Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Passivity Results for Interconnected Systems with Time Delay

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Abstract— **It is well known that a negative feedback interconnection of passive systems is passive. However, the extension of this fundamental property to the case when there are time delays in communication, remains largely unaddressed. In this paper we demonstrate that a negative feedback interconnection of output strictly passive systems, under appropriate assumptions, is passive for non-increasing time delays and may loose passivity for increasing time delays. Passivity can be retained by inserting time-varying gains in the communication path, provided a bound on the maximum rate of change of delay is known. If the dynamical systems are passive, we appeal to the results in bilateral teleoperation [2], [13], to recover passivity of the feedback interconnection. We show that by transforming the two systems into their scattering representation, transmitting the scattering variables as the new outputs, and using time-varying gains in the communication path, passivity of the feedback interconnection can be guaranteed independent of the time-varying delays. Finally we discuss the applicability of the proposed results for networked control of nonlinear mechanical systems.**

I. Introduction

In this paper we study passivity of a feedback interconnection of two passive systems when there are time-varying delays in communication. It is well known that a feedback interconnection of two passive systems is passive [7], [26]. This simple but powerful paradigm for feedback interconnection of linear and nonlinear systems, has led to a wide variety of constructive control designs [3], [23], [22]. Due to the ubiquity of modern communication networks, it is important to extend the standard passivity and dissipativity results for feedback interconnections that may have time delays in communication. Some preliminary results in this direction have been reported in [15], [19], [9], [6], [12], [4], [17], [27], [16].

Inspired by the results in bilateral teleoperation and synchronization of nonlinear systems [13], [5], it was demonstrated in [6] that under appropriate assumptions, a negative feedback interconnection of output strictly passive systems, with constant time delays in communication, is passive. In this note, we extend the aforementioned result to include time-varying delays in communication. Using previous results in the problem of bilateral teleoperation [13], we demonstrate that under appropriate assumptions, a feedback interconnection of output strictly passive systems, with non-increasing time delays, is passive independent of the time-varying delays. In the general case, when the time delay may be increasing or decreasing, we show that passivity of the feedback interconnection can no longer be guaranteed. However, passivity can be recovered provided time-varying gains [13], dependent on the maximum rate of change of delay, are used in the communication path.

We also address the case when the two systems are passive, and not output strictly passive as required in the previous results. If the passive systems are transformed using the scattering representation [7], [26] and the scattering variables are transmitted as the new outputs, then the feedback interconnection is passive independent of the constant time delays. A specific application of this general result [2] is well studied in the telerobotics literature, where the two subsystems are the master-slave robots. However its general formalization, as treated in this paper, is relatively new. In addition, we show that the addition of time-varying gains, dependent on the maximum rate of change of delay, can be used to guarantee passivity independent of the time-varying delays.

The outline of the paper is as follows. We briefly discuss standard the passivity result for a feedback interconnection in Section II, which is followed by the main results in Section III. We point out the relevance of the proposed results to networked control of mechanical systems in Section IV and finally summarize the results in Section V.

II. Background

The concept of passivity is one of the most physically appealing concepts of system theory [23] and, as it is based on input-output behavior of an system, is equally applicable to both linear and nonlinear systems. Most of the ideas presented in this section are from [11]. Consider a dynamical system represented by the state space model

$$\dot{x} = f(x, u) \quad (1)$$

$$y = h(x) \quad (2)$$

where f: $R^n \times R^p \rightarrow R^n$ is locally Lipschitz, h: $R^n \times R^p \rightarrow R^p$ is continuous, $f(0,0) = 0$, $h(0) = 0$ and the system has the same number of inputs and outputs.

Definition The dynamical system (1)-(2) is said to be passive if there exists a continuously differentiable non-negative definite scalar function $S(x)$: $R^n \rightarrow R$ (called the storage function) such that

$$u^T y \geq \dot{S}(x), \qquad \forall (x,u) \in R^n \times R^p$$

Moreover, the system is said to be

- strictly passive if $u^T y \geq \dot{S}(x) + D(x)$ for some positive definite function $D(x)$

This research was supported by a faculty startup grant from the University of Maryland, College Park.

