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A Novel Nonlinear Output Feedback Control Applied to the TORA Benchmark System

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***Abstract*— A novel nonlinear control scheme is applied to the TORA benchmark system. This approach incorporates a continuous implementation of sliding mode control and an extended high-gain observer, and is based on previous work on the stabilization of a non-minimum phase nonlinear system, under the assumption that its associated auxiliary system has a stabilizing controller. The rotor angle is the only measurement required by this controller, and the closed-loop system thus obtained has quite interesting properties. The performance characteristics of a full-order observer-based second order linear system were recovered through an iterative tuning procedure. The final design provides good transient performance and some robustness to perturbations in the masses of the cart and rotor.**

I. Introduction

The translational oscillator with a rotating actuator (TORA) system, or rotational-translational actuator (RTAC), as it is also known, has been widely used in the literature for the past several years as a benchmark problem for testing novel nonlinear control schemes. The problem was introduced in [1] and [2]. In [3], two control laws are presented, the first a cascade controller and the second one a feedback passivating controller. A control law based on an $\mathcal{L}_2$ disturbance attenuation approach was proposed in [8]. Output feedback controllers dependent on measurements of the rotational and translational positions were proposed in [6], while controllers requiring only the measurement of the rotational position were presented in [5] and [7]. In [9], four different controllers were experimentally evaluated on an RTAC testbed.

The controller proposed in this paper is based on a novel approach to the stabilization of a non-minimum phase nonlinear system if its associated auxiliary system can be asymptotically stabilized. This technique is based on previous work by Isidori [11], [13]. It was shown in [4] that the output feedback system can asymptotically recover the performance of the partial state feedback system in the absence of uncertainty in the control coefficient. When uncertainty is present, it recovers the performance of a perturbed version of the state feedback system. This approach is applied to the TORA benchmark problem in this paper.

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II. Robust Stabilization of a Non-Minimum Phase System Using an Extended High-Gain Observer

This section provides a brief overview of an extended high-gain observer-based robust output feedback control scheme for systems in the normal form, which could potentially include unstable zero dynamics. This control methodology incorporates continuous sliding mode control—chosen for its robustness properties as well as its ability to prescribe or constrain the motion of trajectories in the sliding phase—and the aforementioned extended high-gain observer to estimate one of the unknown functions in the plant model. In [4], stabilization in the case of an unknown control coefficient and uncertain constant parameters is shown for the state as well as output feedback cases.

A. Problem Statement

We consider a single-input, single-output system with relative degree ρ, which, under a suitable diffeomorphism, can be expressed in the following normal form.

$$\begin{aligned} \dot{\eta} &= \phi(\eta, \xi, \theta), && (1)\\ \dot{\xi}_i &= \xi_{i+1}, \quad 1 \le i \le \rho - 1, && (2)\\ \dot{\xi}_\rho &= b(\eta, \xi, \theta) + a(\eta, \xi, \theta)u, && (3)\\ y &= \xi_1, && (4) \end{aligned}$$

where $a(\cdot) \neq 0$, $\eta \in D_\eta \subset \mathbb{R}^{n-\rho}$, $\xi \in D_\xi \subset \mathbb{R}^\rho$, and $\theta \in \Theta \subset \mathbb{R}^p$ is a vector of constant parameters. In the technique developed by Isidori [11], the stabilization problem can be solved, provided an auxiliary system—defined below—can be globally stabilized by a dynamic feedback controller. The auxiliary system is defined by ([11], [13]),

$$\begin{aligned} \dot{\eta} &= \phi(\eta, \xi_1, \ldots, \xi_{\rho-1}, u_a, \theta), && (5)\\ \dot{\xi}_i &= \xi_{i+1}, \quad 1 \le i \le \rho - 2, && (6)\\ \dot{\xi}_{\rho-1} &= u_a, && (7)\\ y_a &= b(\eta, \xi_1, \ldots, \xi_{\rho-1}, u_a, \theta). && (8) \end{aligned}$$

Equations (5)–(7) come from (1)–(2) by viewing ξ_ρ as the control input u_a, while the term $b(\eta, \xi, \theta)$ on the right-hand side of (3) is taken as the measured output.

