Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Decentralized Robust Servomechanism Problem for Large Flexible Space Structures under Sensor and Actuator Failures

Samuel T. C. Huang, Edward J. Davison, Raymond H. S. Kwong

Abstract— **The decentralized robust servomechanism problem (DRSP) for large flexible space structures (LFSS) under sensor and actuator failures is considered. Failure conditions are modelled by failure matrices. This permits a unified treatment of sensor and actuator failures. For colocated LFSS, stabilization, tracking of constant set points, and regulating constant disturbances can be simultaneously handled using a decentralized tuning PID output-feedback controller. Necessary and sufficient conditions for solvability of the DRSP for colocated LFSS under sensor and actuator failures are derived. Two detailed examples demonstrate the effectiveness of the controller.**

I. INTRODUCTION

Many systems deployed in space contain large flexible structures. Typical examples include communication satellites and solar panels. This has generated considerable interest in the control of large flexible space structures (LFSS) [1]. Since it is extremely costly, if not impossible to carry out repairs in space, control systems which can tolerate sensor and actuator failures are highly desirable. Previous work in fault-tolerant control design for LFSS includes [2], [3], [4], [5]. In [2], a controller design procedure is proposed to maintain stability of the closed loop system in the presence of actuator failure. In [3]–[6], the controller parameters of the LFSS are "tuned," based on precalculated scenarios to handle anticipated component failures. The fault-tolerant control designs in these works do not consider reference tracking or disturbance rejection.

In this paper, the decentralized robust servomechanism problem (DRSP) [7] for a colocated LFSS under sensor and actuator failures is studied. The control objective is to design a decentralized controller to stabilize the closed loop system, and to track constant set points independent of any unknown constant disturbances. Furthermore, in the case of sensor or/and actuator failures, closed loop stability should be maintained, and reference tracking and disturbance rejection properties continue to hold in the "non-failed" part of the LFSS. This constitutes stronger fault tolerance requirements than those considered previously. It is shown that the existence condition for a solution to the problem can be expressed solely in terms of the rigid body model of the LFSS, and the obtained controller that solves the problem is a decentralized tuning PID controller. The existence conditions provide insight on how structural interconnections of the LFSS can have significant impact on the design of fault tolerant controllers.

This paper is organized as follows: Section II introduces the LFSS model, states the DRSP and develops our fault model. Section III states the main results. Section IV presents numerical examples. Section V concludes this paper.

S. T. C. Huang is with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Canada s.huang@utoronton.ca

E. J. Davison is with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Canada ted@control.utoronto.ca

R. H. S. Kwong is with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Canada kwong@control.utoronto.ca

II. PRELIMINARIES

A. Colocated LFSS System Model

A colocated LFSS that is controlled by $\nu \geq 2$ control agents $\{\mathbf{S}^i\}_{i\in\mathbb{I}}$ with $\mathbb{I} := \{1, 2, \ldots, \nu\}$ is modelled as:

$$\begin{aligned} \dot{x} &= \underbrace{\begin{bmatrix} 0 & I_n \\ -\Omega^2 & -D \end{bmatrix}}_{A} x + \sum_{i=1}^{\nu} \underbrace{\begin{bmatrix} 0 \\ L_i \end{bmatrix}}_{B_i} u_i + \underbrace{\begin{bmatrix} 0 \\ E \end{bmatrix}}_{\mathcal{E}} \omega, \\ y_i &= \underbrace{\begin{bmatrix} L_i^{\mathrm{T}} & 0 \end{bmatrix}}_{C_i} x, \qquad i \in \mathbb{I}, \\ e_i &= y_i - y_i^{\mathrm{ref}}, \qquad i \in \mathbb{I}, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{2n}$ is the system state; $\{u_i \in \mathbb{R}^{m_i}\}_{i\in\mathbb{I}}$ are the control inputs to the system; $\{y_i \in \mathbb{R}^{m_i}\}_{i\in\mathbb{I}}$ are the outputs of the system; $\{y_i^{\mathrm{ref}} \in \mathbb{R}^{m_i}\}_{i\in\mathbb{I}}$ are constant set points; and $\omega \in \mathbb{R}^q$ is a constant disturbance. The matrices $\Omega^2, D \in \mathbb{R}^{n\times n}$ and $\{L_i \in \mathbb{R}^{n\times m_i}\}_{i\in\mathbb{I}}$ take the following forms:

$$\Omega^2 = \begin{bmatrix} 0 & 0 \\ 0 & \bar{\Omega}^2 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & \bar{D} \end{bmatrix}, \quad L_i = \begin{bmatrix} \hat{L}_i \\ \bar{L}_i \end{bmatrix}, \; i \in \mathbb{I}.$$

Ω^2 has $\hat{n} := n - \mathrm{rank}(\Omega^2)$ 0-eigenvalues, which are the **rigid body modes** of the system. $\bar{\Omega}^2, \bar{D} \in \mathbb{R}^{(n-\hat{n})\times(n-\hat{n})}$ are diagonal matrices with strictly positive diagonal elements. $\bar{\Omega}^2$ contains the elastic modes of the LFSS, and $\bar{D}$ contains the matching damping factors. For $i \in \mathbb{I}$, $\hat{L}_i \in \mathbb{R}^{\hat{n}\times m_i}$ are rows in L_i that match the rigid body modes of the system. Let $m = m_1 + m_2 + \ldots + m_\nu$.

We will use superscripts to denote scalar components of a vector. For example, we write $y_i = \begin{bmatrix} y_i^1 & y_i^2 & \ldots & y_i^{m_i} \end{bmatrix}^{\mathrm{T}}$ and $u_i = \begin{bmatrix} u_i^1 & u_i^2 & \ldots & u_i^{m_i} \end{bmatrix}^{\mathrm{T}}$ for $i \in \mathbb{I}$.

The LFSS model can be generalized to include a $F_i\omega$ term in y_i. This will require only minor changes to the results, so the term has been omitted for simplicity.

B. Rigid Body Model of LFSS

In the LFSS mode, the state is actually $x = \begin{bmatrix} d^{\mathrm{T}} & \dot{d}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{2n}$, where $d \in \mathbb{R}^n$ is the displacement vector. $\{d \in \mathbb{R}^n \mid$

$-\Omega^2 d = 0\}$ is the dimension-$\hat{n}$ subspace of displacement vectors in which no elastic deformations occur. The dynamics in this subspace form the **rigid body model of the LFSS** (here $\hat{x} \in \mathbb{R}^{2\hat{n}}$, and $\hat{L}_i \in \mathbb{R}^{\hat{n}\times m_i}$ for $i \in \mathbb{I}$):

$$\begin{aligned} \dot{\hat{x}} &= \begin{bmatrix} 0 & I_{\hat{n}} \\ 0 & 0 \end{bmatrix} \hat{x} + \sum_{i=1}^{\nu} \begin{bmatrix} 0 \\ \hat{L}_i \end{bmatrix} u_i, \\ y_i &= \begin{bmatrix} \hat{L}_i^{\mathrm{T}} & 0 \end{bmatrix} \hat{x}, \qquad i \in \mathbb{I}, \end{aligned} \tag{2}$$

C. Decentralized Robust Servomechanism Problem for LFSS

Given a LFSS (1), the **decentralized robust servomechanism problem for the LFSS** (**DRSP**) is stated as follows:

Find a decentralized controller so that:

A1) Every eigenvalue of the resultant closed loop system is contained in the open left half plane.

