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Variable Structure Adaptive Backstepping Controller

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Abstract— **This paper presents a Variable Structure Adaptive Backstepping Controller (VS-ABC) for plants with relative degree one, using only input/output measurements. Instead of traditional integral adaptive laws for estimating the plant parameters, switching laws are used to increase robustness to parametric uncertainties and disturbances, as well as, to improve transient response. Moreover, the controller design is easier when compared with the original adaptive backstepping controller, since the amplitude relays are related to the plant nominal parameters. Additionally, preliminary simulation results for an unstable second order system are shown.**

I. INTRODUCTION

Traditionally, the main problem of adaptive systems is concerned with their transient performance, which becomes an important issue in real applications. For traditional adaptive controllers, no results about bounds on the transient behavior can be assured by the designer. Furthermore, the traditional adaptive schemes normally present large initial oscillations, since the system is learning about the process through the parameter estimates. In order to solve these drawbacks, a new adaptive technique for linear systems with unknown parameters, namely, adaptive backstepping, was proposed in [1].

When compared with the traditional adaptive controllers for linear systems, e.g., MRAC (Model Reference Adaptive Control) in [2] and [3], and APPC (Adaptive Pole Placement Control) in [4], the adaptive backstepping controller guarantees stability without adaptation and presents a better transient response. However, these new features are obtained by increasing in control law complexity, which is not interesting in practical implementations, particularly in embedded systems. The parameter controller tuning is another drawback, inherent of all adaptive controllers with integral adaptive laws.

In this paper, we propose a Variable Structure Adaptive Backstepping Controller (VS-ABC) where the integral adaptive laws are replaced by switching laws. The aim of this strategy is to aggregate the best features of both techniques: fast transient and robustness to parametric uncertainties and disturbances. In addition, the controller design has been simplified, because the amplitude relays are related to physical parameters (plant nominal parameters), e.g., resistance, capacitance, inertia moments, etc.

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Similar approaches, combining variable structure with adaptive control, have been presented in [5] and [6], where the VS-MRAC (Variable Structure Model Reference Adaptive Control) and VS-APPC (Variable Structure Adaptive Pole Placement Control) techniques were proposed, respectively, in their direct and indirect versions. Moreover, the indirect VS-MRAC has been presented in [7], which the controller design is simplified as the same as mentioned before, due to the amplitude relays simplicity.

A combined backstepping/variable structure control strategy for a class of uncertain nonlinear systems is proposed by the authors in [8]. This approach is characterized by the generation of a second order sliding mode to compensate uncertainty terms. The control algorithm is composed by $n-1$ steps analogous to that presented in [9], and a final step using a second order sliding mode. The VS-ABC scheme differs from it in the variable structure approach, which is proposed here to substitute the integral adaptive laws using appropriate switching laws.

In [10], the authors shown that the original adaptive algorithms get unstable in the presence of unmodeled high-frequency dynamics and unmeasurable output disturbances. Hence, over the last years, several researchers have proposed additional modifications in the adaptive laws, e.g., normalized dead zone [11], parameter projection [12] and σ modification [13]. The main idea behind these modifications is to bound the parameter estimates, avoiding system instability from the traditional adaptive laws. The use of variable structure in the adaptive controllers also bounds the "parameter estimates", since the relay terms are used.

This paper is organized as follows. Some necessary assumptions for the controller designs are described in the next section. The adaptive backstepping controller design and the variable structure adaptive backstepping controller design are presented, respectively, in sections III and IV, both using Lyapunov theory. The simulation results are illustrated in section V and finally, some conclusions are drawn in section VI.

II. THE BACKGROUND DESIGN

Consider the Single-Input Single-Output (SISO) and Linear Time Invariant system (LTI) with relative degree one ($\rho = n - m = 1$), described by

$$y(s) = \frac{B(s)}{A(s)}u(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \quad (1)$$

where the coefficients $b_m \cdots b_0$ and $a_{n-1} \cdots a_0$ are constants but unknown. Introducing the output error variable

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$$z = x_1 - y_r(t), \tag{2}$$

the aim design is to force x_1 to asymptotically track the reference signal $y_r(t)$, by regulating z to zero, while keeping all the closed-loop signals bounded. The reference $y_r(t)$ may be the output of a model reference with a piecewise continuous input $r(t)$ or a signal whose its first derivative is known, bounded and piecewise continuous. Therefore, the reduction in the *output error* z corresponds to the tracking problem or the set-point regulation problem, depending on the kind of $y_r(t)$. Some additional assumptions are necessary:

1) The sign of the high-frequency gain ($sgn(b_m)$) is known.
2) The plant is minimum phase, i. e., the polynomial $B(s) = b_m s^m + \cdots + b_1 s + b_0$ is *Hurwitz*.
3) The model reference relative degree must be equal or greater than the plant relative degree ($\rho_r \geq \rho$).