This auxiliary problem is assumed to have a stabilizing dynamic controller of the form

$$\begin{aligned} \dot{z} &= L(z, \xi_1, \ldots, \xi_{\rho-1}) + M(z, \xi_1, \ldots, \xi_{\rho-1})y_a, && (9)\\ u_a &= N(z, \xi_1, \ldots, \xi_{\rho-1}), && (10) \end{aligned}$$

where $z \in D_z \subset \mathbb{R}^r$. Under this assumption, it was shown by Isidori that a dynamic feedback law exists, which can robustly stabilize the original system [11, equation (27)], [13]. In [4], a combination of a continuous implementation of sliding mode control and an extended high-gain observer was utilized to stabilize the original system using only measurement of the output y. The sliding mode control was designed to force the system trajectories to a manifold within a finite time, along which the system response coincides with a perturbed version of the auxiliary system, and hence the performance of the auxiliary system may be recovered under certain conditions and constraints on the model uncertainties.

B. Output Feedback Design Using an Extended High-Gain Observer

The output feedback design relies upon the estimates of the states ξ and of $b(\eta, \xi, \theta)$ that are obtained using an extended high-gain observer for the system (1)–(3), which is taken as

$$\dot{\hat{\xi}}_i = \hat{\xi}_{i+1} + (\alpha_i/\varepsilon^i)(\xi_1 - \hat{\xi}_1),\ 1 \le i \le \rho - 1, \quad (11)$$
$$\dot{\hat{\xi}}_\rho = \hat{\sigma} + \hat{a}(\hat{\xi})u + (\alpha_\rho/\varepsilon^\rho)(\xi_1 - \hat{\xi}_1), \quad (12)$$
$$\dot{\hat{\sigma}} = (\alpha_{\rho+1}/\varepsilon^{\rho+1})(\xi_1 - \hat{\xi}_1), \quad (13)$$

where ε is a positive constant to be specified, and the positive constants α_i are chosen such that the roots of $\lambda^{\rho+1} + \alpha_1\lambda^\rho + \ldots + \alpha_\rho\lambda + \alpha_{\rho+1} = 0$ are in the open left-half plane. The design of the extended high-gain observer closely follows the work by Freidovich and Khalil [10].

It is apparent from (11)–(13) that the $\hat{\xi}_1, \ldots, \hat{\xi}_\rho$ are used to estimate the output and its first $(\rho - 1)$ derivatives, while $\hat{\sigma}$ is intended to provide an estimate for $b(\cdot)$. With the aid of these estimates, the output feedback controller for the original system can be taken as

$$\begin{aligned} \dot{z} &= L(z, \hat{\xi}_1, \ldots, \hat{\xi}_{\rho-1}) \\ &\quad + M(z, \hat{\xi}_1, \ldots, \hat{\xi}_{\rho-1})\hat{\sigma}, \end{aligned} \quad (14)$$
$$l(\hat{\xi}, z) = -\frac{\beta(\hat{\xi}, z)}{\hat{a}(\hat{\xi})} \operatorname{sat}\left(\frac{s}{\mu}\right), \quad (15)$$
$$u = K \operatorname{sat}(l(\hat{\xi}, z)/K), \quad (16)$$

where

$$s = \hat{\xi}_\rho - N(z, \hat{\xi}_1, \ldots, \hat{\xi}_{\rho-1}), \quad (17)$$
$$K > \max_{(\xi, z, s) \in \Omega} \left| \frac{\beta(\xi, z)}{\hat{a}(\xi)} \right|, \quad (18)$$

and Ω is a compact set of interest containing the initial state and the origin. The control is saturated at $\pm K$ outside Ω in order to protect the system from peaking during the observer's transient response.

In [4], the above output feedback design is shown to be able to recover exponential stability of the origin by appropriately tuning the high-gain parameter ε.

III. Output Feedback Stabilization of the TORA Benchmark System

The TORA problem was originally conceived as a simplified version of the dynamics of dual-spin spacecraft [1]. The interaction between rotation and translation in the oscillating eccentric rotor is analogous to the interaction between spin and nutation in a dual-spin spacecraft.