A2) Asymptotic tracking occurs, i.e., $\lim_{t\to\infty} e_i(t) = 0$, $i \in \mathbb{I}$ for all constant disturbances ω, all constant set points y_i^{ref}, and all initial conditions of the system.

A3) Property A2 holds for all perturbations of the LFSS which do not cause the resultant controlled system to become unstable.

Details on the robustness condition A3 is found in [7].

D. Existence Conditions

Necessary and sufficient conditions for the DRSP to have a solution are given in [1], as follows:

Theorem 1: [1] Given a LFSS (1), the following are equivalent:

- There exists a solution to the DRSP.
- The LFSS has no decentralized fixed modes [1] at 0.
- The rigid body model (2) of the LFSS has no decentralized fixed modes at 0.
- The rigid body model of the LFSS is controllable.
- $\mathrm{rank}(\hat{L}) = \hat{n}$, where $\hat{L} \in \mathbb{R}^{\hat{n}\times m}$ is defined as:

$$\hat{L} := \begin{bmatrix} \hat{L}_1 & \hat{L}_2 & \dots & \hat{L}_\nu \end{bmatrix}. \tag{3}$$

The following result characterizes a controller which solves the DRSP:

Theorem 2: [1] Assume that the DRSP for a LFSS has a solution. Consider a decentralized controller in which each control agent $\mathbf{S}^i$, $i \in \mathbb{I}$ applies a tuning PID controller:

$$\begin{aligned} u_i &= -K_i^P y_i - K_i^D \dot{y}_i - \epsilon K_i^I \eta_i, \\ \dot{\eta}_i &= 0\eta_i + (y_i - y_i^{\mathrm{ref}}), \end{aligned} \tag{4}$$

where *arbitrary* $K_i^P, K_i^D, K_i^I > 0$ are in $\mathbb{R}^{m_i\times m_i}$. Then this controller has the property that there exists $\epsilon^\star > 0$ such that for all $\epsilon \in (0, \epsilon^\star]$, properties A1 to A3 hold.

E. The Closed Loop System

Let $\mathbf{S}^i$, $i \in \mathbb{I}$ be the only active control agent. It applies input u_i from (4) to the LFSS (1), yielding:

$$\begin{aligned} \dot{x} &= Ax - B_i(K_i^P y_i + K_i^D \dot{y}_i + \epsilon K_i^I \eta_i) + \mathcal{E}\omega \\ &= Ax - B_i[K_i^P C_i x + K_i^D C_i A x + \epsilon K_i^I \eta_i] + \mathcal{E}\omega. \end{aligned}$$

For $\dot{y}_i = C_i \dot{x}$ we used $C_i B_j = 0$ for all $j \in \mathbb{I}$, and $C_i \mathcal{E} = 0$. Including $\dot{\eta}_i$ from (4), we can write[1]

$$\begin{bmatrix} \dot{x} \\ \dot{\eta}_i \end{bmatrix} = \begin{bmatrix} \tilde{A}_i & -\epsilon B_i K_i^I \\ C_i & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta_i \end{bmatrix} + \begin{bmatrix} \mathcal{E} \\ 0 \end{bmatrix} \omega - \begin{bmatrix} 0 \\ I \end{bmatrix} y_i^{\mathrm{ref}},$$

$$y_i = C_i x, \quad \text{where}$$

$$\begin{aligned} \tilde{A}_i &:= A - B_i(K_i^P C_i + K_i^D C_i A) \\ &= \begin{bmatrix} 0 & I_n \\ -\Omega^2 - L_i K_i^P L_i^{\mathrm{T}} & -D - L_i K_i^D L_i^{\mathrm{T}} \end{bmatrix}. \end{aligned}$$

Now consider the combined effects of controllers $\{\mathbf{S}^i\}_{i\in\mathbb{I}}$. Define matrices $K^P \in \mathbb{R}^{m\times m}$ and $L \in \mathbb{R}^{n\times m}$ as:

$$\begin{aligned} K^P &:= \text{block diag}(K_1^P, K_2^P, \dots, K_\nu^P) > 0, \\ L &:= \begin{bmatrix} L_1 & L_2 & \dots & L_\nu \end{bmatrix}. \end{aligned} \tag{5}$$

K^D and K^I are defined in a similar fashion as K^P. Using a similar derivation as before, we obtain

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} &= A^{\mathrm{clo}} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} \mathcal{E} \\ 0 \end{bmatrix} \omega - \begin{bmatrix} 0 \\ I \end{bmatrix} y^{\mathrm{ref}}, \\ y &= \left[\begin{array}{cc|c} L^{\mathrm{T}} & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ \eta \end{bmatrix}, \quad \text{where} \end{aligned} \tag{6}$$

$$A^{\mathrm{clo}} := \left[\begin{array}{cc|c} 0 & I_n & 0 \\ -\Omega^2 - LK^P L^{\mathrm{T}} & -D - LK^D L^{\mathrm{T}} & -\epsilon L K^I \\ \hline L^{\mathrm{T}} & 0 & 0 \end{array}\right].$$

This is the **LFSS closed loop system (CLS)**, with output $y := \begin{bmatrix} y_1^{\mathrm{T}} & y_2^{\mathrm{T}} & \dots & y_\nu^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ (y^{ref} and η are similarly defined).

F. The LFSS Model Under Failure Conditions

It is now assumed that **sensor failure** or/and **actuator failure** may occur in (1). Failures are modelled as follows:

1) Sensor Failure: One or more outputs read by a control agent $\mathbf{S}^i$, $i \in \mathbb{I}$ may fail. We assume that failed outputs are unreliable, and should be ignored. From the perspective of $\mathbf{S}^i$, failed output signals are assigned a value of 0.

Sensor failures in $\mathbf{S}^i$ are described by an **output sensor failure matrix** $\mathcal{F}_i^y \in \mathbb{R}^{m_i\times m_i}$, which is a diagonal matrix consisting of either 0 or 1's. For $j \in \{1, \dots, m_i\}$, the j-th diagonal element in $\mathcal{F}_i^y$ is 0 if y_i^j corresponds to a failed output; and 1 otherwise. For example, $\mathcal{F}_1^y = \mathrm{diag}(0, 1, 1, 0)$ means the outputs y_1^1 and y_1^4 both fail. We can then substitute $y_i \leftarrow \mathcal{F}_i^y y_i$ (i.e. replace each y_i with $\mathcal{F}_i^y y_i$) wherever applicable.

