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Experimental Study of Fractional Order Proportional Integral (FOPI) Controller for Water Level Control

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***Abstract*— Based on our previously developed tuning procedure for fractional order proportional integral controller (FO-PI), we present in this paper an extensive comparative experimental study on coupled-tank liquid level controls. Our experimental study consists of four steps, they are mathematical modeling of the plant, identification of plant parameters, water-level controller design and comparisons in Simulink [software (s/w) mode] and final experimental verification and comparisons in real-time [hardware (h/w) mode]. The FO-PI controller is compared with Ziegler Nichol's (ZN) and Modified Ziegler Nichol's (MZN) conventional integer order PI controllers in terms of load disturbance rejection, changes in plant dynamics and setpoint tracking. Experimental results confirmed that FO-PI controller is a promising controller in terms of percentage overshoot and system response in liquid-level control in face of nonlinearities introduced by pumps, valves and sensors.**

***Index Terms*— Fractional calculus, fractional order controller, proportional and integral control, controller tuning, coupled tank, liquid level control.**

I. INTRODUCTION

Process industries play a significant role in economical growth of a nation. Control of liquid level in tanks and fluid flow between tanks is a fundamental requirement in almost all process industries like water including waste water, chemical, petrochemical, pharmaceutical, food, beverages, etc. There exists a variety of methods for liquid level and flow control. This paper attempts to use a fractional order proportional-integral (FO-PI) controller which optimizes the load disturbance rejection, yet with a constraint on the maximum sensitivity, for real-time fluid level control of a coupled-tank. Three different configurations of coupled tank system has been studied namely first order SISO system, second order SISO system and cascaded control system. Each case provides an in-depth information on modeling, system identification, simulations and Real-Time control experiments. The FO-PI results are compared to integer PI controllers in terms of setpoint tracking.

This paper is organized as follows. Section II discusses the preliminaries required to understand the problem and presents the controller tuning methods used. This consists of reviewing FO-PI controller tuning methods, construction and working of KRi control apparatus coupled tank PP-100 and concepts of real-time control and hardware-in loop. This is followed by Section III which provides intensive case studies of experiments on coupled tank in different configurations. This section is subdivided into three main subsections focussed on first order single input single output (SISO), second order SISO system and cascade control plant system in the given order. Each subsection involves brief description of system modeling, real time system identification, controller design, experiments & simulations followed by results and comments. Finally, Section IV concludes the paper by summarizing the results of our experimental study on coupled tank system.

II. PRELIMINARIES

A. Fractional Order PI Controller and Its Practical Tuning Rules

Lots of work on fractional controllers can be found in literature [2], [3], [4], [5], [6]. Expressing in time-domain, if $r(t)$ is the set-point signal, and $u(t)$ is the control input, $y(t)$ is the output, the fractional PI^{α} controller is represented by (1) as:

$$u(t) = K_p(r(t) - y(t)) + K_i D_t^{-\alpha}(r(t) - y(t)), \qquad (1)$$

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where D_t^{α} is the fractional differointegral operator. The following definition is used for the fractional derivative of order α of function $f(t)$ [8], [9]:

$$\frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}} f(t) \doteq \begin{cases} f^{(n)}(t) & \text{if } \alpha = n \in N, \\ \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} * f^{(n)}(t) & \text{if } n-1 < \alpha < n, \end{cases} \qquad (2)$$

where the $*$ denotes the time convolution between two functions.

Expressing in frequency-domain, the FO-PI controller $C(s)$ is simply written as:

$$C(s) = K_p + \frac{K_i}{s^{\alpha}}, \qquad (3)$$

where K_p and K_i are the proportional and integral gain values of the fractional controller and α is the noninteger order of the fractional integrator. The work of [8] shows how to tune the gains K_p, K_i and the non-integer order α whereas [9] experimentally validates the tuning rules. The tuning rules developed in [8] are restated as:

$$K_p = \frac{0.2978}{K(\tau + 0.000307)}, \qquad (4)$$

$$K_i = \frac{K_p(\tau^2 - 3.402\tau + 2.405)}{0.8578T},$$

$$\alpha = \begin{cases} 0.7, & \text{if } \tau < 0.1 \\ 0.9, & \text{if } 0.1 \leq \tau < 0.4 \\ 1.0, & \text{if } 0.4 \leq \tau < 0.6 \\ 1.1, & \text{if} \tau \geq 0.6 \end{cases}.$$

These tuning rules are based on fractional M_s constrained integral gain optimization method (F-MIGO) for generic first order plus delay time (FOPDT) model $Ke^{-Ls}/(Ts+1)$ and $\tau = L/(L+T)$ is the relative delay. More detailed information can be found in [8], [9]. Note that, the tuning rule is only dependent on the τ parameter.