The assumptions described above are the same as presented in traditional model reference adaptive control.

A. State Estimation Filters

In the proposed control schemes, only input/output measurements are considered and then state estimation filters will be used in order to overcome this constraint. The adaptive backstepping design will employ the K-filters developed by Kreisselmeier in [14] for adaptive linear observers.

The system (1) for any relative degree can be represented in the observer canonical form

$$\begin{aligned} \dot{x}_1 &= x_2 - a_{n-1} y \\ &\vdots \\ \dot{x}_{\rho-1} &= x_\rho - a_{m+1} y \\ \dot{x}_\rho &= x_{\rho+1} - a_m y + b_m u \\ &\vdots \\ \dot{x}_{n-1} &= x_n - a_1 y + b_1 u \\ \dot{x}_n &= -a_0 y + b_0 u \\ y &= x_1, \end{aligned} \tag{3}$$

or, more compactly, as

$$\begin{aligned} \dot{x} &= Ax - ya + \begin{bmatrix} 0_{(\rho-1)\times 1} \\ b \end{bmatrix} u \\ y &= e_1^T x, \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} 0 & I_{n-1} \\ \vdots & \\ 0 & \cdots \quad 0 \end{bmatrix}, \; a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix}, \; b = \begin{bmatrix} b_m \\ \vdots \\ b_0 \end{bmatrix}.$$

The system's representation (4) can also be rewritten as

$$\begin{aligned} \dot{x} &= Ax + F(y,u)^T \theta \\ y &= e_1^T x, \end{aligned} \tag{5}$$

where

$$F(y,u)^T = \begin{bmatrix} \begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} u & -I y_n \end{bmatrix},$$

and the parameter vector is

$$\theta = \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} b_m \\ \vdots \\ b_0 \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_{m+1} \\ \theta_{m+2} \\ \vdots \\ \theta_{2n} \end{bmatrix}. \tag{6}$$

For state estimation, the following filters will be used

$$\begin{aligned} \dot{\xi} &= A_0 \xi + ky \\ \dot{\Omega}^T &= A_0 \Omega^T + F(y,u)^T, \end{aligned} \tag{7}$$

where the vector $k = [k_1 \cdots k_n]^T$ is chosen so that the matrix

$$A_0 = A - k e_1^T, \tag{8}$$

is *Hurwitz*, and hence P exists such that

$$PA_0 + A_0^T P = -I, \quad P = P^T > 0. \tag{9}$$

Using (7), the state estimate is

$$\hat{x} = \xi + \Omega^T \theta, \tag{10}$$

and it is easy to demonstrate that the state estimation error

$$\varepsilon = x - \hat{x}, \tag{11}$$

vanishes exponentially, since

$$\dot{\varepsilon} = A_0 \varepsilon. \tag{12}$$

The K-filters are summarized in Table I. More details can be found in [14] and [15].

TABLE I

THE K-FILTERS FOR SISO LINEAR SYTEMS WITH ANY RELATIVE DEGREE.

$\dot{\eta} = A_0 \eta + e_n y$
$\dot{\lambda} = A_0 \lambda + e_n u$
$\Xi = -\left[A_0^{n-1}\eta, \cdots, A_0\eta, \eta\right]$
$\xi = -A_0^n \eta$
$\upsilon_j = A_0^j \lambda, \quad j = 0 \cdots m$
$\Omega^T = [\upsilon_m, \cdots, \upsilon_1, \upsilon_0, \Xi]$