2) Actuator Failure: Similarly, one or more actuators in $\mathbf{S}^i$, $i \in \mathbb{I}$ may fail. We assume that $\mathbf{S}^i$ disables failed actuators without affecting other actuators. This is modelled by assigning 0 to failed actuator signals.

Actuator failures in $\mathbf{S}^i$ are described by an **actuator input failure matrix** $\mathcal{F}_i^u \in \mathbb{R}^{m_i\times m_i}$, which is a diagonal matrix consisting of either 0 or 1's (defined similarly to $\mathcal{F}_i^y$). We can then substitute $u_i \leftarrow \mathcal{F}_i^u u_i$ wherever applicable.

For the overall system, $\mathcal{F}^y \in \mathbb{R}^{m\times m}$ is defined as:

$$\mathcal{F}^y := \text{block diag}(\mathcal{F}_1^y, \mathcal{F}_2^y, \dots, \mathcal{F}_\nu^y). \tag{7}$$

[1]Physical interpretation of $\tilde{A}_i$: $L_i K_i^P L_i^{\mathrm{T}} \ge 0$ (resp. $L_i K_i^D L_i^{\mathrm{T}} \ge 0$) effectively augments the stiffness (resp. damping) of the closed loop system.

$\mathcal{F}^u$ is similarly defined. We assume sensor and actuator failures have been diagnosed, so $\mathcal{F}^y$ and $\mathcal{F}^u$ are known.

G. Colocated LFSS System Model under Output Sensor and Control Actuator Failures

The substitutions $y_i \leftarrow \mathcal{F}_i^y y_i$ and $u_i \leftarrow \mathcal{F}_i^u u_i$ transform the nominal LFSS (1) into the **colocated LFSS system model under sensor and actuator failures (LFSS\F)**:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & I_n \\ -\Omega^2 & -D \end{bmatrix} x + \sum_{i=1}^{\nu} \begin{bmatrix} 0 \\ L_i \mathcal{F}_i^u \end{bmatrix} u_i + \begin{bmatrix} 0 \\ E \end{bmatrix} \omega, \\ y_i &= \begin{bmatrix} \mathcal{F}_i^y L_i^{\mathrm{T}} & 0 \end{bmatrix} x, \qquad i \in \mathbb{I}. \end{aligned} \tag{8}$$

We now present the **decentralized robust servomechanism problem for the LFSS\F (DRSP\F)**, which is the main problem we wish to solve:

H. The Main Problem: DRSP for the LFSS under Failure

Given a LFSS\F specified by (1) and failure matrices ($\mathcal{F}^y$, $\mathcal{F}^u$), find a decentralized controller so that:

- B1) Every eigenvalue of the resultant minimal order closed loop system is contained in the open left half plane.
- B2) Asymptotic tracking occurs for *outputs that correspond to no sensor or actuator failures*, i.e. $\lim_{t\to\infty} \mathcal{F}_i^y \mathcal{F}_i^u e_i(t) = 0$, $i \in \mathbb{I}$ for all constant disturbances ω, all constant set points y_i^{ref}, and all initial conditions of the system.
- B3) Property B2 holds for all perturbations of the LFSS which do not cause the resultant controlled system to become unstable.

I. Controller Adjustments to Failures

We wish to apply the tuning PID controller (4) to the DRSP\F, but controller adjustments are necessary. When failure in y_i^j or u_i^j for $i \in \mathbb{I}$, $j \in \{1, 2, \ldots, m_i\}$ is detected, the control agent $\mathbf{S}^i$ performs the following adjustments:

- C1) If a sensor failure is detected on y_i^j, then the actuator signal u_i^j is deactivated (i.e. u_i^j is set to 0).
- C2) If an actuator failure is detected on u_i^j, then the sensor signal y_i^j is ignored (i.e. y_i^j is set to 0).
- C3) The tracking set point $(y_i^{\mathrm{ref}})^j$ is set to 0.
- C4) The tracking error integral signal η_i^j is set to 0.

For an unified treatment, we define:

$$\begin{aligned} \mathcal{F}_i &:= \mathcal{F}_i^y \mathcal{F}_i^u, \quad i \in \mathbb{I}, \\ \mathcal{F} &:= \mathcal{F}^y \mathcal{F}^u = \text{block diag}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\nu). \end{aligned} \tag{9}$$

Thus adjustments C1 to C4 are represented by substituting $u_i \leftarrow \mathcal{F}_i u_i$, $y_i \leftarrow \mathcal{F}_i y_i$, $y_i^{\mathrm{ref}} \leftarrow \mathcal{F}_i y_i^{\mathrm{ref}}$, $\eta_i \leftarrow \mathcal{F}_i \eta_i$.

If $y_i^j = 0$ due to sensor failure or adjustment C2, then adjustment C3 imposes $\dot{\eta}_i^j = y_i^j - (y_i^{\mathrm{ref}})^j = 0$, and holds the tracking error integral η_i^j constant. Adjustment C4 resets η_i^j, and prevents it from injecting a constant disturbance into the system.

[2] only considers actuator failures, and an entire control agent $\mathbf{S}^i$, $i \in \mathbb{I}$ is deactivated for any actuator failure in $\mathbf{S}^i$. These mean $\mathcal{F}_i^y = I$ and $\mathcal{F}_i^u \in \{0, I\}$ for all $i \in \mathbb{I}$. Therefore our fault model is more general than the model in [2].

Remark: Suppose a failure occurs in u_i^j, and not y_i^j. Adjustment C2 sets y_i^j to 0, and we will use $y_i = \mathcal{F}_i C_i x$ to develop the closed loop system. However, the actual signal y_i^j is available via $y_i^{\mathrm{obs}} = \mathcal{F}_i^y C_i x$, and may be used for evaluating stability (but not for tracking, since B2 explicitly ignores such a $(y_i^{\mathrm{obs}})^j$).

J. Reduced Order LFSS Closed Loop System with Failure

Adjustments C1 to C4 change the controller (4) into:

$$\begin{aligned} u_i &= -\mathcal{F}_i (K_i^P \mathcal{F}_i y_i + K_i^D \mathcal{F}_i \dot{y}_i + \epsilon K_i^I \mathcal{F}_i \eta_i), \\ \dot{\eta}_i &= 0\eta_i + (\mathcal{F}_i y_i - \mathcal{F}_i y_i^{\mathrm{ref}}). \end{aligned} \tag{10}$$

