Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Data-driven controller tuning with integrated stability constraint

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Abstract— **This paper presents a data-driven controller-tuning algorithm that includes a sufficient condition for closed-loop stability. This stability condition is defined by a set of convex constraints on the Fourier transform of specific auto- and cross-correlation functions. The constraints are included in a correlation-based controller-tuning method that solves a model-reference problem. This entirely data-driven method requires a single experiment and can also be applied to nonminimum-phase and unstable systems. The resulting controller is guaranteed to stabilize the plant as the data length tends to infinity. The performance with finite data length is illustrated through a simulation example.**

I. Introduction

In many control problems, the closed-loop specifications are given as a reference model. A model-based solution to these problems requires data from the plant to identify and validate a plant model. The controller is then computed via minimization of the model-reference control criterion, possibly followed by controller-order reduction. In recent years, several data-driven approaches have been proposed as an alternative to the model-based approach. In data-driven approaches, the input-output data from the plant are used directly for the minimization of a control criterion [1], [2], [3], [4].

The data-driven approaches can be classified as iterative and non-iterative techniques. In iterative feedback tuning (IFT) [1] and iterative correlation-based tuning (ICbT) [2], a gradient approach is used to iteratively minimize a control criterion. At each iteration, experiments are needed for criterion evaluation or gradient estimation. Since the control criterion is non-convex, the algorithm converges to a local minimum. In non-iterative approaches, classical parameter estimation algorithms are used to compute a controller that minimizes an approximate control criterion [3], [5]. Using a linearly parameterized controller, the optimization problem becomes convex.

The main difficulty with both the iterative and non-iterative algorithms is that, once a controller is designed, there is no guarantee that this controller actually stabilizes the system. Closed-loop stability for data-driven controller tuning remains a problem that is usually addressed a posteriori. This means that stability is verified after controller computation and before actual implementation on the plant. Several such a posteriori tests have been proposed in the literature. Identification of the resulting closed-loop system is proposed in [6]. The test proposed in [7] is based on the estimation of the infinity norm of a transfer function. In [8], the controller validation test is based on the estimation of the phase of the current closed-loop transfer function.

This paper proposes the first known attempt to integrate a stability constraint in data-driven controller design. Under mild conditions, the resulting controller is guaranteed to stabilize the plant as the data length tends to infinity. The stability constraint is defined as an upper bound on the infinity norm of a certain transfer function. This constraint is then approximated by a set of constraints on the spectral estimate of this transfer function, implemented as the discrete Fourier transform of the corresponding auto- and crosscorrelations. The resulting set of constraints is shown to be convex with respect to the controller parameters.

In Section II the data-driven model-reference problem is defined for stable systems and a non-iterative tuning algorithm using the correlation approach is reviewed. A sufficient condition for closed-loop stability and its implementation in controller design using a spectral estimate is discussed in Section III. In Section IV, this method is adapted to handle unstable and/or nonminimum-phase systems. A simulation example illustrates the approach in Section V. The paper ends with some concluding remarks.

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This research work is financially supported by the Swiss National Science Foundation under Grant No. 200020-105139.

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II. Data-Driven Model-Reference Control

Consider the unknown plant $G(q^{-1})$, where q^{-1} denotes the backward shift operator. The objective is to design a linear fixed-order controller $K(\rho, q^{-1})$ for which the controlled plant resembles the stable reference model $M(q^{-1})$. This can be achieved by minimizing the two-norm of the difference between the reference model and the achieved closed-loop system:

$$J_{mr}(\rho) = \left\| M - \frac{K(\rho)G}{1+K(\rho)G} \right\|_2^2 \tag{1}$$

Minimizing this model-reference criterion is a standard control problem when the plant model G is known. Using Parseval's theorem, a time-domain equivalent of this frequency-domain criterion can be formulated. The time-domain criterion $\|\varepsilon_{cl}(t)\|_2$ can be minimized for a given reference signal $r(t)$ using a data-driven approach as illustrated in Fig. 1.

Since the criterion (1) is non-convex with respect to the controller parameters ρ, only a local minimum can be guaranteed upon convergence. This problem can be circumvented by minimizing a convex approximation of criterion (1).

