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On Modeling and Observer Design of Fluid Flow Dynamics for Petroleum Drilling Operations

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Abstract— **This paper presents a method for constructing an empirical simplified model of the fluid flow in the well during drilling operations. The model is fitted towards a snapshot of data generated by a detailed model. An observer is designed for the simplified model, utilizing the fast updated measurements at the surface, instead of slow updated and time-lagged downhole measurements. The observer is evaluated against the detailed fluid flow model. The results show that the designed observer is capable of following the detailed model sufficiently accurate.**

I. Introduction

Stabilizing the pressure throughout a wellbore is of particular interest in oil well drilling applications so as to take advantage of ensuring safety, increasing the penetration rate of the drill bit, and avoiding formation damage in the reservoir which eventually increases the production rate of the oil. Several different types of equipment can be employed in stabilizing the pressure during drilling [9]. However, such kind of equipment is rather expensive to procure. Moreover, most wells nowadays are still drilled using conventional drilling equipment where the annulus section of the well is open to the atmosphere.

Typically, drilling systems are operated manually by the drilling crew where various actuators of the drilling system, such as pumps and valves, are adjusted independently. In this case, several measurements are available for the drilling crew. However, some of the main parameters are not typically measured, such as the downhole pressure and the flow rate in the annulus section. And even if the downhole measurements are available, the downhole measurements are sampled at a much lower rate. These measurements are also time lagged due to the data transmission method using standard mud pulse telemetry. Moreover, the accuracy of the measurements may not be reliable. This leads to the need of defining an observer for the system.

An observer for the drilling system can be designed in several ways. In [18], an observer in the form of ensemble Kalman filter [3], [4] is designed based on a detailed two-phase fluid flow model. The ensemble Kalman filter calculates the model parameter variance using several instances of the model. The ensemble Kalman filter is designed based on high-order models. Another work by [7] utilizes the unscented Kalman filter [14] for evaluating the frictional fluid flow parameters during drilling applications. This work shows that by tuning the frictional parameters of a detailed fluid flow model, a fairly good match between the measured data and the model can be obtained. This method has also been evaluated in an actual drilling application with promising results [13], where the focus has been on using the model to optimize and safe-guard the drilling process. The ensemble Kalman filter and the unscented Kalman filters have also been used to estimate the reservoir parameters during drilling operations [21], [22].

However, using an observer based on a detailed model is computationally intensive, especially for a model predictive control setting [11], [23]. Another direction will be to design an observer based on a more simplified model [24]. In this case, a model based on a first order approximation is combined with an unscented Kalman filter. The filter is used as an observer for a model predictive control scheme. The resulting controller shows fairly well results in stabilizing the pressure during large fluid flow rate changes. A recent work [31] describes an adaptive observer for a simplified model. The simplified model is tuned with respect to a detailed flow model. The observer shows good convergence when being used towards the same simplified model.

This paper presents a method for constructing another type of simplified model using a data set generated by a detailed flow model. The simplified model is in the form of polynomial differential equations. The polynomial structure of the model gives an advantage from computational point of view. The structure benefits from the use of sum of squares programming [26] which is amenable to computer solution. In short, the model will be easier to handle for analysis and synthesis purposes. As a starting point in this direction, an observer is designed based on the simplified model. The observer is evaluated towards the detailed model using only the surface measurements.

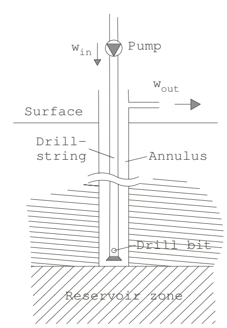

Fig. 1. Drilling of a well into an oil reservoir.

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II. Process Description

An oil well is typically drilled by using a drill string with a drill bit attached to it; see Figure 1. During this process, drilling fluid is circulated through the drill string and the drill bit. The drill bit is equipped with a check valve, which prevents the drilling fluid in the annulus to return into the drill string.

