

Dual-high-order Periodic Adaptive Learning Compensation for State-dependant Periodic Disturbance

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Abstract—State-periodic disturbances are frequently found in motion control systems. Examples include cogging in permanent magnetic linear motor, eccentricity in rotary machines and etc. This paper considers general form of state-dependent periodic disturbance and proposes a new high-order periodic adaptive learning compensation (HO-PALC) method for state-dependent periodic disturbance where the stored information of more than one previous periods is used. The information includes composite tracking error as well as the estimate of the periodic disturbance. This dual HO-PALC (DHO-PALC) scheme offers potential to achieve faster learning convergence. In particular, when the reference signal is also periodically changing, the proposed DHO can achieve much better convergence performance in terms of both convergence speed and final error bound. Asymptotical stability proof of the proposed DHO-PALC is presented. Extensive lab experimental results are presented to illustrate the effectiveness of the proposed DHO-PALC scheme over the first order periodic adaptive learning compensation (FO-PALC).

Index Terms—State-dependent periodic disturbance, adaptive control, dual-high-order periodic adaptive learning control, dynamometer.

I. INTRODUCTION

In practice, the state-dependent periodic disturbances exist in many electromechanical systems. For example, it has been shown that the external disturbance is a state-dependent periodic disturbance for rotary systems [1], [2], [3]; in [4], the friction force is shown to be a state-dependent periodic parasitic effect; in [5], the engine crankshaft speed pulsation was expressed as Fourier series expansion as a periodic function of position; in [6], the tire/road contact friction was represented as a function of the system state variable; in [7], the friction and the eccentricity in the low-cost wheeled mobile robots is treated as the state-dependent periodic disturbance; in [8], [9] and [10], the cogging force in a permanent magnetic motor was defined as a position-dependent disturbance. Since the state-dependent periodic disturbance is almost everywhere in practice, the suppression of this type of disturbance has been paid much attention to in control community. To take advantage of the state-dependent periodicity, adaptive learning control idea has been attempted. For example, an adaptive learning compensator for cogging and coulomb friction in permanent-magnet linear motors was proposed in [8] and [11]; the authors of [12] and [13] proposed an iterative learning control (ILC) algorithm and a variable step-size normalized ILC scheme to reduce periodic torque ripples from cogging and other effects of PMSM, respectively; in [9], a periodic adaptive learning compensation method for cogging was performed on the PMSM servo system. However, all these efforts did not utilize the stored information of more than one previous periods, that is, they are not high-order periodic adaptive learning control scheme for state-dependent periodic disturbance compensation. In view of this, in our previous work [10], a simple high order periodic adaptive learning compensator was proposed for cogging effect in PMSM position servo system, where only the stored tracking

errors of more than one previous periods are utilized in the adaptive updating/learning law. Experimental results reported in [10] confirm that HO-PALC does achieve better compensation performance than the first-order PALC scheme.

In the present work, we propose a new high-order periodic adaptive learning compensation (HO-PALC) method for state-dependent periodic disturbance where the stored information of more than one previous periods is used and, the information includes composite tracking error as well as the estimate of the periodic disturbance. This is called dual HO-PALC (DHO-PALC) scheme which we show offers potential to achieve faster learning convergence. In particular, when the reference signal is also periodically changing, the proposed DHO-PALC can achieve much better convergence performance in terms of both convergence speed and final error bound. Asymptotical stability proof of the proposed DHO-PALC is presented. Extensive lab experimental results are presented to illustrate the effectiveness of the proposed DHO-PALC scheme over the first-order periodic adaptive learning compensation (FO-PALC).

The major contributions of this paper include 1) A new dual-high-order periodic adaptive learning compensation method for state-dependent periodic disturbance and the proof of the asymptotical stability of the system with the DHO-PALC; 2) Experimental study of the DHO-PALC for state-dependent periodic disturbance on a dynamometer position control system; 3) Experimental demonstration of the advantages of the DHO-PALC over the FO-PALC scheme.

II. THE GENERAL FORM OF STATE-DEPENDENT PERIODIC DISTURBANCE

This paper is mainly concerned with the general state-dependent periodic disturbance similar to [1], [2], [3], [8], [9], [10], [14]. The disturbance could be any type of nonlinear periodic function depending on a state variable x which usually represents the linear displacement or rotational angle. In Fourier series, the general state-dependent periodic disturbance can be expressed by

$$F_{disturbance} = \sum_{i=1}^{\infty} A_i \sin(\omega_i x + \varphi_i), \quad (1)$$

where A_i is the amplitude, ω_i is the state-dependent periodic disturbance frequency, and φ_i is the phase angle. Note that, this general form can well represent state-dependent periodic disturbance in real world, for example, the state-dependent friction in [2], the position-dependent cogging force in permanent magnetic linear motor [8] or permanent magnetic synchronous motor [9] [10], the eccentricity in the wheeled mobile robots [7] and the experiment apparatus [1], and so on.

