

# Model reference adaptive control of nonlinear plant with dead time

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**Abstract**—State feedback Lyapunov-based design of direct model reference adaptive control (MRAC) for a class of nonlinear systems with input and state delays based only on the lumped-delays without so-called distributed-delay (DD) blocks are developed. The design procedure is based on the concept of reference trajectories prediction, and on the formulation of an augmented error. We propose a controller parametrization which attempts to anticipate the future states. An appropriate Lyapunov-Krasovskii type functional is introduced.

**Index Terms**—Adaptive control, time-delay systems.

## I. INTRODUCTION

The adaptive control problem for delay systems has been investigated by many authors. Most of the developed results are developed for *state delayed* systems.

Adaptive signal tracking based on state feedback can be found in, e.g. [1], and in the papers [2], [3], [4], [5], based on output feedback. The problem of output reference model signal tracking by state feedback for linear systems with bounded multiple delayed nonlinear state perturbations, and disturbance was considered in [1], where the reference model was an autonomous dynamic system. The output adaptive tracking control of a class of linear, minimum-phase, single-input-single-output (SISO) and MIMO systems of relative degree one described by functional differential equations was considered in the framework of functional differential inclusions in [2], [3]. The author in [4] used tools of differential geometry for a tracking problem of a special type of nonlinear plants. In [5] the backstepping technique was used to form an adaptive control scheme for a class of SISO parametric strict-feedback nonlinear systems with unknown state time delays.

Recently a new approach, [6], [7], was developed for the output model reference adaptive control of linear continuous-time SISO and MIMO plants with state delay. The main idea is to treat the state delay element not as a part of the plant but rather as the input to the system without delay and then decompose the control law into two components. The first base component is designed by a standard MRAC procedure, [8], as for a plant without delay but applied to the time-delay plant. The second component is formed by a special adaptively adjusted dynamic system as a function of the reference signals. This makes it possible to use the well-understood MRAC design technique.

For *input delayed* plants, adaptive following control problems were investigated in a small number of papers: A glob-

ally stable MRAC Smith-predictor-like solution for SISO input delayed plants was developed by Ortega and Lozano [9]. It was assumed that the process is minimum phase with arbitrary relative degree, though not necessarily stable. The controller structure was similar to the one proposed by Ichikawa [10]. More recently, in [11], [12] MRAC controllers were proposed, also with a Smith-predictor-like structure. In [12] adaptive control laws were generated by using a high-order Morse tuner and a Lyapunov-Krasovskii functional. A model reference adaptive control scheme which can simultaneously achieve adaptive decoupling and model matching, is proposed in [13] for a class of multi-output plants with input delay by employing the decoupling principle of MIMO delay-free systems. Variable structure adaptive model predictive control was considered in [14].

The scope of applicability of these results is mainly limited because all the adaptive control laws involve finite-time integrals of the delayed control signals, so-called distributed-delay (DD) blocks in the form  $\int_{-h}^0 \lambda(t,s)u(s)ds$ , where  $\lambda(t,s)$  is an unknown parameter vector which is adaptively tuned by some dynamic system described by a differential equation,  $\dot{\lambda}(t,s) = f(*)$ . For implementation purposes this integral is *discretized*. However even for the much easier nonadaptive control case, the DD approximation leads to many difficulties. The problem of safe implementation of controllers with DD was included in a recent survey paper [15] as one of the open problems in the control of time-delay systems.

The present note expands our results from in [16] in which state feedback tracking of *linear* systems with input and state delays, but without external disturbances, is proposed. The present paper uses methods similar to those in [16], but the result is stronger since non-linear dynamics, as well as an external disturbance are handled.

## II. PLANT MODEL AND PROBLEM STATEMENT

The class of nonlinear delayed plants with parametric uncertainty considered in this paper is of the form

$$\dot{x}(t) = Ax(t) + A_\tau x(t - \tau) + bu(t - h) + bf(x) + bd(t) \quad (1)$$

where,  $x \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}$  is the control input. The constant matrices  $A, A_\tau \in \mathbb{R}^{n \times n}$  and  $b \in \mathbb{R}^{n \times 1}$  have unknown elements.  $f(x)$  and  $d(t)$  are unknown functions which represents the nonlinearity and external disturbance. The state and input time delays  $\tau \in \mathbb{R}^+$  and  $h \in \mathbb{R}^+$  are assumed to be known.