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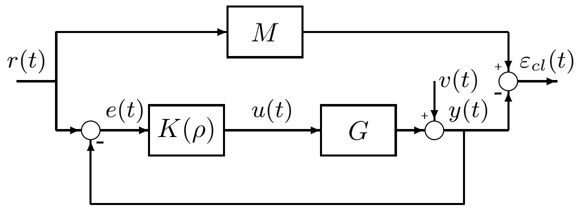

Fig. 1. Model-reference control problem

A. Convex Approximation

The reference model $M(q^{-1})$ can be expressed as

$$M(q^{-1}) = \frac{K^*(q^{-1})G(q^{-1})}{1+K^*(q^{-1})G(q^{-1})} \tag{2}$$

where $K^*(q^{-1})$ is the ideal controller. Note that neither the order nor the structure of this ideal controller are specified explicitly; they are given implicitly through the specification of M. Furthermore, if the reference model is inappropriate, the ideal controller K^* might not ensure internal stability.

The model-reference criterion can be approximated as

$$J_{app}(\rho) = \left\| \frac{K^*G - K(\rho)G}{(1+K^*G)^2} \right\|_2^2 . \tag{3}$$

This approximation is good if the difference between $K(\rho)$ and the ideal controller K^* can be made small. If the controller $K(\rho)$ is linear in the parameters ρ, the approximate criterion in (3) is convex with respect to ρ. This approximation has been used in model reduction and controller reduction (see [9] for an overview) as well as in data-driven controller tuning [3], [5].

A controller-tuning scheme that uses only one experiment was proposed in [5] for linear SISO systems. The time-domain equivalent of criterion (3) is illustrated in Fig. 2. It results in an identification problem that has a noisy input in contrast to classical identification problems where the output is affected by noise. In order to find an unbiased controller, the correlation approach is used in which the correlation between the error $\varepsilon(t)$ and the reference $r(t)$ is minimized.

In the following, the implementation of this scheme for stable systems using a periodic input is discussed. Although the asymptotic results presented in this paper are also valid for non-periodic signals, the use of periodic signals is advantageous for finite data length. This can be explained as follows. The estimation of the correlation functions and Fourier transform of signals used in the proposed approach contains two major sources of errors: the truncation error and the noise term. For non-periodic signals, both errors reduce when the number of data goes to infinity, but they can be important for finite data length. When using a periodic input, the truncation error can be removed completely for the (periodic) deterministic part of the signals. Hence, only the noise term will affect the estimates. This remaining error can be reduced by averaging over a number of periods.

The power spectrum of the periodic signal $r(t)$ is defined as

$$\Phi_r(\omega_k) = \frac{1}{T}\sum_{\tau=0}^{T-1} R_r(\tau)e^{-i\tau\omega_k},$$

where $\omega_k = 2\pi k/T, k = 0, \ldots, \lfloor (T-1)/2 \rfloor$ and $\lfloor\cdot\rfloor$ denotes the closest integer below. $R_r(\tau)$ is the autocorrelation of $r(t)$:

$$R_r(\tau) = \frac{1}{T}\sum_{t=1}^{T} r(t-\tau)r(t) \quad \text{for } \tau = 0, \ldots, T-1$$

If the input signal is a Pseudo-Random Binary Signal (PRBS) with period T and total length $N = pT$, i.e. for p periods, the spectrum $\Phi_r(\omega)$ is nonzero for ω_k and as $T \to \infty, \Phi_r(\omega) \neq 0, \forall\omega \in [0, 2\pi)$. This characteristic makes PRBS very attractive for the proposed approach.