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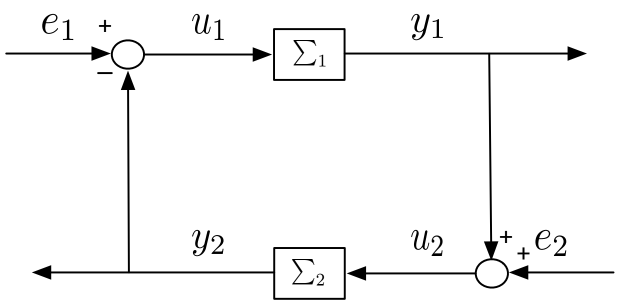

Fig. 1. A feedback interconnection of passive systems

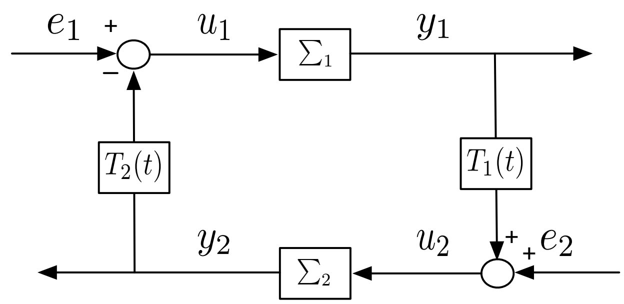

Fig. 2. A feedback interconnection of passive systems with time delays

- lossless if $u^T y = \dot{S}(x)$
- input strictly passive if $u^T y \geq \ \dot{S}(x) + u^T \psi(u)$, where $u^T \psi(u) \ > 0$ for some function ψ and $\forall u \ \neq 0$
- output strictly passive if $u^T y \geq \ \dot{S}(x) + y^T \rho(y)$, where $y^T \rho(y) \ > 0$ for some function ρ and $\forall y \ \neq 0$

A. Feedback Interconnection of Passive Systems

At this point we recall a fundamental property of interconnection of passive systems. Consider the feedback connection of Figure 1, where each of the feedback components is a time-invariant dynamical system represented by the state model

$$\begin{array}{rcl} \dot{x}_i & = & f_i(x_i, e_i) \\ y_i & = & h_i(x_i) \end{array} \tag{3}$$

The closed-loop (composed of the components $\sum_1$ and $\sum_2$) then takes the form

$$\begin{array}{rcl} \dot{x} & = & f(x, u) \\ y & = & h(x) \end{array} \tag{4}$$

where $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, $u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$, $y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$. A fundamental result on the feedback interconnection of passive system is the following

Theorem 2.1: The feedback connection of two passive systems is passive.

We refer the reader to [11] for a proof of this result. A similar property follows when the two systems are output strictly passive with

$$e_i^T y_i \geq \dot{S}_i(x_i) + \delta_i y_i^T y_i \quad \delta_i > 0 \quad i = 1, 2 \tag{5}$$

In this case it is possible to show that

$$u^T y \geq \dot{S}(x) + \delta y^T y$$

where $S(x) = S_1(x_1) + S_2(x_2)$ and $\delta = \min\{\delta_1, \delta_2\}$.

Consider the feedback interconnection with time-varying delays in communication, as shown in Figure 2, The time delays are assumed to satisfy $0 \leq T_i(t) < h, \quad i = 1, 2$, where h is an unknown constant. However, the subsequent results are independent of the upper bound of the time delay. The delays are also assumed to be continuously differentiable with

$$\dot{T}_i(t) \leq \dot{T}_i^{\max} < 1, \quad i = 1, 2 \tag{6}$$

The above condition implies that the time delays cannot grow faster than time itself, and hence is a statement about the causality of the system. An upper bound on the rate of change of delay is often assumed [18] in the analysis of continuous time plants and controllers with input,output or state delays. However, it is to be noted that several other variants of this assumption have also been used in the literature [8]. Denote by $\mathcal{C} = \mathcal{C}([-h, 0], R^{2n})$, the Banach space of continuous functions mapping the interval $[-h, 0]$ into R^{2n}, with the topology of uniform convergence. Define $x_t = x(t + \phi) \in \mathcal{C}, -h < \phi < 0$ as the state of the system [10]. We assume in this note that $x(\phi) = \eta(\phi), \eta \in \mathcal{C}$ and that all signals belong to $\mathcal{L}_{2e}$, the extended $\mathcal{L}_2$ space.