The problem is to design an adaptive feedback control, and tune, on-line, the controller parameters in order to achieve desired closed loop specifications.

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The desired specification in this paper is that all signals of the closed loop system remain bounded and the following is achieved:

- *approximate* tracking with small enough asymptotic error.
- *exact* asymptotic tracking, i.e. the tracking errors  $e(t) = x(t) - x_r(t) \rightarrow 0$  as  $t \rightarrow \infty$  if the input signal of the reference model  $r(t)$  is such that the plant is identifiable.  $x_r(t)$  is the state of the stable reference model

$$\dot{x}_r(t) = A_r x_r(t) + b_r r(t-h) \quad (2)$$

and  $A_r \in \mathbb{R}^{n \times n}$ ,  $b_r \in \mathbb{R}^n$  are known constant matrices, and  $r(t) \in \mathbb{R}$  is a bounded reference input signal.

To meet this control specification, we assume

**(A1)** There exist constant vectors  $\theta_x^* \in \mathbb{R}^n$ ,  $\theta_\tau^* \in \mathbb{R}^n$  and a nonzero constant scalar  $\theta_r^*$  such that the following equations are satisfied:

$$A - A_r + b\theta_x^{*T} = 0, A_\tau + b\theta_\tau^{*T} = 0, b\theta_r^* - b_r = 0 \quad (3)$$

**(A2)** The sign of  $\theta_r^*$ ,  $\text{sign}[\theta_r^*]$ , is known.

**(A3)**  $f(x) = \theta_g^{*T} g(x(t))$  for some unknown constant vector,  $\theta_g^*$ , of arbitrary and constant dimension.  $g(x(t))$  which is of the same dimension as  $\theta_g^*$ , is a vector of known bounded basis functions.

**(A4)**  $d(t) = \theta_d^{*T} \phi(t)$  for some unknown constant vector,  $\theta_d^*$ , of arbitrary and constant dimension.  $\phi(t)$  which is of the same dimension as  $\theta_d^*$ , is a vector of known bounded functions of time.

**(A5)** The plant is asymptotically stable.

### III. CONTROL LAW AND ERROR EQUATION PARAMETRIZATION

To develop an adaptive law we need to express the closed-loop system in terms of the tracking error  $e(t) = x(t) - x_r(t)$ . In view of the assumptions (A1)-(A4) we have from (1) and (2) the following error equation

$$\begin{aligned} \dot{e}(t) = & A_r e(t) - b\theta_x^{*T} x(t) - b\theta_\tau^{*T} x(t-\tau) \\ & + b\theta_g^{*T} g(x(t)) + b\theta_d^{*T} \phi(t) - b_r r(t-h) + bu(t-h) \end{aligned} \quad (4)$$

By adding and subtracting the term  $\Theta^T(t)\Omega(t)$  with the signal vector

$$\Omega(t) = [x^T(t), x^T(t-\tau), -g^T(x(t)), -\phi^T(t), r(t-h)]^T, \quad (5)$$

and some time varying vector

$$\Theta^T(t) = [\theta_x^T(t), \theta_\tau^T(t), \theta_g^T(t), \theta_d^T(t), \theta_r(t)]^T \quad (6)$$

we obtain from (4)

$$\dot{e}(t) = A_r e(t) + b\tilde{\Theta}^T(t)\Omega(t) - b\Theta^T(t)\Omega(t) + bu(t-h) \quad (7)$$

where we introduce, as usually is done in model reference adaptive control, the parameter error vector

$$\begin{aligned} \tilde{\Theta}^T(t) = & [(\theta_x(t) - \theta_x^*)^T, (\theta_\tau(t) - \theta_\tau^*)^T, -(\theta_g(t) - \theta_g^*)^T, \\ & -(\theta_d(t) - \theta_d^*)^T, (\theta_r(t) - \theta_r^*)^T]^T. \end{aligned} \quad (8)$$

