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Frequency Domain Iterative Tuning for the Control of Nonlinear Vibrations

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Abstract— **A general gradient estimation method is proposed that relies on the computation of system dynamics and signals in the frequency domain. With the theory presented the computation of the gradient of an average linear quadratic (LQ) performance criterion can be simplified using only 'partial-modelling' with respect a finite signals spectrum. Using gradient estimate a new iterative tuning (IT) method is presented for nonlinear Active Noise and Vibration Control (ANVC) problems. The most important novelty of the method is that it performs one experiment per iteration that makes it suitable for implementation as a self-tuning and adaptive controller. Apart from the reduced number of experiments, relative to time domain IFT methods, the new method has the added advantage of its suitability for enhanced disturbance rejection tuning in the frequency domain. The effectiveness of the method is demonstrated on physical laboratory hardware.**

Iterative controller tuning, frequency domain methods, active noise and vibration control, nonlinear control

I. INTRODUCTION

Iterative feedback tuning in the time domain (TD-IFT) that relies on measured input-output data to tune the controller without estimating a model of the plant, had been investigated [1], [2], [3], [4] and it turned out to be an effective iterative tuning (IT) control method. It does not need explicit modelling but requires additional signal injection path and extra manual experiments in each tuning iteration.

This paper investigates iterative tuning of controllers for vibration attenuation of nonlinear plants when all disturbance signals are periodic. Prior to this research a frequency domain iterative tuning (FD-IT) method has been developed for linear ANVC problems with periodic disturbances in [5]. Although FD-IT has been initially developed for linear problems, its idea can be extended to the control of some nonlinear plants.

This paper also proposes a new gradient estimation theory derived from the gradient estimation theory in [5] by extending it to nonlinear plants. The new method relies on local linearisation and analysis in the frequency domain. Within this approach the nonlinear dynamics is presented as a local linear mapping from one multi-dimensional space (representing the input spectrum) to another multi-dimensional space (representing the output spectrum). When the performance function, that is an averaged quadratic criterion, can be represented with the spectra of the signals, the gradient of the performance can be completely represented using the local linear mapping in the frequency domain.

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In case of finite discrete spectrum disturbances that result in an essentially finite spectrum of all signal, only partial modelling with respect to the contained frequencies is required to compute the gradient of average performance with respect to tunable control parameters. Based on the gradient estimation method presented here, a new self-tuning method, namely the Nonlinear Frequency Domain Iterative Tuning (NL-FD-IT) method, is developed to solve control problems with approximately finite-spectrum signals. The new method only requires one experiment per iteration apart from some extra manual experiments needed at startup for initialization. Tuning simultaneously takes place while TD-IFT in earlier publications have had to perform multiple experiments for feedback and feed-forward controllers separately.

While active noise and vibration control (ANVC) often deals with periodic disturbances, it is one of the most important target areas of FD-IT's application. The initial idea of ANVC has been reported in the early 1930s [6], and the underlying physical theory is well established [7]. One of the most commonly used and well understood methods is the filtered-x LMS algorithm [8], which requires the model of the plant and is generally used to tune a feed-forward controller. The FSF-IFT method [9] and frequency domain tuning (FDT) method [10] all require additional injection paths and extra experiments in each tuning iteration. This paper discussed the extension of gradient estimation theory in the frequency domain to nonlinear cases, and gives a very prototype of an iterative tuning method for nonlinear ANVC problems. Like FD-IT for linear systems, NL-FD-IT has some advantages (over time domain methods) for applications: no parametric modelling is needed and only partial frequency response modelling is needed to deal with periodic disturbances. According to the simulation example and laboratory experimental example, the feasibility of NL-FD-IT is illustrated. Although there are some further improvement to complete NL-FD-IT, the fundamental tuning strategy of FD-IT is proved to be an effective and simple adaptive method to solve both linear and nonlinear ANVC problems.

The remainder of this paper is organized as follows. In Section II the problem of gradient-based tuning control for ANVC is shortly reviewed in both the time domain and the frequency domain. In Section III a gradient estimation theory is proposed that operates in the frequency domain. NL-FD-IT is developed and some implementation issues are discussed in Section IV. In section V experimental work is presented to show the effectiveness of NL-FD-IT in nonlinear ANVC problems, respectively. Finally conclusions are drawn in the last section.

II. Gradient Based Iterative Tuning for ANVC

This section presents the ANVC problem addressed in this paper, the fundamental equations and performance function will be defined and the essence of iterative tuning (IT) is formulated in the context of the ANVC problem.

A. Problem setting in the time domain

Fig. 1 displays a schematic description of the control system considered in this paper.