Combining these with the LFSS\F (8) results in the following substitutions: $L \leftarrow L\mathcal{F}$, $L^{\mathrm{T}} \leftarrow \mathcal{F} L^{\mathrm{T}}$, $K^P \leftarrow \mathcal{F} K^P \mathcal{F}$, $K^D \leftarrow \mathcal{F} K^D \mathcal{F}$, $K^I \leftarrow \mathcal{F} K^I \mathcal{F}$ and $y^{\mathrm{ref}} \leftarrow \mathcal{F} y^{\mathrm{ref}}$. Using these and $\mathcal{F}\mathcal{F} = \mathcal{F}$ to generalize the CLS (6) results in the **LFSS closed loop system under failures (CLS\F)**:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} &= A^{\mathrm{clo}} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} \mathcal{E} \\ 0 \end{bmatrix} \omega - \begin{bmatrix} 0 \\ \mathcal{F} \end{bmatrix} y^{\mathrm{ref}}, \\ y &= \left[\begin{array}{cc|c} \mathcal{F} L^{\mathrm{T}} & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ \eta \end{bmatrix}, \quad \text{where} \end{aligned} \tag{11}$$

$$\begin{aligned} A^{\mathrm{clo}} &:= \left[\begin{array}{cc|c} 0 & I_n & 0 \\ -K^{\mathrm{eff}} & -D^{\mathrm{eff}} & -\epsilon L\mathcal{F} K^I \mathcal{F} \\ \hline \mathcal{F} L^{\mathrm{T}} & 0 & 0 \end{array}\right], \\ K^{\mathrm{eff}} &:= \Omega^2 + L\mathcal{F} K^P \mathcal{F} L^{\mathrm{T}}, \\ D^{\mathrm{eff}} &:= D + L\mathcal{F} K^D \mathcal{F} L^{\mathrm{T}}. \end{aligned} \tag{12}$$

Incorporating $\mathcal{F}$ introduces new 0-eigenvalues into A^{clo}, but these 0-eigenvalues do not affect system stability, because they correspond to η_i^j that are held at 0 by adjustment C4. We wish to remove these "benign" 0-eigenvalues, and simplify the CLS\F into a reduced system.

Let us remove 0-columns in $\mathcal{F} \in \mathbb{R}^{m\times m}$ to form $\mathcal{R} \in \mathbb{R}^{m\times m'}$ for $m' \le m$, and let $L^\star := L\mathcal{R} \in \mathbb{R}^{n\times m'}$. Now assume $\mathcal{F} = \begin{bmatrix} I_{m'} & 0 \\ 0 & 0 \end{bmatrix}$, which means $\mathcal{R} = \begin{bmatrix} I_{m'} \\ 0 \end{bmatrix}$ and

$$\begin{aligned} \mathcal{F} &= \begin{bmatrix} \mathcal{R}^{\mathrm{T}} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{R} & 0 \end{bmatrix}, \\ L\mathcal{F} &= L \begin{bmatrix} \mathcal{R} & 0 \end{bmatrix} = \begin{bmatrix} L^\star & 0 \end{bmatrix}, \\ \mathcal{F} L^{\mathrm{T}} &= \begin{bmatrix} L^{\star\mathrm{T}} \\ 0 \end{bmatrix}. \end{aligned}$$

Noting that $L\mathcal{F} = L\mathcal{F}\mathcal{F}$, we obtain:

$$\begin{aligned} -\epsilon L\mathcal{F} K^I \mathcal{F} &= -\epsilon \begin{bmatrix} L^\star & 0 \end{bmatrix} \begin{bmatrix} \mathcal{R}^{\mathrm{T}} \\ 0 \end{bmatrix} K^I \begin{bmatrix} \mathcal{R} & 0 \end{bmatrix} \\ &= \begin{bmatrix} -\epsilon L^\star (\mathcal{R}^{\mathrm{T}} K^I \mathcal{R}) & 0 \end{bmatrix}. \end{aligned}$$

Applying all these to A^{clo} in (12) results in

$$A^{\mathrm{clo}} = \left[\begin{array}{cc|cc} 0 & I_n & 0 & 0 \\ -K^{\mathrm{eff}} & -D^{\mathrm{eff}} & -\epsilon L^\star (\mathcal{R}^{\mathrm{T}} K^I \mathcal{R}) & 0 \\ \hline L^{\star\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Next, we simplify K^{eff}, D^{eff} and the term with y^{ref} in (11):

$$\begin{aligned} L\mathcal{F}K^P\mathcal{F}L^{\text{T}} &= \begin{bmatrix} L^\star & 0 \end{bmatrix} \begin{bmatrix} \mathcal{R}^{\text{T}} \\ 0 \end{bmatrix} K^P \begin{bmatrix} \mathcal{R} & 0 \end{bmatrix} \begin{bmatrix} L^{\star\text{T}} \\ 0 \end{bmatrix} \\ &= L^\star(\mathcal{R}^{\text{T}}K^P\mathcal{R})L^{\star\text{T}}, \\ L\mathcal{F}K^D\mathcal{F}L^{\text{T}} &= L^\star(\mathcal{R}^{\text{T}}K^D\mathcal{R})L^{\star\text{T}}, \\ \left[\begin{array}{c} 0 \\ \hline \mathcal{F} \end{array}\right] y^{\text{ref}} &= \left[\begin{array}{c} 0 \\ \hline \mathcal{R}^{\text{T}} \\ 0 \end{array}\right] y^{\text{ref}} = \left[\begin{array}{c} 0 \\ \hline I \\ 0 \end{array}\right] \mathcal{R}^{\text{T}} y^{\text{ref}}. \end{aligned}$$

Finally, note that the bottom block-row of A^{clo} and $\mathcal{F}y^{\text{ref}}$, and the rightmost block-column of A^{clo} are 0. We remove these to form the **reduced order LFSS closed loop system with failure (Red(CLS\F))**:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\eta}^\star \end{bmatrix} &= A^{\text{clo}\star} \begin{bmatrix} x \\ \eta^\star \end{bmatrix} + \begin{bmatrix} \mathcal{E} \\ 0 \end{bmatrix} \omega - \begin{bmatrix} 0 \\ I \end{bmatrix} y^{\text{ref}\star}, \\ y^\star &= \left[\begin{array}{cc|c} L^{\star\text{T}} & 0 & 0 \end{array} \right] \begin{bmatrix} x \\ \eta^\star \end{bmatrix}, \quad \text{where} \end{aligned} \tag{13}$$

$$\begin{aligned} A^{\text{clo}\star} &:= \left[\begin{array}{cc|c} 0 & I_n & 0 \\ -K^{\text{eff}\star} & -D^{\text{eff}\star} & -\epsilon L^\star(\mathcal{R}^{\text{T}}K^I\mathcal{R}) \\ \hline L^{\star\text{T}} & 0 & 0 \end{array} \right], \\ L^\star &:= L\mathcal{R}, \\ K^{\text{eff}\star} &:= \Omega^2 + L^\star(\mathcal{R}^{\text{T}}K^P\mathcal{R})L^{\star\text{T}}, \\ D^{\text{eff}\star} &:= D + L^\star(\mathcal{R}^{\text{T}}K^D\mathcal{R})L^{\star\text{T}}, \\ y^{\text{ref}\star} &:= \mathcal{R}^{\text{T}}y^{\text{ref}}. \end{aligned}$$

The same result is obtained from a general $\mathcal{F} \neq \begin{bmatrix} I_{m'} & 0 \\ 0 & 0 \end{bmatrix}$. The zero rows and columns in A^{clo} and $\mathcal{F}y^{\text{ref}}$ are scattered, but these are ultimately removed.