B. Coupled-Tank System PP-100

Coupled Tank PP-100 is a compact, bench top instrument made of two water tanks made of perspex seated on a water reservoir [7], [1]. The reservoir contains two PWM operated motor pumps. These pumps use either $0-5$V analog voltage (internal signal conditioning system covert analog to PWM (digital) signals) or external PWM sources for their operation. Flow rates of water into tanks can be varied by change of these pump voltages. There is a baffle plate separating the two tanks which can be slided up and down to vary interaction or coupling dynamics between the two tanks. Further there are two capacitive probes, one in each tank, provided to measure water level. Output signals from these probes are conditioned to give $0-5$V analog output. A water outlet at side near base of each tank connected by a flexible tube returns water to reservoir. Also there are two potentiometers at back side of CTS are provided for manual operation of motors. The back side and front side of CTS at CSOIS is shown in Fig. 1.

C. Real time control & Hardware in loop experiment platform

Schematic of the real-time, hardware-in-loop configuration provided by Quanser is shown in fig. 2.

A Data Acquisition and Control Board (DACB) provided by Quanser is used to provide feedback to the digital controller (computer) from the plant in the appropriate format. This hardware in loop (HIL) multi-$Q4$ board has 4 analog inputs, 4 voltage outputs, 4 quadrature encoder inputs, 16 programmable digital I/O channels, 2 counters/timers and several other useful features. Analog sensor signals from the plant are converted to digital form and then sent back to the computer by these HIL boards. The next set of control

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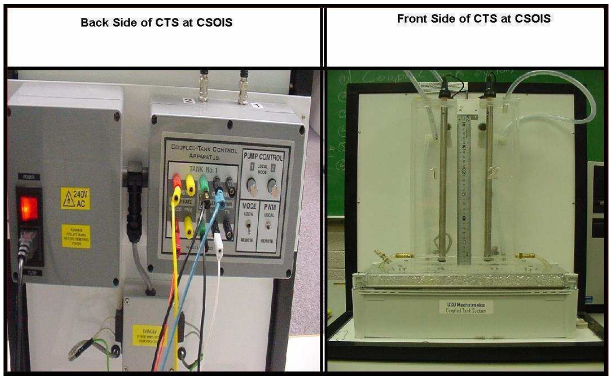

Fig. 1. Front and back side of coupled tank at CSOIS.

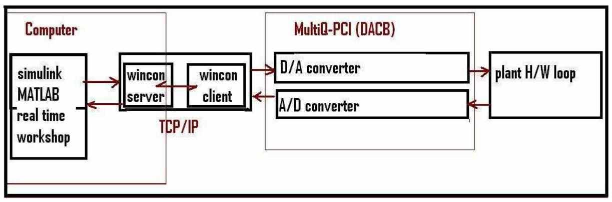

Fig. 2. Schematic of the real-time, hardware-in-loop experiment platform.

signals obtained as result of user program implementation on data from sensor sends digital data to the DACB which converts it to an analog signal, sent to the actuators. This is implemented in WinCon in real-time. WinCon is a real-time Windows2000/XP application that runs in real-time from the C code generated for the control law implemented in MATLAB/Simulink Real Time Workshop. WinCon has two basic parts: WinCon Client - the real component of the software running at a period specified by the user; WinCon Server - the interface with the DACB unit with a graphical user interface for the user to record the signals returned from the sensors.

III. Experimental study on coupled tank system

This section has been subdivided into three subsections, each focussing on a specific coupled tank configuration. The three configurations of coupled tank system which will be considered in following subsections are as:

- Case I - First order SISO coupled tank system
- Case II - Second order SISO coupled tank system
- Case III - Cascaded control coupled tank system

Each subsection provides in-depth explanation of how system identification of coupled tank system is done using frequency response or step response, how controller is designed and gain parameters computed using tuning rules listed in Section II. All this is done in real time and then compared with simulation results.

A. *Case I - First order SISO system*

When the baffle plate is lowered completely, two tanks operate independently as two decoupled first order single input single output (SISO) systems.

1) Mathematical modeling of the system plant: The system model for first order SISO coupled tank is shown in Fig. 3.