III. Main Results

Consider the feedback interconnection with time-varying delays in communication, as shown in Figure 2. The time delay in communication networks, for example the Internet, are often time-varying and may be increasing or are non-increasing at any particular time instant. As we demonstrate in the subsequent results, the passivity of the closed loop system crucially depends on the time-varying nature of the delay. Our first result in this section studies passivity of the feedback interconnection for non-increasing time delays.

Theorem 3.1: Consider two output strictly passive systems described by Σ_1, Σ_2, (5), (6) and Figure 2. Assuming that the feedback interconnection is well defined, if the time-varying delays are non-increasing, then the feedback interconnection is

1) Passive if $\delta_1 = \delta_2 = 1$.
2) Strictly output passive if $\delta_1, \delta_2 > 1$

Proof: Consider the case when $\delta_1 = \delta_2 = 1$. The output strict passivity condition (5) for the individual systems can then be written as

$$\dot{S}_i(x_i) \leq u_i^T y_i - y_i^T y_i \quad i = 1, 2 \tag{7}$$

Adding the above inequalities for $i = 1, 2$ and using the feedback interconnection in Figure 2 yields

$$\begin{aligned}
&\dot{S}_1(x_1) + \dot{S}_2(x_2) \leq u_1^T y_1 + u_2^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&\leq (e_1 - y_2(t - T_2(t)))^T y_1 + (e_2 + y_1(t - T_1(t)))^T y_2 \\
&-||y_1||^2 - ||y_2||^2 \\
&\leq e_1^T y_1 + e_2^T y_2 - y_2(t - T_2(t))^T y_1 + y_1(t - T_1(t))^T y_2 \\
&-||y_1||^2 - ||y_2||^2 \\
&\leq e_1^T y_1 + e_2^T y_2 + \frac{1}{2}(||y_2(t - T_2(t))||^2 + ||y_1||^2) \\
&+\frac{1}{2}(||y_1(t - T_1(t))||^2 + ||y_2||^2) - ||y_1||^2 - ||y_2||^2 \\
&\leq e_1^T y_1 + e_2^T y_2 - \frac{1}{2}(||y_2||^2 - ||y_2(t - T_2(t))||^2) \\
&-\frac{1}{2}(||y_1||^2 - ||y_1(t - T_1(t))||^2) \\
&\leq e_1^T y_1 + e_2^T y_2 - \phi(t) \qquad (8)
\end{aligned}$$

where $||\cdot||$ is the Euclidean norm of the enclosed signal and $\phi(t) := \frac{1}{2}(||y_2||^2 - ||y_2(t-T_2(t))||^2) + \frac{1}{2}(||y_1||^2 - ||y_1(t - T_1(t))||^2)$. Integrating $\phi(t)$ from $[0, t]$ we get

$$\begin{aligned}
&\int_0^t \phi(\tau)d\tau = \int_0^t \Big(\frac{1}{2}(||y_2(\tau)||^2 - ||y_2(\tau - T_2(\tau))||^2) \\
&+\frac{1}{2}(||y_1(\tau)||^2 - ||y_1(\tau - T_1(\tau))||^2)\Big)d\tau
\end{aligned}$$

Rewriting the non-delayed terms in the above equation yields

$$\begin{aligned}
&= \frac{1}{2}\Big(\int_{t-T_1(t)}^t ||y_1(\tau)||^2 d\tau + \int_{t-T_2(t)}^t ||y_2(\tau)||^2 d\tau \\
&+ \int_0^{t-T_1(t)} ||y_1(\tau)||^2 d\tau + \int_0^{t-T_2(t)} ||y_2(\tau)||^2 d\tau \\
&- \int_0^t (||y_1(\tau - T_1(\tau))||^2 + ||y_2(\tau - T_2(\tau))||^2)d\tau\Big)
\end{aligned}$$

Performing a change of variables [13], $\sigma = \tau - T_i(\tau) := g_i(\tau)$ in the last term in the above equations, we note from (6) that

$$g_i' = 1 - \frac{dT_i}{d\tau} \geq 0 \ ; \ i = 1, 2 \qquad (9)$$

which is a statement that the change of variables is causal and (by the Implicit Function Theorem) invertible [13]. Performing this change of variables it can be shown after some calculations that