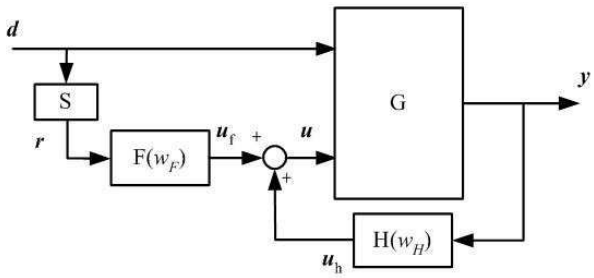

Fig. 1. Block diagram of a linear feedforward feedback system of ANVC system

The measured output, which is affected by the disturbance $\boldsymbol{d} \in \mathbb{R}^{n_d}$, is represented by $\boldsymbol{y} \in \mathbb{R}^{n_y}$. G is the unknown plant dynamics with inputs $\boldsymbol{d}$ and $\boldsymbol{u}$ and produce $\boldsymbol{y}$. It can be described as

$$\boldsymbol{y} = G(\boldsymbol{d}, \boldsymbol{u}) \tag{1}$$

The control signals from the feedforward controller F and feedback controller H are denoted by $\boldsymbol{u}_f \in \mathbb{R}^{n_u}$ and $\boldsymbol{u}_h \in \mathbb{R}^{n_u}$, respectively. The tunable control system C comprises the parameterized feedforward controller F and the feedback controller H:

$$C(\boldsymbol{w}, \boldsymbol{r}, \boldsymbol{y}): \quad \begin{array}{rcl} F: \boldsymbol{u}_f &=& F(\boldsymbol{w}_F, \boldsymbol{r}) \\ H: \boldsymbol{u}_h &=& H(\boldsymbol{w}_H, \boldsymbol{y}) \\ \boldsymbol{u} &=& \boldsymbol{u}_f + \boldsymbol{u}_h \end{array} \tag{2}$$

which can be tuned by adjusting their parameter vectors in $\boldsymbol{w} := \{\boldsymbol{w}_F, \boldsymbol{w}_H\} \in \mathbb{R}^{n_w}$.

It is assumed that the disturbance-reference signal $\boldsymbol{r} \in \mathbb{R}^{n_r}$ is obtained through an unknown dynamics S from $\boldsymbol{d}$ while the output signal $\boldsymbol{y}(t)$ is measurable and recordable. The disturbance signal $\boldsymbol{d}$ cannot be measured directly.

In order to apply iterative tuning method, it is always assumed the system states can be repeated after a certain iteration period under stationary conditions. In this case, the steady input and output of the system are repeated as well after a certain common period.

If the ANVC system has steady output $\boldsymbol{y}$ with period N, the control performance criterion is defined as an average quadratic performance with respect to a length N output sequence:

$$J(\boldsymbol{w}) := \sum_{t=0}^{N-1} \boldsymbol{y}^{\mathrm{T}}(t) Q \boldsymbol{y}(t) \tag{3}$$

where Q is a priori known weighting matrix.

The objective of controller tuning is to tune the controller parameters $\boldsymbol{w}_F$ and $\boldsymbol{w}_H$ to minimize performance (3), which can be represented as an optimization problem in mathematics as:

$$\begin{array}{rl} \min: & J(\boldsymbol{w}) \text{ in (3)}, \\ s.t. & \text{Eqn. (1)}, \\ & \text{Eqn. (2)}. \end{array} \tag{4}$$

In general the problem of minimizing $J(\boldsymbol{w}_F, \boldsymbol{w}_H)$ is not necessarily convex. The tuning method only finds a suboptimal solution at a local minimum. This suboptimal solution of the problem is given as a solution of

$$\nabla J(\boldsymbol{w}_F^o, \boldsymbol{w}_H^o) = \mathbf{0} \tag{5}$$

For further interest about global optimization, some work [11], [12], [13], [14] can be referred.

In general a practically efficient method to solve this kind of optimization problem is a negative gradient-based algorithm such as Newton's optimization method. The gradient-based tuning is the one of the most efficient and popular approaches in adaptive control.

B. Problem setting in the frequency domain

In this subsection the ANVC problem as in Fig.1 with cost function (3) will be addressed. First some frequency domain notations are defined for further discussion.

Consider the SISO discrete system as described by Fig.1 with repeated iteration of a common period N. There is an N-length output data set $\boldsymbol{y} := \{y(0); \ldots; y(N-1)\} \in \mathrm{R}^N$. Let $\omega_m := \frac{2\pi m}{N}, m = 0, \ldots, N-1$ denote discrete frequencies. $\boldsymbol{\phi}_y := \{\phi_y(\omega_0); \ldots; \phi_y(\omega_{N-1})\} \in \mathbb{C}^N$ is the discrete spectrum of the N-length time sequence $\boldsymbol{y}$ and can be estimated through the Discrete Fourier Transform (DFT) i.e. $\boldsymbol{\phi}_y \doteq \mathrm{DFT}(\boldsymbol{y})$. There are similar notations of $\boldsymbol{\phi}_d$, $\boldsymbol{\phi}_r$, $\boldsymbol{\phi}_{uf}$ and $\boldsymbol{\phi}_{uh}$.