The drilling fluid is non-Newtonian where one of the important properties is the ability to transport cuttings from the drill bit and up through the annulus. The fluid flow through the drill string is typically in a turbulent regime. The drilling fluid has then a low viscosity and has reduced frictional effects. When the drilling fluid is flowing through the drill bit nozzles, then the velocity is being reduced. This is due to the fact that the flow area in the annulus is larger than the flow area in the drill string. When the drilling fluid changes velocity, the viscosity of the drilling fluid changes, giving better cuttings transportation properties, and also increased frictional effects.

In a drilling operation, the drilling crew might need to adjust the cuttings transportation properties. This is normally done by changing the velocity of the drilling fluid at the bit. However, this causes fluctuations in the downhole pressure. The pressure itself is one of the most important factors to determine the success of a drilling process into a formation. If the pressure is too low, then the fluid from the reservoir might enter the wellbore. On the other hand, if the wellbore pressure is too high, then the pressure might fracture the formation. These limitations leads to a need for a coordinated control of both the downhole pressure and the cuttings transportation properties.

III. Review of Drilling Fluid Flow Models Used for Observer Designs

This section presents different modeling efforts for describing the dynamic pressure variations in drilling operations, ranging from detailed models where the focus is to model the process as detail as possible, to more simplified models that are able to represent the most important dynamic behaviour of the well.

A. Detailed flow modeling

In principle, the dynamics of the downhole pressure should be modeled accurately to describe a more realistic drilling process as a whole. However, since the downhole pressure is very dependent on other parameters such as the density and friction pressure losses, the modeling efforts can be very complex. This is especially true when the pressure in the well is below the reservoir pore pressure, resulting in an influx of gas when drilling in gas reservoirs. The annulus part of the well will then have two-phase flow conditions which, in this case, add even more complexity to the modeling of the well fluid flow.

To this end, the research on dynamic well modeling had been mainly focused on model designs for accurately describing the fluid flow along a well, using multi-phase, multi component dynamic models. The model is derived from momentum and mass balance principles which come in the form of partial differential equations. A numerical scheme, which takes multi-phase fluid dynamics into account, has been developed over several years by [10], [29], [30] and verified with several experimental tests by [12], [16], [17]. The type of models used here calculates the behaviour of the fluids in more detail. The spatial discretization divides the well into several boxes, and the mass balance for each box is calculated along with the pressure balance, giving mass for each phase and velocity of the mixture as well as the pressure in the box. This results in a high-order state vector for a model with several boxes.

B. Low-order model based on mass balance and pressure balance

In the model of [20], the flow pattern in both the drill string and the annulus is assumed to be uniform. Therefore, the well is divided into two compartments with different dynamics, the drill string and the annulus. The inter-connection between these two compartments is modeled using mass balances and pressure balances. In addition to the mass balances, a pressure balance is set up. This is because the friction pressure in the well is strongly related to the flow rates in the well, causing unsteady flow conditions [19]. The pressure balance is set up at the drill bit at the bottom of the well, and at the choke valve at the exit of the well. The closure relations between masses, flow rates and pressures are further described in [23]. This type of simple modeling methodology can be used for design and analysis of a control system for the pressure in the well.

C. Simplified model using direct pressure modeling

Another drilling model has been developed to be used for control purposes during drilling [15]. The idea of this approach is to model the pressure dynamics directly, and design the model sufficiently simple so that analysis and synthesis can be performed by utilizing Lyapunov theory. An observer scheme designed for this model is further described in [31].

D. Simplified model using first order approximations of flow velocity

The model in [24] is based on a first order approximation of the fluid flow velocity in a pipe segment. The pressures in the well are influenced by the frictional pressure losses in the drillstring, the annulus and the choke line. This model layout is able to capture the most important dynamics that are present during a pipe connection operation. In [24] an observer scheme in the form of an unscented Kalman filter is designed for this type of model.