III. THE ADAPTIVE BACKSTEPPING DESIGN

The adaptive backstepping design for plants with relative degree one is deduced from the general case described in [15]. Due to the minimum phase assumption, this design is restricted to the equation

$$\dot{x}_1 = x_2 - a_{n-1}y + b_m u = x_2 - ye_1^T a + b_m u. \quad (13)$$

From (10) and (11), the x_2 variable can be obtained as

$$\begin{aligned} x_2 &= \xi_2 + \Omega_{(2)}^T \theta + \varepsilon_2 \\ &= \xi_2 + \left[\upsilon_{m,2},\, \upsilon_{m-1,2}, \cdots, \upsilon_{0,2},\, \Xi_{(2)} \right] \theta + \varepsilon_2. \end{aligned} \quad (14)$$

Substituting the above result in (13), it yields

$$\begin{aligned} \dot{x}_1 &= \xi_2 + \left[\upsilon_{m,2}, \cdots, \upsilon_{0,2},\, \Xi_{(2)} - ye_1^T \right] \theta + \varepsilon_2 + b_m u \\ &= \xi_2 + \left[w_1 \cdots w_{2n} \right] \theta + \varepsilon_2 + b_m u \\ &= \xi_2 + w^T \theta + \varepsilon_2 + b_m u, \end{aligned} \quad (15)$$

where w is the "regressor" vector. Then, the first time derivative of the output error (2) using (15) is given by

$$\dot{z} = \dot{x}_1 - \dot{y}_r = \xi_2 + w^T\theta + \varepsilon_2 + b_m u - \dot{y}_r. \quad (16)$$

Scaling the control law $u(t)$ as

$$u = \hat{\varrho}\bar{u}, \quad (17)$$

where $\hat{\varrho}$ is an estimate of $\varrho = 1/b_m$ and defining

$$\begin{aligned} \tilde{\theta} &= \theta - \hat{\theta} \\ \tilde{\varrho} &= \varrho - \hat{\varrho}, \end{aligned} \quad (18)$$

we obtain

$$\begin{aligned} \dot{z} &= \xi_2 + w^T\theta + \varepsilon_2 + b_m\hat{\varrho}\bar{u} - \dot{y}_r \\ &= \xi_2 + w^T\theta + \varepsilon_2 - b_m\tilde{\varrho}\bar{u} + \bar{u} - \dot{y}_r. \end{aligned} \quad (19)$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2}z^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{|b_m|}{2\gamma}\tilde{\varrho}^2 + \frac{1}{4d_1}\varepsilon^T P\varepsilon > 0, \quad (20)$$

and its first time derivative using (9) and (12)

$$\dot{V} = z\dot{z} - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \frac{|b_m|}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} - \frac{1}{4d_1}\varepsilon^T\varepsilon. \quad (21)$$

By substituting (19) in (21),

$$\begin{aligned} \dot{V} &= z(\xi_2 + w^T\theta + \varepsilon_2 - b_m\tilde{\varrho}\bar{u} + \bar{u} - \dot{y}_r) \\ &\quad -\tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \frac{|b_m|}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} - \frac{1}{4d_1}\varepsilon^T\varepsilon, \end{aligned} \quad (22)$$

and selecting the auxiliary control law as

$$\bar{u} = -c_1 z - d_1 z - \xi_2 - w^T\hat{\theta} + \dot{y}_r, \quad (23)$$

we have

$$\begin{aligned} \dot{V} &= -c_1z^2 - d_1z^2 + z\varepsilon_2 - \frac{1}{4d_1}\varepsilon^T\varepsilon \\ &\quad +\tilde{\theta}^T\Gamma^{-1}\left[\Gamma wz - \dot{\hat{\theta}}\right] - \frac{|b_m|}{\gamma}\tilde{\varrho}\left[\gamma sgn(b_m)\bar{u}z + \dot{\hat{\varrho}}\right]. \end{aligned} \quad (24)$$