Inspection of (7) shows that for the input delay free case ( $h = 0$ ), the introduction of the conventional control parametrization  $u(t) = \Theta^T(t)\Omega(t)$  leads to the classical error equation for the design of the adaptation algorithms, see e.g. [8]. The presence of the input delay  $h$  makes a control parametrization of this type impossible, because there is a need for the prediction of plant signals. To overcome the difficulties of predicting plant signals, we look for a control law parametrization of the form

$$u(t) = \Theta^T(t)\Omega_r(t+h) \quad (9)$$

where the signal vector

$$\Omega_r(t) = [x_r^T(t), x_r^T(t-\tau), -g^T(x_r(t)), -\phi^T(t), r(t-h)]^T \quad (10)$$

now includes only the known signals which are easily predictable, and  $\Theta(t)$ , an adaptive gain which is the same as in (6).

Applying the control (9) to (7) we obtain

$$\dot{e}(t) = A_r e(t) + b\tilde{\Theta}^T(t)\Omega(t) - b\omega(t) \quad (11)$$

where, for convenience, we define the scalar signal

$$\omega(t) = \Theta^T(t)\Omega(t) - \Theta^T(t-h)\Omega_r(t) \quad (12)$$

Let us now introduce the new scalar adaptive parameter  $\theta_\omega(t)$  while adding and subtracting, in the right part of (11), the term  $\theta_\omega(t)\omega(t)$ . Then by using (3), and defining the parameter error  $\tilde{\theta}_\omega(t) = \theta_\omega(t) - \theta_r^{*-1}$  we obtain

$$\begin{aligned} \dot{e}(t) = & A_r e(t) + b_r \theta_r^{*-1} \tilde{\Theta}^T(t)\Omega(t) \\ & + b_r \tilde{\theta}_\omega(t)\omega(t) - b_r \theta_\omega(t)\omega(t) \end{aligned} \quad (13)$$

We now introduce the variable  $x_a(t)$  as the output of the *auxiliary model*, which is the stable dynamic system driven by the input signal  $u_a(t) = \theta_\omega(t)\omega(t)$  with adjusted gain  $\theta_\omega(t)$  on the input,

$$\dot{x}_a(t) = A_r x_a(t) + b_r \theta_\omega(t)\omega(t). \quad (14)$$

*Remark 1:* We note, that the dynamics of the auxiliary model (14) is exactly the same as the dynamics of the reference model (2), and the difference is only in the input signal.

Using  $x_a(t)$  and defining the signal of the *augmented error*  $e_a(t)$  as

$$e_a(t) = e(t) + x_a(t) \quad (15)$$

we transform equation (13) as follows,

$$\dot{e}_a(t) = A_r e_a(t) + b_r \frac{1}{\theta_r^*} \tilde{\Theta}^T(t)\Omega(t) + b_r \tilde{\theta}_\omega(t)\omega(t). \quad (16)$$

*Remark 2:* The augmented error equation (16) has the “standard” form of error parametrization in adaptive control theory for plant without delays, so that well-understood adaptive control methods can now be used for the design of the adaptive algorithms.

In view of (16) and (14) it is clear that the control objective now transforms as follows:

*The case of approximate tracking:* The augmented error and the auxiliary signal must be bounded with small enough asymptotic error.

*The case of exact tracking:* The augmented error  $e_a(t)$  and auxiliary signal  $x_a(t)$  must tend to zero asymptotically;

#### IV. ADAPTATION ALGORITHMS AND STABILITY

##### A. Adaptation algorithms

In order to update  $\Theta(t)$  and  $\theta_\omega(t)$  we choose the adaptation algorithms as

$$\begin{aligned}\dot{\Theta}(t) &= -\Lambda(0) - \Lambda(t) - \Lambda(t - v_1) - \int_0^t \Lambda(s) ds, \\ \Lambda(t) &= \text{sign}[\theta_r^*] \Gamma b_r^T P e_a(t) \Omega(t) \\ \dot{\theta}_\omega(t) &= -\lambda(0) - \lambda(t) - \lambda(t - v_2) - \int_0^t \lambda(s) ds, \\ \lambda(t) &= \gamma b_r^T P e_a(t) \omega(t)\end{aligned}\quad (17)$$