In the frequency domain the plant G can be described as a function $\{\boldsymbol{\phi}_d, \boldsymbol{\phi}_u\} \in \mathbb{C}^{2N} \mapsto \boldsymbol{\phi}_y \in \mathbb{C}^N$:

$$\boldsymbol{\phi}_y = \Phi_G(\boldsymbol{\phi}_d, \boldsymbol{\phi}_u), \tag{6}$$

and controller system C can be also described as function $\{\boldsymbol{w}, \boldsymbol{\phi}_r, \boldsymbol{\phi}_y\} \in \mathbb{C}^{n_w + 2N} \mapsto \boldsymbol{\phi}_u \in \mathbb{C}^N$:

$$\Phi_C(\boldsymbol{w}, \boldsymbol{\phi}_r, \boldsymbol{\phi}_y): \quad \begin{array}{rcl} \Phi_F: \boldsymbol{\phi}_{uf} &=& \Phi_F(\boldsymbol{w}_F, \boldsymbol{\phi}_r) \\ \Phi_H: \boldsymbol{\phi}_{uh} &=& \Phi_H(\boldsymbol{w}_H, \boldsymbol{\phi}_y) \\ \boldsymbol{\phi}_u &=& \boldsymbol{\phi}_{uf} + \boldsymbol{\phi}_{uh} \end{array} \tag{7}$$

According to Parseval's Theorem [15], it is straightforward to rewrite (3) in the frequency domain format as

$$J = \frac{1}{N} \sum_{i=0}^{N-1} \phi_y^*(\omega_i) Q_F(\omega_i) \phi_y(\omega_i) = \frac{1}{N^2} \boldsymbol{\phi}_y^* Q_F \boldsymbol{\phi}_y \tag{8}$$

where Q_F is the representation of Q in the frequency domain.

Similarly, the optimization problem in mathematics can be written as:

$$
\begin{aligned}
\min: \quad & J(\boldsymbol{w}) \text{ in (8)}, \\
s.t. \quad & \text{Eqn. (6)}, \\
& \text{Eqn. (7)}.
\end{aligned} \tag{9}
$$

It is noted that the system as Fig.1 is represented in the frequency domain as general mappings between two multi-dimension spaces, which is not limited in linear or nonlinear dynamics. While the mapping of linear dynamics can be illustrated by a Frequency Response Function (FRF) matrix, the mapping of nonlinear dynamics in the frequency domain can be only explained by Generalized Frequency Response Function (GFRF) [16], [17] that is based on Volterra/Wiener series function [18], [19], [20].

Therefore, when the average LQ cost function (3) can be represented in the frequency domain as multiplication of discrete spectra, the above representation in the frequency domain is suitable to demonstrate a general system including nonlinear systems.

III. Gradient Estimate in the Frequency Domain

In this section, a new gradient estimation theory is proposed from the aspect of the frequency domain, which is based on the reformatted ANVC problem setting in the frequency domain in the above section.

A. Gradient Estimation in the Frequency Domain

While discussing gradient-based algorithms, the differentiable condition is always necessary. Similarly, considering iterative tuning in the frequency domain, it is assumed that Φ_G, Φ_F and Φ_H are differentiable functions with respect to their input spectra ($\boldsymbol{\phi}_u, \boldsymbol{\phi}_r, \boldsymbol{\phi}_y$) and tunable parameters ($\boldsymbol{w}$).

In order to discover the relationship between the performance (8) and control parameters $\boldsymbol{w}$, local linearization can be performed and computed using infinitesimal increments.

Considering the ANVC system as Fig. 1, the local linearization of Φ_G can be described as:

$$
\Delta\boldsymbol{\phi}_y \approx \frac{\mathrm{d}\boldsymbol{\phi}_y}{\mathrm{d}\{\boldsymbol{\phi}_d, \boldsymbol{\phi}_u\}} \begin{bmatrix} \Delta\boldsymbol{\phi}_d \\ \Delta\boldsymbol{\phi}_u \end{bmatrix} \tag{10}
$$

Since $\boldsymbol{\phi}_d$ is fixed due to stationary disturbance, only the case $\Delta\boldsymbol{\phi}_d = \mathbf{0}$ is needed to be discussed.