A. TORA Dynamic Equations

The oscillating eccentric rotor being considered (a figure can be found in [12, Figure 1.25]) consists of a cart of mass M connected to a fixed reference frame via a linear spring with spring constant k. The cart is constrained to only move horizontally. Attached to the cart is an unbalanced mass m with moment of inertia I about its center of mass, located at a distance L from its rotational axis. Let x_1 be the rotational angle of the unbalanced mass, x_2 denote the translational position of the center of mass of the proof mass, $x_3 = \dot{x}_1$ and $x_4 = \dot{x}_2$. The control torque applied to the proof mass is denoted by u. The dynamics are then given by [1], [12]

$$\dot{x}_1 = x_3, \quad (19)$$
$$\dot{x}_2 = x_4, \quad (20)$$
$$\begin{aligned} \dot{x}_3 &= [(m+M)u - mL(\cos x_1) \\ &\quad \times \left(mLx_3^2 \sin x_1 - kx_2\right)] / \Delta(x_1), \end{aligned} \quad (21)$$
$$\begin{aligned} \dot{x}_4 &= \left[\left(I + mL^2\right)\left(mLx_3^2 \sin x_1 - kx_2\right)\right. \\ &\quad \left. - mLu\cos x_1\right] / \Delta(x_1), \end{aligned} \quad (22)$$
$$y = x_1, \quad (23)$$

where $\Delta(x_1) = (I + mL^2)(m + M) - m^2L^2\cos^2 x_1 \ge (I + mL)^2 M + mI > 0$.

Our objective is to design an output feedback controller to stabilize the origin. The controller will be designed under the assumption that all the parameters are known, but we will use simulations to investigate its robustness to perturbations in the masses m and M.

We now introduce the change of variables

$$\eta_1 = x_2 + \frac{mL\sin x_1}{m+M}, \quad \eta_2 = x_4 + \frac{mLx_3\cos x_1}{m+M},$$
$$\xi_1 = x_1, \quad \xi_2 = x_3,$$

which transforms the system into the normal form

$$\dot{\eta}_1 = \eta_2, \quad (24)$$
$$\dot{\eta}_2 = \frac{k}{m+M}\left(\frac{mL\sin\xi_1}{m+M} - \eta_1\right), \quad (25)$$
$$\dot{\xi}_1 = \xi_2, \quad (26)$$
$$\begin{aligned} \dot{\xi}_2 &= \frac{1}{\Delta(\xi_1)}\left\{(m+M)u - mL(\cos\xi_1)\right. \\ &\quad \left. \times\left[mL\xi_2^2 \sin\xi_1 - k\left(\eta_1 - \frac{mL\sin\xi_1}{m+M}\right)\right]\right\}, \end{aligned} \quad (27)$$
$$y = \xi_1. \quad (28)$$

We see from the above equations that the internal dynamics are given by (24) and (25). Setting $\xi_1 = 0$ in these equations results in the zero dynamics

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{-k}{m+M} & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix},$$

which have eigenvalues at $\pm j\sqrt{k/(m+M)}$, meaning the system is non-minimum phase. Consequently, we need a control scheme that is applicable to non-minimum phase systems, and in this paper, we apply the technique of [4].

B. Auxiliary Problem

Following [11], [4], we first consider the auxiliary system associated with this problem, and this is given by

$$\begin{aligned} \dot{\eta}_1 &= \eta_2, & (29)\\ \dot{\eta}_2 &= \frac{k}{m+M}\left(\frac{mL\sin\xi_1}{m+M} - \eta_1\right), & (30)\\ \dot{\xi}_1 &= u_a, & (31)\\ y_a &= \frac{mL\cos\xi_1}{\Delta(\xi_1)} \\ &\quad \times\left[k\left(\eta_1 - \frac{mL\sin\xi_1}{m+M}\right) - mLu_a^2\sin\xi_1\right]. & (32) \end{aligned}$$

In order to stabilize this system, we shall adopt a strategy of first linearizing the input-output map and designing a linear compensator for the transformed system, and then implementing the inverse transformations to obtain the nonlinear control in the original coordinates. Hence, we define a new output for the auxiliary system as

$$\begin{aligned} \tilde{y}_a &= \frac{\Delta(\xi_1)y_a}{mLk\cos\xi_1} + \frac{mL}{k}u_a^2\sin\xi_1 + \frac{mL\sin\xi_1}{m+M} \\ &\triangleq \tilde{h}_a(y_a, \xi_1, u_a), & (33) \end{aligned}$$