For simplicity, in the sequel, we denote the above state-dependent periodic disturbance as $a(x)$. From physical limit consideration, it is reasonable to believe that $a(x)$ is bounded, that is,

$$|a(x)| \leq b_0. \quad (2)$$

Moreover, based on the same physical reason, the profile shape change in $a(x)$ can also be regarded as bounded, that is,

$$|\partial a(x)/\partial x| \leq b_a < \infty$$

where b_a is an unknown positive real number. Note that, however, it is not correct to assume that $|\dot{a}(x)|$ is bounded since this amounts to assuming that $|\dot{x}|$ is bounded, which is yet to be proved.

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III. DUAL-HIGH-ORDER PERIODIC ADAPTIVE LEARNING COMPENSATION OF STATE-PERIODIC DISTURBANCE

A. Problem Formulation

In this paper, to present our ideas clearly, without loss of generality, we consider the following canonical motion control system

$$\dot{x}(t) = v(t), \quad (3)$$

$$\dot{v}(t) = u(t) - a(x)/J, \quad (4)$$

where $x(t)$ is the state (displacement); $v(t)$ is the velocity; $u(t)$ is the control input signal; J is a constant which can be the moment of inertia of the motor when rotary motion system is considered; and $a(x)$ is the unknown state-dependent periodic disturbance with known state periodicity.

First, before proceeding our main results, the following definitions and assumptions are necessary which are adapted from [8] for self-containing purpose.

Definition 3.1: The total passed trajectory is given as:

$$s(t) = \int_0^t \left| \frac{dx}{d\tau} \right| d\tau = \int_0^t |v(\tau)| d\tau,$$

where x is the position, and v is the velocity. Physically, $s(t)$ is the total passed trajectory length, hence it has the following property:

$$s(t_1) \geq s(t_2), \text{ if } t_1 \geq t_2.$$

With notation $s(t)$, the position corresponding to $s(t)$ is denoted as $x(s(t))$ and the disturbance corresponding to $s(t)$ is denoted as $a(s(t))$. In our definition, since $s(t)$ is the summation of the absolute position increasing along the time axis, just like t , $s(t)$ is also monotonous growing signal, so we have

$$a(x(t)) = a(x(s)) = a(s(t)) = a(t). \quad (5)$$

Definition 3.2: Since the disturbance is periodic with respect to position, based on Definition 3.1, the following relationship can be derived:

$$x(s(t)) = x(s(t) - s_p), \quad a(s(t)) = a(s(t) - s_p). \quad (6)$$

where s_p is the known periodicity of the trajectory. Note that, in the rotary machine case, s_p is simply 2π while in the permanent magnetic linear motor case, s_p is the pitch distance [8].

Definition 3.3: In Definition 3.2, s_p was defined as the period of the periodic trajectory. So, $s(t) - s_p$ is one past trajectory point from $s(t)$ on the s -axis. Let us denote the time corresponding to $s(t) - s_p$ as τ_{k-1} . Then, $t - \tau_{k-1} := P_k$ is the time-elapsed to complete one periodic trajectory from the time τ_{k-1} to time t . This time-elapsed is called "cycle". k is the integer part of the quotient s/s_p , and we denote $P_0 = t - \sum_{j=1}^k P_j$. When considering n passed cycles from the current time t , let us denote the time at the " $(k-n+1)$ -th past trajectory cycle" as τ_{k-n} and denote P_k the time-elapsed to complete the first past cycle. So, $\tau_{k-n} = t - \sum_{j=0}^{n-1} P_{k-j}$. We can use the so-called "search process" to find P_k at time instant t by interpolating the stored data array in the memory as in [8]. Note P_k is depended on t , so in fact it should be $P_k(t)$.

Definition 3.4: The first trajectory cycle P_p is the elapsed time to complete the first repetitive trajectory from the initial starting time t_0 . In other words, P_p is the time corresponding to the total passed trajectory when $s(t) = s_p$.

From now on, for accurate notation, the state (position) corresponding to time t is denoted as $x(t)$ and its total passed trajectory by the time t is denoted as $s(t)$. Henceforward, the time instant for one past trajectory from the time instant t is denoted as τ_{k-1} , and its corresponding cycle is completed in $P_k(t)$ amount of time.

Assumption 3.1: Throughout the paper, it is assumed that the current position and current time of the motion system are measured. Let us denote the current position as $x(t)$ at time t . Then,

τ_{k-1} is always calculated, hence P_k is calculated at time instant t .

With the above definitions and assumption, the following property is observed.

Remark 3.1: As will be shown in the following theorem, the actual state-dependent disturbance a is not estimated on the state-axis. In our adaptation law, a is estimated on the time-axis. So, to find $a(s(t) - s_p)$, the following formula is used:

$$a(s(t) - s_p) = a(\tau_{k-1}) = a(t - P_k). \quad (7)$$

Here, P_k is calculated in Assumption 3.1 (recall that P_k can be used to indicate exactly one past trajectory position).

