P_a(i,k) > 0$ for some a). The latter information can be obtained using local communication.

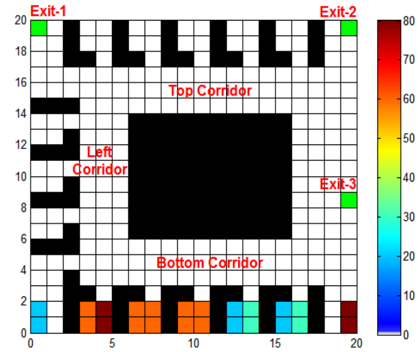

Fig. 2. Displays the initial distribution of the agents in the whole building where different colors indicate different number of agents contained on each cell.

IV. Simulation Results

A. Simulation Setup

The simulation is initialized with 1,160 agents in the bottom offices of the building (see Fig. 2). By solving the linear program (11), we obtained the minimal evacuation time $T^* = 386$ for the fluid model as well as workload vectors. These workload vectors are used to obtain the h-MaxWeight evacuation policy.

B. Simulation Results and Discussion

1) *Best route policy*. With this policy there is no feedback control and each agent tries to leave the building via their own closest exit. The matrix P_0 is used as the transition matrix. This policy performs poorly because it does not efficiently utilize all of the available resources (the corridors and exits shown in Fig. 2). After a short transient of relatively rapid decrease in the number of agents, both Exit 1 and Exit 3 become congested as do the left and bottom corridors. In contrast, the top corridor and Exit 2 are never used. Fig. 3 (left) depicts the flow direction for the movement of agents. The evacuation time using this policy is almost three times the lower bound $T^*(x) = 386$ obtained in Prop. 1 for the mean evacuation time, see Fig. 4.

These results provide a motivation for the use of optimization and control for the purposes of better resource utilization.

2) *h-MaxWeight policy*. In Fig. 4, h-MaxWeight-1 shows simulation results of h-MaxWight policy based on the 1st workload vector (see (30) and (31)). The 1st workload vector is depicted on the right hand side in Fig. 3. It defines a partition for the building into two regions: the region $A^c = \{i : \xi_i = 0\}$ contains the three exit nodes while the region $A = \{i : \xi_i > 0\}$ contains the remaining transient nodes. In the simulation, the h-MaxWeight policy is implemented according to (38).

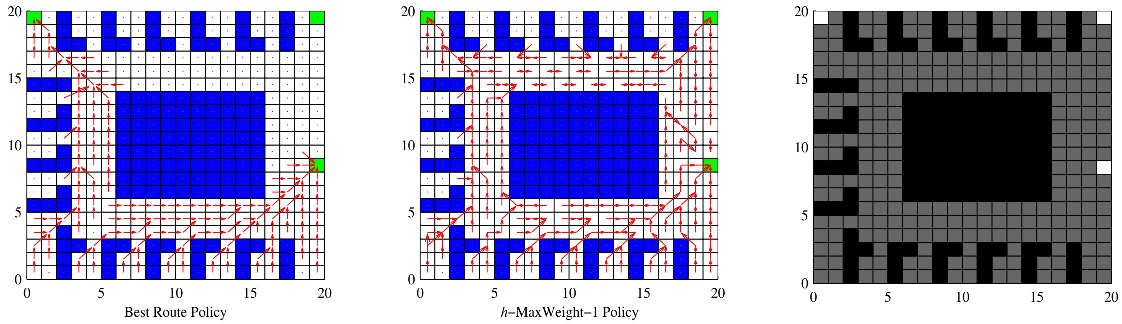

Fig. 3. Depicts the most possible flow direction of agent movement during the whole evacuation process for Best Route Policy (left) and h-MaxWeight-1 Policy (middle). Also depicts 1st Workload Vector (right) where gray grid spaces (region A) indicate nonzero entries of the workload vector, white grid spaces (region A^c) indicate zero entries of the workload vector.

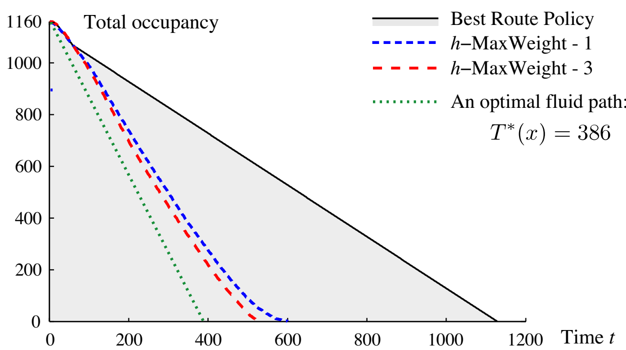

Fig. 4. Comparison of the evacuation times obtained using three policies for the CRW model. Also shown is an optimal trajectory for the fluid model. The h-MaxWeight-3 policy most closely reaches the performance of the optimal fluid trajectory, with $T = 528$ (approximately).

Fig. 3 (middle) depicts the flow direction for the movement of agents obtained using this policy. The agents starting in the left region of the building move towards Exit 1 while the remaining agents move towards Exit 3. This is just like the best route policy as shown in Fig. 3 (left). The main difference is that as the simulation progresses, the agents begin to move towards Exit 2 as the first and third exits become congested. Inefficiencies were also observed in simulations: the myopic nature of the policy leads to multiple streams of agents crossing each other at the top corridor.

Multiple workload vectors (i.e. the first three ones that capture the dominant characteristics of this building problem) can also be considered in h-MaxWeight policy. But the average behavior of agent movement is very similar to the single workload vector case (see h-MaxWeight-3 of Fig. 4).

In closing, we note that either h-MaxWeight policy provides a much better evacuation performance than obtained with the best route policy. In fact, more than fifty percent of evacuation time can be saved by h-MaxWeight policy than the best route policy under this simulation setup (see Fig. 4).

V. Conclusions

We have demonstrated that workload relaxation techniques from queueing theory can be applied to address performance bounds and optimization problems in the context of building evacuation. Two models were considered: a stochastic queueing model based on the Controlled Random Walk model of [2], and a fluid model that describes the average behavior of the stochastic model. The h-MaxWeight policy showed considerable improvement in evacuation time over the baseline where each agent uses their best route.

In future work, we will consider the stability of the h-MaxWeight policy for the stochastic model. From an implementation standpoint, we will also consider consensus based methods for distributed implementation of these policies.

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