$$\begin{aligned}
&\int_0^t \phi(\tau)d\tau = \frac{1}{2}\Big(\int_{t-T_1(t)}^t ||y_1(\tau)||^2 d\tau + \int_{t-T_2(t)}^t ||y_2(\tau)||^2 d\tau \\
&- \int_0^{t-T_1(t)} \frac{T'_1(\sigma)}{1 - T'_1(\sigma)} ||y_1(\sigma)||^2 d\sigma \\
&- \int_0^{t-T_2(t)} \frac{T'_2(\sigma)}{1 - T'_2(\sigma)} ||y_2(\sigma)||^2 d\sigma\Big) \qquad (10)
\end{aligned}$$

where

$$T_i'(\sigma) := \frac{dT_i}{d\tau}_{|_{\tau=g^{-1}(\sigma)}}$$

We note that the last two terms in (10) are negative when the time-delay is increasing ($T_i' > 0$) and are non-positive for non-increasing delays. Thus, $\int_0^t \phi(\tau)d\tau \geq 0$ for non-increasing time delays. Integrating (8) and from the above discussion, passivity of the feedback interconnection follows for non-increasing time-varying delays.

To prove the second claim, using the above calculations, for non-increasing delays and $\delta_i > 1$, $i = 1, 2$, the inequality (8) can be rewritten as

$$\begin{aligned}
\dot{S}_1(x_1) + \dot{S}_2(x_2) \ &\leq \ e_1^T y_1 + e_2^T y_2 - \phi(t) - (\delta_1 - 1)y_1^T y_1 \\
&- \ (\delta_2 - 1)y_2^T y_2
\end{aligned}$$

Integrating the above inequality from $[0, t]$, and using (10) it follows that

$$S(x(t)) - S(x(0)) \leq \int_0^t e(\tau)^T y(\tau)d\tau - \delta_c \int_0^t y(\tau)^T y(\tau)d\tau$$

where $S(x) = S(x_1) + S(x_2)$ and $\delta_c = \min\{(\delta_1 - 1), (\delta_2 - 1)\}$. Hence the feedback interconnection is output strictly passive for non-increasing time delays. ■

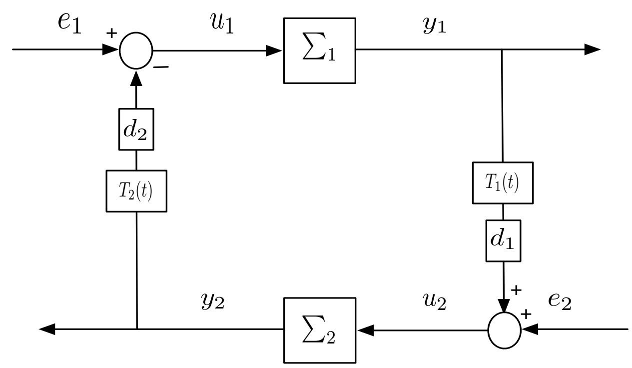

Fig. 3. A feedback interconnection with time-varying delays and gains

In Theorem 3.1, passivity of the feedback interconnection was shown for non-increasing time delays. In a practical scenario, it is important to ensure passivity of the feedback interconnection independent of the time-varying delays. To this end, we propose the addition of time-varying gains in the communications as shown in Figure 3. The closed loop system is now given as

$$\Sigma_1 = \begin{array}{rcl} \dot{x}_1 &=& f_1(x_1, u_1) \\ y_1 &=& h_1(x_1, u_1) \\ u_1(t) &=& e_1(t) - d_2(t)y_2(t - T_2(t)) \end{array} \qquad (11)$$

$$\Sigma_2 = \begin{array}{rcl} \dot{x}_2 &=& f_2(x_2, u_2) \\ y_2 &=& h_2(x_2, u_2) \\ u_2(t) &=& e_2(t) + d_1(t)y_1(t - T_1(t)) \end{array} \qquad (12)$$

Theorem 3.2: Consider the feedback interconnection $\sum_{1,2}$ described by (11), (12), (5), (6) and Figure 3. Assuming that the feedback interconnection is well posed, if $d_i^2(t) \leq 1 - \dot{T}_i^{\max}$, $i = 1, 2$, where $\dot{T}_i^{\max}$ is the maximum rate of change of the delay $T_i(t)$, then the feedback interconnection $\sum_{1,2}$, with $(e_1, y_1), (e_2, y_2)$ as the input-output pairs, is

1) Passive if $\delta_1 = \delta_2 = 1$.
2) Strictly output passive if $\delta_1, \delta_2 > 1$