To simplify the presentation in the following discussions, some frequently used notations are defined as

$$
\Phi_{G'} := \frac{\partial \Phi_G(\boldsymbol{\phi}_d, \boldsymbol{\phi}_u)}{\partial \boldsymbol{\phi}_u} \in \mathbb{C}^{N\times N}, \tag{11}
$$

and similarly

$$
\Phi_{H'} := \frac{\partial \Phi_H(\boldsymbol{w}_H, \boldsymbol{\phi}_y)}{\partial \boldsymbol{\phi}_y} \in \mathbb{C}^{N\times N} \tag{12}
$$

Given $\boldsymbol{\phi}_y$, $\boldsymbol{\phi}_u$ and $\Phi_{G'}$, the infinitesimal increment of plant dynamics G in the frequency domain with respect to $\Delta\boldsymbol{\phi}_u$ can be written as

$$
\Delta\boldsymbol{\phi}_y = \Phi_{G'}(\Delta\boldsymbol{\phi}_{uf} + \Delta\boldsymbol{\phi}_{uh}) \tag{13}
$$

Considering small increment $\Delta\boldsymbol{\phi}_u$ caused by the small update of parameter $\boldsymbol{w}$, i.e., $\boldsymbol{w}_F \to \boldsymbol{w}_F + \Delta\boldsymbol{w}_F$ and $\boldsymbol{w}_H \to \boldsymbol{w}_H + \Delta\boldsymbol{w}_H$, it is straightforward to write

$$
\begin{aligned}
\Delta\boldsymbol{\phi}_y \quad = \quad & \Phi_{G'}\big(\tfrac{\partial \Phi_F(\boldsymbol{\phi}_r, \boldsymbol{w}_F)}{\partial \boldsymbol{w}_F}\Delta\boldsymbol{w}_F + \tfrac{\partial \Phi_H(\boldsymbol{\phi}_y, \boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\Delta\boldsymbol{w}_H \\
& + \Phi_{H'}\Delta\boldsymbol{\phi}_y\big)
\end{aligned} \tag{14}
$$

Denoting $\Delta\boldsymbol{\phi}^w_{uf} := \frac{\partial \Phi_F(\boldsymbol{\phi}_r, \boldsymbol{w}_F)}{\partial \boldsymbol{w}_F}\Delta\boldsymbol{w}_F, \Delta\boldsymbol{\phi}^w_{uh} := \frac{\partial \Phi_H(\boldsymbol{\phi}_y, \boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\Delta\boldsymbol{w}_H, \Delta\boldsymbol{\phi}^y_{uh} := \Phi_{H'}\Delta\boldsymbol{\phi}_y$, (14) can be graphically described by following Fig. 2.

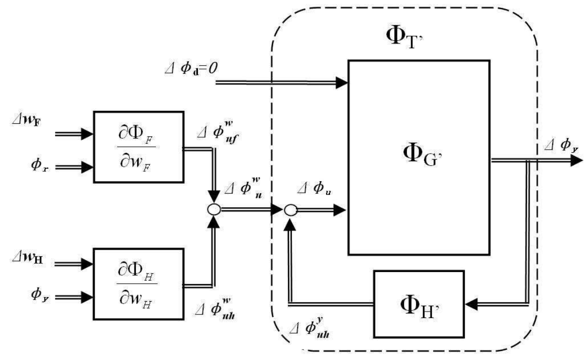

Fig. 2. Block diagram of small increment in frequency domain

According to (14), when considering the plant G as the unknown control object, the mapping is $\Delta\boldsymbol{\phi}_u \mapsto \Delta\boldsymbol{\phi}_y$.

The change in the feed forward path $\boldsymbol{\phi}^w_{uf}$ is straight forward, which is caused by the change of parameters Δw_F. At the same time, the increment in feedback path comprises two parts: $\boldsymbol{\phi}^w_{uh}$, caused by the change of controller parameter $\Delta\boldsymbol{w}_H$; $\boldsymbol{\phi}^y_{uh}$, caused by the change of system output $\Delta\boldsymbol{\phi}_y$.

Considering the part in $\Delta\boldsymbol{\phi}_u$ caused directly by the Δw, if $(I - \Phi_{G'}\Phi_{H'})^{-1}$ exists, the mapping $\{\Delta\boldsymbol{\phi}^w_{uf} + \Delta\boldsymbol{\phi}^w_{uh}\} \mapsto \Delta\boldsymbol{\phi}_y$ can be rewritten from (14) as

$$
\Delta\boldsymbol{\phi}_y = (I - \Phi_{G'}\Phi_{H'})^{-1}\Phi_{G'}(\Delta\boldsymbol{\phi}^w_{uf} + \Delta\boldsymbol{\phi}^w_{uh}) \tag{15}
$$

Considering the closed-loop system $T := \{G, H\}$ as the unknown control object, $(I - \Phi_{G'}\Phi_{H'})^{-1}\Phi_{G'}$ is the partial derivative of Φ_T with respect of the change of the spectrum of input signals $\Delta\phi_u$ when fixing controllers.