To eliminate the unknown indefinite terms $\tilde{\theta}$ and $\tilde{\varrho}$ in (24), the update laws can be chosen as

$$\dot{\hat{\theta}} = \Gamma wz, \quad (25)$$

$$\dot{\hat{\varrho}} = -\gamma sgn(b_m)\bar{u}z, \quad (26)$$

where Γ and γ are the adaptive gains. Then,

$$\begin{aligned} \dot{V} &= -c_1z^2 - d_1z^2 + z\varepsilon_2 - \frac{1}{4d_1}\varepsilon^T\varepsilon \\ &= -c_1z^2 - d_1\left(z - \frac{1}{2d_1}\varepsilon_2\right)^2 \\ &\quad -\frac{1}{4d_1}({\varepsilon_1}^2 + {\varepsilon_3}^2 + \cdots + {\varepsilon_n}^2), \end{aligned} \quad (27)$$

which yields

$$\dot{V}(z, \tilde{\theta}, \tilde{\varrho}, \varepsilon) \leq c_1z^2 \leq 0. \quad (28)$$

The above result renders $[z, \tilde{\theta}, \tilde{\varrho}, \varepsilon]^T = [0, 0, 0, 0]^T$ a stable equilibrium point. From LaSalle-Yoshizawa theorem [15], we can show that $z(t) \to 0$ as $t \to \infty$.

IV. THE VARIABLE STRUCTURE ADAPTIVE BACKSTEPPING DESIGN

Following the steps described in the previous section, switching laws will be proposed to replace the integral adaptive laws (25-26), which were required to guarantee the origin stability as shown in (28). Now, consider the Lyapunov function candidate

$$V = \frac{1}{2}z^2 + \frac{1}{2d_1}\varepsilon^T P\varepsilon > 0, \quad (29)$$

and its first time derivative

$$\dot{V} = z\dot{z} - \frac{1}{2d_1}\varepsilon^T\varepsilon. \quad (30)$$

By substituting (19) and (23) in (30), we obtain

$$\begin{aligned} \dot{V} &= -c_1z^2 + w^T\tilde{\theta}z - b_m\tilde{\varrho}\bar{u}z - \frac{1}{4d_1}\varepsilon^T\varepsilon \\ &\quad -d_1z^2 + z\varepsilon_2 - \frac{1}{4d_1}\varepsilon^T\varepsilon \\ &= -c_1z^2 + w^T\tilde{\theta}z - b_m\tilde{\varrho}\bar{u}z - \frac{1}{4d_1}\varepsilon^T\varepsilon \\ &\quad -d_1\left(z - \frac{1}{2d_1}\varepsilon_2\right)^2 - \frac{1}{4d_1}({\varepsilon_1}^2 + {\varepsilon_3}^2 + \cdots + {\varepsilon_n}^2). \end{aligned} \quad (31)$$

Therefore,

$$\dot{V} \leq -c_1z^2 + \sum_{i=1}^{2_n}\tilde{\theta}_i w_i z - b_m\tilde{\varrho}\bar{u}z - \frac{1}{4d_1}, \quad (32)$$

and using the switching laws

$$\hat{\theta}_i = \bar{\theta}_i sgn(w_i z),\ \hat{\theta}_i > \theta_i \quad (33)$$

$$\hat{\varrho} = -\bar{\varrho} sgn(b_m) sgn(\bar{u}z),\ \hat{\varrho} > \frac{1}{|b_m|}, \quad (34)$$

in (32), it yields

$$\begin{aligned}\dot{V} \quad &\leq -c_1 z^2 - \frac{1}{4d_1}\varepsilon^T\varepsilon + \sum_{i=1}^{2n}(\theta_i w_i z - \bar{\theta}_i|w_i z|) \\ &-b_m(\varrho\bar{u}z + \bar{\varrho}|\bar{u}z|).\end{aligned} \quad (35)$$

The new result is

$$\dot{V} \leq -c_1 z^2 - \frac{1}{4d_1}\varepsilon^T\varepsilon < 0, \quad (36)$$

which guarantees that $[z, \varepsilon]^T = [0, 0]^T$ is globally asymptotically stable (GAS), because (36) is a negative definite function.

The switching laws proposed (33-34) simplify the control algorithm implementation, since the "parameter estimation" is now obtained by using relays instead of integral adaptive laws (25-26). For instance, the $\hat{\theta}_i$ calculation in (33) does not require w_i times z, but only their signal analysis which is a very simple task in digital systems. Therefore, the number of calculations is reduced, and consequently, the number of instructions used. In embedded systems, this new feature is welcome due to hardware and software constraints, such as, reduced number of peripherals and limited Arithmetic Logic Units (ALU).

