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Left-inversion of nonlinear fading memory systems from data

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***Abstract*— A method for the left-inversion of nonlinear fading memory systems from data is proposed. The method is based on the identification of a model of the system to invert, and the computation of the left-inverse directly from this model. It is not required to identify an inverse system. Such an identification is in general more difficult than the identification of the "direct" system. The invertibility of the regression function defining the system is also not required. The inversion error, defined as the difference between the desired output and the actual system output, is shown to be bounded by the identification error, measured by the L_∞ norm of the difference between the system and the model. The Nonlinear Set Membership identification approach is used for the identification of the model. This approach provides models with minimal identification error. A simulation example on the inversion of a nonlinear dynamic semi-active suspension shows the effectiveness of the method.**

I. Introduction

Consider a nonlinear discrete-time dynamic system described in regression form:

$$y^t = f_o\left(\mathbf{y}^{t-1}, \mathbf{u}^t\right), \quad t \in \mathbb{Z} \tag{1}$$

$$\begin{array}{l}\mathbf{y}^{t-1} = [y^{t-1}; ...; y^{t-n_y}] \\ \mathbf{u}^t = [u^t; ...; u^{t-n_u}]\end{array}$$

where $u^t, y^t \in \mathbb{R}$, $f_o : W \subset \mathbb{R}^n \to \mathbb{R}$, $n = n_y + n_u$. The domain W of f_o is a compact convex set. The function f_o is differentiable. The notation $[...; ...; ...]$ is used to indicate vertical concatenation, the notation $[..., ..., ...]$ is used to indicate horizontal concatenation.

Suppose that the function f_o is not known, but a set of noise-corrupted measurements of y^t and u^t is available.

The problem considered in this paper is to find a *left-inverse* of system (1). A left-inverse of (1) is a system with input y^t and output u^t. Suppose that a solution y^t_{des} of (1) is used as the input of the left-inverse system. The output of the left-inverse is then a signal $\widehat{u}^t$ that, used as input in (1), yields $\widehat{y}^t = y^t_{des}$, $\forall t$, where $\widehat{y}^t = f_o\left(\widehat{\mathbf{y}}^{t-1}, \widehat{\mathbf{u}}^t\right)$. Note that, since f_o is unknown and the measurements are noise-corrupted, only an approximate left-inverse can be obtained.

Solving this problem is useful in several applications. A typical application is open-loop tracking: A certain trajectory has to be tracked by a system and an input sequence yielding this trajectory has to be found. Another application is block-oriented system identification: A system composed of several subsystems has to be identified, but some of the signals exchanged between the subsystems are not measured. An iterative identification algorithm can be used and system inversion can be performed at each iteration to estimate the unknown signals. A third application is internal model control (IMC) design for nonlinear systems: this design technique requires an (approximate) inverse of the system to control. A fourth application, that we consider in Section V, is actuator inversion: A control law is usually applied to a plant through an actuator, which in general can be dynamic and nonlinear. The actuator inversion can be performed to obtain an actuator output signal (nearly) equal to the desired control law.

In general, inversion problems play a significant role in the field of automatic controls. In the case of linear systems, these problems can be analyzed and, when possible, solved in a systematic way [21]. In the case of nonlinear systems, these problems are considerably more difficult. The difficulties may derive, for instance, from the non invertibility of the function f_o or from the complex nonlinear dynamics which may characterize a system of the form (1).

The problem of inversion of nonlinear systems has been addressed in the literature both from the theoretical point of view, see e.g. [17], [3], [4], [9], [22], and in several applications, see e.g. [7], [11], [20], [25]. However, the system to invert is usually assumed known, and very few works consider the case where it is unknown and has to be identified from the data [8], [1].