Let the stable linear SISO system G be excited in open loop with a PRBS (see Fig. 2). The noisy output $y(t)$ is measured. The noise is assumed to be zero mean and uncorrelated with the input signal, i.e. the cross-correlation vanishes, $R_{rv}(\tau) = 0, \forall\tau$. The error signal $\varepsilon(t,\rho)$ in Fig. 2 can be expressed in terms of the signals $r(t)$ and $y(t)$:

$$\varepsilon(t,\rho) = Mr(t) - K(\rho)(1-M)y(t) \tag{4}$$

Let the controller be linearly parameterized in ρ:

$$K(\rho) = \beta^T(q^{-1})\rho \tag{5}$$

where $\beta(q^{-1})$ is a vector of linear stable discrete-time transfer operators:

$$\beta(q^{-1}) = [\beta_1(q^{-1}), \beta_2(q^{-1}), \ldots, \beta_{n_\rho}(q^{-1})]^T \tag{6}$$

with n_ρ the number of controller parameters. Then, the error signal $\varepsilon(t,\rho)$ can be obtained by linear regression:

$$\varepsilon(t,\rho) = r_M(t) - \phi^T(t)\rho \tag{7}$$

where $r_M(t) = Mr(t)$ and $\phi(t) = \beta(1-M)y(t)$.

The correlation function $f_T(\rho)$ for periodic signals is also periodic with the same period:

$$f_T(\rho) = \frac{1}{T}\sum_{t=1}^{T} E\{\zeta_w(t)\varepsilon(t,\rho)\}$$
$$= \frac{1}{T}\sum_{t=1}^{T} \zeta_w(t)\big[r_M(t) - E\{\phi^T(t)\}\rho\big] \tag{8}$$

where $E\{\cdot\}$ is the mathematical expectation and

$$\zeta_w(t) = [r_w(t), r_w(t-1), \ldots, r_w(t-T+1)]^T \tag{9}$$

with $r_w(t) = W(q^{-1})r(t)$. The choice of $W(q^{-1})$ will be discussed later.

An unbiased estimate of the cross-correlation function between $r_W(t)$ and $\varepsilon(t,\rho)$ can be obtained if $E\{\phi^T(t)\}$ is replaced by $\phi^T(t)$. The variance of the estimate can be reduced if the cross-correlation function is computed over all periods:

$$\hat{f}_N(\rho) = \frac{1}{N}\sum_{t=1}^{N} \zeta_w(t)\big[r_M(t) - \phi^T(t)\rho\big] \tag{10}$$

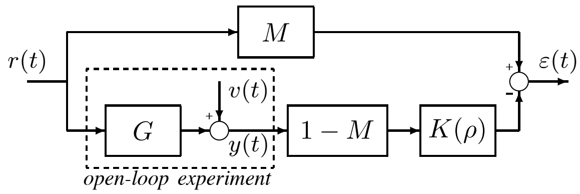

Fig. 2. Approximation of the model-reference control problem for stable systems. This set-up allows the non-iterative computation of the controller parameters ρ

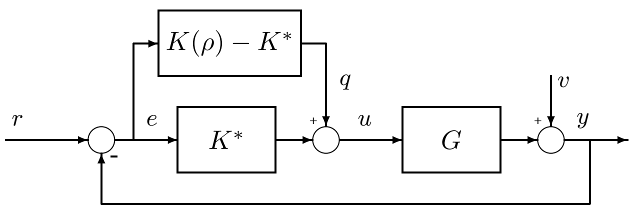

Fig. 3. Closed-loop system using explicit representation of the controller error $K(\rho) - K^*$

The controller $K(\rho)$ that minimizes the approximate model-reference criterion (3) is then given by

$$\hat{\rho} = \min_{\rho} \hat{f}_N^T(\rho)\hat{f}_N(\rho).$$

This criterion is convex in ρ and its minimizer is the least-squares solution.

It can be shown that with the weighting filter

$$W(e^{-j\omega_k}) = \frac{1 - M(e^{-j\omega_k})}{\Phi_r(\omega_k)}, \tag{11}$$

the time-domain $\hat{f}_N^T(\rho)\hat{f}_N(\rho)$ criterion and J_{app} are asymptotically equivalent, i.e. :

$$\lim_{T,N\to\infty, T/N\to 0} \hat{f}_N^T(\rho)\hat{f}_N(\rho) = J_{app}(\rho)$$

The proof is similar to that given in [5] for non-periodic signals and is omitted here. Note that the filter is defined for the frequencies where the input spectrum $\Phi_r(\omega_k) \neq 0$.