With the newly defined output $\tilde{y}_a$, the output equation is given by

$$\tilde{y}_a = \eta_1. \tag{34}$$

The design of a stabilizing feedback controller for the system (29)–(31) and (34) can be simplified by viewing $\sin\xi_1$ as a virtual control input to the system (29)–(30), and then using either backstepping or high-gain feedback to determine the control u_a. We shall use the high-gain feedback approach. To this end, let $u_a = (v_a - \xi_1)/\varepsilon_a$, where v_a is a control input to be designed and ε_a is a positive number. The auxiliary system can now be expressed in terms of (29), (30), (34) and

$$\varepsilon_a\dot{\xi}_1 = v_a - \xi_1. \tag{35}$$

For ε_a small enough, (29), (30) and (35) will exhibit a two time-scale behavior and can thus be regarded as a singularly perturbed system. The reduced system, obtained by setting $\varepsilon_a = 0 \Rightarrow v_a = \xi_1$, is given by (29), (30) and (34), and for this reduced system, the quantity

$$\sin\xi_1 = \sin v_a \triangleq w_a \tag{36}$$

can be regarded as the control input, and can now be designed as a linear control. To summarize, the control problem for the auxiliary system reduces to a problem of finding a compensator with a strictly proper transfer function (due to the assumptions made about the dynamic controller in [4]) for the linear system

$$\begin{aligned} \dot{\eta}_1 &= \eta_2, & (37)\\ \dot{\eta}_2 &= \frac{k}{m+M}\left(\frac{mL}{m+M}w_a - \eta_1\right), & (38)\\ \tilde{y}_a &= \eta_1. & (39) \end{aligned}$$

It can be shown that the above system is observable and controllable, and a state feedback controller $w_a = k_1\eta_1 + k_2\eta_2$ was found using LQR theory. The values of k_1 and k_2 providing near-optimal trade-off between the transient performance of the states and the control effort were computed in terms of a parameter ρ_1 used to minimize the cost function

$$J = \int_0^\infty \left[\left(\eta_1 - \frac{mL}{m+M}w_a\right)^2 + \rho_1 w_a^2\right]\mathrm{d}t,$$

subject to the constraint $|w_a| \le 1$. This parameter was tuned to ensure that w_a just meets the constraint. This yielded the parameter values of $k_1 = -1$ and $k_2 = -8$.

Next, a full-order observer was designed for (37)–(39) as

$$\begin{aligned} \dot{z}_1 &= z_2 + \tilde{\alpha}_1(\tilde{y}_a - z_1), & (40)\\ \dot{z}_2 &= \tilde{\alpha}_2(\tilde{y}_a - z_1) - \frac{kz_1}{m+M} + \frac{kmL}{(m+M)^2}w_a, & (41) \end{aligned}$$

where $\tilde{\alpha}_1 = 2$ and $\tilde{\alpha}_2 = 1$. The observer-based control law can now be taken as

$$w_a = k_1z_1 + k_2z_2, \tag{42}$$

and (40)–(42) now provide the strictly proper dynamic compensator alluded to earlier.

We now substitute the transformations into the compensator equations above and find the stabilizing controller for the auxiliary system (29)–(32) to be

$$\begin{aligned} \dot{z}_1 &= z_2 + \tilde{\alpha}_1(\tilde{h}_a(y_a, \xi_1, N(z,\xi_1)) - z_1), & (43)\\ \dot{z}_2 &= \tilde{\alpha}_2(\tilde{h}_a(y_a, \xi_1, N(z,\xi_1)) - z_1) - \frac{kz_1}{m+M} \\ &\quad + \frac{kmL}{(m+M)^2}\,\mathrm{sat}(k_1z_1 + k_2z_2), & (44)\\ u_a &= N(z,\xi_1), & (45) \end{aligned}$$

where $N(z,\xi_1) = \left\{\sin^{-1}\left[\mathrm{sat}(k_1z_1 + k_2z_2)\right] - \xi_1\right\}/\varepsilon_a$. We note that we needed to use the saturated control $\mathrm{sat}(w_a) = \mathrm{sat}(k_1z_1 + k_2z_2)$ in order to ensure that its contribution to the control effort does not exceed unity magnitude in the presence of peaking. The partial state feedback controller for the system (24)–(27) is now obtained as [4]

$$u = -\frac{(m+M)\beta}{\Delta(\xi_1)}\,\mathrm{sat}\left(\frac{\xi_2 - N(z,\xi_1)}{\mu}\right). \tag{46}$$

C. Output Feedback Controller for the TORA

The output feedback controller for the original system (19)–(23) can now be designed using an extended high-gain observer in accordance with [4] and as outlined in §IIB.