IV. Simplified Model from Empirical Data

The model in [15] describes the dynamics of the pressures in the topside and in the bottomhole during drilling. The model consists of nonlinear differential equations with dimension three in the case of closed annulus, and dimension two in the case of open annulus. The model parameters are

TABLE I
WELL GEOMETRY AND FLUID DATA

Parameter	Value
Well length, h_a	2580 m
Well vertical depth, L_a	1950 m
Casing inner diameter, D_a	0.2169 m
Drill string outer diameter, D_d	0.1270 m
Drill string inner diameter, D_d	0.1087 m
Initial main pump flow rate, q_p	1000 l/min
Oil density, ρ	854 kg/m^3
Water density, ρ	1000 kg/m^3
Baryte density, ρ	4027 kg/m^3
Oil-Water Ratio	3.11
Drilling fluid mixture density, ρ	1610 kg/m^3

tuned to fit the trajectories generated by the detailed model of drilling in the simulator WeMod [25] for adaptive observer design [28].

In this section we construct an empirical simplified model based on the snapshot of data which are also generated by the detailed model in WeMod [25]. As an example, we consider the detailed model representing an actual off-shore drilling operation in the North Sea. The well is partly vertical and partly inclined, that is 1950 m deep and 2580 m long. The fluid used in the simulations is a mixture of oil, water and baryte. The various simulation parameters are found in Table I.

The input to the simulator is the main pump rate in the topside. From practical point of view in real drilling operations, there are some restrictions on how the rate of the topside pump should be fed to enhance safety during drilling. In our case, since we are working with a simulator, we have more freedom to apply a wider possibilities of inputs to excite the detailed model in order to obtain a better identification result. However, as a starting point, this paper mainly concentrates on the construction of the empirical model. The subject of persistent excitation of the inputs is beyond the scope of the paper and will be considered in the future. Thus, we will only show how to construct an empirical model based on the data generated by the sinusoidal inputs to the simulator WeMod. Similarly, the method may also be used for the data generated by other types of inputs.

The empirical simplified model describes a particular dynamics of the pressure in the topside and the pressure in the bottomhole during drilling process. The model is in the form of polynomial differential equations where the number of the monomial terms of the vector fields depends on the accuracy and purpose of the model. The model is given by

$$\dot{x}_1 = \sum_{j=1}^{m_1} a_{1j} p_{1j}(x) + \sum_{j=1}^{r_1} b_{1j} q_{1j}(x) u \tag{1a}$$

$$\dot{x}_2 = \sum_{j=1}^{m_2} a_{2j} p_{2j}(x) + \sum_{j=1}^{r_2} b_{2j} q_{2j}(x) u \tag{1b}$$

where x_1 is the mudpump pressure in the topside, x_2 is the downhole pressure, p_{ij} and q_{ij} are monomials in x and u is the main pump rate. The parameters $a_1^T = \begin{bmatrix} a_{11} & \dots & a_{1m_1} \end{bmatrix}$, $a_2^T = \begin{bmatrix} a_{21} & \dots & a_{2m_2} \end{bmatrix}$, $b_1^T = \begin{bmatrix} b_{11} & \dots & b_{1r_1} \end{bmatrix}$, $b_2^T = \begin{bmatrix} b_{21} & \dots & b_{2r_2} \end{bmatrix}$ determine the characteristics of the drilling system.

The problem of estimating the parameters

$$\beta = \begin{bmatrix} a_1^T & a_2^T & b_1^T & b_2^T \end{bmatrix} \in \mathbb{R}^{n_\theta}$$

is the center of this section. To be more precise, the problem of estimating the parameters is formulated as follows.