V. Simulation Results

In this section, simulation results for an unstable second order system with relative degree one will be presented. Robustness tests in the presence of parametric uncertainties and disturbances for adaptive schemes from sections III and IV are included. Consider the system described by

$$y(s) = \frac{s+1}{s^2 - 3s + 2}u(s), \quad (37)$$

and a reference model by

$$y_r(s) = \frac{s+1}{s^2 + 4s + 4}r(s). \quad (38)$$

The K-filters were implemented as described in Table 1

$$\begin{aligned}\dot{\eta} &= A_0\eta + e_2 y \\ \dot{\lambda} &= A_0\lambda + e_2 u\end{aligned} \quad (39)$$

$$\begin{aligned}\Xi &= -[A_0\eta,\, \eta] \\ \xi &= -A_0^2\eta \\ \upsilon_1 &= A_0\lambda \\ \upsilon_0 &= \lambda \\ \Omega^T &= [\upsilon_1,\, \upsilon_0,\, \Xi],\end{aligned} \quad (40)$$

where the matrix

$$A_0 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}, \quad (41)$$

is *Hurwitz*, due to the choice of the vector

$$k = [k_1\ k_2]^T = [2\ 1]^T. \quad (42)$$

The system's behavior without parametric uncertainties and disturbances is shown in Figs. 1 and 2, respectively, for the adaptive backstepping controller and the VS-ABC. The adaptive gains used in the former case were

$$\Gamma = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix},\ \gamma = 100, \quad (43)$$

and the auxiliary constants, $c_1 = d_1 = 18$. In the latter, the relay amplitudes were $\bar{\theta}_1 = 1.5$, $\bar{\theta}_2 = 1.5$, $\bar{\theta}_3 = 3.5$, and $\bar{\theta}_4 = 2.5$, while the auxiliary constants, $c_1 = d_1 = 18$. In both situations, the reference input was $r(t) = 1$, the plant initial condition was $x_1(0) = 0.15$ and other initial conditions were zero. The control signals are shown in Figs. 3 and 4. Notice that the VS-ABC presents a transient improvement when compared with the adaptive backstepping controller. On the other hand, this result is achieved with a higher control input $u(t)$.

Figs. 5 and 6 show the simulation results for the same system, however with an input additive disturbance ($d = 2$), from $t = 7s$, and a parameter deviation of 20% in the nominal values, from $t > 13s$. The control signals for the adaptive backstepping and VS-ABC schemes can be analyzed through Figs. 7 and 8. As can be observed, the VS-ABC presents a better performance in the presence of parametric uncertainties and disturbances, when compared with traditional adaptive backstepping control. The parameter estimates for the backstepping adaptive controller with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values are shown in Fig. 9, while the parameter estimates without parametric uncertainties and disturbances are presented in Fig. 10.

VI. Conclusions

In this paper, a Variable Structure Adaptive Backstepping Controller (VS-ABC) was shown for plants with relative degree one, using only input/output measurements. Simulation results were presented for an unstable second order system in order to corroborate the theoretical studies. As previewed, the union of both techniques has improved the transient performance and the robustness to parametric uncertainties and disturbances, when compared with traditional adaptive backstepping control. Even though they have the same number of parameters, the VS-ABC design was easier, since tuning process of the auxiliary constants and adaptive gains required several preliminary tests in the backstepping scheme.

In future papers, results for plants with arbitrary relative degree will be presented as well as practical applications in motion control, current control loop and process control where industrial embedded components (FPGAs, MCUs and DSPs) will be used. The Variable Structure Adaptive Backstepping technique is not limited to controllers and can be also applied to other areas in control systems as state observers, allowing the same benefits as described here, in particular inherent fast transient response and robustness to parametric uncertainties and disturbances. In addition, comparisons with similar adaptive controllers, e.g., VS-MRAC and VS-APPC, will be discussed.

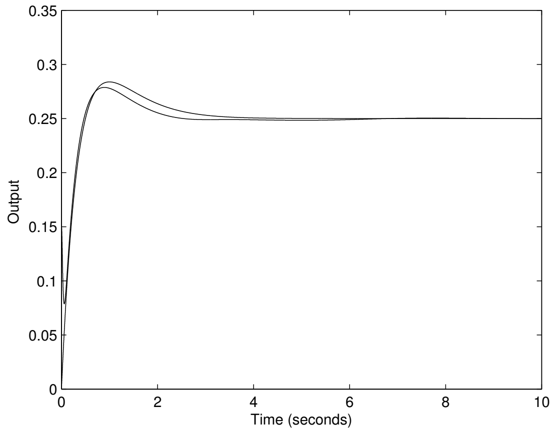

Fig. 1. Output system for the adaptive backstepping controller without parametric uncertainties and disturbances.