or in differential form

$$\begin{aligned}\dot{\Theta}(t) &= -\Lambda(t) - \dot{\Lambda}(t) - \dot{\Lambda}(t - v_1), \\ \Lambda(t) &= \text{sign}[\theta_r^*] \Gamma b_r^T P e_a(t) \Omega(t) \\ \dot{\theta}_\omega(t) &= -\lambda(t) - \dot{\lambda}(t) - \dot{\lambda}(t - v_2), \\ \lambda(t) &= \gamma b_r^T P e_a(t) \omega(t)\end{aligned}\quad (18)$$

where the matrix  $P \in \mathbb{R}^{n \times n}$  is a constant matrix such that  $P = P^T > 0$  and satisfies

$$PA_r + A_r^T P + Q + Q_e = 0 \quad (19)$$

for any chosen constant matrix  $Q, Q_e \in \mathbb{R}^{n \times n}$  such that  $Q = Q^T > 0, Q_e = Q_e^T > 0$ , with  $\Gamma \in \mathbb{R}^{n \times n}$  a constant matrix such that  $\Gamma = \Gamma^T > 0$ . The time-varying vectors  $\Lambda(t)$  and  $\lambda(t)$  are ‘‘artificial’’. These parameters are used only in the process of the stability proof. The parameters  $\gamma > 0, v_1 > 0$  and  $v_2 > 0$  are design parameters.

*Remark 3:* Although only the integral components  $\Lambda(t)$  and  $\lambda(t)$  ( $\dot{\Lambda}(t) = 0, \dot{\lambda}(t) = 0$  and  $\dot{\Lambda}(t - v_1) = 0, \dot{\lambda}(t - v_2) = 0$  in (18)) of the adaptation algorithm are needed for stability and exact asymptotic tracking, the use of the proportional  $\dot{\Lambda}(t), \dot{\lambda}(t)$  and the proportional delayed  $\dot{\Lambda}(t - v_1), \dot{\lambda}(t - v_2)$  terms in the adaptation algorithm (18) makes it possible to achieve better adaptation performance than with the traditional I and PI schemes [18]. Our adaptation algorithms include the traditional I and PI schemes as special cases. The design parameters  $v_1$  and  $v_2$  are chosen in the same way as the traditional gains  $\Gamma$  and  $\gamma$  in (17), (18).

##### B. Stability proof

To design the update laws for the parameter vectors  $\Theta(t)$  and  $\theta_\omega(t)$  we use the following Lyapunov-Krasovskii type

functional

$$\begin{aligned}V &= V_1 + V_2 + V_3 \\ V_1 &= e_a^T(t) P e_a(t) + \int_{t-\tau}^t e_a^T(s) Q_e e_a(s) ds \\ V_2 &= |\theta_r^*|^{-1} \left( \tilde{\Lambda}^T \Gamma^{-1} \tilde{\Lambda} + \int_{t-v_1}^t \Lambda(s)^T \Gamma^{-1} \Lambda(s) ds \right) \\ V_3 &= \gamma^{-1} \left( \tilde{\lambda}^2 + \int_{t-v_2}^t \lambda^2(s) ds \right)\end{aligned}\quad (20)$$

where

$$\begin{aligned}\tilde{\Lambda}(t) &= \tilde{\Theta}(t) + \Lambda(t) + \Lambda(t - v_1) \\ \tilde{\lambda}(t) &= \tilde{\theta}_\omega(t) + \lambda(t) + \lambda(t - v_2)\end{aligned}\quad (21)$$