K. Reduced Order Colocated LFSS Model

Given a LFSS model (1) and fault matrices $\{\mathcal{F}_i\}_{i\in\mathbb{I}}$, let us remove zeros-columns in $\mathcal{F}_i$ to form $\mathcal{R}_i \in \mathbb{R}^{m_i \times m'_i}$, with $m'_i \leq m_i$ for each $i \in \mathbb{I}$. Also, let $L^\star_i := L_i\mathcal{R}_i$. The **reduced order colocated LFSS model (RedLFSS)** is defined as:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & I_n \\ -\Omega^2 & -D \end{bmatrix} x + \sum_{i=1}^{\nu} \begin{bmatrix} 0 \\ L^\star_i \end{bmatrix} u^\star_i + \begin{bmatrix} 0 \\ E \end{bmatrix} \omega, \\ y^\star_i &= \begin{bmatrix} L^{\star\text{T}}_i & 0 \end{bmatrix} x, \qquad i \in \mathbb{I}, \\ e^\star_i &= y^\star_i - y^{\text{ref}\star}_i, \qquad i \in \mathbb{I}, \end{aligned} \tag{14}$$

with $u^\star_i \in \mathbb{R}^{m'_i}$, $y^\star_i \in \mathbb{R}^{m'_i}$ and $y^{\text{ref}\star}_i := \mathcal{R}^{\text{T}}_i y^{\text{ref}}_i \in \mathbb{R}^{m'_i}$. Note that $m'_i = 0$ is possible, but this degenerate case poses no problem. The RedLFSS does not alter Ω^2 and D, so it has $\hat{n}$ rigid body modes like the original LFSS.

We now apply a tuning PID controller that has the same form as (4), with $K^{P\star}_i, K^{D\star}_i, K^{I\star}_i > 0$, $i \in \mathbb{I}$ as gains:

$$\begin{aligned} u^\star_i &= -K^{P\star}_i y^\star_i - K^{D\star}_i \dot{y}^\star_i - \epsilon K^{I\star}_i \eta^\star_i, \\ \dot{\eta}^\star_i &= 0\eta^\star_i + (y^\star_i - y^{\text{ref}\star}_i). \end{aligned} \tag{15}$$

The resultant closed loop system is denoted **RedCLS**. It has the same form as the CLS (6), and is omitted here.

We now relate B1 to B3 to the RedLFSS and RedCLS:

Lemma 1: Given the LFSS (1) and failure matrices $\mathcal{F}^y$ and $\mathcal{F}^u$ (resulting in $\mathcal{F}$ and $\mathcal{R}$), consider the following:

D1) There exist $K^P, K^D, K^I > 0$ and $\epsilon > 0$ such that the resulting PID controller (10) (which applies C1 to C4) satisfies conditions B1 to B3 for the LFSS\F.

D2) There exist $K^{P\star}, K^{D\star}, K^{I\star} > 0$ and $\epsilon > 0$ such that the resulting PID controller (15) satisfies conditions A1 to A3 for the RedLFSS.

The lemma states that D1 holds if and only if D2 holds.

Proof: D1 defines the Red(CLS\F) (13); D2 defines the RedCLS, which is obtained by applying (6) to (14). By direct comparison, Red(CLS\F) and RedCLS are identical for a common ϵ, *assuming that* $\mathcal{R}^{\text{T}}K^P\mathcal{R} = K^{P\star}$, $\mathcal{R}^{\text{T}}K^D\mathcal{R} = K^{D\star}$ and $\mathcal{R}^{\text{T}}K^I\mathcal{R} = K^{I\star}$. With these, B1 holds iff A1 holds. WOLOG, for $i \in \mathbb{I}$ let $m'_i > 0$, $\mathcal{F}_i = \begin{bmatrix} I_{m'_i} & 0 \\ 0 & 0 \end{bmatrix}$ and $\mathcal{R}_i = \begin{bmatrix} I_{m'_i} \\ 0 \end{bmatrix}$. Then $\mathcal{F}_i e_i = 0$ holds iff $e^\star_i = 0$, since $\mathcal{F}_i(y_i - y^{\text{ref}}_i) = \begin{bmatrix} L^{\star\text{T}}_i & 0 \\ 0 & 0 \end{bmatrix} x - \begin{bmatrix} I \\ 0 \end{bmatrix} \mathcal{R}^{\text{T}}_i y^{\text{ref}}_i$. Therefore B2 and B3 hold iff A2 and A3 hold. Now we need to justify our assumptions.

For sufficiency, assume D1 holds for $K^P, K^D, K^I > 0$. Choosing $K^{P\star} := \mathcal{R}^{\text{T}}K^P\mathcal{R}$ trivially satisfies $K^{P\star} > 0$. For necessity, assume D2 holds for $K^{P\star}, K^{D\star}, K^{I\star} > 0$. WOLOG, for $i \in \mathbb{I}$, choosing $K^P_i := \begin{bmatrix} K^{P\star}_i & 0 \\ 0 & I_{m_i - m'_i} \end{bmatrix}$ satisfies $K^P_i > 0$ and $\mathcal{R}^{\text{T}}_i K^P_i \mathcal{R}_i = K^{P\star}_i$. The above arguments hold for K^D and K^I, etc. □

III. RESULTS

A. Existence of Solution

Let the LFSS (1) and failure matrices $(\mathcal{F}^y, \mathcal{F}^u)$ be given. Our main result for solving the DRSP\F is presented below.

Theorem 3: (Main result)

E1) The DRSP\F can be solved using the decentralized controller (4) if and only if the **rigid body model of the LFSS\F**, defined by

$$\begin{aligned} \dot{\hat{x}} &= \begin{bmatrix} 0 & I_{\hat{n}} \\ 0 & 0 \end{bmatrix} \hat{x} + \sum_{i=1}^{\nu} \begin{bmatrix} 0 \\ \hat{L}_i\mathcal{F}_i \end{bmatrix} u_i, \\ y_i &= \begin{bmatrix} \mathcal{F}_i\hat{L}^{\text{T}}_i & 0 \end{bmatrix} \hat{x}, \qquad i \in \mathbb{I}, \end{aligned} \tag{16}$$

has no decentralized fixed modes at 0.

E2) Assume that condition E1 holds; then there exists $\epsilon^\star > 0$ so that for all $\epsilon \in (0, \epsilon^\star]$, the DRSP\F is solved using the controller (4).

Proof: For E1, stating that the DRSP\F can be solved with (4) is equivalent to statement D1, which is equivalent to statement D2 (Lemma 1). E1 follows from applying Theorem 1 to D2 (which has the structure of a regular LFSS). E2 is obtained by applying Lemma 1 to Theorem 2, provided that $\epsilon > 0$ is common, using the same (K^P, K^D, K^I) to $(K^{P\star}, K^{D\star}, K^{I\star})$ mapping as in the proof of Lemma 1. □

Analogues to the equivalent conditions in Theorem 1 can be readily obtained. One of these conditions is very succinct, and deserves special attention:

Corollary 1: The DRSP\F can be solved if and only if

$$\operatorname{rank}(\hat{L}\mathcal{F}) = \hat{n}, \tag{17}$$

where $\hat{L}$ is defined in (3), $\mathcal{F}$ is defined in (9), and $\hat{n}$ is the number of rigid body modes in (1).