From Definition 3.2, Remark 3.1 and (5), we also have the following property:

Property 3.1: The current state periodic disturbance is equal to the disturbance of one past trajectory. From the relationship:

$$a(t) = a(s(t)) = a(s(t) - s_p) = a(\tau_{k-1}), \quad (8)$$

from (7), the following equality is derived: $a(t) = a(t - P_k)$.

Then we define

$$e_a(s(t)) = a(s(t)) - \hat{a}(s(t)),$$

where $\hat{a}(s(t)) = \hat{a}(t)$ (note: t is the current time corresponding to the current total passed trajectory $s(t)$). Here, let us change $e_a(s(t)) = a(s(t)) - \hat{a}(s(t))$ into time-domain such as:

$$e_a(s(t)) = a(s(t)) - \hat{a}(s(t)) = a(t) - \hat{a}(t) = e_a(t). \quad (9)$$

In the same way, the following relationships are true:

$$v_d(s(t)) = v_d(t), \quad v(s(t)) = v(t),$$

and the following notations are also defined

$$e_x(t) = x_d(t) - x(t), \quad e_v(t) = v_d(t) - v(t).$$

The control objective is to track or servo the given desired position $x_d(t)$ and the corresponding desired velocity $v_d(t)$ with tracking errors as small as possible. In practice, it is reasonable to assume that $x_d(t)$, $v_d(t)$ and $\dot{v}_d(t)$ are all bounded signals.

From now on, based on relationship: $a(x(t)) = a(t - P_k) = a(t)$, $a(x(t))$ is equal to $a(t)$ as in (5); So, $a(x)$ is replaced by $a(t)$ in the following theorems.

The feedback controller is designed as:

$$u(t) = \dot{v}_d(t) + \hat{a}(t)/J + \alpha m(t) + \gamma e_v(t), \quad (10)$$

with

$$m(t) := \gamma e_x(t) + e_v(t), \quad (11)$$

where α and γ are positive gains; $\hat{a}(t)$ is an estimated state-dependent periodic disturbance from an adaptation mechanism to be specified later; $\dot{v}_d(t)$ is the desired acceleration; and $e_x(s(t)) = e_x(t)$; and $m(s(t)) = m(t)$.

Our adaptation law is designed as follows:

$$\hat{a}(t) = \begin{cases} \delta A(t) + \frac{K}{J} S(t) & \text{if } s \geq s_p \\ z - \mu v & \text{if } s < s_p \end{cases} \quad (12)$$

with

$$A(t) := \sum_{i=1}^N h_i \hat{a}_i(t), \quad S(t) := \sum_{i=1}^N \beta_i m_i(t), \quad (13)$$

where

$$\hat{a}_i(t) = \hat{a}(t - \sum_{j=1}^i P_{k+1-j}), \quad \sum_{i=1}^N h_i = 1, \quad 0 \leq |h_i| \leq 1,$$

$$m_i(t) = m(t - \sum_{j=1}^i P_{k+1-j}), \quad (i = 1, 2, \dots, N)$$

P_k is the trajectory cycle defined in Definition 3.2; δ is a weighting coefficient and $0 < \delta < 1$, K is a positive design parameter called the periodic adaptation gain; μ is also a positive design parameter; h_i are the weight coefficients of the high-order disturbance estimates; β_i are the weight coefficients of the high-order composite feedback errors, β_i are chosen to be the bounded with upper bound denoted by b_β , that is

$$b_\beta = \max_{1 \leq i \leq N} |\beta_i|; \quad (14)$$

in our analysis part, the following tuning mechanism is proposed for z:

$$\dot{z} = \mu[\dot{v}_d(t) + \alpha m(t) + \gamma e_v(t)] + \frac{e_v(t)}{J}. \quad (15)$$

B. Stability Analysis

Now, based on the above discussions, the following stability analysis of the proposed DHO-PALC scheme is performed. Our N -th order periodic adaptive learning compensation approach is summarized as follows:

- When $s(t) < s_p$, the system is controlled to be bounded input bounded output.
- When $s(t) \geq s_p$, the system is stabilized to follow the desired speed at the desired position. By trajectory periodic adaptation, the unknown disturbance is estimated.

Consider two cases: 1) when $0 \leq t < P_p$ ($0 \leq s < s_p$) and 2) when $t \geq P_p$ ($s \geq s_p$). The key idea is that, for case 1), it is required to show the finite time boundedness of equilibrium points; for case 2), it is necessary to show the asymptotic stability of equilibrium points.

First, let us consider the case 1) when $t < P_p$ ($s < s_p$).

Our major results are summarized in the following theorems with Remark 3.2.

Remark 3.2: From the relationship (9), it can be said that if $e_a(t) \rightarrow 0$ as $t \rightarrow \infty$, then $e_a(s) \rightarrow 0$ as $s \rightarrow \infty$. Thus, in what follows, the stability analysis of $\hat{a}(x)$ is performed on the time-axis.