In this paper, a method for left-inversion of unknown nonlinear systems from data is proposed. In order to ensure the boundedness of the inversion error $|y^t_{des} - \widehat{y}^t|$ in open-loop, a fading memory assumption is made. The method is based on the identification of a model $\widehat{f}$ of f_o, and on the computation of the left-inverse directly from $\widehat{f}$. It is not required to identify an inverse system. In general, such an identification is more difficult than the identification of the "direct" system, because an inverse system may be not differentiable, and even not continuous. The invertibility of f_o is also not required. The inversion error is shown to be bounded by the identification error, measured by the L_∞ norm of $f_o - \widehat{f}$. The Nonlinear Set Membership (NSM) approach [13] is used for the identification of $\widehat{f}$. This approach allows to derive models with minimal identification error.

A simulation example regarding the inversion of a nonlinear dynamic semi-active suspension is introduced to show the effectiveness of the proposed approach.

II. Problem formulation

Let us consider $t \in [1, \infty]$. The regression system (1) is a nonlinear operator $\mathbf{f}_o$ mapping the initial condition $\mathbf{y}^0 \in \mathbb{R}^{n_y}$, and the input sequence $u = [u^1; u^2; ...] \in \mathbb{R}^\infty$ into an output sequence $y = [y^1; y^2; ...] \in \mathbb{R}^\infty$. The operator $\mathbf{f}_o$ is defined as

$$y = \mathbf{f}_o\left(\mathbf{y}^0, u\right) = \left[\mathbf{f}^1_o\left(\mathbf{y}^0, u\right); \mathbf{f}^2_o\left(\mathbf{y}^0, u\right); ...\right] \tag{2}$$

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where $\mathbf{f}_o : \mathbb{R}^\infty \to \mathbb{R}^\infty$, $\mathbf{f}_o^t : \mathbb{R}^{n_y+t} \to \mathbb{R}$. The output at a time τ is given by

$$y^\tau = \mathbf{f}_o^\tau\left(\mathbf{y}^0, u\right) = \mathbf{f}_o^\tau\left(\mathbf{y}^0, [u^1; ...; u^\tau]\right)$$

where $\mathbf{f}_o^\tau\left(\mathbf{y}^0, u\right)$ is computed by iterating τ times the difference equation (1), starting from the initial condition $\mathbf{y}^0$. The notation $y = \mathbf{f}_o(u)$ will be used when not necessary to explicit the dependence on initial conditions.

The sequence y is called *solution* of system $\mathbf{f}_o$ corresponding to the initial condition $\mathbf{y}^0$ and input sequence u. A generic sequence z is a solution of system $\mathbf{f}_o$ if an initial condition $\mathbf{y}^0$ and an input sequence u exist such that $z = \mathbf{f}_o\left(\mathbf{y}^0, u\right)$.

Suppose that the function f_o in (1) is not known, but a set of noise-corrupted measurements of y^t and u^t is available.

Problem 1: Find an approximate *left-inverse* of system $\mathbf{f}_o$, i.e. find a system

$$u = \widehat{\mathbf{f}}_{inv}\left(\mathbf{u}^0, y\right) \tag{3}$$

and an initial condition $\mathbf{u}^0$ for which a $N < \infty$ exists such that the *inversion error*

$$\begin{aligned} &|y_{des}^t - \widehat{y}^t|, \quad t \geq N \\ &\widehat{y} = \mathbf{f}_o\left(\widehat{\mathbf{y}}^0, \widehat{\mathbf{f}}_{inv}(y_{des})\right) \end{aligned} \tag{4}$$

is "small", for any initial condition $\widehat{\mathbf{y}}^0$ and any solution $y_{des} = [y_{des}^1; y_{des}^2; ...]$ of the system $\mathbf{f}_o$. ■

III. Inversion algorithm

Let us consider a function

$$z = f(\mathbf{x}, u)$$

where $z \in \mathbb{R}$, $\mathbf{x} \in X \subseteq \mathbb{R}^{n_x}$, $u \in \mathbb{R}$, $f : \mathbb{R}^n \to \mathbb{R}$, $n = n_x + 1$.