Proof: Consider the first case when $\delta_1 = \delta_2 = 1$. Summing the storage functions (5) for the individual systems $\sum_1, \sum_2$ yields

$$
\begin{aligned}
&\dot{S}_1(x_1) + \dot{S}_2(x_2) \le u_1^T y_1 + u_2^T y_2 - y_1^T y_1 - y_2^T y_2 \\
&\le (e_1 - d_2(t) y_2(t - T_2(t)))^T y_1 + (e_2 \\
&+ d_1(t) y_1(t - T_1(t)))^T y_2 - ||y_1||^2 - ||y_2||^2 \\
&\le e_1^T y_1 + e_2^T y_2 - d_2(t) y_2(t - T_2(t))^T y_1 \\
&+ d_1(t) y_1(t - T_1(t))^T y_2 - ||y_1||^2 - ||y_2||^2 \\
&\le e_1^T y_1 + e_2^T y_2 + \frac{1}{2}(d_2^2(t)||y_2(t - T_2(t))||^2 + ||y_1||^2) \\
&+ \frac{1}{2}(d_1^2(t)||y_1(t - T_1(t))||^2 + ||y_2||^2) - ||y_1||^2 - ||y_2||^2 \\
&\le e_1^T y_1 + e_2^T y_2 - \frac{1}{2}(||y_2||^2 - d_2^2(t)||y_2(t - T_2(t))||^2) \\
&- \frac{1}{2}(||y_1||^2 - d_1^2(t)||y_1(t - T_1(t))||^2) \\
&\le e_1^T y_1 + e_2^T y_2 - \frac{1}{2}(||y_2||^2 - (1 - \dot{T}_2^{\max})||y_2(t - T_2(t))||^2) \\
&- \frac{1}{2}(||y_1||^2 - (1 - \dot{T}_1^{\max})||y_1(t - T_1(t))||^2) \\
&\le e_1^T y_1 + e_2^T y_2 - \frac{1}{2}(||y_2||^2 - (1 - \dot{T}_2(t))||y_2(t - T_2(t))||^2) \\
&- \frac{1}{2}(||y_1||^2 - (1 - \dot{T}_1(t))||y_1(t - T_1(t))||^2)
\end{aligned}
$$

As before, integrating the above inequality from $[0, t]$ we get

$$
\begin{aligned}
S(x(t)) - S(x(0)) \quad &\le \quad \int_0^t e(\tau)^T y(\tau) d\tau - \int_{t-T_1(t)}^t ||y_1(\tau)||^2 d\tau \\
&- \quad \int_{t-T_2(t)}^t ||y_2(\tau)||^2 d\tau
\end{aligned}
$$

Hence, the feedback interconnection $\sum_{1,2}$ is passive with $(e_1 \quad y_1), (e_2 \quad y_2)$ as the input-output pairs.

If $\delta_1, \delta_2 > 1$, output strict passivity of the feedback interconnection follows from the above discussion and the proof developed in Theorem 3.1 ∎

We next consider the case when the individual systems are passive in comparison to the stronger notion of output-strict passivity that has been used in the previous results. It is well known [1] [1], [7], [26] that passive systems, under appropriate assumptions [7], can be transformed into their scattering representation. We study passivity of the feedback interconnection when scattering variables (defined subsequently) are transmitted between the two systems. It turns out in this case that passivity of the feedback interconnection can be guaranteed for constant time delays. In the time-varying delay case, as before, time-varying gains are used to guarantee passivity of the feedback interconnection. We first investigate the constant time delay case.

The two systems $\Sigma_i \quad i = 1, 2$ are assumed to be passive. Therefore, $\exists S_i(x_i) \quad i = 1, 2$ such that

$$\dot{S}_i(x_i) \le u_i^T y_i$$

[1]The author would like to thank Dr. M. Vidyasagar for pointing to the work on scattering representation [1] by Dr. B.D.O. Anderson.