Theorem 3.1: If $\mu > \frac{1}{4}J(1 + b_a^2)$ and $(\alpha + \gamma) > 1$, the equilibrium points of e_x , e_v , and e_a are bounded, when $t < P_p$ ($s < s_p$).

Proof: The proof of this theorem can be completed by base on the proof of Theorem 3.1 of [10]. Due to a page limitation we omit the proof. ■

Now, let us investigate the case 2) when $t \geq P_p$ ($s \geq s_p$).

First of all, the following lemma is needed for the proof of Theorem 3.2.

Lemma 3.1: Suppose a real position series $[a_n]_1^\infty$ satisfies $a_n \leq \rho_1 a_{n-1} + \rho_2 a_{n-2} + \dots + \rho_N a_{n-N} + \epsilon$, ($n = N+1, N+2, \dots$), where $\rho_i \geq 0$, ($i = 1, 2, \dots, N$), $\epsilon \geq 0$ and

$$\rho = \sum_{i=1}^N \rho_i < 1, \quad (16)$$

then the following holds:

$$\lim_{n \rightarrow \infty} a_n \leq \epsilon / (1 - \rho). \quad (17)$$

For a proof of Lemma 3.1, see Chapter 2 in [15].

Theorem 3.2: When $t \geq P_p$ ($s \geq s_p$), the control law (10) and the periodic adaptation law (12) guarantee the asymptotically stability of the equilibrium points $e_x(t)$, $e_v(t)$ and $e_a(t)$, as $t \rightarrow \infty$ ($s \rightarrow \infty$), with the initial condition, $e_{xi}(t) = e_{vi}(t) = e_{ai}(t) = 0$, as $(t - \sum_{j=1}^i P_{k+1-j}) \leq 0$, where

$$\eta_i(t) = \eta(t - \sum_{j=1}^i P_{k+1-j}), \quad i = 1, 2, \dots, N,$$

$$\eta \in \{x_d, x, v_d, v, a, \hat{a}, e_x, e_v, e_a, m, S\}.$$

(This is a very important notation in this paper).

Proof: From (4) and (10), using

$$\begin{aligned} \dot{e}_{xi}(t) &= \dot{x}_{di}(t) - \dot{x}_i(t) = e_{vi}(t), \\ \dot{e}_{vi}(t) &= \dot{v}_{di}(t) - \dot{v}_i(t) \\ &= \dot{v}_{di}(t) - [u_i(t) - \frac{a_i(t)}{J}] \\ &= \dot{v}_{di}(t) - [\dot{v}_{di}(t) + \frac{\hat{a}_i(t)}{J} + \alpha m_i(t) \\ &\quad + \gamma e_{vi}(t)] + \frac{a_i(t)}{J} \\ &= -\alpha m_i(t) - \gamma e_{vi}(t) + \frac{a_i(t)}{J} - \frac{\hat{a}_i(t)}{J} \\ &= -\alpha \gamma e_{xi}(t) - (\alpha + \gamma) e_{vi}(t) + \frac{e_{ai}(t)}{J}, \end{aligned} \quad (18)$$

and $e_{xi}(t) = e_x(t - \sum_{j=1}^i P_{k+1-j})$.

Let us denote

$$P_{si} = \sum_{j=1}^i P_{k+1-j}, \quad P_{sk} = \sum_{j=1}^k P_{k+1-j},$$

As $P_0 = t - \sum_{j=1}^k P_{k+1-j}$, where k is the integer part of the quotient s/s_p , and from Definition 3.4, so we can get

$$0 \leq P_0 < P_p. \quad (20)$$

Taking norms yields

$$\begin{aligned} \|e_{xi}(t)\| &= \|e_{xi}(P_0 + P_{si}) + \int_{P_0 + P_{si}}^t \dot{e}_{xi}(\tau) d\tau\| \\ &= \|e_{xi}(P_0 + P_{si}) + \int_{P_0 + P_{si}}^t e_{vi}(\tau) d\tau\| \\ &\leq \|e_x(P_0)\| + \int_{P_0 + P_{si}}^t \|e_{vi}(\tau)\| d\tau, \\ \|e_{vi}(t)\| &= \|e_{vi}(P_0 + P_{si}) + \int_{P_0 + P_{si}}^t \dot{e}_{vi}(\tau) d\tau\| \\ &= \|e_{vi}(P_0 + P_{si}) + \int_{P_0 + P_{si}}^t [-\alpha \gamma e_{xi}(\tau) \\ &\quad - (\alpha + \gamma) e_{vi}(\tau) + \frac{e_{ai}(\tau)}{J}] d\tau\| \\ &\leq \|e_v(P_0)\| + \int_{P_0 + P_{si}}^t [\alpha \gamma \|e_{xi}(\tau)\| \\ &\quad + (\alpha + \gamma) \|e_{vi}(\tau)\| + \frac{\|e_{ai}(\tau)\|}{J}] d\tau. \end{aligned} \quad (21)$$