$$\dot{\hat{\xi}}_1 = \hat{\xi}_2 + (\alpha_1/\varepsilon)(y - \hat{\xi}_1), \quad (47)$$

$$\dot{\hat{\xi}}_2 = \hat{\sigma} + \hat{a}u + (\alpha_2/\varepsilon^2)(y - \hat{\xi}_1), \quad (48)$$

$$\dot{\hat{\sigma}} = (\alpha_3/\varepsilon^3)(y - \hat{\xi}_1), \quad (49)$$

$$\dot{z}_1 = z_2 + \tilde{\alpha}_1(\tilde{h}_a(\hat{\sigma}, y, N(z,y)) - z_1), \quad (50)$$

$$\dot{z}_2 = \tilde{\alpha}_2(\tilde{h}_a(\hat{\sigma}, y, N(z,y)) - z_1) - \frac{kz_1}{m+M} + \frac{kmL}{(m+M)^2}\,\mathrm{sat}(k_1 z_1 + k_2 z_2), \quad (51)$$

$$u = -\frac{(m+M)\beta}{\Delta(y)}\,\mathrm{sat}\left(\frac{\hat{\xi}_2 - N(z,y)}{\mu}\right). \quad (52)$$

D. Numerical Simulations

Numerical simulations were performed using the following system and controller parameters, with the former taken from [1]: $M = 1.3608$ kg, $m = 0.096$ kg, $L = 0.0592$ m, $I = 0.0002175$ kg m^2, $k = 186.3$ N/m, $\tilde{\alpha}_1 = 2$, $\tilde{\alpha}_2 = 1$, $k_1 = -1$, $k_2 = -8$, $\beta = 500$, $\varepsilon_a \in \{0.1, 0.01\}$, $\mu \in \{1, 0.1\}$, $\varepsilon \in \{0.01, 0.001\}$, $x_1(0) = 0$ and $x_2(0) = 0.025$ m. Figures 1 through 7 show the effects of tuning certain design parameters and of perturbations in the masses of the cart and the proof mass.

E. Discussion of the Results

Figures 1 through 7 show the results of a step-by-step tuning of the design parameters, starting with matching the performance of the fourth-order reduced system in the first step and ending with the tuning of the extended high-gain observer parameter to match the partial state feedback case. In the process, the full output feedback system is made to match the performance of the reduced system, whose state equation is linear and fourth order—however, since the compensator's eigenvalues are located quite far into the left half-plane relative to those of the reduced system's plant, the state equation for the latter is essentially second order and linear. Therefore, this design methodology enabled us to enforce linear second order-like response characteristics on the controlled TORA system. We explain the tuning procedure to recover the reduced system's performance below. The errors in each step are computed using the system from the previous step as the new target.

1) The top panel in Figure 1 shows the target response trajectory we seek to recover. The solid line is the response of the reduced system (29)–(30), (34), (40)–(42), the dashed line the response of the auxiliary system (29)–(32), (43)–(45), with the latter utilizing high-gain feedback parameter values of $\varepsilon_a = 0.1$ and 0.01. The bottom panel shows how reducing ε_a from 0.1 to 0.01 allows for the auxiliary system to better approximate the response of the reduced system—indeed, the settling time was reduced drastically. The ε_a parameter is now fixed at 0.01 and we move to the next step.
2) Now, we try and match the performance of the auxiliary system with the partial state feedback system (24)–(27), (43)–(44), (46). Figure 2 shows the results. The reference trajectory used to compute the error in this step is the auxiliary system state ξ_1. The switch was made from η_1 to ξ_1 because the former is not an output of the full system. A μ value of 0.1 was deemed to provide an acceptably small error, so it was fixed at this value.
3) We now tune the extended high-gain observer parameter ε to recover the performance of the partial state feedback system from the previous step. Figures 3 and 4 show the recovery of the performance of the state feedback system by reducing ε from 0.01 to 0.001, in the output feedback system (19)–(23), (47)–(52). We note from the bottom panels of these two figures that reducing ε down to 10^{-3} results in a significant reduction in the error. We therefore choose the final design parameter as $\varepsilon = 10^{-3}$.

The above three-step tuning procedure suggests that we pick the design parameters $\varepsilon_a = 0.01$, $\mu = 0.1$ and $\varepsilon = 10^{-3}$ so that the performance of the reduced system is recovered by the output feedback system.

Figure 5 shows that the maximum control effort required to stabilize the output feedback system is under 0.05 Nm.