The rank test in Corollary 1 reveals much insight on the structure of a LFSS, and helps us identify control redundancies that enable tolerance to sensor and actuator failures. This test can also find application in the design and evaluation of a LFSS.

B. Remarks Regarding Choice of ϵ

In the tuning PID controller (4) proposed to be used, Theorems 2 and 3 give conditions for the existence of $\epsilon^\star$, so that for all $\epsilon \in (0, \epsilon^\star]$, the controller solves the DRSP or DRSP\F. For methods to find the optimal ϵ, see [8], [10] and [11]. The numerical examples studied in this paper include the maximum value of $\epsilon^\star$ which can be used in a given problem before instability occurs.

IV. EXAMPLES

This section provides examples to illustrate and interpret our results.

A. 1D System

Our first example is a system of 3 masses connected by springs and dampers, as shown in Fig. 1, operating in 1-dimension (horizontal). For $i \in \{1, 2, 3\}$, the control agent $\mathbf{S}^i$ observes the displacement ξ_i of mass M_i, and applies force along ξ_i. Let $[M_1, M_2, M_3] = [1, 10, 1]$; $[K_{12}, K_{23}] = [1, 1]$;

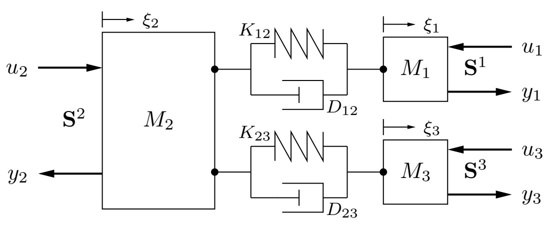


Fig. 1. 1D example with 3 control agents.

$[D_{12}, D_{23}] = [0.1, 0.1]$. The model is given by:

$$\hat{M}\ddot{\xi} + \hat{D}\dot{\xi} + \hat{K}\xi = L_0 u, \qquad y = L_0^{\mathrm{T}}\xi,$$

where

$$\hat{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix},$$

$$L_0 = I_3, \qquad \hat{D} = \begin{bmatrix} 0.1 & -0.1 & 0 \\ -0.1 & 0.2 & -0.1 \\ 0 & -0.1 & 0.1 \end{bmatrix}.$$

TABLE I

CLOSED LOOP EIGENVALUES FOR THE 1D SYSTEM EXAMPLE

Nominal	$\mathbf{S}^1$ fails
-8.8612	-8.8612
-8.8612	-1.2129
-1.2203	$-0.4437 \pm j0.9583$
-1.2057	$-0.1106 \pm j0.9772$
$-0.5085 \pm j0.9586$	-0.0204
-0.0204	-0.0169
-0.0185	0
-0.0156	

The colocated LFSS model with unknown disturbance $\omega \in \mathbb{R}^3$ can be written in the form of (1):

$$\dot{x} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -0.1 & 0 \\ 0 & 0 & -1.2 & 0 & 0 & -0.12 \end{array}\right] x + \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0.2887 & 0.2887 & 0.2887 \\ -0.7071 & 0.0000 & 0.7071 \\ -0.6455 & 0.1291 & -0.6455 \end{array}\right] u + \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \omega,$$

$$y = \left[\begin{array}{ccc|ccc} 0.2887 & -0.7071 & -0.6455 & 0 & 0 & 0 \\ 0.2887 & 0.0000 & 0.1291 & 0 & 0 & 0 \\ 0.2887 & 0.7071 & -0.6455 & 0 & 0 & 0 \end{array}\right] x.$$

Note that we choose $E = I_3$. The rigid body model (2) is:

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ \hat{L} \end{bmatrix} u, \qquad y = \begin{bmatrix} \hat{L}^{\mathrm{T}} & 0 \end{bmatrix} \hat{x}, \quad \text{where} \qquad \hat{L} = \begin{bmatrix} 0.2887 & 0.2887 & 0.2887 \end{bmatrix}. \tag{18}$$

Clearly $\hat{n} = 1$ and $\operatorname{rank}(\hat{L}) = 1$. By Theorems 1 and 2, the DRSP is solved with a tuning PID controller (4). Choosing $K^P = K^D = 10I_3$, $K^I = I_3$, $\epsilon = 0.2$ results in asymptotically stable CLS eigenvalues, as shown in Table I. So A1 is satisfied.

To evaluate tracking and disturbance rejection, we express the steady-state output as a linear function of y^{ref} and ω:

$$\lim_{t\to\infty} y(t) = S^{\mathrm{ref}} y^{\mathrm{ref}} + S^{\omega}\omega. \tag{19}$$

For the nominal system, A2 implies $S^{\mathrm{ref}}_{\mathrm{nom}} = I$ and $S^{\omega}_{\mathrm{nom}} = 0$. This is verified in our simulations.

Now consider failures. By Corollary 1, the DRSP\F is solved if $\operatorname{rank}(\hat{L}\mathcal{F}) = \hat{n} = 1$. In this example, this means $\mathcal{F} \neq 0$, i.e. whenever any $\mathbf{S}^i$ has working sensor and actuator. We verify this by assuming an actuator failure in u_1: $\mathcal{F} = \mathcal{F}^u = \operatorname{diag}(0, 1, 1)$. B1 holds, since Table I shows that the eigenvalues of the CLS\F (using the same gains) are all

TABLE II
VARIOUS FAILURE SCENARIOS FOR THE 1D SYSTEM.

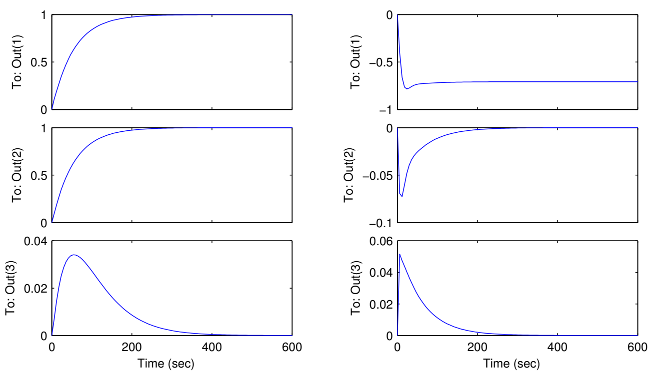
Failed controller	Maximum $\epsilon^\star$	$\mathrm{rank}(\hat{L}\mathcal{F})$
Nominal	11.9865	1
$\{\mathbf{S}^1\}; \{\mathbf{S}^3\}$	8.654	1
$\{\mathbf{S}^2\}$	111.100	1
$\{\mathbf{S}^1, \mathbf{S}^2\}; \{\mathbf{S}^2, \mathbf{S}^3\}$	111.203	1
$\{\mathbf{S}^1, \mathbf{S}^3\}$	6.873	1
$\{\mathbf{S}^1, \mathbf{S}^2, \mathbf{S}^3\}$	(no solution)	0

asymptotically stable, except for a benign 0-eigenvalue that corresponds to η_1,

The steady-state output (19) for the u_1-failure case becomes $S_1^{\mathrm{ref}} y^{\mathrm{ref}} + S_1^{\omega}\omega$, where ($\star$ are non-zero values)

$$S_1^{\mathrm{ref}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad S_1^{\omega} = \begin{bmatrix} \star & \star & \star \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The u_1-failure step responses for $y^{\mathrm{ref}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$ ($\omega = 0$) and for $\omega = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$ ($y^{\mathrm{ref}} = 0$) are shown in Fig. 2 (a) and Fig. 2 (b). Note that y_1 is available for evaluation.