Definition 1: A function $f_{inv} : \mathbb{R}^n \to \mathbb{R}$ is a left-inverse of f with respect to u if, for any fixed $\overline{\mathbf{x}} \in X$ and for any z_{des} in the codomain of the function $f(\overline{\mathbf{x}}, \cdot) : \mathbb{R} \to \mathbb{R}$, the following equality holds:

$$z_{des} = f\left(\overline{\mathbf{x}}, f_{inv}(\overline{\mathbf{x}}, z_{des})\right). \tag{5}$$

■

Let us now define the function

$$f^{-1}(\mathbf{x}, z) \doteq \max(\mathcal{U}) \tag{6}$$

$$\mathcal{U} = \arg\min_{\mathrm{u} \in \mathbb{R}} |z - f(\mathbf{x}, \mathrm{u})|. \tag{7}$$

This function is implicitly defined by means of the optimization problem (7). Since (7) is an optimization problem in $\mathbb{R}$, it can be easily solved using any scalar optimization technique. Note that the set $\mathcal{U}$ of the minimizers of $|z - f(\mathbf{x}, \mathrm{u})|$ may be composed of several elements. The $\max(\mathcal{U})$ is performed to select a unique value, so that the function f^{-1} is properly defined.

For any $\overline{\mathbf{x}} \in X$ and any z_{des} in the codomain of $f(\overline{\mathbf{x}}, \cdot)$, we have $\min_{\mathrm{u}} |z_{des} - f(\overline{\mathbf{x}}, \mathrm{u})| = 0$ and then $f\left(\overline{\mathbf{x}}, f^{-1}(\overline{\mathbf{x}}, z_{des})\right) = z_{des}$. The function f^{-1} is thus a left-inverse of f with respect to u. If z_{des} is not in the codomain of $f(\overline{\mathbf{x}}, \cdot)$, $f\left(\overline{\mathbf{x}}, f^{-1}(\overline{\mathbf{x}}, z_{des})\right)$ is nevertheless the best approximation of z_{des}. Note that the invertibility of f is not required.

Let us now suppose that the function f is not know and it is of interest to find a left-inverse of it. Suppose that an approximation $\widehat{f}$ of f is available. A possible approach is to compute the left-inverse $\widehat{f}^{-1}$ of $\widehat{f}$ by means of (6) and to use $\widehat{f}^{-1}$ as approximate inverse of f. For any $\overline{\mathbf{x}} \in X$ and any z_{des} in the codomain of $f(\overline{\mathbf{x}}, \cdot)$, we have

$$z_{des} = \widehat{f}\left(\overline{\mathbf{x}}, \widehat{f}^{-1}(\overline{\mathbf{x}}, z_{des})\right), \ \widehat{z} = f\left(\overline{\mathbf{x}}, \widehat{f}^{-1}(\overline{\mathbf{x}}, z_{des})\right).$$

Clearly, $\widehat{z} \neq z_{des}$, since $f \neq \widehat{f}$, but the inversion error is bounded as

$$|z_{des} - \widehat{z}| \leq \left\| f - \widehat{f} \right\|_\infty$$

where $\|f\|_\infty \doteq \text{ess-sup}_{w \in W} |f(w)|$ is the standard L_∞ norm. This inequality shows that the accuracy of the inversion depends on the accuracy of the "direct" approximation $\widehat{f}$.

Up to now, static nonlinear functions have been considered, and the left-inversion problem has been shown quite simple to solve. More difficult is to solve the inversion Problem 1, where dynamic nonlinear systems are considered. We propose the following algorithm for its solution.

Suppose that the function f_o in (1) is not known, but a set of noise-corrupted measurements $(\widetilde{y}^t, \widetilde{u}^t)$ of (y^t, u^t), $t = -T+1, -T+2, ..., 0$, is available.

Algorithm

1. From the available data $(\widetilde{y}^t, \widetilde{u}^t)$, $t = -T+1, -T+2, ..., 0$, identify a model of the system (1) of the form

$$y^t = \widehat{f}\left(\mathbf{y}^{t-1}, \mathbf{u}^t\right)$$

where $\mathbf{y}^{t-1} = [y^{t-1}; ...; y^{t-n_y}]$, $\mathbf{u}^t = [u^t; ...; u^{t-n_u}]$ and $\widehat{f}$ is a continuous function approximating f_o. This model is represented by the nonlinear operator $\widehat{\mathbf{f}}$, defined according to (2).