Using (19), the time derivative of the components of (20) along the trajectories of the error system (16) is

$$\begin{aligned}\dot{V}_1|_{(16)} &= -e_a^T(t) Q(t) e_a(t) - e_a^T(t - \tau) Q_e e_a(t - \tau) \\ &\quad + 2e_a^T(t) P b_r \theta_r^{*-1} \tilde{\Theta}^T(t) \Omega(t) \\ &\quad + 2e_a^T(t) P b_r \tilde{\theta}_\omega^T(t) \omega(t)\end{aligned}\quad (22)$$

$$\begin{aligned}\dot{V}_2|_{(16)} &= -[\Lambda(t) + \Lambda(t - v_1)]^T \Gamma^{-1} [\Lambda(t) + \Lambda(t - v_1)] \\ &\quad + 2\tilde{\Theta}^T(t) \Gamma^{-1} \dot{\tilde{\Lambda}}(t)\end{aligned}\quad (23)$$

$$\dot{V}_3|_{(16)} = -\gamma^{-1} [\lambda(t) + \lambda(t - v_1)]^2 + \gamma^{-1} \tilde{\theta}_\omega^T(t) \dot{\tilde{\lambda}}(t) \quad (24)$$

In view of (17), (18) and using (22)-(24) we obtain the time derivative for  $V$

$$\begin{aligned}\dot{V}|_{(16)} &= -e_a^T(t) Q(t) e_a(t) - e_a^T(t - \tau) Q_e e_a(t - \tau) \\ &\quad - [\Lambda(t) + \Lambda(t - v_1)]^T \Gamma^{-1} [\Lambda(t) + \Lambda(t - v_1)] \\ &\quad - \gamma^{-1} [\lambda(t) + \lambda(t - v_1)]^2 \\ &\leq -e_a^T(t) Q(t) e_a(t) - e_a^T(t - \tau) Q_e e_a(t - \tau) \leq 0\end{aligned}\quad (25)$$

i.e.  $\dot{V}|_{(16)}$  is (negative) semi-definite. Thus [8], we have proved that the adaptive control (9) and the update algorithms (17), (18) guarantee that  $V(t)$  and, therefore,  $e_a(t), \tilde{\Theta}(t), \Theta(t), \tilde{\theta}_\omega(t), \theta_\omega(t) \in L_\infty$ . From (20) and (25) we establish that  $e_a(t) \in L_2$ . From (16) and closed-loop signals boundedness, we have that  $\dot{e}_a(t) \in L_\infty$  so that  $e_a(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Using (12) and the closed-loop signals boundedness, i.e.

$$\begin{aligned}\lim_{t \rightarrow \infty} \tilde{\Theta}(t) &= \Theta^o - \Theta^* = \tilde{\Theta}^o \\ \lim_{t \rightarrow \infty} \tilde{\theta}_\omega(t) &= \theta_\omega^o - \theta_\omega^* = \tilde{\theta}_\omega^o\end{aligned}\quad (26)$$

where  $\Theta^o$  and  $\theta_\omega^o$  are some constant vector and scalar, we get from (11) the following error equation

$$\dot{e}(t) = A_r e(t) + b \tilde{\Theta}^o \Omega(t) - b \Theta^{oT} (\Omega(t) - \Omega_r(t)) \quad (27)$$

In view of (5) and (10) we have

$$\Theta^{oT} (\Omega(t) - \Omega_r(t)) = \theta_x^{oT} e(t) + \theta_\tau^{oT} e(t - \tau) \quad (28)$$

Then using (3), (26) and (28) we obtain from (27)

$$\begin{aligned}\dot{e}(t) &= A_r e(t) + b [-\theta_x^{*T} e(t) - \theta_\tau^{*T} e(t - \tau) + \theta_g^{*T} g(e)] \\ &\quad + b \xi(t)\end{aligned}\quad (29)$$

where the signal  $\xi(t)$  is

$$\begin{aligned} \xi(t) = & (\theta_x^o - \theta_x^*)^T x_r(t) + (\theta_\tau^o - \theta_\tau^*)^T x_r(t - \tau) \\ & + (\theta_g^o - \theta_g^*)^T g(x_r(t)) + (\theta_d^o - \theta_d^*)^T \phi(t) \\ & + (\theta_r^o - \theta_r^*) r(t - h) \end{aligned} \quad (30)$$

Note that the signal  $\xi(t)$  is bounded due to Assumptions (A3) and (A4), the boundedness of  $r(t)$  and  $x_r(t)$  and the boundedness of  $\tilde{\Theta}^o$  from (26).