For any function $x(t) \in R^n$, $t \in [P_0 + P_{si}, P_{sk} + P_{si}]$, The λ -norm for $\int_{P_0}^{t-P_{si}} \|x(\tau)\| d\tau$ is

$$\begin{aligned} &\sup_{t \in [P_0 + P_{si}, P_{sk} + P_{si}]} e^{-\lambda t} \int_{P_0}^{t-P_{si}} \|x(\tau)\| d\tau \\ &= \sup_{t \in [P_0 + P_{si}, P_{sk} + P_{si}]} e^{-\lambda t} \int_{P_0}^{t-P_{si}} \|x(\tau)\| e^{-\lambda \tau} e^{\lambda \tau} d\tau \\ &\leq \|x(t)\|_\lambda \sup_{t \in [P_0 + P_{si}, P_{sk} + P_{si}]} e^{-\lambda t} \int_{P_0}^{t-P_{si}} e^{\lambda \tau} d\tau \\ &= \|x(t)\|_\lambda \phi, \end{aligned} \quad (23)$$

where

$$\phi = \frac{e^{-\lambda P_{si}} - e^{-\lambda(P_{si} + P_{sk} - P_0)}}{\lambda}. \quad (24)$$

Thus, from (21), (22) and (23), we get

$$\|e_{xi}(t)\|_\lambda \leq \|e_x(P_0)\|_\lambda + \|e_{vi}(t)\|_\lambda \phi, \quad (25)$$

$$\|e_{vi}(t)\|_\lambda \leq \|e_v(P_0)\|_\lambda + [\alpha\gamma\|e_{xi}(t)\|_\lambda + (\alpha + \gamma)\|e_{vi}(t)\|_\lambda + \frac{\|e_{ai}(t)\|_\lambda}{J}] \phi, \quad (26)$$

solving the above inequalities (25) and (26), we have

$$\begin{aligned} & [1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi]\|e_{vi}(t)\|_\lambda \\ & \leq \alpha\gamma\phi\|e_x(P_0)\|_\lambda + \|e_v(P_0)\|_\lambda + \frac{\phi}{J}\|e_{ai}(t)\|_\lambda. \end{aligned} \quad (27)$$

Clearly, $\exists \lambda$, such that

$$\alpha\gamma\phi^2 + (\alpha + \gamma)\phi < 1, \text{ namely, } 1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi > 0, \quad (28)$$

from (25) and (27), we can get

$$\begin{aligned} & [1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi]\|e_{xi}(t)\|_\lambda \\ & \leq [1 - (\alpha + \gamma)\phi]\|e_x(P_0)\|_\lambda + \phi\|e_v(P_0)\|_\lambda \\ & \quad + \frac{\phi^2}{J}\|e_{ai}(t)\|_\lambda, \end{aligned} \quad (29)$$

thus, (29) and (27) can be changed as

$$\begin{aligned} & \|e_{xi}(t)\|_\lambda \\ & \leq \frac{[1 - (\alpha + \gamma)\phi]\|e_x(P_0)\|_\lambda + \phi\|e_v(P_0)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi} \\ & \quad + \frac{\frac{\phi^2}{J}\|e_{ai}(t)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi}, \end{aligned} \quad (30)$$

$$\begin{aligned} & \|e_{vi}(t)\|_\lambda \\ & \leq \frac{\alpha\gamma\phi\|e_x(P_0)\|_\lambda + \|e_v(P_0)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi} \\ & \quad + \frac{\frac{\phi}{J}\|e_{ai}(t)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi}. \end{aligned} \quad (31)$$

Remark 3.3: In (28), even if $\gamma = 0$ or $\alpha = 0$, the inequality is still satisfied. Thus, we always have the inequalities (30) and (31).

From (9), the adaption law (12) and the relationship $a(t) = a(t - P_k)$, we can get

$$a(t) = a_i(t), \quad (32)$$

$$e_a(t) = a(t) - \hat{a}(t)$$

$$= a(t) - [\delta \sum_{i=0}^N h_i \hat{a}_i(t) + \frac{K}{J} S(t)]$$

$$= \sum_{i=1}^N h_i a_i(t) - \delta \sum_{i=0}^N h_i \hat{a}_i(t) - \frac{K}{J} S(t)$$

$$= \delta \sum_{i=1}^N h_i e_{ai}(t) - \frac{K}{J} \sum_{i=1}^N \beta_i m_i(t) + (1 - \delta)a(t)$$

$$\begin{aligned} & = \delta \sum_{i=1}^N h_i e_{ai}(t) - \frac{K}{J} \sum_{i=1}^N \beta_i [\gamma e_{xi}(t) + e_{vi}(t)] \\ & \quad + (1 - \delta)a(t), \end{aligned} \quad (33)$$

thus

$$\begin{aligned} & \|e_a(t)\|_\lambda \\ & \leq \delta \sum_{i=1}^N |h_i| \|e_{ai}(t)\|_\lambda + \frac{K}{J} \sum_{i=1}^N |\beta_i| [\gamma \|e_{xi}(t)\|_\lambda \\ & \quad + \|e_{vi}(t)\|_\lambda] + (1 - \delta) \|a(t)\|_\lambda, \end{aligned} \quad (34)$$