Here the relation between water entering and leaving tank is expressed as:

$$Q_i - Q_o = A\frac{dH}{dt}, \tag{5}$$

where Q_i is rate of water flow in tank, Q_o is rate of water flow out of tank, A is cross-sectional area of tank and H is height of water in tank. Substituting value of Q_o in (5), we get:

$$Q_i - Ca\sqrt{2gH} = A\frac{dH}{dt}, \tag{6}$$

where C is discharge coefficient of the outlet valve, a is cross-sectional area of orifice and g is gravitational constant equal to 9.8 m/s^2. The above non-linear equation describes the system behavior of first order SISO decoupled tank system.

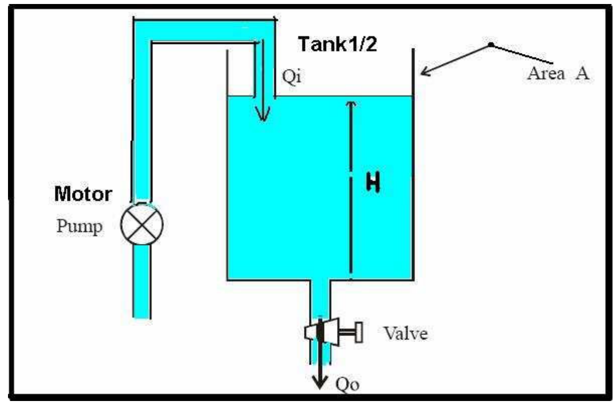

Fig. 3. System model for first order SISO coupled tank.

2) Identification of system plant parameters: In terms of transfer function, in real time, the manipulated variable/plant input is pump input voltage and process variable/plant output is water level in the tank. The transfer function of first order SISO system is given by (7).

$$G(s) = \frac{K}{Ts+1}e^{-Ls}. \tag{7}$$

System parameters as the time constant L and the DC gain K can be found using the step response or frequency response (Bode plot). In the present case we consider frequency response analysis because of the reliability and accuracy of method. Frequency response is the measure of any system's spectrum response at the output to a signal of varying frequency (but constant amplitude) at its input. Different frequencies are considered and gain in decibels and phase shift in degrees of the respective sinusoidal output is noted at steady state. The results of frequency response are shown in Fig. 4 and summarized in Table I.

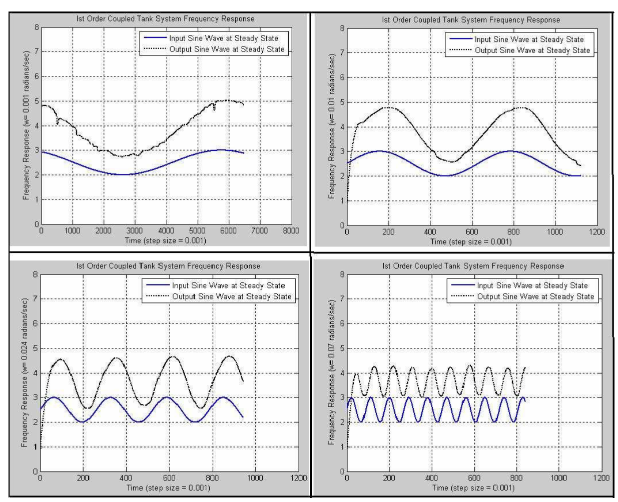

Fig. 4. Frequency response of first order SISO coupled tank system.

The system parameters are computed from the frequency response data as:

$$K = 10^{\frac{M(\omega_{min})}{20}} = 10^{\frac{7.224}{20}} = 2.2972, \tag{8}$$

$$T = \frac{1}{\omega(-45)} = \frac{1}{0.024} = 41.667sec, \tag{9}$$

$$L = 1.75sec, \tag{10}$$

where,

TABLE I
FREQUENCY RESPONSE OF FIRST ORDER SISO COUPLED TANK SYSTEM.

Frequency (rad/sec)	Magnitude (decibels)	Angle (degrees)
0.001	7.224	-0
0.005	6.7408	-8
0.024	6.4208	-45
0.07	1.8852	-80

TABLE II
CONTROLLER GAIN PARAMETERS FOR WATER LEVEL CONTROL IN FIRST ORDER SISO FEEDBACK SYSTEM.