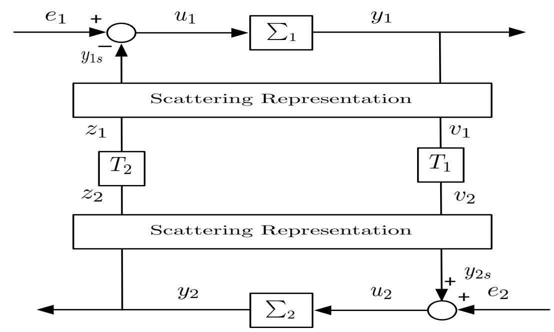

Fig. 4. A feedback interconnection with delays and the scattering transformation

The feedback interconnection of Σ_1, Σ_2 with the scattering representation (to be defined) is shown in Figure 4, and hence,

$$u_1(t) = e_1(t) - y_{1s}(t) \quad , \quad u_2(t) = e_2(t) + y_{2s}(t) \qquad (13)$$

Passivity of the individual systems then dictates that

$$
\begin{aligned}
\dot{S}_1(x_1) &\le (e_1 - y_{1s})^T y_1 \\
\dot{S}_2(x_2) &\le (e_2 + y_{2s})^T y_2
\end{aligned} \qquad (14)
$$

Assuming well-posedness [7], the scattering representation [26] of the dynamical system in Figure 4 is defined as

$$
\begin{aligned}
v_1 &= \tfrac{1}{\sqrt{2}}(y_{1s} + y_1) \quad ; \; z_1 = \tfrac{1}{\sqrt{2}}(y_{1s} - y_1) \\
v_2 &= \tfrac{1}{\sqrt{2}}(y_{2s} + y_2) \quad ; \; z_2 = \tfrac{1}{\sqrt{2}}(y_{2s} - y_2)
\end{aligned} \qquad (15)
$$

In the transformed system, the scattering variables v_1, z_2 are transmitted between the two systems. The dynamical systems $\Sigma_i \quad i = 1, 2$, coupled using the scattering variables, can then be represented as

$$
\Sigma_{1s} = \begin{cases} \dot{x}_1 = f_1(x_1, u_1) \\ y_1 = h_1(x_1, u_1) \\ v_1 = \tfrac{1}{\sqrt{2}}(y_{1s} + y_1) \\ z_1 = \tfrac{1}{\sqrt{2}}(y_{1s} - y_1) \end{cases} \qquad (16)
$$

$$
\Sigma_{2s} = \begin{cases} \dot{x}_2 = f_2(x_2, u_2) \\ y_2 = h_2(x_2, u_2) \\ v_2 = \tfrac{1}{\sqrt{2}}(y_{2s} + y_2) \\ z_2 = \tfrac{1}{\sqrt{2}}(y_{2s} - y_2) \end{cases} \qquad (17)
$$

Theorem 3.3: Assuming well-posedness, consider the feedback interconnection $\Sigma_{1s,2s}$ described by (13), (16) and (17). Then $\Sigma_{1s,2s}$ is passive independent of the constant time delay with $(e_1, y_1), (e_2, y_2)$ as the input-output pairs.

Proof: The sum of the storage functions (14) for the individual systems can be rewritten using (15) as,

$$
\begin{aligned}
&\dot{S}_1(x_1) + \dot{S}_2(x_2) \le (e_1 - y_{1s})^T y_1 + (e_2 + y_{2s})^T y_2 \\
&\le e_1^T y_1 + e_2^T y_2 - y_{1s}^T y_1 + y_{2s}^T y_2 \\
&\le e^T y + \frac{1}{2}\Big(||z_1||^2 - ||v_1||^2 + ||v_2||^2 - ||z_2||^2\Big)
\end{aligned} \qquad (18)
$$

The transmission equations dictate that

$$z_1(t) = z_2(t - T_2); \quad v_2(t) = v_1(t - T_1) \qquad (19)$$

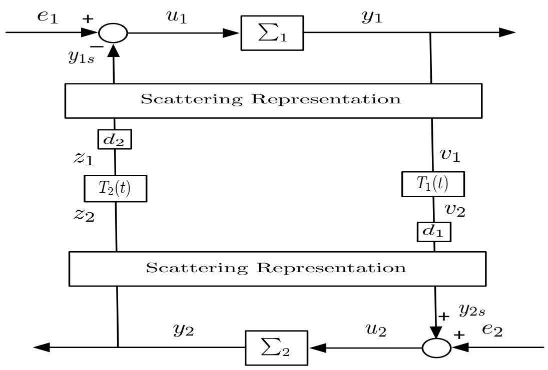

Fig. 5. Scattering transformation and the delay-rate dependent gains d_1, d_2 are used to ensure passivity of the feedback interconnection