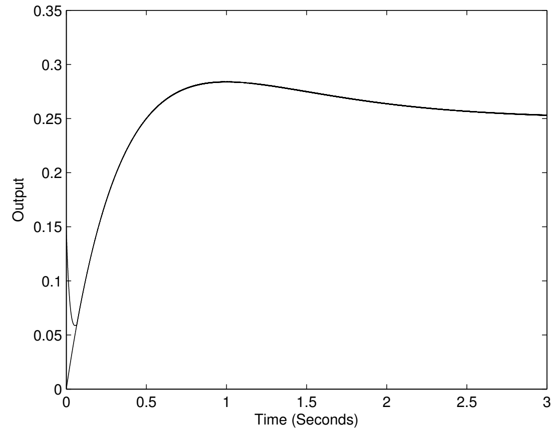

Fig. 2. Output system for the VS-ABC without parametric uncertainties and disturbances.

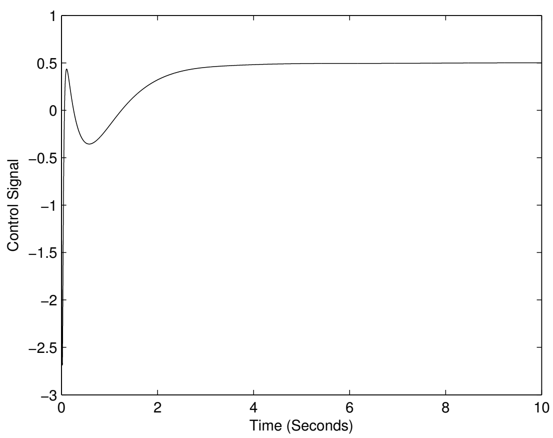

Fig. 3. Control signal for the adaptive backstepping controller without parametric uncertainties and disturbances.

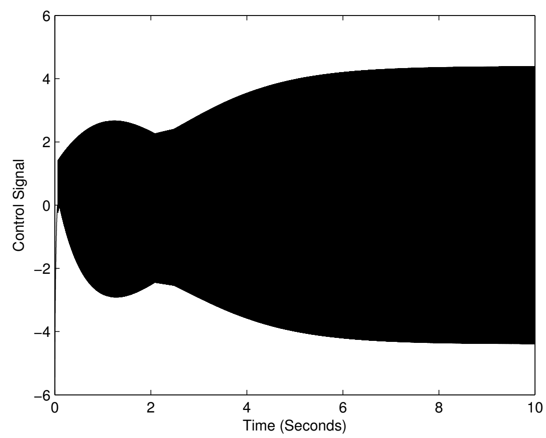

Fig. 4. Control signal for the VS-ABC without parametric uncertainties and disturbances.

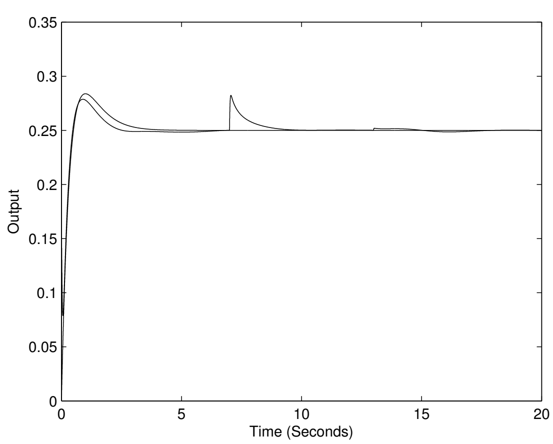

Fig. 5. Output system for the adaptive backstepping controller with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values.