	K_p	T_i	α
PI-ZN	6.6245	5.5071	
PI-MZN	1.4641	6.9443	
FOPI-FMIGO	3.1919	15.7488	0.7

- K is the DC gain of the plant system;
- $M(\omega_{min})$ is the gain at the minimum frequency;.
- T is the time constant;
- $\omega(-45)$ is the frequency when the phase is -45^o;
- L is the time lag of the plant.

Substituting values of K, T and L obtained experimentally as described above in transfer function of first order system, we identify our first order SISO coupled tank system as:

$$G = \frac{2.2972}{41.667s + 1} e^{-1.75s}. \tag{11}$$

3) Water level controller design and comparison in Simulink [s/w mode]: The next step is controller design and controller tuning. Table II shows the gain parameters for different controllers for controlling water level and fluid flow for the SISO configuration. The three controller tuning methods compared are FO-PI/FMIGO, PI/ZN and PI/MZN (modified Ziegler-Nichols). The simulation block diagram of water level control in first order coupled tank system is shown in Fig. 5. In Fig. 6 is shown the result of a

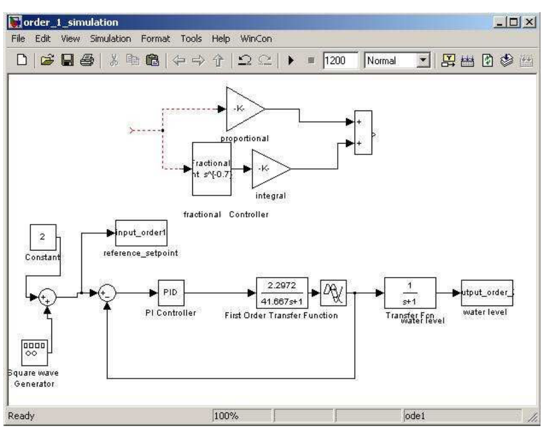

Fig. 5. Simulink block diagram of water level control in first order SISO configuration.

simulation of different controllers performing water level control in tank system. As can be seen from the simulation results, PI-ZN accounts for the largest amount of overshoot, but has a fast response. PI-MZN has a relatively less percentage overshoot whereas the new controller i.e. the FOPI-FMIGO controller results in very less percentage overshoot. In general, quick response and small overshoot are desirable in most of the reference tracking control problems. However, it is well known that quick response may result in a large overshoot. Thus, most of the design schemes have to make a trade-off between these two transient performance indices and in fact from the simulation results, it can be said that FOPI-FMIGO controller is a promising controller in fluid level and flow control in tanks.

4) Experimental Verification in real time [h/w mode]: The final step in design process of any control system is the control experiment. The Quanser HIL platform makes the experimental verification almost in no time to get real-time results as shown in Fig. 7. As can be seen from Fig. 7, the controller performances are in confirmation with the simulation results based on the identified model. Though a little deviation from the simulated results can be accounted due to uncertain real time environmental disturbances like dynamics of fluid, modeling uncertainties, etc.

B. Case II: Second order SISO system

Coupled tank can be configured as a second order SISO system by raising the baffle plate so that water flows from one tank to another. The objective of this experiment is to design a controller to maintain a fixed water level in tank 2 by varying voltage input to motor of tank 1.

1) Mathematical modeling of the system plant: The system model for second order SISO coupled tank is shown in Fig. 8. In this case, the control system has two states, the water levels in two tanks, i.e. H_1 in tank 1 and H_2 in tank 2. The control input to system is the rate of pump flow to tank 1, represented as Q_i, and the output is the water level H_2 in tank 2. Also there are two valves for flow of water. Valve B allows flow of water between tank 1 and tank 2 and valve C allows fluid out of tank 2. Variation in rate of flow of water through these two valves account for load disturbances. Then the rate of change of volume of water in tank 1 is given by (12) as:

$$Q_i - Q_b = A\frac{dH_1}{dt}, \tag{12}$$

where Q_b is the rate of flow of water through valve B and A is cross-sectional area of tank 1. Similarly, the water flow balance equation for tank 2 is given by:

$$Q_b - Q_c = A\frac{dH_2}{dt}, \tag{13}$$

where Q_c is flow of water out of tank 2 through the valve C. Assuming orifices to be ideal, the non-linearities are computed by square law and substituted in (12) and (13) to give the following equations:

$$Q_i - C_b a_b \sqrt{2g(H_1 - H2)} = A\frac{dH_1}{dt}, \tag{14}$$

$$C_b a_b \sqrt{2g(H_1 - H_2)} - C_c a_c \sqrt{2gH_2} = A\frac{dH_1}{dt}$$

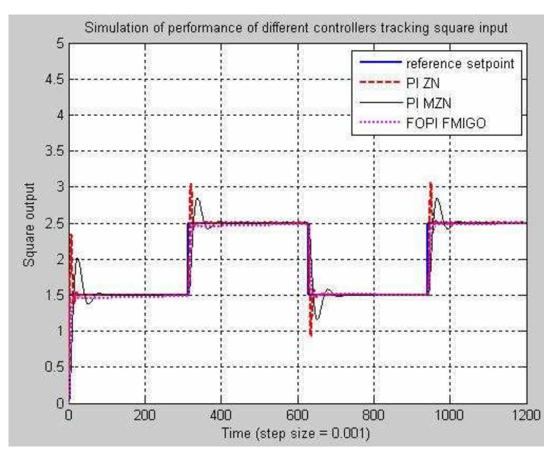

Fig. 6. Simulated control of water level in first order SISO configuration.

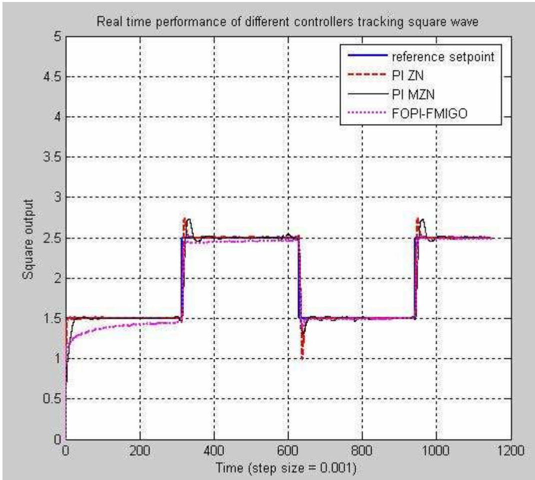

Fig. 7. Real-Time control of water level in Case-I configuration.

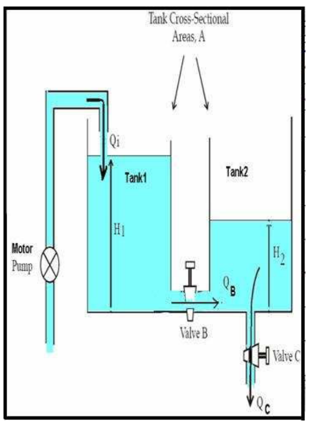

Fig. 8. System model for second order SISO coupled tank.

The above non-linear equations describe the system behavior of second order SISO coupled tank system and can be linearized further to obtain state equation of the coupled tank system.

2) Identification of system plant parameters: The transfer function for second order coupled tank system [1] is given by (15) as:

$$G = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}, \tag{15}$$

where,

- K is the process gain of the system plant;
- ζ is the damping ratio and is defined as degree of oscillation in the process response after a perturbation;
- ω_n is the natural frequency of the system and is the inverse of time constant τ which determines the speed of response of the system.

Again we perform frequency response analysis by feeding sinusoidal signals at different frequencies to second order coupled tank configuration and recording output at steady state. Different frequencies are considered and gain in decibels and phase shift in degrees of the respective sinusoidal output is noted at steady state. This is shown in Fig. 9 and results are listed in Table III. Now the second order plant parameters are obtained as:

$$K = 10^{\frac{M(\omega_{min})}{20}} = 10^{\frac{6.3}{20}} = 2.06, \tag{16}$$

$$T = \frac{1}{\omega(-90)} = T = \frac{1}{0.04}, \tag{17}$$

$$\zeta = \frac{K}{2 * 10^{\frac{M(-90)}{20}}} = \frac{2.06}{2 * 10^{\frac{1.93}{20}}} = 0.824, \tag{18}$$

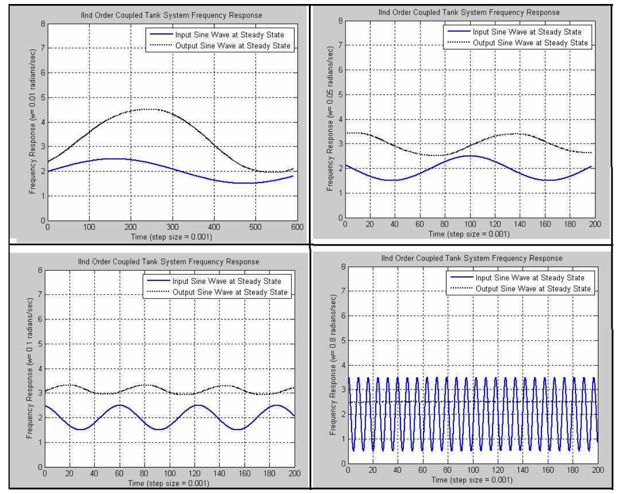

Fig. 9. Frequency response of second order SISO coupled tank system.