Using (19), the inequality (18) can be rewritten as

$$\dot{S}_1(x_1) + \dot{S}_2(x_2) \le e^T y + \frac{1}{2}\Big(||z_2(t-T_2)||^2 - ||z_2||^2 + ||v_1(t-T_1)||^2 - ||v_1||^2\Big)$$

Integrating the above equation from $[0,t]$ yields

$$S(x(t)) - S(x(0)) \le \int_0^t e(\tau)^T y(\tau) d\tau - \frac{1}{2}\Big(\int_{t-T_1}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2}^{t} ||z_2(\tau)||^2 d\tau\Big) \tag{20}$$

where $S(x) = S_1(x_1) + S_2(x_2)$. Therefore, $\Sigma_{1s,2s}$ is passive independent of the constant time delay with $(e_1, y_1), (e_2, y_2)$ as the input-output pairs. ■

A similar result was recently shown in the context of networked control systems in [15]. However, the authors demonstrated $\mathcal{L}_2$ stability of the controller-plant interconnection using the scattering representation.

If the delays are time-varying, then as before, the time-varying delays are assumed to continuously differentiable and $\dot{T}_i(t) \le \dot{T}_i^{\max} < 1, \quad i = 1, 2$. As shown in Figure 5, the scattering variables $z_1(t), v_2(t)$ are scaled by the two gains $d_2(t), d_1(t) > 0$ respectively, and the new scattering equations can be written as

$$\begin{array}{ll} v_1 = \frac{1}{\sqrt{2}}(y_{1s} + y_1) & ; \ d_2 z_1 = \frac{1}{\sqrt{2}}(y_{1s} - y_1) \\ d_1 v_2 = \frac{1}{\sqrt{2}}(y_{2s} + y_2) & ; \ z_2 = \frac{1}{\sqrt{2}}(y_{2s} - y_2) \end{array} \tag{21}$$

The delay-dependent gains $d_1(t), d_2(t)$ are selected as

$$d_1^2(t) \le (1 - \dot{T}_1^{\max}) \ ; \ d_2^2(t) \le (1 - \dot{T}_2^{\max}) \tag{22}$$

The feedback interconnection $\Sigma_{is} \quad i = 1, 2$ is then defined as

$$\Sigma_{1s} = \begin{cases} \dot{x}_1 & = f_1(x_1, u_1) \\ y_1 & = h_1(x_1, u_1) \\ v_1 & = \frac{1}{\sqrt{2}}(y_{1s} + y_1) \\ d_2 z_1 & = \frac{1}{\sqrt{2}}(y_{1s} - y_1) \end{cases} \tag{23}$$

$$\Sigma_{2s} = \begin{cases} \dot{x}_2 & = f_2(x_2, u_2) \\ y_2 & = h_2(x_2, u_2) \\ d_1 v_2 & = \frac{1}{\sqrt{2}}(y_{2s} + y_2) \\ z_2 & = \frac{1}{\sqrt{2}}(y_{2s} - y_2) \end{cases} \tag{24}$$

The next claim follows

Theorem 3.4: Assuming well-posedness of the scattering representation and the feedback interconnection, consider the closed loop systems described by (23), (24), (14), (13), (6) and (22). Then the feedback interconnection is passive independent of the time-varying delays with $(e_1, y_1), (e_2, y_2)$ as the input-output pairs.

Proof: The new transmission equations can be written as

$$z_1(t) = \ \ z_2(t - T_2(t)) \ ; \ v_2(t) = \ \ v_1(t - T_1(t)) \tag{25}$$

Following the proof of Theorem 3.3, we have

$$\begin{aligned} &\dot{S}_1(x_1) + \dot{S}_2(x_2) \le (e_1 - y_{1s})^T y_1 + (e_2 + y_{2s})^T y_2 \\ &\le e^T y + \frac{1}{2}\Big(d_2^2(t)||z_1||^2 - ||v_1||^2 + d_1^2(t)||v_2||^2 - ||z_2||^2\Big) \\ &\le e^T y + \frac{1}{2}\Big((1 - \dot{T}_2^{\max})||z_2(t - T_2(t))||^2 - ||v_1||^2 \\ &+ (1 - \dot{T}_1^{\max})||v_1(t - T_1(t))||^2 - ||z_2||^2\Big) \\ &\le e^T y + \frac{1}{2}\Big((1 - \dot{T}_2(t))||z_2(t - T_2(t))||^2 - ||v_1||^2 \\ &+ (1 - \dot{T}_1(t))||v_1(t - T_1(t))||^2 - ||z_2||^2\Big) \end{aligned}$$