2. Compute the left-inverse $\widehat{\mathbf{f}}^{-1}$ of the model $\widehat{\mathbf{f}}$ by means of the following regression equation:

$$u^t = \widehat{f}^{-1}\left(\mathbf{u}^{t-1}, \mathbf{y}^t\right), \quad t > 0 \tag{8}$$

where $\mathbf{y}^t = [y^t; ...; y^{t-n_y}]$, $\mathbf{u}^{t-1} = [u^{t-1}; ...; u^{t-n_u}]$, $\mathbf{u}^0 = [0; ...; 0]$, and $\widehat{f}^{-1}$ is the left-inverse of $\widehat{f}$ w.r.t. u^t, defined according to (6):

$$\widehat{f}^{-1}\left(\mathbf{u}^{t-1}, \mathbf{y}^t\right) \doteq \max(\mathcal{U}) \tag{9}$$

$$\mathcal{U} = \arg\min_{\mathrm{u} \in \mathbb{R}} \left| y^t - \widehat{f}\left(\mathbf{y}^{t-1}, [\mathrm{u}; u^{t-1}; ...; u^{t-n_u}]\right) \right|. \tag{10}$$

3. Use $\widehat{\mathbf{f}}^{-1}$ as left-inverse of $\mathbf{f}_o$.

■

In order to analyze the properties of this inversion algorithm, let us recall the notion of fading memory system (see e.g. [2]).

Definition 2: A system $\mathbf{f}$ has fading memory if, for any $\epsilon > 0$, a $N > 0$ exists such that, for every $k \geq 0$, every $t \geq N$, every initial conditions $\mathbf{y}^0, \widehat{\mathbf{y}}^0$, and every sequences $q = [q^1; q^2; ...; q^k]$, $\widehat{q} = [\widehat{q}^1; \widehat{q}^2; ...; \widehat{q}^k]$, $u = [u^1; u^2; ...; u^t]$,

$$\left| \mathbf{f}^{k+t}\left(\mathbf{y}^0, [q; u]\right) - \mathbf{f}^{k+t}\left(\widehat{\mathbf{y}}^0, [\widehat{q}; u]\right) \right| < \epsilon.$$

■

We are now in the position of presenting the main result of this paper. Let y_{des} be a solution of both systems $\mathbf{f}_o$ and $\widehat{\mathbf{f}}$ and let

$$\widehat{y} = \mathbf{f}_o\left(\widehat{\mathbf{y}}^0, \widehat{\mathbf{f}}^{-1}\left(y_{des}\right)\right)$$

where $\widehat{\mathbf{y}}^0$ is a generic initial condition.

Theorem 1: Assume that $\mathbf{f}_o$ and $\widehat{\mathbf{f}}^{-1}$ have fading memory. Then, for any $\epsilon > 0$, there exist $N, K < \infty$ such that

$$\left|y_{des}^t - \widehat{y}^t\right| \le \epsilon + K\left\|f_o - \widehat{f}\right\|_\infty, \quad \forall t \ge N$$

for any initial condition $\widehat{\mathbf{y}}^0$ and any sequence $y_{des} = [y_{des}^1; y_{des}^2; ...]$ that is solution of both the systems $\mathbf{f}_o$ and $\widehat{\mathbf{f}}$.