Finally, in order to investigate the robustness of this design, the masses of the cart and the rotor were perturbed and some simulations were run. Figure 6 shows the result when the cart and eccentric rotor masses were perturbed to $M = 1.7$ kg (+24.9% error) and $m = 0.14$ kg (+45.8% error). Figure 7 shows the result when the cart and eccentric rotor masses were perturbed to $M = 1.1$ kg (-23.7% error) and $m = 0.06$ kg (-37.5% error).

IV. Concluding Remarks

A novel extended high-gain observer based output feedback control technique for non-minimum phase systems developed in [4] was employed to stabilize the translational oscillator with rotating actuator (TORA) benchmark system. A key feature of this technique is to initially simplify the problem to one of finding a stabilizing controller for an associated auxiliary system—in order to find this controller, a high-gain feedback approach was employed, which reduced the problem further to a constrained linear control problem for a simple second order linear system. A linear dynamic controller satisfying the control constraint was obtained by optimizing a quadratic cost function. This immediately led to an equation for the nonlinear controller that stabilized the auxiliary system, and the remainder of the problem was then solved in accordance with the design procedure developed in [4] on the basis of the knowledge of the controller for the auxiliary system. This led to an extended high-gain-observer-based five-state dynamic controller for the four-state TORA system, resulting in a ninth order closed-loop system. Nevertheless, the simulation results show that the response of this system is not overly complex, and provides a solution with good settling time and some robustness to parameter perturbations.

The simulations also show that the performance of the closed-loop reduced system, which is only fourth order, can be asymptotically recovered by appropriate tuning of a

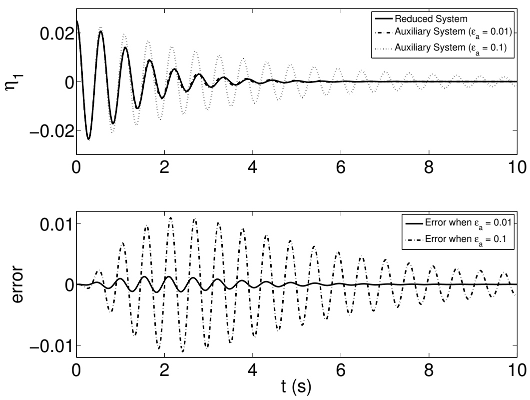

Fig. 1. Recovery of the Reduced System performance by decreasing ε_a.

single high-gain feedback parameter (ε_a). In the full output-feedback system, the performance can be fine-tuned further by reducing the width of the boundary layer μ in the sliding mode control, and also by reducing the high-gain observer parameter ε. However, it was observed that reducing μ did not have a significant impact on the system performance.

Through simulations, it was observed that this controller design allows for uncertainties between -23.7% and +24.9% for the cart mass and from -37.5% to +45.8% for the rotor mass. The output feedback system became unstable when $\varepsilon = 0.01$ and the masses were reduced to the lower extremes of the aforementioned ranges, but was stable for $\varepsilon = 10^{-3}$ or less. It must be noted, however, that decreasing the ε parameter in the extended high-gain observer too much could potentially pose practical difficulties during implementation.

In conclusion, the stabilizing controller obtained in this paper for the TORA system provides good transient performance and exhibits some robustness to perturbations of the cart mass. The nonlinear design equations for the fifth-order extended high-gain observer based controller may look complicated, but the design procedure itself is actually fairly simple and systematic, in that it provides a framework for order reduction, with the design being contingent upon a stabilizing controller for an auxiliary system of order one less than that of the original problem. Furthermore, this design approach was ultimately able to recover the performance characteristics of a full-order observer-based second order linear system.

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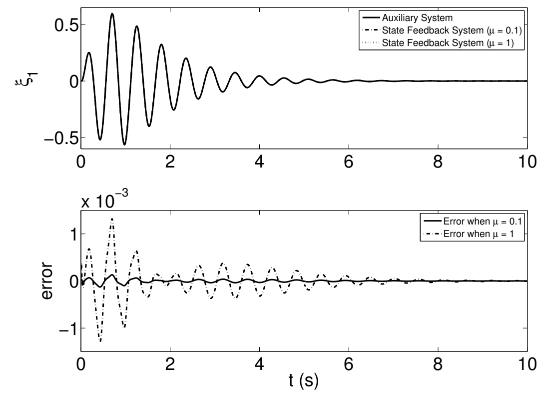

Fig. 2. Recovery of the Auxiliary System performance by decreasing μ ($\varepsilon_a = 0.01$).