TABLE III
FREQUENCY RESPONSE OF SECOND ORDER SISO COUPLED TANK SYSTEM.

Frequency (rad/sec)	Magnitude (decibels)	Angle (degrees)
0.01	6.3	-23.8
0.02	5.3	-48
0.04	1.93	-90
0.05	-0.283	-108
0.1	-10.5	-143
0.8	-30.0063	-180

where,

- K is the DC Gain of a system;
- $M(\omega_{min})$ is the gain at minimum frequency;
- T is the reciprocal of the natural frequency ω_n;
- $\omega(-90)$ is the frequency at which phase shift is -90^o;
- ζ is the damping ratio;
- $M(-90)$ is magnitude of the bode plot when phase is -90^o.

Since $\zeta < 1$, it is an underdamped system. Substituting values of K, ω_n and ζ obtained experimentally as described above in transfer function of second order system, we identify our second order SISO coupled tank system as:

$$G = \frac{2.06 * 0.04^2}{s^2 + 2 * 0.824 * 0.04 + 0.04^2}, \tag{19}$$

Now since a first order plant system is required for computing ZN, MZN and FMIGO tuning parameters, a MATLAB file called "`getfod.m`" is used to approximate a second order system by first order plant system. The approximated first order SISO system obtained is given by (20) as[1]:

$$G = \frac{2.06}{21.1528s + 1} e^{-20.0472s}. \tag{20}$$

3) Water Level controller design and comparison in Simulink [s/w mode]: Based on the approximated first order transfer function (20)for the Case-II second order system, Table IV shows the gain parameters for different controllers for controlling water level and fluid flow for second order SISO configuration.

The Simulink block diagram for water level control in second order coupled tank system is shown in Fig. 10. In Fig. 11 is shown

[1]"For more details on "`getfod.m`" file and derivation of (20) from (19), please refer to [10]"

TABLE IV
CONTROLLER TUNING PARAMETERS FOR SECOND ORDER SISO COUPLED TANK SYSTEM.

	K_p	T_i	α
PI-ZN	0.4556	50.0966	
PI-MZN	0.4692	63.171	
FOPI-FMIGO	0.2969	18.3949	1

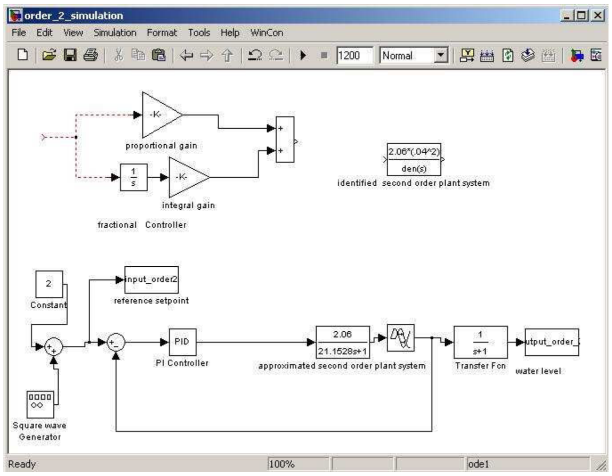

Fig. 10. Simulink block diagram of water level control in second order SISO system configuration.

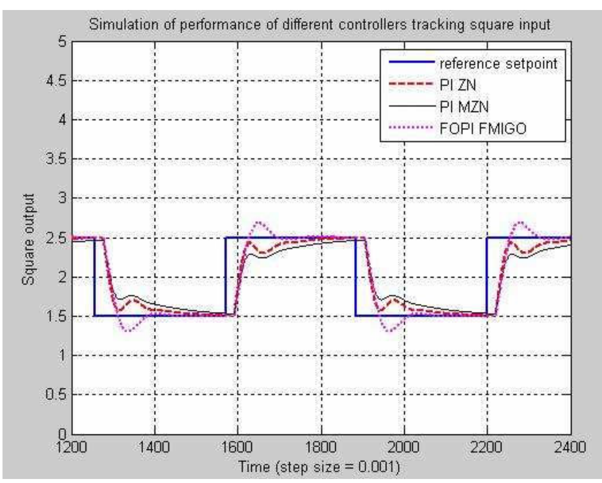

Fig. 11. Simulated control of water level in second order SISO system configuration.

the result of a simulation of different controllers performing water level control in tank system. As can be seen from the simulation results, PI-ZN accounts for relatively less amount of overshoot as compared to fractional controller, but this is on the account of slow system performance. PI-MZN has minimum overshoot and slowest response among the three controller designs. Thus we see there is a trade off between percentage overshoot and system response.