Introducing the notation

$$
\Phi_{T'} := (I - \Phi_{G'}\Phi_{H'})^{-1}\Phi_{G'}, \tag{16}
$$

and using (15), the partial derivative of $\boldsymbol{\phi}_y$ with respect to controller parameters $\boldsymbol{w}_H$ and $\boldsymbol{w}_F$ can be written as

$$
\frac{\partial \boldsymbol{\phi}_y}{\partial \boldsymbol{w}_H} = \Phi_{T'}\frac{\partial \Phi_H(\boldsymbol{\phi}_y, \boldsymbol{w}_H)}{\partial \boldsymbol{w}_H} \tag{17}
$$

and

$$
\frac{\partial \boldsymbol{\phi}_y}{\partial \boldsymbol{w}_F} = \Phi_{T'}\frac{\partial \Phi_F(\boldsymbol{\phi}_r, \boldsymbol{w}_F)}{\partial \boldsymbol{w}_F} \tag{18}
$$

The derivative of performance J with respect to the controller parameters can be written in the frequency domain format as

$$
\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}_i} = \frac{2}{N^2}\boldsymbol{\phi}^*_y Q_F \Phi_{T'}\frac{\partial \Phi_C(\boldsymbol{w}, \boldsymbol{\phi}_y, \boldsymbol{\phi}_r)}{\partial \boldsymbol{w}_i} \tag{19}
$$

In ANVC, while the output $\boldsymbol{y}$, the controller H and F are all known, $\boldsymbol{\phi}_y$ and $\frac{\partial \Phi_C(\boldsymbol{w},\boldsymbol{\phi}_y,\boldsymbol{\phi}_r)}{\partial \boldsymbol{w}_i}$ are both available in (19). The key to gradient estimation turns out to be to find an estimate of $\Phi_{T'}$.

B. Gradient estimation in systems with periodic disturbance

In the above subsection, (19) gives a full-bandwidth format in the frequency domain including the full-bandwidth from ω_0 to ω_{N-1}. It is not convenient to compute the gradient because N frequencies are required to proceed in the full band. Fortunately, thanks to the computations in the frequency domain, the estimation of the performance gradient can be greatly simplified in systems with periodic input $\boldsymbol{u}$ and output $\boldsymbol{y}$.

Note that $\boldsymbol{\phi}_y^*$ can be considered as a weighted factor vector when (19) is rewritten as

$$\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}_i} = \frac{2}{N^2}\sum_{n=0}^{N-1}\phi_y^*(\omega_n)\frac{\partial \phi_y(\omega_n)}{\partial \boldsymbol{w}_i} \tag{20}$$

Considering periodic output $\boldsymbol{y}$ with common period N, only the finite frequency set $\Omega_y := \{\omega_0,\ldots,\omega_{n\Omega_y}\}$ are included in $\boldsymbol{\phi}_y$ that is denoted by $\boldsymbol{\phi}_y|_{\Omega_y}$, and the other elements in $\boldsymbol{\phi}_y$ are 0. To solve $\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}_i}$ in (19), only the elements in $\frac{\partial \boldsymbol{\phi}_y}{\partial \boldsymbol{w}_i}$ with respect to $\boldsymbol{\phi}_y|_{\Omega_y}$ are required to consider, i.e.,

$$\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}_i} = \frac{2}{N^2}\sum_{\omega\in\Omega_y}\phi_y^*(\omega)\frac{\partial \phi_y(\omega)}{\partial \boldsymbol{w}_i} \tag{21}$$

Compared with the gradient estimate in the LTI system [5], the gradient estimate is somewhat more complicated in the nonlinear case as the frequency responses can be coupled. Given $\boldsymbol{\phi}_r$,$\boldsymbol{\phi}_u$ and $\boldsymbol{\phi}_y$ with finite frequency sets Ω_r, Ω_u and Ω_y respectively, (19) can be rewritten as

$$\begin{array}{rcl}\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}_i} & = & \frac{2}{N^2}\boldsymbol{\phi}_y^*|_{\Omega_y}\Phi_Q\Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}} \\ & & \frac{\partial \Phi_C(\boldsymbol{\phi}_y|_{\Omega_y},\boldsymbol{\phi}_r|_{\Omega_r},\boldsymbol{w})}{\partial \boldsymbol{w}_i}\end{array} \tag{22}$$

where $\Omega_{\Delta u}$ denotes the frequency set of the change of the control action spectrum $\Delta\boldsymbol{\phi}_u$ caused by the change of control parameters $\Delta\boldsymbol{w}$ at $\{\boldsymbol{\phi}_y|_{\Omega_y},\boldsymbol{\phi}_r|_{\Omega_r},\boldsymbol{w}\}$.