Problem 1 *Suppose that the parameters in β are time invariant. Given are the collection of data, in the form of the inputs u and the outputs $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$. Determine an appropriate scheme to estimate the parameters.*

As the dynamics of the drilling system are much slower than the sampling time T_s we could consider the case when the forward difference method, i.e. $\dot{x}(t) \approx \frac{x(t+T_s)-x(t)}{T_s}$, is applied to solve (1). The sampling time T_s is considered to be constant. The discrete-time version of (1) can be written as

$$z_i(k) = \phi_i(k)\beta_i, \quad i = 1, 2 \tag{2}$$

where

$$\begin{aligned} z_i(k) &= \frac{x_i(k+1) - x_i(k)}{T_s}, \\ \psi_i(k) &= \big[\; p_{i1}(x(k)) \;\; \dots \;\; p_{im_i}(x(k)) \\ &\qquad q_{i1}(x(k))\,u(k) \;\; \dots \;\; q_{ir_i}(x(k))\,u(k) \;\big] \\ \beta_i &= \begin{bmatrix} a_i^T & b_i^T \end{bmatrix}. \end{aligned}$$

Let the data be the sets of input $\{u(k)\}_{k=1}^{N-1}$ with output response $\{x(k)\}_{k=1}^{N}$. For the number of data measurement N, (2) gives

$$Z_i = \Upsilon_i \beta_i, \quad i = 1, 2$$

where

$$\begin{aligned} Z_i &= \begin{bmatrix} z_i(1) & \dots & z_i(N-1) \end{bmatrix}^T, \\ \Upsilon_i &= \begin{bmatrix} \psi_i^T(1) & \dots & \psi_i^T(N-1) \end{bmatrix}^T. \end{aligned}$$

Notice that $Z_i \in \mathbb{R}^{N-1}$ and $\Upsilon_i \in \mathbb{R}^{(N-1)\times n_{\theta_i}}$. Computing the solution β_i from the data Z_i and Υ_i might result in an ill-posed problem if the matrix Υ_i is ill conditioned. This problem arises if the data are not rich enough to excite the modes representing the behaviour of the system. One way to solve this problem is by seeking an approximate solution by regularization so that the solution is smooth.

To improve the condition of the matrix Υ_i we define

$$\begin{aligned} v_{ij}^p &= \begin{bmatrix} p_{ij}(x(1)) & \dots & p_{ij}(x(N-1)) \end{bmatrix}^T, \\ v_{ij}^q &= \big[\; q_{ij}(x(1))\,u(1) \;\; \dots \\ &\qquad q_{ij}(x(N-1))\,u(N-1) \big]^T, \\ \phi_i(k) &= \Bigg[\; \frac{p_{i1}(x(k))}{\|v_{i1}^p\|} \;\; \dots \;\; \frac{p_{im_i}(x(k))}{\|v_{im_i}^p\|} \\ &\qquad \frac{q_{i1}(x(k))u(k)}{\|v_{i1}^q\|} \;\; \dots \;\; \frac{q_{ir_i}(x(k))u(k)}{\|v_{ir_i}^q\|} \;\Bigg], \end{aligned}$$

$$\Phi_i = \left[\begin{array}{ccc} \phi_i^T(1) & \dots & \phi_i^T(N-1) \end{array} \right]^T,$$

where

$$Z_i = \Phi_i \theta_i,$$

with

$$\begin{aligned} \theta_i = \Big[& a_{i1}\left\|v_{i1}^p\right\| \quad \dots \quad a_{im_i}\left\|v_{im_i}^p\right\| \\ & b_{i1}\left\|v_{i1}^q\right\| \quad \dots \quad b_{ir_i}\left\|v_{ir_i}^q\right\| \Big]^T. \end{aligned}$$

In this case, the condition number of Φ_i is better than that of Υ_i. It is standard in the textbooks [1], [6], [27] that the regressor matrix Φ_i can be written in terms of the singular value decomposition, that is

$$\Phi_i = U_i S_i V_i'.$$

This decomposition is one of the most celebrated and is called SVD (Singular Value Decomposition). This decomposition can be used for solving the least squares method, that is

$$\hat{\theta}_i = \sum_{j=1}^{p} \frac{U_{ij}' Z_i}{\sigma_{ij}} V_{ij} \tag{3}$$

where $p \leq \min(m, N-1)$ is the rank of the matrix Φ, the vectors U_{ij} and V_{ij} are the j-thorhonormal column of the orthonormal matrices U_i and V_i, respectively and the singular value σ_{ij} is the j-th diagonal element of S_i.