As before, integrating the above inequality yields

$$S(x(t)) - S(x(0)) \le \int_0^t e(\tau)^T y(\tau) d\tau - \frac{1}{2}\Big(\int_{t-T_1(t)}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2(t)}^{t} ||z_2(\tau)||^2 d\tau\Big) \tag{26}$$

Therefore, the feedback interconnection $\Sigma_{1s,2s}$ is passive with $(e_1, y_1), (e_2, y_2)$ as the input-output pairs. ■

IV. NETWORKED CONTROL OF MECHANICAL SYSTEMS

Starting with the work of [25], passivity-based control [21] has emerged as a powerful paradigm for control design in mechanical systems. Several algorithms have been developed [20], [14] where the controller and the mechanical system can be represented as a negative feedback interconnection of passive systems. Invoking the fundamental passivity theorem, it is then possible to guarantee passivity of the closed loop system.

The passivity results in the previous section demonstrate a constructive methodology to guarantee passivity of the closed loop system when there are time delays in communication between the mechanical system and the controller. To this end, we revisit the set-point control problem for mechanical systems. Following [24], in the absence of gravitational forces, the Euler-Lagrange equation of motion for an n-degree-of-freedom system is given as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = u_p + u_g = u \tag{27}$$

where $q \in R^n$ is the vector of generalized configuration coordinates, $u_p \in R^n$ is the motor torque acting on the system, $u_g \in R^n$ is the set of external forces acting on the system, $M(q) \in R^{n \times n}$ is a symmetric, positive definite inertia matrix and $C(q, \dot{q})\dot{q} \in R^n$ is the vector of Coriolis/Centrifugal forces.

It is well known [14] that the PI control strategy (assuming zero initial conditions)

$$u_p(t) = -K_P\dot{q}(t) - K_I(q(t) - q_d)$$

where $K_P, K_I > 0$, drives the system to the desired configuration given by $q(t) = q_d \in R^n$. The coupled mechanical system and the controller can be represented as a feedback interconnection of passive systems where the mechanical system (27) is passive with $(u, \dot{q})$ as the input-output pair and the controller

$$\text{Controller} = \begin{cases} \dot{x}_c = u_c = \dot{q} \\ y_c = K_P u_c + K_I(x_c - q_d) \end{cases} \tag{28}$$

is passive from $u_c \rightarrow y_c$. It can be shown [14] that the negative feedback interconnection with $u_p(t) = -y_c(t)$ is passive from $u_g \rightarrow \dot{q}$ with

$$\begin{aligned} S(q,\dot{q}) &= S_p + S_c \\ &= \frac{1}{2}\Big(\dot{q}^T M(q)\dot{q} + K_I(x_c - q_d)^T(x_c - q_d)\Big) \end{aligned} \tag{29}$$

as the storage function. Therefore, in the presence of time-varying delays, using the scattering representation and the time-varying gains (see Theorem 3.4), the feedback interconnection recovers the passivity property from $u_g \rightarrow \dot{q}$. This can be demonstrated by following the proof of Theorem 3.4 with (29) as the storage function. A detailed study on networked control of mechanical systems will be addressed in a sequel.

V. Conclusions

In this paper we studied input-output passivity of a negative feedback interconnection of passive systems with time delays in communication. Extending the previously developed results in [6], we demonstrated that a negative feedback interconnection of output strictly passive systems, under appropriate assumptions, is passive for non-increasing time delays and may loose passivity for increasing time delays. Passivity can be recovered by inserting time-varying gains [13], that depend on the maximum rate of change of delay, in the communication path. If the dynamical systems are passive instead of the output strict passivity property required for the above results, inspired by previous algorithms for bilateral teleoperation [2], [13] we demonstrated that by transforming the two systems into the scattering representation, transmitting the scattering variables, and using time-varying gains in the communication path, passivity of the feedback interconnection can be guaranteed independent of the time-varying delays. Finally, using the remote set-point control problem for mechanical systems, we presented a brief overview of the relevance of the results for networked control of nonlinear mechanical systems.

VI. Acknowledgements

This work developed directly out of previous collaborations with Professor M. W. Spong and Dr. R. Lozano. The author would like to thank them for useful discussions on the topic.

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