B. Stability Issues

There is no guarantee that the controller designed using the approach described above – or any other data-driven approach – stabilizes the system. Instability can occur if the reference model M is chosen inappropriately or if the measurements are corrupted by noise. Even the ideal controller may destabilize the closed-loop system. Stability can only be guaranteed if G is minimum phase or M contained the unstable zeros of G. Clearly, this last condition makes an appropriate choice of M difficult.

An example of an inappropriately chosen reference model is shown in [10], where the stabilizing ideal controller K^* does not belong to the set of possible controllers $K(\rho)$. As a consequence, the optimal controller $K(\rho_{opt})$ minimizing (3) destabilizes the system.

III. Convex Constraint for Stability Using Spectral Estimation

A way of integrating stability constraints in data-driven controller-design methods is proposed next. The set of constraints represents a sufficient condition for closed-loop stability, defined as the H_∞-norm of a certain transfer function. This norm is estimated using power spectral density functions. The constraints are implemented using the discrete Fourier transform (DFT) of the corresponding auto- and crosscorrelation functions and are convex with respect to the controller parameters. It should be noted that this estimate for noise-free periodic signals is exact for a finite number of frequencies. In the presence of noise the estimate is unbiased and its variance decreases as the number of data increases.

Minimization of the H_∞-norm of a transfer function has already been considered in some data-driven approaches using Toeplitz matrices [4], [11], but it has never been used to define a convex set of stabilizing controllers.

A. Sufficient Condition for Closed-Loop Stability

A sufficient condition for closed-loop stability with the controller $K(\rho)$ has been proposed in [7]. The closed-loop system $\frac{K(\rho)G}{1+K(\rho)G}$ can be represented as the interconnection of a reference model M and the difference between $K(\rho)$ and K^* as illustrated in Fig. 3.

If the loop is opened at q, a plant is obtained that is stable if the optimal controller K^* internally stabilizes the system and $K(\rho)$ is stable. If this is the case, the small-gain theorem can be used to define a sufficient condition for closed-loop stability. The stable controller $K(\rho)$ is guaranteed to stabilize G if K^* internally stabilizes G and

$$\delta_0 = \left\| \frac{-(K(\rho) - K^*)G}{1 + K^*G} \right\|_\infty < 1. \tag{12}$$

δ_0 is the H_∞-norm of the loop transfer function from q back to q. Replacing $\frac{K^*G}{1+K^*G}$ by M and $\frac{1}{1+K^*G}$ by $(1-M)$ gives:

$$\delta_0 = \|M - K(\rho)(1 - M)G\|_\infty < 1 \tag{13}$$

This H_∞-norm can be estimated if a set of data consisting of the corresponding input and output signals is available or can be constructed.

It turns out that $r(t)$ and $\varepsilon(t,\rho)$ used in the control criterion give exactly the input and output signals needed to estimate δ_0. Hence, the same set of data can be used to define both a convex control objective and a set of convex constraints on $K(\rho)$ such that $\hat{\delta} < 1$, where $\hat{\delta}$ is an estimate of δ_0 based on spectral analysis.

B. Convex Constraint for Stability using DFT

Let the stable minimum-phase system G be excited in open-loop mode by a PRBS as illustrated in Fig. 2. The resulting error $\varepsilon(t,\rho)$ is corrupted by noise. δ_0 can be estimated using the power spectral density function $\Phi_r(\omega_k)$ of $r(t)$ and the power cross-spectral density function $\Phi_{r\varepsilon}(\omega_k,\rho)$ between $r(t)$ and $\varepsilon(r,\rho)$, defined as

$$\Phi_{r\varepsilon}(\omega_k,\rho) = \frac{1}{T}\sum_{\tau=0}^{T-1} R_{r\varepsilon}(\tau,\rho)e^{-i\tau\omega_k},$$

using the cross-correlation between $r(t)$ and $\varepsilon(t,\rho)$:

$$R_{r\varepsilon}(\tau,\rho)=\frac{1}{T}\sum_{t=1}^{T}E\{r(t-\tau)\varepsilon(t,\rho)\}.$$