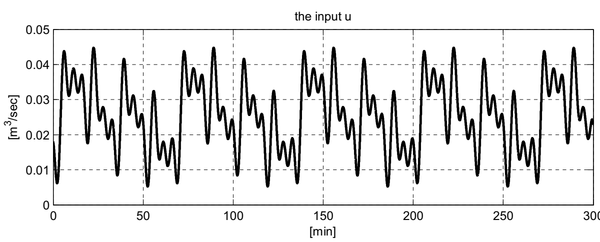

Fig. 2. Sinusoidal input to the detailed model

In most practical cases, numerically computing the pseudoinverse of the matrix Φ_i will, at best, produce the pseudoinverse of a perturbed matrix $\Phi_i + E_i$. In this case the mathematical notion of rank is not appropriate [1]. Therefore, a definition of numerical rank is used as follows.

Definition 1 *[1] A matrix Φ_i has numerical δ-rank equal to k if*

$$\sigma_{i1} \geq \ldots \geq \sigma_{ik} > \delta \geq \sigma_{i,k+1} \geq \ldots \geq \sigma_{ip}$$

where $\sigma_{i1} \geq \ldots \geq \sigma_{ip} > 0$ are the singular values of Φ_i.

When there is a well-defined gap between σ_{ik} and $\sigma_{i,k+1}$, Definition 1 can be used to determine the rank of Φ_i [1]. Apparently this is not in the situation for some datasets we encounter. By way of example we consider a set of data generated by the detailed model representing the drilling process in the North Sea. The data are with respect to the inputs in the form of sinusoidal as shown in Fig. 2. The response of the detailed model to the sinusoidal input can

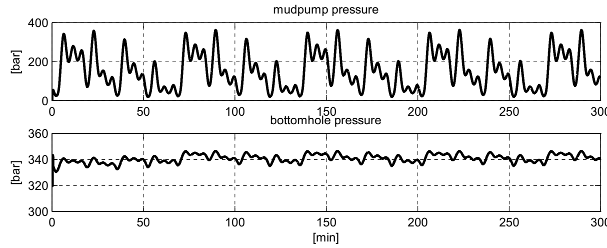

Fig. 3. The response of the detailed model to the sinusoidal input

be seen in Fig. 3. The set contains 18,000 time-steps of data ($N = 18,000$) with sampling time $T_s = 1$ second. Suppose we consider

$$\begin{aligned} \psi_i = \Big[& 1 \quad x_1 \quad x_2 \quad x_1^2 \quad x_1x_2 \quad x_2^2 \quad x_1^3 \quad x_2^3 \quad u \\ & x_1^2u \quad x_2^2u \Big] \end{aligned}$$

for $i = 1, 2$. The singular values of the matrix Φ_i with respect to the data can be seen in Fig. 4. The singular values decay gradually to zero and there is no gap in its singular value spectrum. In this case, it is difficult to define the numerical rank because we deal with a discrete ill-posed problem which is defined as follows.

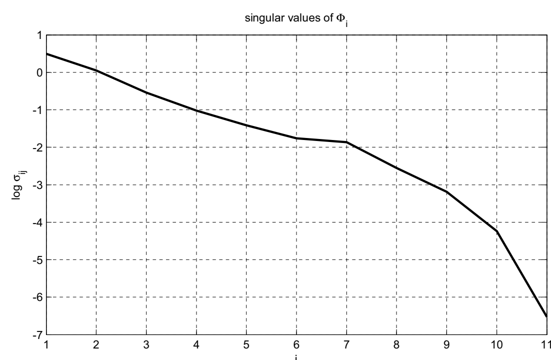

Fig. 4. Singular values of the matrix Φ_i for $i = 1, 2$

Definition 2 *[8] Consider a system in the form*

$$Z_i = \Phi_i \theta_i, \;\; \Phi_i \in \mathbb{R}^{(N-1)\times n_{\theta_i}}, \; N-1 > n_{\theta_i},$$

the problem of determining θ_i from the given Φ_i and Z_i is a discrete ill-posed problem if both of the following criteria are satisfied:

1) *the singular values of Φ_i decay gradually to zero,*
2) *the ratio between the largest and the smallest nonzero singular values is large.*

Criterion 2 indicates that the condition number of the matrix Φ_i is large which means that the matrix is ill-conditioned.