(a) Tracking resp. on $y_2^{\mathrm{ref}} = 1$ (b) Disturbance resp. on $\omega_2 = 1$

Fig. 2. $\mathbf{S}^1$-Failure responses of 1D benchmark.

Thus we have tracking and disturbance rejection for y_2 and y_3, and not y_1; this agrees with B2. In fact, y_1 tracks y_2^{ref}, and this has a physical explanation: Actuator failure $u_1 = 0$ means M_1 is uncontrolled; M_1 becomes "limp" and is "towed" by M_2 through a spring-damper connection, so $y_2(t) \to y_1(t) \to y_1^{\mathrm{ref}}$ as $t \to \infty$. Finally, note that disturbance ω affects y_1.

Table II summarizes stability results of CLS\F for every failure scenario.[2] Maximum $\epsilon^\star$ (from E2) is found numerically. Corollary 1 is verified, since our controller can stabilize the LFSS\F ($\epsilon^\star > 0$) whenever $\mathrm{rank}(\hat{L}\mathcal{F}) = 1$. With the given gains, ϵ is chosen to be less than every maximum $\epsilon^\star$ from Table II, then the CLS\F would be stable for under failure scenario.

[2]The notation "$\{\mathbf{S}^1, \mathbf{S}^2\}; \{\mathbf{S}^2, \mathbf{S}^3\}$" represents "the case in which both $\mathbf{S}^1$ and $\mathbf{S}^2$ fail; and the case in which both $\mathbf{S}^2$ and $\mathbf{S}^3$ fail," etc.

B. 4-Component System

Our second example, taken from [2], is shown in Fig. 3. The LFSS consists of 4 interconnected subsystems arranged in a plane. For $i \in \{1, 2, 3, 4\}$, the states of subsystem i are displacements $(\xi_i, \zeta_i, \theta_i)$ and velocities $(\dot{\xi}_i, \dot{\zeta}_i, \dot{\theta}_i)$. Each control agent $\mathbf{S}^i$ reads $y_i \in \mathbb{R}^3$, and produces $u_i \in \mathbb{R}^3$.

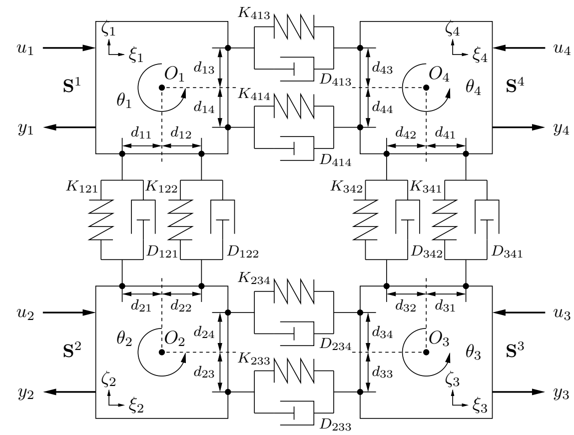


Fig. 3. Example from [2], with 4 subsystems.

For the simulation, we chose subsystems [2] to have identical masses $M_i = 25$, moment of inertia $J_i = 4$ and connector radii $d_{ij} = 0.3$. Interconnections are also identical, with stiffness $K_{ijk} = 250$ and damping $D_{ijk} = 75$.

The colocated LFSS has $n = 24$ states and $m = 12$ input/output pairs. Expressing the LFSS in the standard form (1) reveals $\hat{n} = 5$ rigid body modes. $\hat{L}$ as defined by (3) is:

$$\hat{L} = \left[\begin{array}{ccc|ccc|ccc|ccc} . & \alpha & . & . & \alpha & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & \alpha & . & . & \alpha & . \\ \alpha & . & . & . & . & . & . & . & . & \alpha & . & . \\ . & . & . & \alpha & . & . & \alpha & . & . & . & . & . \\ . & . & \beta & . & . & \beta & . & . & \beta & . & . & \beta \end{array}\right], \tag{20}$$

where $\alpha = -0.1414$, $\beta = 0.2500$, and periods denote 0. The columns are arranged by $(\xi_1, \zeta_1, \theta_1, \ldots, \xi_4, \zeta_4, \theta_4)$.

[2] effectively assumes $\mathcal{F}_i \in \{0, I\}$, and uses a dynamic displacement feedback controller [9] to stabilize the system. The resulting closed loop system is shown to be stable in the nominal, $\mathbf{S}^1$-failure and $\{\mathbf{S}^1, \mathbf{S}^3\}$-failure cases, but unstable in the $\{\mathbf{S}^1, \mathbf{S}^2\}$-failure case.

We reproduced the stability results in [2] using the tuning PID controller (10), with $K^P = K^D = 100I$ and $K^I = I$. Table III addresses all 16 actuator failure cases under the fault model in [2]. Again, we list the maximum $\epsilon^\star$ and $\mathrm{rank}(\hat{L}\mathcal{F})$ from (17) for each case. The CLS\F is stable ($\epsilon^\star > 0$) iff the rank is $\mathrm{rank}(\hat{L}\mathcal{F}) = \hat{n} = 5$; this verifies Corollary 1. Note that the tuning PID controller also tracks y^{ref}, and regulates disturbances.

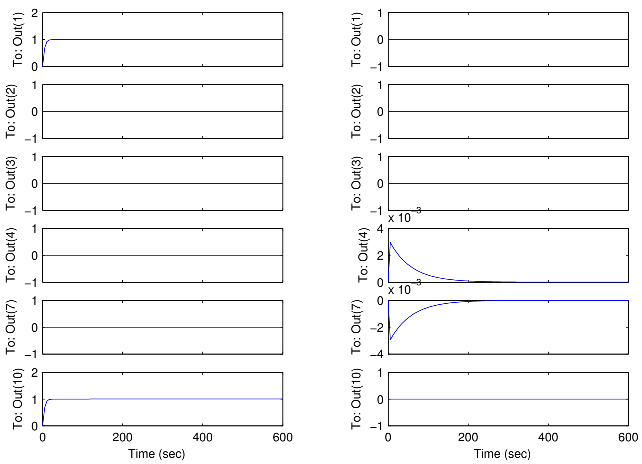
We simulated the transient responses for actuator failure in u_1^1, using the same gains with $\epsilon = 20$ and $E = I_{12}$. Step response for $(y_4^{\mathrm{ref}})^1 = 1$ and $\omega^{10} = 1$ (both corresponding to ξ_4) are plotted in Fig. 4 (a) and Fig. 4 (b). Only outputs $(y_1^1, y_1^2, y_1^3, y_2^1, y_3^1, y_4^1) = (\xi_1, \zeta_1, \theta_1, \xi_2, \xi_3, \xi_4)$ are displayed.