Compared with the gradient estimate in the LTI system, (22) has three important difference.

1) In (22) for a nonlinear system, Ω_r, Ω_u and $\Omega_{\Delta u}$ are often different from Ω_y; In a LTI system, Ω_r, Ω_u and $\Omega_{\Delta u}$ are always identical with Ω_y and all substituted with Ω.
2) In (22) for a nonlinear system, $\Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$ is a $(n_{\Omega_y}\times n_{\Omega_{\Delta u}})$-dimension matrix; In a LTI system, $\Phi_{T'}|_{\Omega_y}$ is a n_{Ω_y}-dimension diagonal square matrix.
3) In a nonlinear system, the partial closed dynamics $\Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$ is presented as

$$\begin{array}{rl} & \Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}} \\ = & (I-\Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}\Phi_{H'}|_{\{\Omega_y\mapsto\Omega_{\Delta u}\}})^{-1} \\ & \Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}\end{array} \tag{23}$$

In a linear system, the partial closed dynamics has simpler format without considering the difference between $\Omega_{\Delta u}$ and Ω_y.

As discussed in the above subsection, the key to estimate $\nabla J(\boldsymbol{w})$ is to solve Φ'_T. In the system with periodic $\boldsymbol{y}$, the key of gradient estimation in control can be simplified to estimating a linear mapping from a $n_{\Omega_{\Delta u}}$-dimension $\boldsymbol{\phi}_u$ space to a n_{Ω_y}-dimension $\boldsymbol{\phi}_y$, which can be further simplified to n_{Ω_y} sub-problems of linear one-to-one mapping in the linear case.

Considering the case of $N >> n_{\Omega_y}$, the advantage of a gradient estimate in the frequency domain is obvious: while the problem in time domain [3] is to solve N sub-problems of $\frac{\partial y(t)}{\partial \boldsymbol{w}}, t=0,\ldots,N-1$, the problem in the frequency domain is to solve n_{Ω_y} sub-problems of estimation of $\frac{\partial \phi_y(\omega_i)}{\partial \boldsymbol{w}}, i = 0,\ldots,n_{\Omega_y}$.

IV. Frequency Domain Iterative Tuning in the Nonlinear System

In this section, based on the previously proposed gradient estimation theory, a new adaptive control method, named here Iterative Tuning in the Frequency Domain (NL-FD-IT), will be developed to solve ANVC problem in the nonlinear system with periodic disturbances .

In (22) controller C is known by the designer, $\frac{\partial \Phi_C(\boldsymbol{\phi}_y|_{\Omega_y},\boldsymbol{\phi}_r|_{\Omega_r},\boldsymbol{w})}{\partial \boldsymbol{w}_i}$ and $\Delta\phi_u$ caused by $\Delta\boldsymbol{w}$ is computable given $\{\phi_y|_{\Omega_y},\boldsymbol{\phi}_r|_{\Omega_r},\boldsymbol{w}\}$. The only unknown item is $\Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$, which is the key in the iterative tuning in the frequency domain in nonlinear system as well.

Recalling (23), $\Phi_{T'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$ can be computed out through estimating $\Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$, which can be estimated through input-output difference pairs (i.e., $\{\Delta\boldsymbol{u},\Delta\boldsymbol{y}\}$) in different experiments. It is called the indirect approach to estimation.

Considering one spectrum difference pair $\{\Delta\boldsymbol{\phi}_u|_{\Omega_{\Delta u}},\Delta\boldsymbol{\phi}_y|_{\Omega_y}\}$ from two experiments with different inputs $\boldsymbol{u}$, one infinitesimal increment equation set is given by

$$\Delta\boldsymbol{\phi}_y|_{\Omega_y} \approx \Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}\Delta\boldsymbol{\phi}_u|_{\Omega_{\Delta u}} \tag{24}$$

which contains n_{Ω_y} equations.

Since $\Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$ has $n_{\Omega_y}\times n_{\Omega_{\Delta u}}$ unknown variables, it requires $n_{\Omega_{\Delta u}}$ such equation sets as (24) to solve $\Phi_{G'}|_{\{\Omega_{\Delta u}\mapsto\Omega_y\}}$.

As above discussed, $n_{\Omega_{\Delta u}}$ difference pairs $\{\Delta\boldsymbol{\phi}_u,\Delta\boldsymbol{\phi}_y\}$ can be obtained through $(n_{\Omega_{\Delta u}}+1)$ experiments with different inputs, which can be produced either by injecting extra signals as TD-IFT or by updating control parameters as FD-IT in [5].