Discrete ill-posed problems mostly arise from discretization of ill-posed problems and the solutions to discrete ill-posed problems are very sensitive to errors [5], [8]. As shown by (3), division by a very small singular value σ_{ij} might dominate the solution of $\hat{\theta}_i$ with the errors in Z_i. To limit the effect of the noisy data, the solution is to truncate the sum at $k \leq p$, that is

$$\hat{\theta}_i^{trunc,k} = \sum_{j=1}^{k} \frac{U_{ij}^T Z_i}{\sigma_{ij}} V_{ij}. \tag{4}$$

Equation (4) is the least squares solution of

$$Z_i = \Phi_{i,k}\theta_i,$$

where

$$\Phi_{i,k} = \sum_{j=1}^{k} \sigma_{ij} U_{ij} V_{ij}^T.$$

This means that by truncating at k, the matrix Φ_i is approximated by the lower-rank matrix $\Phi_{i,k}$.

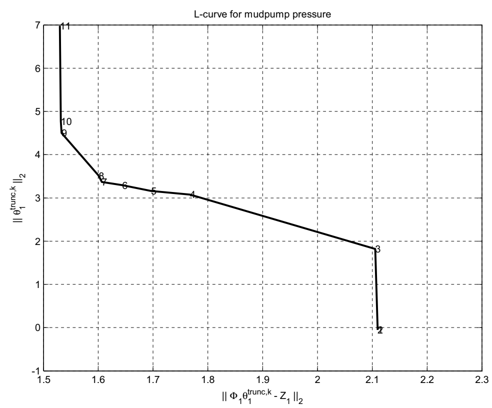

Fig. 5. The $L-curve$ of the mudpump pressure based on the data generated by the detailed model

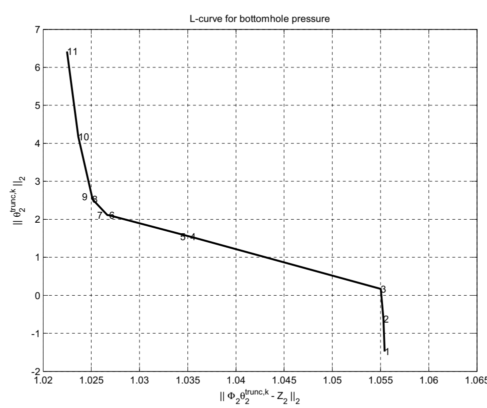

Fig. 6. The $L-curve$ of the downhole pressure based on the data generated by the detailed model

Truncating the SVD at k as in (4) leads to the question how to choose the appropriate k. Rather than observing the shape of the singular values spectrum of Φ_i, Hansen proposes the $L-curve$ to deal with the problem of ill-determined numerical rank [8]. The curve is a logarithmic plot of the norm of the truncated solution $\left(\left\|\hat{\theta}_i^{trunc,k}\right\|_2\right)$ against the corresponding residual norm $\left(\left\|\Phi_i\hat{\theta}_i^{trunc,k} - Z_i\right\|_2\right)$. The curve displays the compromise between minimization of these two quantities. With the compromise in mind, the number k can be chosen based on the corner of the curve. The idea behind this choice is that the corner separates the flat and vertical parts of the curve where the solution is dominated by truncation errors and perturbation errors, respectively.

We take again the set of data from the previous case. The plots of the $L-curve$ for the mudpump pressure and the downhole pressure can be seen in Fig. 5 and 6, respectively. It is quite clear that the corner is around 7 and 6, respectively. Thus we can choose k around 7 for the mudpump pressure and around 6 for the downhole pressure. By truncating the solution at k we obtain the estimate of the parameters for our empirical model.