4) Experimental Verification in real time [h/w mode]: To validate the simulation results for water level controller design for second order system, real-time control system is a must. The real-time experimental results are compared in Fig. 12 where we can observe that the control performance is in confirmation with the simulated results. Note also that a little deviation from the simulated results exists due to real time environmental disturbances like

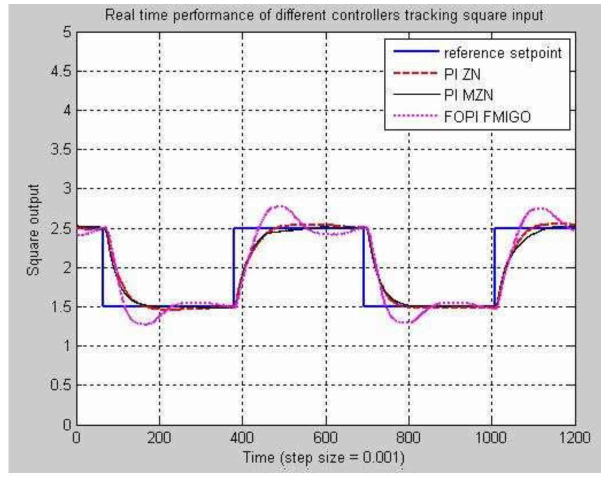

Fig. 12. Real-Time control of water level in second order SISO system configuration.

dynamics of fluid, modeling uncertainties, etc.

C. Case III: Cascade control system

The baffle plate is raised so that water flows from one tank to another and there is dynamic coupling between the two tanks. This type of control system has two cascaded controllers namely primary and secondary controllers.

1) Mathematical modeling of the system plant: For the cascade control coupled tank configuration, the controlled variable is water flow to tank 1. The master controller decides the set point of the slave controller. The slave controller tries to track the set point. The master controller uses water level in tank 2 as process variable by varying water level in tank 1. Suitable baffle opening between two tanks introduces significant time separation between the two controllers which minimizes the effect of disturbance in water level of tank 1 to water level of tank 2. The system model for cascaded control coupled tank is shown in Fig. 13.

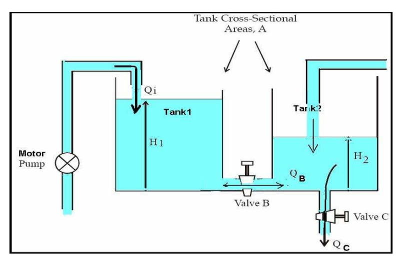

Fig. 13. System model for cascade control coupled tank.

2) Identification of system plant parameters: Both cascaded plants are configured as first order transfer function namely primary and secondary plants having transfer function in general form as:

$$G = \frac{K}{Ts+1}e^{-Ls}, \tag{21}$$

where, K is the gain of the system, T is the time constant and L is the delay of the system. One can either do frequency response analysis or step response analysis to identify transfer function. Instead of doing frequency response twice for each plant (which is time consuming), one can do step response in which step input is applied to the plant and the response recorded. This is performed in Real-Time. The step response for the two tanks so obtained are plotted as shown in Fig. 14. This system appears to be a first order system, because the response does not oscillate and has a non-zero slope when $t = 0$. For this reason we will model this system as a first order system.

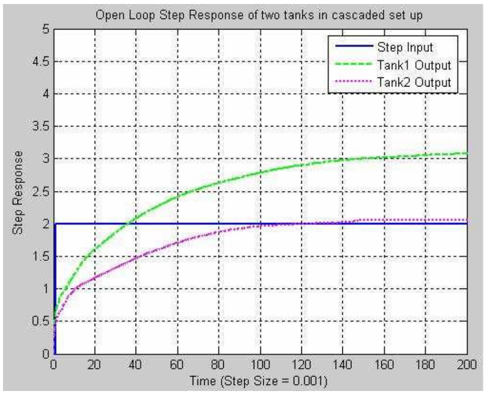

Fig. 14. Open loop step response of two cascaded tanks.