TABLE III

VARIOUS FAILURE SCENARIOS FOR THE 4-COMPONENT SYSTEM

Failed controller	Maximum $\epsilon^\star$	$\text{rank}(\hat{L}\mathcal{F})$
Nominal	400.000	5
Any 1 fails	194.3	5
$\{\mathbf{S}^1, \mathbf{S}^3\}$; $\{\mathbf{S}^2, \mathbf{S}^4\}$	194.3	5
$\{\mathbf{S}^1, \mathbf{S}^2\}$; $\{\mathbf{S}^3, \mathbf{S}^4\}$	(no solution)	4
$\{\mathbf{S}^1, \mathbf{S}^4\}$; $\{\mathbf{S}^2, \mathbf{S}^3\}$	(no solution)	4
Any 3 fail	(no solution)	3
All fail	(no solution)	0

We see that $y_4^1 = \xi_4$ tracks properly, but $y_1^1 = \xi_1$ (corresponds to failed u_1^1) tracks $(y_4^{\text{ref}})^1 = 1$, not $(y_1^{\text{ref}})^1 = 0$. As before, this can be explained by the horizontal connection between subsystems 1 and 4.



(a) Tracking resp. on $(y_4^{\text{ref}})^1 = 1$ (b) Disturb. resp. on $\omega^{10} = 1$

Fig. 4. u_1^1-Failure responses of 4-component benchmark.

Finally, applying Corollary 1 to $\hat{L}$ in (20) reveals much insight for this example. In order to decrease $\text{rank}(\hat{L}\mathcal{F})$ by nullifying row 1 if $\hat{L}$, both u^2 (for ζ_1) and u^5 (for ζ_2) need to fail. Physically, this reflects the system redundancy (w.r.t. stabilization) that arises from the vertical connection between subsystems 1 and 2. In the linear model, as long as one of ζ_1 or ζ_2 is controlled, the other would not grow unbounded as $t \to \infty$. The same explanation can be given for rows 2, 3 and 4 in $\hat{L}$. In row 5, $(u^3, u^6, u^9, u^{12}) = (\theta_1, \theta_2, \theta_3, \theta_4)$ need to all fail to decrease $\text{rank}(\hat{L}\mathcal{F})$. Physically, this means we only need to apply control to the angle in one subsystem in order to keep the angles in other subsystem bounded.

V. CONCLUSIONS

In this paper, we augment the LFSS model in [1] with a sensor and actuator failure model, which is a generalization of the fault model in [2]. In addition to stabilization, we treat also tracking of constant set points and regulation of constant disturbances in a unified framework. The LFSS\F and the DRSP\F are direct extensions of the LFSS and the DRSP for systems subject to failures. In defining the DRSP\F, we use controller adjustments to remove the distinction between sensor and actuator failures. This allows Red(CLS\F) to be defined, using a decentralized tuning PID controller with controller adjustments C1 to C4. We also define RedLFSS, to which Theorems 1 and 2 can be directly applied. Lemma 1 shows that the resulting RedCLS is equivalent to the Red(CLS\F); this leads to our main result Theorem 3. If the rigid body model is known, then the rank test in Corollary 1 provides a powerful and insightful way to test the existence of a solution to the DRSP\F.

The two examples provided demonstrate the effectiveness of our results for checking solvability of the fault-tolerant control problem. In addition, the existence conditions show the importance of physical connections in the LFSS. The insight gained can be quite useful for structural design of LFSS, and sensor and actuator placement.

REFERENCES

[1] G. S. West-Vukovich, E. J. Davison, P. C. Hughs, "The Decentralized Control of Large Flexible Space Structures", *IEEE Trans. Automat. Contr.*, vol. AC-29, no. 10, Oct. 1984, pp. 866–879.

[2] Y. Kobayashi, M. Ikeda, Y. Fujisaki, "Stability of Large Space Structures Preserved Under Failures of Local Controllers", *IEEE Trans. Automat. Contr.*, vol. 52, no. 2, Feb. 2007, pp. 318–322.

[3] G. G. Yen, "Reconfigurable learning control in large space structures", *IEEE Trans. Control Syst. Technol.*, vol. 2, no. 4, Apr. 1994, pp. 362–370.

[4] S. S. Ahmad, J. S. Lew, L. H. Keel, "Fault tolerant controller design for large space structures", *Proc. 1999 IEEE Int. Conf. Control Applications*, 1999, pp. 63–68

[5] S. S. Ahmad, J. S. Lew, L. H. Keel, "Robust control of flexible structures against structural damage", *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 1, Jan. 2000, pp. 170–182.

[6] J. N. Aubrun, "Theory of the control of structures by low-authority controllers", *J. Guid. Contr.*, vol. 3, no. 5, 1980, pp.444–451.

[7] E. J. Davison, "The robust decentralized control of a general servomechanism problem", *IEEE Trans. Automat. Contr.*, vol. AC-21, no. 1, Feb. 1976, pp. 14–24

[8] E. J. Davison, "Multivariable Tuning Regulators: The Feedforward and Robust Control of a General Servomechanism Problem", *IEEE Trans. Automat. Contr.*, vol. AC-21, no. 1, Feb. 1976, pp. 35–47.

[9] Y. Fujisaki, M. Ikeda, K. Miki, "Robust stabilization of large space structures via displacement feedback", *IEEE Trans. Autom. Control*, vol. 46, no. 12, Dec. 2001, pp. 1993–1996.

[10] D. Miller, E. J. Davison, "The Self-Tuning Robust Servomechanism Problem", *IEEE Trans. Automat. Contr.*, vol. 34, no. 5, May. 1989, pp. 511–523.

[11] E. J. Davison, "Decentralized Robust Control of Unknown Systems Using Tuning Regulators", *IEEE Trans. Automat. Contr.*, vol. AC-23, no. 2, Apr. 1978, pp. 276–289.

[12] M. Witczak M.(2007): *Modelling and Estimation Strategies for Fault Diagnosis of Non-Linear Systems*, Series: Lecture Notes in Contr. and Info. Sci., Vol. 354, Springer; 2007

[13] J. Korbicz, J. M. Kościelny, Z. Kowalczuk, and W. Cholewa (Eds.), *Fault Diagnosis: Models, Artificial Intelligence, Applications*, Springer; 2004.