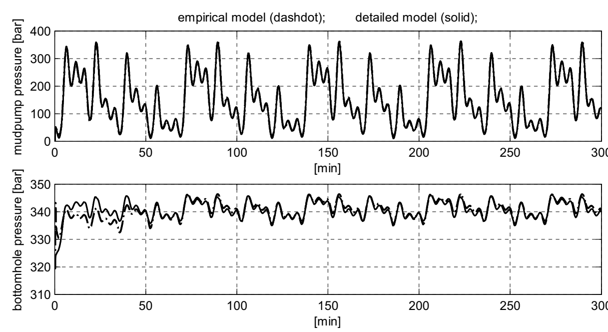

Fig. 7. Comparison between the response of the detailed model and that of the empirical model

To see how well the response of the empirical model fits to that of the detailed model we perform a simulation using the empirical model. The results can be seen in Fig. 7 where the empirical model gives good accuracy for the mudpump pressure in the topside (x_1) while for the downhole pressure (x_2) the result is quite accurate after the transient period.

The same procedure can be carried out for other types of representation of ψ_i or for other specific windows of trajectories (for example during transient time). We need to point out that not all type of data will result in the discrete ill-posed problem. Nevertheless, the $L-curve$ can still provide indication for selecting the truncation number leading to more stable solutions on the least mean squares problems. An important issue worth mentioning on the method of constructing a simplified model in this section is on choosing the right representation of the vector fields. There is no systematic means yet on choosing which monomial terms and how many monomial terms needed to accurately capture a specific behaviour of the drilling processes. One possible direction will be on the use of the Akaike's information criterion (AIC) for choosing the number of monomial terms.

V. Observer Design

In this section, we consider the problem of designing observer as follows.

Problem 2 *Given the system (1), design an observer for the system based on the measurement of the mudpump pressure in the topside, i.e.* $y = x_1$.

For the general structure of (1), this is a difficult task. However, there exist several observer designs from the literatures which can fit to several classes of the system (1). The easiest one is for linear representation

$$\psi_i = \begin{bmatrix} x_1 & x_2 & 1 & u \end{bmatrix}$$

for $i = 1, 2$ where a Luenberger observer can be designed provided a certain observability condition is satisfied. However, this observer might perform poorly especially in the region with high nonlinearity. Indeed, an observer for representation which captures nonlinearity is an advantage.

In this section, we consider another type of observer designed for nonlinear representation

$$\psi_i = \begin{bmatrix} x_1 & x_2 & x_1 x_2 & 1 & u \end{bmatrix}$$

for $i = 1, 2$ which can capture the transient dynamics of the drilling process. The observer is given as follows.

Proposition 1 *Consider the system*

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_1x_2 + a_{14} + b_{11}u, && \text{(5a)} \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_1x_2 + a_{24} + b_{21}u, && \text{(5b)} \\ y &= x_1. && \text{(5c)} \end{aligned}$$

If there exist a positive-definite symmetric matrix $Q \in \mathbb{R}^{2\times 2}$ *and a matrix* $M \in \mathbb{R}^{2\times 1}$ *such that*

$$\begin{aligned} QA^T + MC + AQ + C^T M^T &\prec 0 \\ Q \quad G = & \; C^T \end{aligned}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; G = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix},$$

then for $L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}^T$ *given by*

$$L = Q^{-1}M - \frac{y^2}{\varepsilon} Q^{-1} C^T$$

where $\varepsilon > 0$ *is sufficiently small we can guarantee that the system*

$$\begin{aligned} \dot{\hat{x}}_1 &= a_{11}\hat{x}_1 + a_{12}\hat{x}_2 + a_{13}y\hat{x}_2 + a_{14} + b_{11}u + L_1(\hat{y} - y), \\ \dot{\hat{x}}_2 &= a_{21}\hat{x}_1 + a_{22}\hat{x}_2 + a_{23}y\hat{x}_2 + a_{24} + b_{21}u + L_2(\hat{y} - y), \\ \hat{y} &= \hat{x}_1. \end{aligned}$$

is a globally asymptotically stabilizing observer for the system (5).