Proof. See [16]. ■

Remarks

1) The proposed left-inversion algorithm does not require the invertibility of the function f_o. According to the optimization problem (6), if z is in the codomain of $f(\mathbf{x}, \cdot)$, it is sufficient to find a $\mathfrak{u}$ such that $f(\mathbf{x}, \mathfrak{u}) = z$ to compute the left-inverse. On the other hand, if f_o is invertible, the set $\mathcal{U}$ in (10) is composed of a unique element. This implies that the sequence $\widehat{u}^t = \widehat{f}^{-1}\left(\widehat{\mathbf{u}}^{t-1}, \mathbf{y}_{des}^t\right)$ is univocally determined. Therefore, in the case that $\widehat{\mathbf{f}} = \mathbf{f}_o$, the algorithm provides the (left-right-) inverse of the system (1): for any solution y_{des} of $\mathbf{f}_o$, $\widehat{u}$ is the unique sequence such that $y_{des} = \mathbf{f}_o(\mathbf{f}_o^{-1}(y_{des}))$ and $\widehat{u} = \mathbf{f}_o^{-1}(\mathbf{f}_o(\widehat{u}))$. In the case that $\widehat{\mathbf{f}} \simeq \mathbf{f}_o$, we have $y_{des} \simeq \mathbf{f}_o(\widehat{\mathbf{f}}^{-1}(y_{des}))$ and $\widehat{u} \simeq \widehat{\mathbf{f}}^{-1}(\mathbf{f}_o(\widehat{u}))$, where the inversion errors $|y_{des}^t - \mathbf{f}_o^t(\widehat{\mathbf{f}}^{-1}(y_{des}))|$ and $|\widehat{u}^t - \widehat{\mathbf{f}}^{-1t}(\mathbf{f}_o(\widehat{u}))|$ are bounded.

2) While the regression function f_o of the system (1) is differentiable, a left-inverse of this function may be not differentiable and even not continuous. It is well known that approximating a discontinuous function is in general more difficult than approximating a continuous one. The system inversion approach proposed here, overcomes this problem, since it does not require to identify an inverse system.

3) Consider a system of the form

$$y^t = f_o\left(\mathbf{y}^{t-1}, \mathbf{v}^t, \mathbf{u}^t\right)$$

where $\mathbf{v}^t = [v^t; ...; v^{t-n_v}]$, $v^t \in \mathbb{R}^{m_v}$, is an additional input. The inversion method proposed in this section can be used to invert this system w.r.t. the input $\mathbf{u}^t$ with no significant modifications.

IV. Computation of $\widehat{f}$ with Nonlinear Set Membership identification

Theorem 1 gives a bound on the inversion error $\left|y_{des}^t - \widehat{y}^t\right|$ which depends on the identification error $\left\|f_o - \widehat{f}\right\|_\infty$. Therefore, to obtain a small inversion error, it is important to have a small identification error. In this section, we summarize the Nonlinear Set Membership (NSM) method [13], which allows the identification of models with minimal identification error.

Let $\widetilde{w}^t \doteq [\widetilde{\mathbf{y}}^{t-1}; \widetilde{\mathbf{u}}^t]$ and $\mathfrak{T} \doteq \{-T+1, -T+2, ..., 0\}$. Consider that a set of noise corrupted data $\widetilde{Y}^T = \{\widetilde{y}^t,\ t \in \mathfrak{T}\}$, $\widetilde{W}^T = \{\widetilde{w}^t,\ t \in \mathfrak{T}\}$ generated by (1) is available. Then:

$$\widetilde{y}^t = f_o(\widetilde{w}^t) + d^t,\ t \in \mathfrak{T} \tag{11}$$

where the term d^t accounts for the fact y^t and w^t are not exactly known.

The aim is to derive an estimate $\widehat{f}$ of f_o from available measurements $(\widetilde{Y}^T, \widetilde{W}^T)$.

An identification algorithm ϕ is an operator mapping the available data $(\widetilde{Y}^T, \widetilde{W}^T)$ into an estimate $\widehat{f}$ of f_o: $\phi(\widetilde{Y}^T, \widetilde{W}^T) = \widehat{f} \simeq f_o$. The algorithm ϕ should be chosen to give small (possibly minimal) L_p error $||f_o - \widehat{f}||_p$, where:

$$||f||_p \doteq \left[\int_W |f(w)|^p dw\right]^{\frac{1}{p}},\ p \in [1, \infty)$$
$$||f||_\infty \doteq \text{ess-sup}_{w \in W} |f(w)|,$$

and W is a bounded convex set in $\mathbb{R}^n$.