V. Experimental work in an air-duct system

This section illustrates the the feasibility of NL-FD-IT through experimental work in an air-duct system. As shown in Fig. 3, a duct system is set up to test the FD-IT method for ANC in a semi-closed environment.

The duct is made of tin plate and comprises two parts. The speaker (EuroTec 520CO 4Ω, 10w), i.e., Spk0, fixed in left part is disturbance speaker. In the middle of of the whole duct, the speaker (EuroTec 520CO), i.e., Spk1, is fixed in the right part as a control speaker. The error microphone

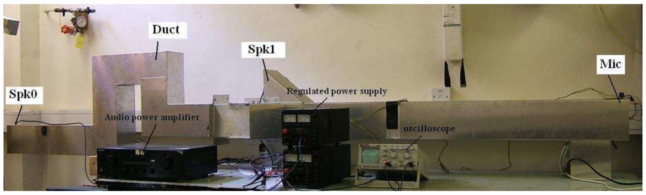

Fig. 3. Duct system

(Mic1, L160, 10Ω) is fixed to the right end of the duct, which acquires the error signal as system output $\boldsymbol{y}$.

- The disturbance speaker (Spk0) and control speaker (Spk1) are driven by the signal $\boldsymbol{d}$ and $\boldsymbol{u}$ from an audio power amplifier (SONY TA-FE570).
- The error signal $\boldsymbol{y}$ collected by Mic1 is amplified by an audio power amplifier (Philips TD1015). The reference signal $\boldsymbol{r}$ directly uses low-pass filtered stimulus disturbance source $\boldsymbol{d}_0$. $\boldsymbol{y}$ and $\boldsymbol{r}$ are both filtered through a band-pass RC filter.
- The source control signal $\boldsymbol{u}_0$ is directly recorded by the PC as control actions used in FD-IT.

The above experimental platform is controlled by a PC-based control system that comprise three parts:

1) A/D and D/A interface: Blue Wave System PC/16IO8 multichannel I/O board acquires analogue measurement data and provides analogue disturbance and control signals derived from DSP control unit.
2) DSP control unit: Blue Wave System PCI/C44S-60-1 is a DSP board based on Texas InstrumentsTMS320C4x Digital Signal Processors (DSPs) acting as disturbance source generator and real controllers.
3) PC host unit: Compatible PC system has AMD Althlon 1G Hz CPU and 384M RAM, and the operation system is Windows 2000. The iterative tuning work is implemented by Matlab R12.

As well known, nonlinearities exist in real dynamics. The speaker present strong nonlinearity when working in low frequency conditions. According to some offline test about the single frequency FRF of the contained frequencies, there are some nonlinearity in the actuator-sensor path in the duct system:

1) When the input signal is lower than some threshold, the amplitude response of the single frequency FRF present some 'dead zone' nonlinearity;
2) When the input signal is hight than some threshold, the amplitude response of the single frequency FRF present some 'saturation' nonlinearity.

In the following ANC experiment in the duct system, the periodic source disturbances is produced by the DSP board, which are given by

$$\begin{aligned} \boldsymbol{d} &= 0.33 \times [sin(2\pi \times 200t) + sin(2\pi \times 400t) \\ &\quad + sin(2\pi \times 500t)] \end{aligned} \tag{25}$$

Due to the memory limitation of the DSP board, FIR structure was selected to be used in the control systems while the FSF-based FD-IT requires to dynamically add FSF path containing IIR filters. In all experiments, the initial controller is to set $F^0 = 1.2$ and $H^0 = 0$. The typical initial states of the ANC system is illustrated as following Fig. 4, that have initial performance as $J^0 = 0.5288$.

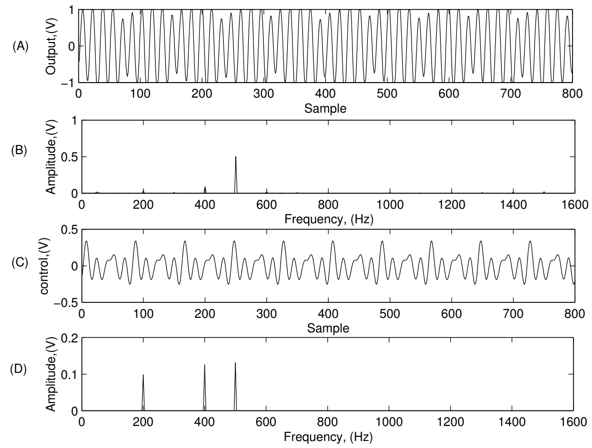

Fig. 4. Initial state in duct system, (A) error output, (B) power spectrum of output, (C) control actions, (D) power spectrum of control

In the initial manual experiments, the manually set controllers are given by $F^i = F^0 + \Delta F^i, i \neq 0$, where the random parameter change $||\Delta F^i||_2 < 0.1||F^0||_2$.