Proof: Denote

$$\Phi(x, y) = yx_2, \; \phi(u) = \begin{bmatrix} a_{14} + b_{11}u \\ a_{24} + b_{21}u \end{bmatrix},$$

where

$$\|\Phi(x, y) - \Phi(\hat{x}, y)\| \le |y| \, . \, \|x - \hat{x}\| \, .$$

The result follows from Theorem 7 of [2]. ■

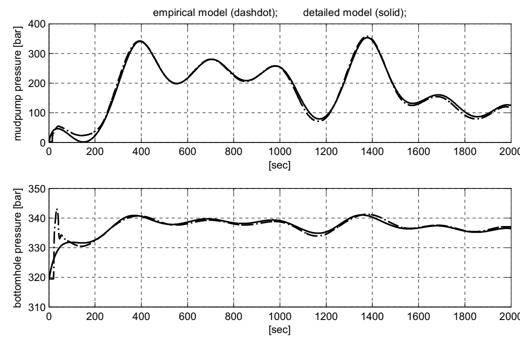

Fig. 8. Comparison between the response of the detailed model and that of the empirical model based on the first 2000 steps of the dataset

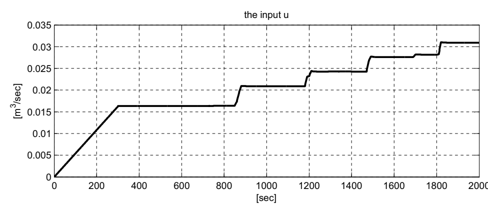

Fig. 9. Step input to the detailed model

As an example, we consider the first 2000 steps of the dataset from the previous section. Using the scheme from the previous section we construct the empirical model (5). From Fig. 8 we can see how well the response of the empirical model is with respect to the dataset generated from the detailed model. The observer from Proposition 1 can be applied to estimate the state x_2 based on the measurement $y = x_1$.

To asses the observer against the model error between the detailed model and the simplified model (5) we generate a set of different trajectories from the detailed model using a series of inputs related to a realistic drilling operation, see Fig. 9. In this case, initially, the drilling fluid is flowing through the main pump at volume flow rate of 1000 l/min. Then the flow rate is gradually increased in steps until 2000 l/min is reached. The downhole pressure is gradually increasing due to the frictional pressure losses in the well. The simulations are performed using the Matlab-interface described in [25]. Instead of using the measurement $y = x_1$ from (5) we use the measurement from the detailed model to feed the observer. The results are shown in Fig. 10 where the observer can estimate the downhole pressure from the measurement of the mudpump pressure of the detailed model, up to a certain degree of accuracy.

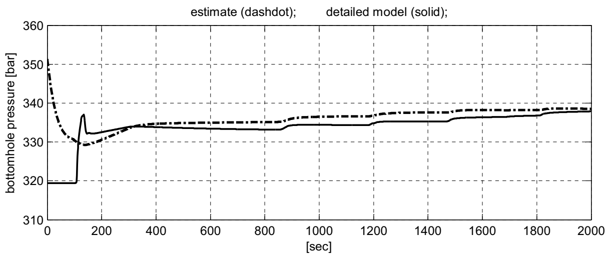

Fig. 10. The estimate and the real downhole pressure from the detailed model

VI. CONCLUSION

This paper presents a dynamical model of the fluid flow in drilling operations. This model is based on empirical evaluation of the pressure dynamics during flow rate changes, and is suitable to be used for observer design.

The results show that the model representation presented is able to describe the dynamics during flow rate changes sufficiently accurate to fit the pressure measurements. The results also indicate the designed observer for the downhole pressure might use only the surface pressure measurements.

For future research, we will focus on observer design for a wider class of system. This is of interest to cover broader operational changes during a drilling operation, such as drillstring movements. The observer will also be evaluated using real drilling data, to verify the observer in an actual drilling operation.

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