Substituting (30) and (31) into (34) and from (2) yield

$$\|e_a(t)\|_\lambda \leq \sum_{i=1}^N (\delta |h_i| + \frac{K}{J^2} \xi_i) \|e_{ai}(t)\|_\lambda + \varepsilon, \quad (35)$$

where

$$\xi_i = b_\beta \frac{\gamma\phi^2 + \phi}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi}, \quad (36)$$

$$\begin{aligned} \varepsilon & = \frac{K}{J} \sum_{i=1}^N b_\beta \left[\frac{[\gamma - \gamma(\alpha + \gamma)\phi + \alpha\gamma\phi] \|e_x(P_0)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi} \right. \\ & \quad \left. + \frac{[1 + \gamma\phi] \|e_v(P_0)\|_\lambda}{1 - \alpha\gamma\phi^2 - (\alpha + \gamma)\phi} \right] + (1 - \delta) \|b_0\|_\lambda. \end{aligned} \quad (37)$$

As $t = \sum_{j=1}^k P_{k+1-j} + P_0$, where $P_0 \in [0, P_p]$ then (35) can be changed as

$$\begin{aligned} & \|e_a(\sum_{j=1}^k P_{k+1-j} + P_0)\|_\lambda \\ & \leq \sum_{i=1}^N (\delta + \frac{K}{J^2} \xi_i) \|e_a(\sum_{j=i+1}^k P_{k+1-j} + P_0)\|_\lambda + \varepsilon, \end{aligned} \quad (38)$$

In (36), as $0 < \delta < 1$ and $0 \leq |h_i| \leq 1$, so we can find a sufficiently large λ such that $(\delta |h_i| + \frac{K}{J^2} \xi_i) < 1$, and $\xi = \sum_{i=1}^N (\delta |h_i| + \frac{K}{J^2} \xi_i) < 1$. Then, according to Lemma 3.1 and (38), we can obtain that

$$\lim_{k \rightarrow \infty} \|e_a(\sum_{j=1}^k P_{k+1-j} + P_0)\|_\lambda \leq \frac{\varepsilon}{1 - \xi}, \quad (39)$$

Thus, we can get the result

$$\lim_{t \rightarrow \infty} \|e_a(t)\|_\lambda \leq \frac{\varepsilon}{1 - \xi}. \quad (40)$$

From Theorem 3.1 and (20), $\|e_x(P_0)\|$ and $\|e_v(P_0)\|$ should be bounded, so from (37) and (40), ε and $e_a(t)$ are bounded, if $(1 - \delta)$ tend to zero and λ tend to infinite, ε and $e_a(t)$ bound tend to zero asymptotically as $t \rightarrow \infty$. Then, from (30) and (31), we can conclude that the estimated disturbance error $e_a(t)$ and the tracking errors $e_x(t)$, $e_v(t)$ bound tend to be zero asymptotically as $t \rightarrow \infty$.

So the system (3)-(4) can be asymptotically stabilized by the control law (10) and the adaptation law (12) as $t \rightarrow \infty$. This completes the proof. ■

IV. EXPERIMENTS

A. Introduction to the Experiment Platform

A fractional horsepower dynamometer was developed as a general purpose experiment platform to emulate mechanical nonlinearities such as time-dependent disturbances, state-dependent disturbances, etc. This lab system can be used as a research platform to test various nonlinear control schemes [16].

1) *Architecture of the Dynamometer:* The architecture of the dynamometer control system is shown in Fig. 1. The Dynamometer includes the DC motor to be tested, a hysteresis brake for applying load to the motor, a load cell to provide force feedback, an optical encoder for position feedback and a tachometer for velocity feedback. The dynamometer was modified to connect to a Quanser MultiQ4 terminal board in order to control the system through Matlab/Simulink Real-Time Workshop (RTW) based software. This terminal board connects with the Quanser MultiQ4 data acquisition card. Then, using the Matlab/Simulink environment, which uses the WinCon application, from Quanser, to communicate with the data acquisition card, thus the complex nonlinear control schemes were tested. This brings rapid prototyping and experiment capabilities to many nonlinear models.

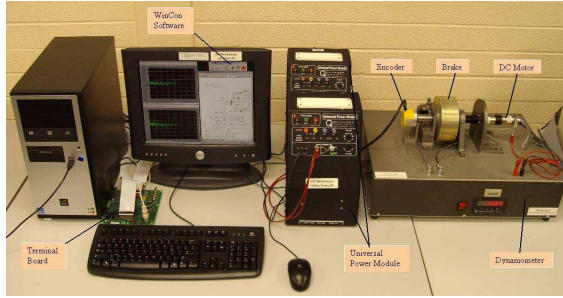


Fig. 1. The dynamometer setup used in PALC experiments

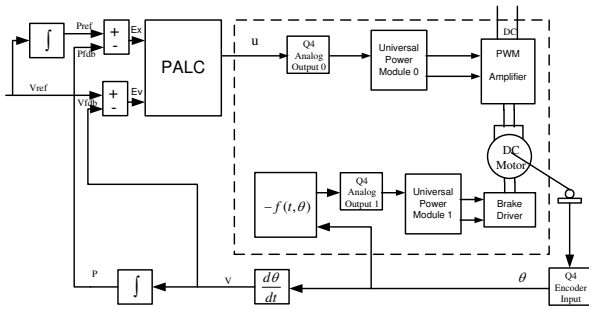


Fig. 2. Block diagram of the cogging-like disturbance PALC in the Dynamometer position control system