For periodic signals $R_r(\tau)$ can be calculated but $R_{r\varepsilon}(\tau,\rho)$ needs to be estimated when only a finite number of noise corrupted data is available. The effect of noise can be reduced by the use of several periods in the calculation of this cross-correlation function. An unbiased estimate $\hat{R}_{r\varepsilon}(\tau,\rho)$ can be found using:

$$\hat{R}_{r\varepsilon}(\tau,\rho)=\frac{1}{N}\sum_{t=1}^{N}r(t-\tau)\varepsilon(t,\rho),\quad \tau=0,1,\ldots,T-1$$

An estimate of δ_0 can then be bounded using

$$\hat{\delta}(\rho)=\max_{\{\omega_k|\Phi_r(\omega_k)\neq 0\}}\left|\frac{\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)}{\Phi_r(\omega_k)}\right|<1 \qquad (14)$$

Since the controller $K(\rho)$ is linearly parametrized, $\varepsilon(t,\rho)$ and $\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)$ are linear in the controller parameters ρ. Consequently, $\left|\frac{\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)}{\Phi_r(\omega_k)}\right|$ is convex in ρ for each frequency ω_k. The sufficient condition (13) can thus be approximated as a set of convex constraints on the controller parameters ρ, by imposing $\hat{\delta}<1$. This allows the stability condition to be integrated in the controller design. Note that $\hat{\delta}<1$ does not necessarily imply that $\delta_0<1$. However, since both the input signal $r(t)$ and the deterministic part of $\varepsilon(t,\rho)$ are periodic, well-known results on power spectra of periodic signals are applicable to this estimate $\hat{\delta}$ [12]. $\Phi_r(\omega_k)$ is non-zero at the specific T frequencies. Due to symmetry only $\lfloor (T-1)/2\rfloor$ frequencies need to be considered. The variance of $\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)$ decreases as the number of periods p increases and tends to zero as $p\to\infty$. Asymptotically the estimate using (14) is therefore unbiased. If the length of the period $T\to\infty$, the frequency grid becomes continuous. Since the input $r(t)$ is a PRBS its spectrum $\Phi_r(\omega_k)\neq 0,\forall\omega$ as $T\to\infty$.

The solution of the following convex optimization problem provides asymptotically a stabilizing controller for the stable minimum-phase plant G:

$$\hat{\rho}=\min_{\rho}\hat{f}_N^T(\rho)\hat{f}_N(\rho)$$
subject to
$$\left|\frac{1}{T}\sum_{\tau=0}^{T-1}\hat{R}_{r\varepsilon}(\tau,\rho)e^{-i\tau\omega_k}\right|<\left|\frac{1}{T}\sum_{\tau=0}^{T-1}R_r(\tau)e^{-i\tau\omega_k}\right| \qquad (15)$$
$$\omega_k=2\pi k/T,\quad k=0,\ldots,\lfloor (T-1)/2\rfloor$$

In this controller-tuning algorithm, the approximate model-reference criterion (3) is minimized over a subset of the set of stabilizing controllers for which (13) is satisfied.

C. Convex Constraint for stability using LMI

H_∞ specifications have been used in system identification [13] and data-driven controller tuning [4] as well as in the stability test as introduced in [7]. In these methods, non-periodic signals are considered and the constraint on the H_∞-norm is defined using Toeplitz matrices, which leads to a Linear Matrix Inequality (LMI). The constraints proposed in this paper can also be implemented using an LMI.

Consider a circulant matrix defined for $x(t)$ as

$$C(x)=\begin{bmatrix} x(1) & x(2) & \ldots & x(N-1) & x(N)\\ x(N) & x(1) & \ldots & x(N-2) & x(N-1)\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ x(3) & x(4) & \ldots & x(1) & x(2)\\ x(2) & x(3) & \ldots & x(N) & x(1)\end{bmatrix}$$

where each row is a cyclic shift of the row above it. Some characteristics of circulant matrices are as follows [14]:

1) The eigenvalues of a circulant matrix are given by :
$$\lambda_k(C(x))=\sum_{t=1}^{N}x(t)e^{-it\omega_k},$$
$$\omega_k=2\pi k/N,\quad k=0,\ldots,N-1 \qquad (16)$$
2) The eigenvectors of each circulant matrix of size N are given by:
$$U_k=\frac{1}{\sqrt{N}}\left(1,e^{-i\omega_k},e^{-i2\omega_k},\ldots,e^{-i(N-1)\omega_k}\right) \qquad (17)$$
The eigenvectors are independent of the elements of the matrix.
3) Define the matrix U, which has the eigenvectors U_k, $k=0,\ldots,N-1$, as columns, and define $\Lambda(\cdot)=\text{diag}(\lambda_k(C(\cdot))$. U is full rank and unitary, i.e. $UU^*=I$ and $U^*U=I$. For each circulant matrix $C(\cdot)$:
$$\Lambda(\cdot)=U^*C(\cdot)U \qquad (18)$$

To proceed, we need the following lemma:

Lemma 1: Consider $C(x)$ and $C(z)$ two $N\times N$ circulant matrices. One has:

$$C^T(x)C(x)-C^T(z)C(z)<0\Longleftrightarrow$$
$$|\lambda_k(C(x))|-|\lambda_k(C(z))|<0,\quad k=0,\ldots,N-1 \qquad (19)$$

Proof: Since U is full rank:

$$C^T(x)C(x)-C^T(z)C(z)<0\Longleftrightarrow$$
$$U^*\big(C^T(x)C(x)-C^T(z)C(z)\big)U<0\Longleftrightarrow$$
$$\Lambda(x)^*\Lambda(x)-\Lambda(z)^*\Lambda(z)<0\Longleftrightarrow$$
$$|\lambda_k(C(x))|-|\lambda_k(C(z))|<0,\quad k=0,\ldots,N-1$$

The third expression follows from (18) and the last one from the definition of $\Lambda(\cdot)$. ■

The main result of this subsection is presented in the following theorem:

Theorem 1: The convex constraints given in (14) are equivalent to the following LMI:

$$\begin{bmatrix} -C_t^T(r)C_t(r)C_t^T(r)C_t(r) & C_t^T(r)C_t(\varepsilon(\rho))\\ C_t^T(\varepsilon(\rho))C_t(r) & -I\end{bmatrix}<0 \qquad (20)$$

where $C_t(\cdot)$ is a truncated circulant matrix of size $(N\times T)$.

Proof: The DFT of the periodic input signal has only T non-zero values. In order to avoid problems with eigenvalues equal to zero, only one period will be used in the LMI. For this reason truncated circulant matrices of size $(N \times T)$ are used. The multiplication of two truncated matrices $C_t^T(\varepsilon(\rho))C_t(r)$ is a circulant matrix of size $(T \times T)$ of the unbiased estimate $\hat{R}_{r\varepsilon}(\tau,\rho)$ for $\tau = 0,\ldots,T-1$ and its eigenvalues are $T\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)$. In the same way, $C_t^T(r)C_t(r)$ is a circulant matrix of size $(T \times T)$ of $R_r(\tau)$ with positive eigenvalues equal to $T\Phi_r(\omega_k)$. Then, using Lemma 1 we have :

$$\begin{aligned} &C_t^T(\varepsilon(\rho))C_t(r)C_t^T(r)C_t(\varepsilon(\rho)) \\ &\qquad - C_t^T(r)C_t(r)C_t^T(r)C_t(r) < 0 \\ &\iff |\hat{\Phi}_{r\varepsilon}(\omega_k,\rho)| - |\Phi_r(\omega_k)| < 0 \\ &\qquad \omega_k = 2\pi k/T, \quad k = 0,\ldots,(T-1) \end{aligned} \tag{21}$$

Using the Schur complement, the LMI in (20) is obtained. ■

Remark: Constraint (20) can be seen as a periodic version of the norm proposed in [10]. The direct use of the DFT instead of these circulant matrices has two advantages. First of all the computational load is much smaller. Secondly the frequencies considered can be chosen in a straightforward manner. For example, only frequencies where the signal-to-noise ratio is reasonable could be selected. A comparison is given in the example of section V.