For the memory limitation of a DSP board, F is 40-th order FIR and H is 10-th order FIR. The common periods is set as $N = 1600$ in NL-FD-IT.

Each iteration lasts $3N$ sampling periods including $2N$ sampling periods to let the system reach steady state.

In NL-FIR-FD-IT, the tuning step sizes are are set as $\mu_F = 0.16, \mu_H = 0.016$. After 100 experiments, the tuned performance is $J^{100} = 0.0051$ with 21.2dB cancellation after 105.53s of tuning. The final outputs and control actions of the duct system are illustrated in Fig. 5, and the performance updating during the tuning is shown in Fig. 6.

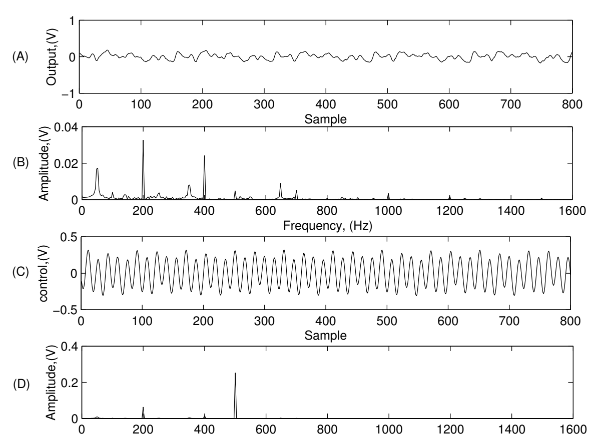

Fig. 5. Final state using NL-FIR-FD-IT, (A) error output, (B) power spectrum of output, (C) control action, (D) power spectrum of control

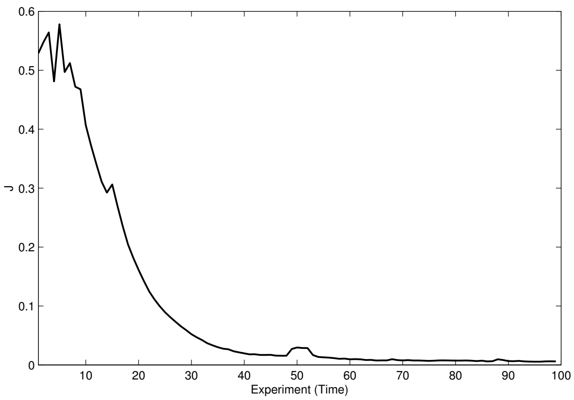

Fig. 6. Performance updating in NL-FIR-FD-IT

As shown in (B) of Fig. 5, the nonlinearity of the system is very notable, which includes more significant frequencies than the original spectrum with 500Hz, 400Hz and 200Hz shown in (B) of Fig. 4. The first three dominant frequencies turn out to be 200Hz, 400Hz and 50Hz in the end of the tuning.

At the same time, the linear FD-IT algorithm [5] is also tested in the same platform and produces only 16.4dB final cancelation due to its working in the range of linear dynamics.

Compare (D) of Fig. 5 with (D) of Fig. 4, the range of the change of control action amplitude is from 0.08 to 0.25 which is in the nonlinear range of 'dead zone' of the system. NL-FD-IT gives extra 5dB performance over the 'dead zone'.

From the aspect of tuning process as shown in Fig. 6, compared with the tuning process in linear case [5],

- It is clear that there are more than two manual extra experiments in NL-FIR-FD-IT, while there are only two in linear case.
- The tuning process has some notable fluctuations while it is much more smooth in the linear case. They illustrate tuning's overcoming the impact of bad SNR conditions, and demonstrate the global robustness of the algorithm.

VI. CONCLUSIONS

An innovative way to estimate the control performance criterion's gradient is proposed in the frequency domain. The method can help to solve iterative tuning problems with nonlinear dynamics. The system dynamics is represented as mappings between two multi-dimension spaces (spectrum space), and gradient-based tuning problem is represented as multiplication of spectrum vectors and FRF derivative matrixes. It has an inherent advantage being able deal with control problems with periodic inputs and outputs due to its simplified presentation in the frequency domain.

Based on the presented gradient estimation theory, a new iterative tuning method has been developed to solve nonlinear ANVC problems with periodic disturbances.

An experimental air-duct system, that was found to contain some nonlinearity of its dynamics, has been used to test the feasibility of NL-FD-IT in ANC. The results demonstrate the applicability of NL-FD-IT to nonlinear systems.

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