2) *Proposed Application*: Without loss of generality, consider the servo control system modeled by:

$$\dot{x}(t) = v(t), \quad (41)$$

$$\dot{v}(t) = f(t, x) + u(t). \quad (42)$$

where x is the position state, $f(t, x)$ is the unknown disturbance, which may be state-dependent or time-dependent, v is the velocity, and u is the control input. The system under consideration, i.e. the DC motor in the dynamometer, has a transfer function $1.52/(1.01s + 1)$. Moreover, the presence of the hysteresis brake allows us to add a time-dependent or state-dependent disturbance (load) to the motor. These factors combined can emulate a system similar to the one given by (41) and (42). A nonlinear controller can be designed for such a problem and can be tested in the presence of the real disturbance as introduced through the dynamometer.

B. Experiments on the Dynamometer

The proposed method is verified on the real-time dynamometer position control system. The hysteresis brake force is designed as multi-harmonics state-dependent disturbance

$$f(t, x) = a(x)/J = F_{disturbance}, \quad (43)$$

where

$$F_{disturbance} = 10 \cos(x) + 5 \cos(2x) + 2.5 \cos(3x).$$

when we substitute (43) into (42), then the system (41) and (42) is the same format with (3) and (4). So we can validate the DHO-PALC for state-dependent disturbance on the dynamometer platform, Fig. 2 shows the block diagram. The control gains in (10) were selected as: $\alpha = 5$, $\gamma = 10$ and $\mu = 0.05$. The periodic adaptation gain K was selected as 0.015.

Two experimental cases are performed.

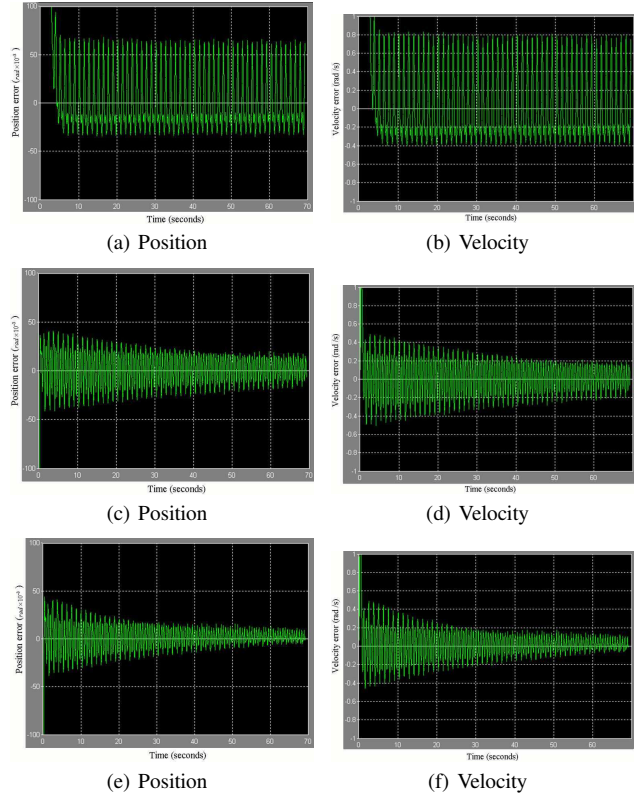


Fig. 3. Experiment tracking errors without compensation, with compensation using FO-PALC and using S-SO-PALC.

1) *Case-1: Convergence speed comparison*: For this case experiment test, the following reference trajectory and velocity signals are used:

$$\begin{aligned} s_d(t) &= 5t \text{ (rad)}, \\ v_d(t) &= 5 \text{ (rad/s)}. \end{aligned}$$

First, we use the first order (FO) PALC, the adaptation law (12) is presented as:

$$\hat{a}(t) = \begin{cases} \hat{a}_1(t) + \frac{K}{J} m_1(t) & \text{if } s \geq s_p \\ z - \mu v & \text{if } s < s_p. \end{cases} \quad (44)$$

Figures 3(c) and 3(d) show the position/speed tracking errors with compensation of using the FO-PALC. We can observe that, as time increases, the positive/speed tracking errors become smaller and smaller. The FO-PALC works efficiently comparing with the tracking errors without compensation in Figures 3(a) and 3(b).