IV. Approach for nonminimum-phase and/or unstable systems

For nonminimum-phase and for unstable systems, it is difficult to specify an appropriate reference model. An M that leads to a stabilizing optimal controller K^* can in general only be found if the unstable poles and zeros of the plant are known. For this reason, the use of model-based control design methods is often preferred over data-driven model-reference approaches for such systems. However, with the addition of a stability constraint in the controller design step, this method becomes applicable to nonminimum-phase and unstable systems as well. In the following, the approach of Section III is extended to nonminimum-phase and unstable systems.

Since the stability constraint (12) is based on the small-gain theorem, it is applicable only if K^* stabilizes the system internally. The stability condition is thus subject to the same difficulties as the initial model-reference problem. However, the reference model M used in the approximate model-reference criterion (3) does not need to be the same as that in the stability criterion (14). If a stabilizing controller K_s is available and used to control the plant, it can be used to specify the reference model

$$M_s = \frac{K_s G}{1 + K_s G}$$

for the stability condition. It should be noted that, since G is unknown, M_s will be unknown as well. However, in order to estimate δ_0 in (13) and define the constraints for the optimization problem, a set of input/output data of the transfer function

$$M_s - K(\rho)(1 - M_s)G \tag{22}$$

is sufficient. These signals as well as the signals necessary to minimize the model-reference criterion (3) are available from the tuning scheme shown in Fig. 4.

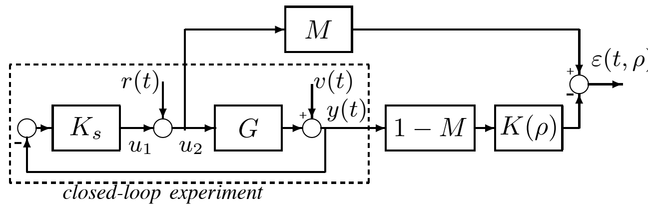

Fig. 4. Model-reference control problem using one closed-loop experiment

The plant is controlled using K_s, the excitation signal is applied directly to the input of the plant. The available data include the exogeneous excitation signal $r(t)$, the output of the controller $u_1(t)$, the resulting input to the plant $u_2(t) = u_1(t) + r(t)$ and the output of the controlled plant $y(t)$.

The stability constraints are defined using $r(t)$ as input to the transfer function in (22). The corresponding (noise-corrupted) output $\varepsilon_s(t,\rho)$ can be found as follows:

$$\begin{aligned} \varepsilon_s(t,\rho) = -u_1(t) - K(\rho)y(t) = \big(M_s - K(\rho)(1-M_s)G\big)r(t) \\ + (K_s - K(\rho))(1 - M_s)v(t) \end{aligned} \tag{23}$$

This output is no longer equal to the error signal $\varepsilon(t,\rho)$ needed for the model-reference criterion. $\varepsilon(t,\rho)$ is shown in Fig. 4 and can be expressed as a function of ρ:

$$\varepsilon(t,\rho) = Mu_2(t) - K(\rho)(1 - M)y(t) \tag{24}$$

The correlation function and the instrumental variables are the same as those defined in (10) and in (9).

The filter W necessary for asymptotic equivalence of $\hat{f}_N^T(\rho)\hat{f}_N(\rho)$ and J_{app} now becomes:

$$W(e^{-j\omega_k}) = \frac{1 - M(e^{-j\omega_k})}{\Phi_{u_2 r}(\omega_k)}, \tag{25}$$

where

$$\Phi_{u_2 r}(\omega_k) = \frac{1}{1 + K_s G}\Phi_r(\omega_k)$$

is the cross-spectrum between $u_2(t)$ and $r(t)$, [5].

The solution of the following convex optimization problem provides asymptotically a stabilizing controller for the unstable and/or nonminimum-phase plant G:

$$\begin{aligned} &\hat{\rho} = \min_{\rho} \hat{f}_N^T(\rho)\hat{f}_N(\rho) \\ &\qquad \text{subject to} \\ &\left| \frac{1}{T}\sum_{\tau=0}^{T-1} \hat{R}_{r\varepsilon_s}(\tau,\rho)e^{-i\tau\omega_k} \right| < \left| \frac{1}{T}\sum_{\tau=0}^{T-1} R_r(\tau)e^{-i\tau\omega_k} \right| \\ &\qquad \omega_k = 2\pi k/T, \quad k = 0,\ldots,\lfloor (T-1)/2 \rfloor \end{aligned} \tag{26}$$

In this controller tuning algorithm the approximate model-reference criterion is minimized for M, while stability is guaranteed using M_s.