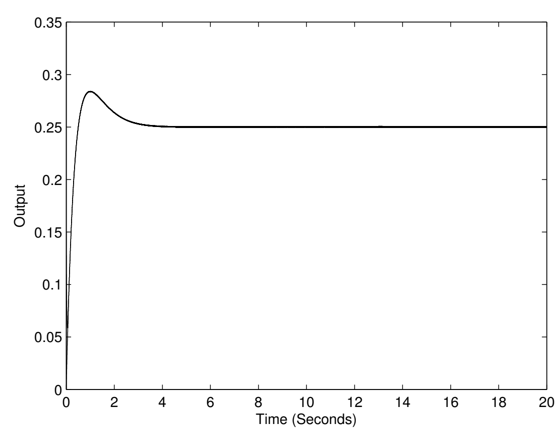

Fig. 6. Output system for the VS-ABC with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values.

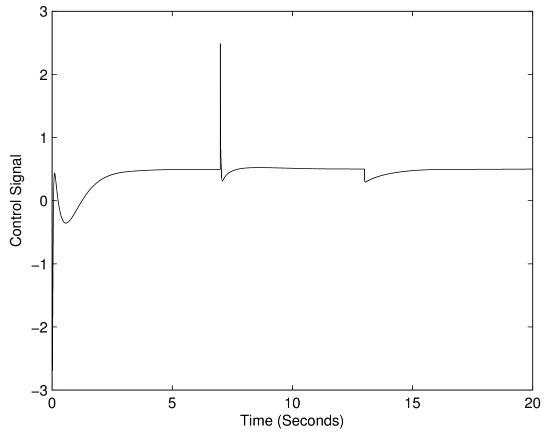

Fig. 7. Control signal for the backstepping adaptive controller with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values.

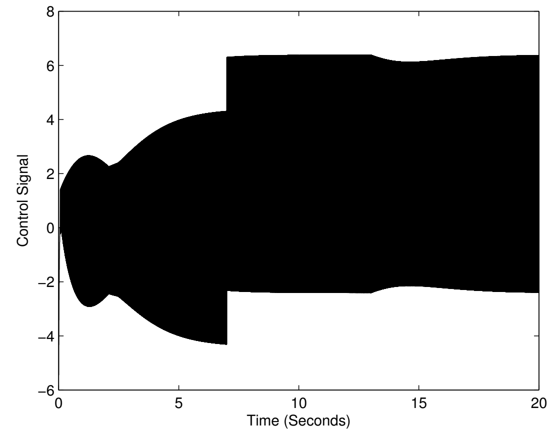

Fig. 8. Control signal for the VS-ABC with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values

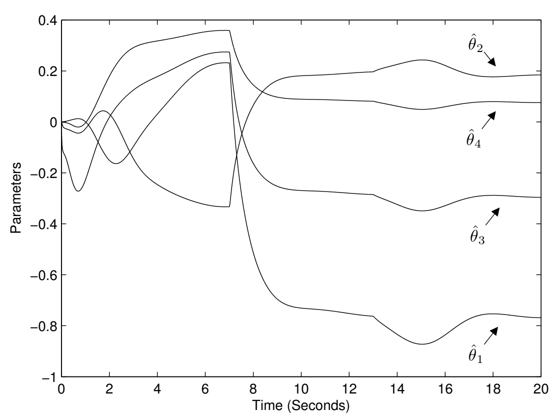

Fig. 9. Parameter estimates for the backstepping adaptive controller with an input additive disturbance ($d = 2$) and a parameter deviation of 20% in the nominal values.

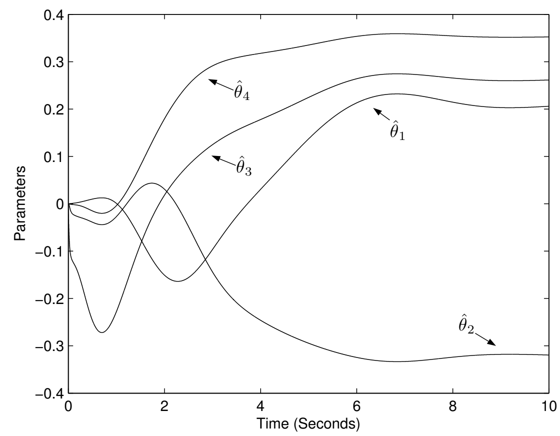

Fig. 10. Parameter estimates for the backstepping adaptive controller without parametric uncertainties and disturbances.

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