Second, we use the second order (SO) of the composite feedback errors S (S-SO-PALC) to test the HO-PALC. At the same time, in order to compare with the FO-PALC fairly, we design $\beta_1 = \beta_2 = 0.5$, so the adaptation law (12) is presented as:

$$\hat{a}(t) = \begin{cases} A(t) + \frac{K}{J} S(t) & \text{if } s \geq s_p \\ z - \mu v & \text{if } s < s_p \end{cases} \quad (45)$$

with $A(t) = \hat{a}_1(t)$, $S(t) = 0.5m_0(t) + 0.5m_1(t)$.

Figures 3(e) and 3(f) show the positive/speed tracking errors when using the S-SO-PALC method. Comparing with Fig. 3(c) and Fig. 3(d), the convergence speed of the position/speed tracking errors using the S-SO-PALC is obviously faster than that using the FO-PALC.

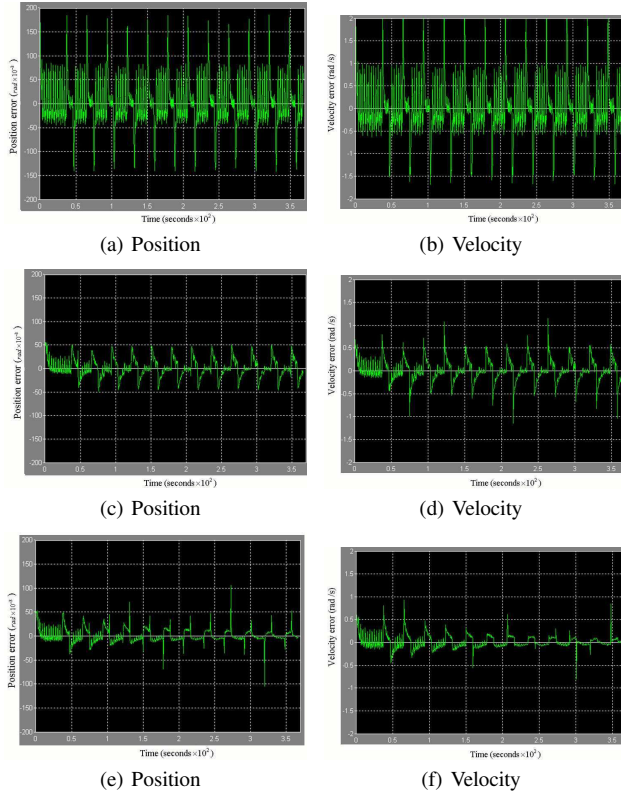


Fig. 4. Varying reference tracking errors without compensation, with compensation using FO-PALC and using A-SO-PALC.

2) *Case-2: Performance comparison with varying reference:* In this case, the following varying reference trajectory and velocity signals are used:

$$s_d(t) = \int_0^t v_d(\tau) d\tau,$$

$$v_d(t) = \begin{cases} 2 & (\text{rad/s}) \text{ if } j s_p \leq s < (j+1) s_p \\ 4 & (\text{rad/s}) \text{ if } (j+1) s_p \leq s < (j+2) s_p \end{cases}$$

where $j = 0, 2, 4, \dots$

First, we apply the first order PALC, the adaptation law (12) is presented as:

$$\hat{a}(t) = \begin{cases} \hat{a}_1(t) + \frac{K}{J} m_1(t) & \text{if } s \geq s_p \\ z - \mu v & \text{if } s < s_p. \end{cases} \quad (46)$$

Figures 4(c) and 4(d) show the position/speed tracking errors with FO-PALC, where we can observe that, the FO-PALC works comparing with the tracking errors without compensation in Figures 4(a) and 4(b), but the compensation residual is not satisfactory.

Now, we use the second order information of the estimate of A for state-dependent disturbance (A-HO-PALC) to test the HO-PALC. We choose $h_1=0$ and $h_2=1$, so the adaptation law (12) is presented as:

$$\hat{a}(t) = \begin{cases} \hat{a}_2(t) + \frac{K}{J} m_1(t) & \text{if } s \geq s_p \\ z - \mu v & \text{if } s < s_p. \end{cases} \quad (47)$$

Figure 4 shows the positive/speed tracking errors using the above A-SO-PALC. Comparing with Fig. 4(c) and Fig. 4(d), we can clearly observe that the performance of using the A-SO-PALC is much better than that using the FO-PALC with alternatively varying reference.

V. CONCLUDING REMARKS

In this paper, a new high-order state-dependent periodic disturbance compensation method is proposed. The key idea of this method is to use two types of past information of more than one period along the state-axis in the current adaptation learning law. From the experimental results, we can conclude that, the proposed DHO-PALC method for state-dependent disturbance works effectively, and performs much better than the FO-PALC scheme. The convergence speed of the position/speed tracking errors with the S-HO-PALC is faster than that with the FO-PALC. In particular, the compensation performance using A-HO-PALC is much better than that using the FO-PALC when a varying reference is considered. Furthermore, our suggested DHO-PALC method has been tested for the general form of the state-dependent disturbances, which include, state-dependent friction, cogging effect, eccentricity and so on.

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