In view of Assumptions (A1) and (A3) we have from (29)

$$\dot{e}(t) = Ae(t) + A_\tau e(t - \tau) + f(e(t)) + \xi(t) \quad (31)$$

Then the assumption (A5) and the boundedness of  $\xi(t)$  guarantee, that  $e(t) \in L_\infty$ . From (15) and the fact that  $e_a(t) \in L_\infty$  it also follows that  $x_a(t) \in L_\infty$ , and therefore all the signals in the closed loop system are bounded.

If the signal  $r(t)$  is such that the plant is identifiable [17], then  $\Theta^o = \Theta^*$  and  $\theta_\omega^o = \theta_\omega^*$ . Hence, we obtain from (29)

$$\dot{e}(t) = Ae(t) + A_\tau e(t - \tau) + f(e(t)), \quad (32)$$

and then, in view of (A5), we have that  $e(t) \rightarrow 0$ , when  $t \rightarrow \infty$ .

### C. Main Result

We summarize the main result as

*Theorem 1:* Consider the closed-loop system defined by the plant in (1) and the controller in (9), (5), (12), (14), (15) and (17), (18) with Assumptions (A1)-(A5). Then the following two properties hold:

- (i) all signals of the closed-loop system are bounded;
- (ii)  $\lim_{t \rightarrow \infty} e(t) = 0$  if the reference signal  $r(t)$  is such, that the plant is identifiable.

The general adaptive controller structure is depicted in Figure 1. From the figure it is clear that unlike the standard MRAC structure which is widely used in the adaptive control literature for plants without delay, in our feedback path in Figure 1 the augmented error is used instead of the standard error  $e(t) = x(t) - x_r(t)$ . Moreover, in the controller structure the ‘‘Adjusted auxiliary model’’ and ‘‘Predictor’’ blocks are added. The control generator generates the control signal  $u(t)$  to the plant and the input  $u_a(t)$  to the auxiliary model.

### V. CONCLUDING REMARKS

For a class of *nonlinear* systems with an *external disturbance* and with *both input and state delays* we utilize the new structure in [16] of adaptive input and state delay compensation based only on lumped delay blocks without so-called distributed-delay (DD) blocks. The design procedure is based on the concept of reference trajectories prediction and the formulation of an augmented error. The model output matching between plant and reference model is accomplished by an adaptive compensator based on the delayed and predicted reference signals and some auxiliary model driven by the signal  $u_a(t) = \theta_\omega(t)\omega(t)$ . Note, that the dynamics of the auxiliary model (14) is exactly the same as the reference model dynamics (2). An appropriate Lyapunov-Krasovskii

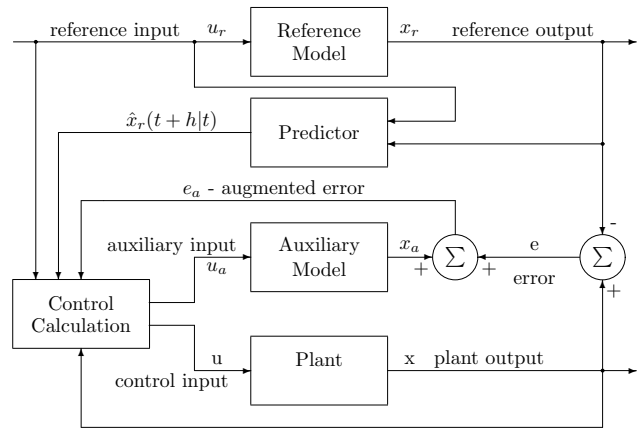


Fig. 1. The general adaptive controller structure

type functional is introduced for updating the controller parameters and for the stability analysis.

Much more difficult than the problem treated in this paper is the case of nonstable plants. The difficulty is mainly the necessary prediction in the closed loop. One of several possible approaches is to split the overall design in two distinct phases. The first phase addresses the so-called ‘‘robust stabilization problem’’, which consists of designing a memoryless state feedback which stabilizes the uncertain system, i.e. the design of a some gain  $\theta^\#$  in order to stabilize (1), see e.g. [15]. In the second ‘‘adaptive control design’’ phase, only the fact that  $\theta^\#$  exists is required, see Section 2-4; the exact value of  $\theta^\#$  is not necessary to know.

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