V. Simulation Example

A simple example was used in [10] to show that stability problems occur "for the class of identification-for-control methods that use arbitrary data in the identification". The same example will be used here to show that the method proposed in this paper leads to stabilizing controllers.

The pure time-delay system $G(q^{-1}) = q^{-1}$ is considered. The proportional controller $K = \rho$ is used to control the plant. The controlled system is unstable for $|\rho| > 1$. The reference model is $M = 1 - \alpha + \alpha q^{-1}$, where α is a parameter controlling the bandwidth. The model-reference control problem is minimized by $K_{opt} = \frac{4\alpha-1}{6\alpha}$. For $0 < \alpha < 0.1$, $|K_{opt}| > 1$ and the controlled system will be unstable.

The system is excited by a periodic PRBS with period $T = 63$, $p = 4$ periods and $\beta = 1$. The reference model is chosen as $M = 0.95 + 0.05q^{-1}$, i.e. with $\alpha = 0.05$ for which the optimal controller $K_{opt} = -2.67$ destabilizes the plant. Two controllers are calculated using noise-free simulation data. The first controller is calculated without the constraints in (15) and thus minimizes $\hat{f}_N^T \hat{f}_N$. The controller found is $K(\rho_1) = -2.67$, which destabilizes the system. The second controller is calculated using the stability constraints in (15). In order to avoid numerical problems, constraints (14) are bounded by 0.999. This optimization is infeasible. A closer look at the bound (13) shows that $\|M - K(1-M)G\|_\infty = 1$ for all stabilizing controllers and the problem is indeed infeasible. The controller design was poorly formulated through an inappropriate choice of M.

In order to show the effectiveness of the method in the presence of noise, the reference model used for the stability constraints is slightly altered, $M_s = 1 - \alpha + 0.95\alpha q^{-1}$. The reference model used in the control objective remains unchanged. For this problem $\|M_s - K(\rho)(1-M_s)G\|_\infty < 1$ for a subset of the stabilizing controllers and the problem is feasible. The output of the system is perturbed by a white noise such that the signal-to-noise ratio is about 10 in terms of variance. The controller obtained without using the constraints in (15) is $K(\rho_1) = -2.34$, which again destabilizes the system. The second controller calculated using (15) is $K(\rho_2) = -0.33$. Clearly, $|K(\rho_2)| < 1$ and it stabilizes the system. The difference between $K(\rho_1)$ and $K(\rho_2)$ indicates a poor problem formulation.

Constraints in (15) can also be implemented using circulant matrices. However, this leads to a large LMI, which becomes expensive to compute for large data length. The following comparison is found using Matlab V 7.4 on a Mac with a 3 GHz processor and 5 GB memory. The optimization is implemented using Yalmip and Sedumi. The aforementioned problem for $N = 252$ leads to exactly the same result using both implementations. The DFT approach is more expensive to formulate but faster to run. The difference is small for small data lengths, e.g. for $T = 63, N = 252$ the DFT approach takes $0.7s$ vs. $2s$ for the LMI. For $T = 127, N = 1016$, the DFT approach takes $2.2s$ vs. $13s$ for the LMI. For $T = 255, N = 2040$, the LMI cannot be solved (memory problems) whereas the DFT approach only takes $3s$. When using the DFT approach the data length can be increased to at least $T = 1023, N = 8184$, for which the optimization is solved within $10s$.

VI. Conclusions

A data-driven controller tuning approach that asymptotically guarantees closed-loop stability is presented. The approach combines minimization of an approximate H_2 model-reference problem with an H_∞ constraint that represents a sufficient condition for closed-loop stability. The H_∞ constraint is implemented using the DFT of auto- and cross-correlation functions. This corresponds to an estimate of the H_∞-norm using power spectral densities. A periodic input signal is used to improve the quality of this estimate. The approach leads to a set of convex constraints that can be added to any data-driven controller tuning scheme with a linearly parametrized controller. The stability of the closed-loop is guaranteed if the data length tends to infinity.

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