The DC gain is the ratio of the steady state step response to the magnitude of a step input. From the Fig. 14, we have:

$$K_1 = \frac{3.14}{2} = 1.57, \tag{22}$$

$$K_2 = \frac{2.06}{2} = 1.03$$

Further time constant T is the time when output reaches 63% of the final steady state value, which is 35 seconds for tank 1 (secondary controller) and 25 seconds for tank 2 (primary controller).

Delay L is the time after which system responds, once the input is provided. This is 2 seconds for each tank. Summarizing, the tank systems have following transfer function:

$$G_1 = \frac{1.57}{35s+1}e^{-2s}; G_2 = \frac{1.03}{25s+1}e^{-2s}, \tag{23}$$

3) Water Level controller design and comparison in Simulink [s/w mode]: We leave the details of this subsection due to space limitation. The readers can check [10] for the details.

Figure 15 shows the result of a simulation of different controllers performing water level control in tank system. As seen from the simulation results, PI-ZN accounts for the largest amount of overshoot, but has a fast response. PI-MZN has a relatively less percentage overshoot whereas the new controller i.e. the FOPI-FMIGO controller result in very less percentage overshoot. From these simulation results, it can again be said that FOPI-FMIGO controller is a promising controller in fluid level and flow control in tanks.

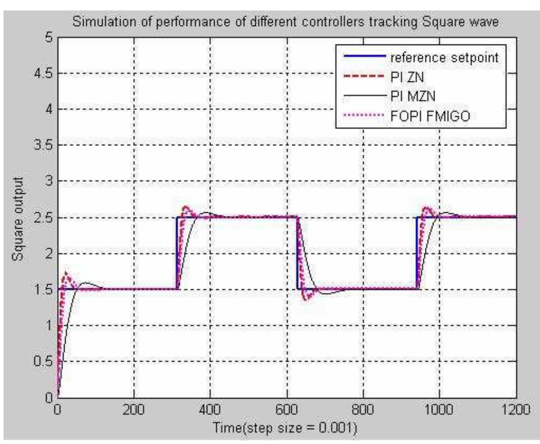

Fig. 15. Simulated control of water level in cascaded control configuration.

4) Experimental Verification in real time [h/w mode]: As seen from Fig. 16, the control performance is in confirmation with the simulated results.

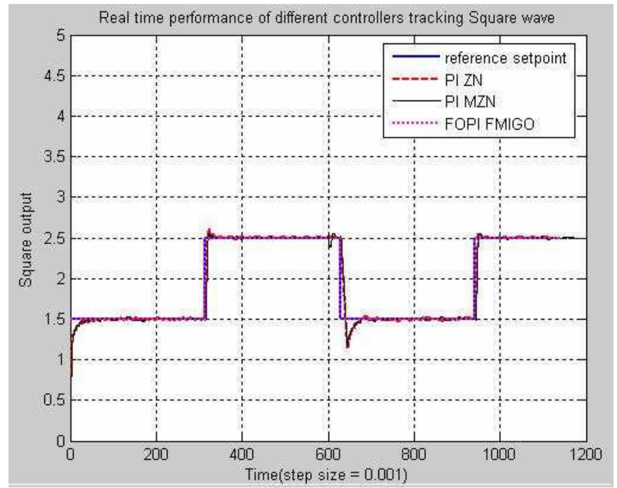

Fig. 16. Real-Time control of water level in cascaded control configuration.

IV. Conclusion

This paper presents an intensive study and experimental work on water level control in coupled-tank PP-100 in three different configurations. For each tank configuration, a comparison between different PI controller tuning methods like Ziegler Nichol's (ZN), Modified Ziegler Nichol's (MZN) and Fractional M_s Constrained Integral Gain Optimization (FMIGO) has been made. Our experimental study consists of four steps, they are mathematical modeling of the plant, identification of plant parameters, water-level controller design and comparisons in Simulink [software (s/w) mode] and finally experimental verification and comparisons in real-time [hardware (h/w) mode]. For each case, simulation results agree well with real-time control results even though with real disturbances due to water dynamics, modeling uncertainties, nonlinearities in the coupled-tank system, etc. responsible for the differences observed. It can be concluded that FO-PI tuned by F-MIGO method can be a promising controller in process industries and can even perform better than its integer-order counterpart.

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