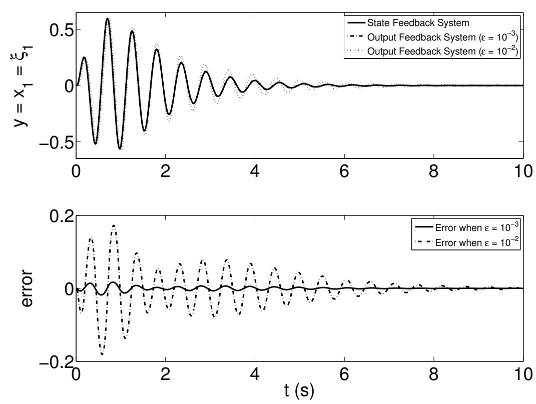

Fig. 3. Recovery of the Partial State Feedback System's performance by decreasing ε ($\mu = 0.1$).

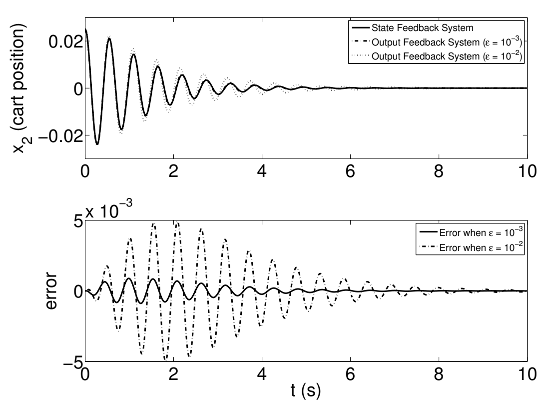

Fig. 4. Recovery of the Partial State Feedback System's performance by decreasing ε ($\mu = 0.1$).

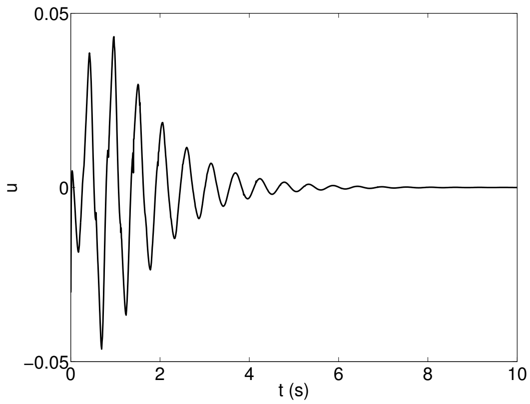

Fig. 5. The control effort for the output feedback system.

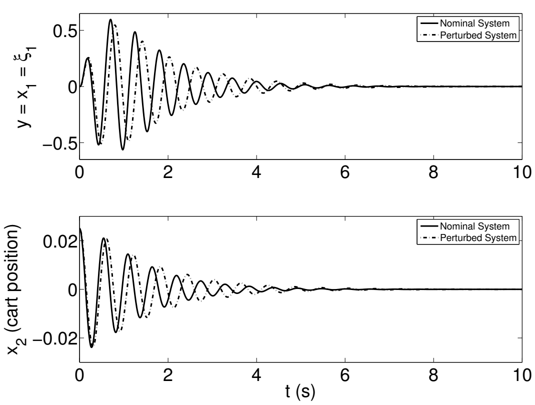

Fig. 6. Response under +24.9% perturbation in M and +45.8% perturbation in m.

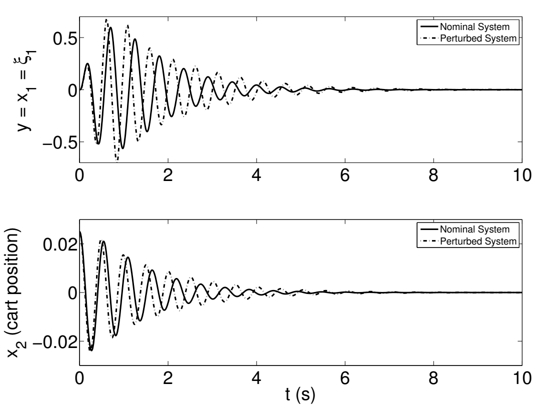

Fig. 7. Response under −23.7% perturbation in M and −37.5% perturbation in m.

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