Whatever algorithm ϕ is chosen, no information on the identification error can be derived, unless some assumptions are made on the function f_o and the noise d. The typical approach in the literature is to assume a finitely parametrized functional form for f_o (linear, bilinear, neural network, etc.) and statistical models for the noise [5], [12], [15], [10]. In the NSM approach, different and somewhat weaker assumptions are taken, not requiring the selection of a parametric form for f_o, but related to its derivatives. Moreover, the noise sequence $\{d^t, t = 1, .., T\}$ is supposed bounded.

Prior assumptions on f_o:

$$f_o \in K \doteq \left\{f \in C^1(W) : \|f'(w)\| \le \gamma,\ \forall w \in W\right\}.$$

Prior assumptions on noise: $|d^t| \le \varepsilon,\ t \in \mathfrak{T}$. ■

Here, $f'(w)$ denotes the gradient of $f(w)$ and $\|x\| \doteq \sqrt{\sum_{i=1}^n x_i^2}$ is the Euclidean norm.

As typical in any estimation theory, the problem of checking the validity of prior assumptions arises. This problem is considered in [13], where a validation analysis is provided, which also allows to properly choose the values of the bounds γ and ε.

A key role in this Set Membership framework is played by the Feasible Systems Set, often called "unfalsified systems set", i.e. the set of all systems consistent with prior information and measured data.

Definition 3: Feasible Systems Set:

$$FSS^T \doteq \{f \in K\text{: } \left|\widetilde{y}^t - f\left(\widetilde{w}^t\right)\right| \le \varepsilon^t, t \in \mathfrak{T}\}. \tag{12}$$

■

The Feasible Systems Set FSS^T summarizes all the information on the mechanism generating the data that is available up to time T. If prior assumptions are "true", then $f_o \in FSS^T$, an important property for evaluating the accuracy of identification.

Using the notion of Feasible Systems Set, we can define an identification algorithm ϕ as an operator mapping all available information about function f_o, noise d, data $(\widetilde{Y}^T, \widetilde{W}^T)$ until time T, summarized by FSS^T, into an estimate $\widehat{f}$ of f_o:

$$\phi\left(FSS^T\right) = \widehat{f} \simeq f_o.$$

For given estimate $\phi\left(FSS^T\right) = \widehat{f}$, the related L_p error $\left\|f_o - \widehat{f}\right\|_p$ cannot be exactly computed, but its tightest bound is given by $\left\|f_o - \widehat{f}\right\|_p \le \sup_{f \in FSS^T}\left\|f - \widehat{f}\right\|_p$. This motivates the following definition of worst-case identification error.

Definition 4: The worst-case identification error of the estimate $\widehat{f} = \phi\left(FSS^T\right)$ *is:*

$$E\left[\phi\left(FSS^T\right)\right] = E(\widehat{f}) \doteq \sup_{f \in FSS^T} \left\| f - \widehat{f} \right\|_p .$$

■

Looking for algorithms that minimize the worst-case identification error, leads to the following optimality concepts.

Definition 5: An algorithm ϕ^* *is optimal if:*

$$E\left[\phi^*\left(FSS^T\right)\right] = \inf_{\phi} E\left[\phi\left(FSS^T\right)\right] = r_I.$$

■

The quantity r_I, called *radius of information*, gives the minimal worst-case identification error that can be guaranteed by any estimate based on the available information up to time T.

Define the functions:

$$\begin{aligned} \overline{f}\left(w\right) &\doteq \min_{t \in \mathfrak{T}} \left(\overline{h}^t + \gamma \left\| w - \widetilde{w}^t \right\|\right) \\ \underline{f}\left(w\right) &\doteq \max_{t \in \mathfrak{T}} \left(\underline{h}^t - \gamma \left\| w - \widetilde{w}^t \right\|\right) \end{aligned} \quad (13)$$

where $\overline{h}^t \doteq \widetilde{y}^t + \varepsilon$, $\underline{h}^t \doteq \widetilde{y}^t - \varepsilon$. The next result shows that the algorithm:

$$\phi_c(FSS^T) = f_c \doteq \frac{1}{2}\left(\underline{f} + \overline{f}\right)$$

is optimal for any L_p norm.

Theorem 2: [13] For any $L_p(W)$ norm, with $p \in [1, \infty]$:
i) The identification algorithm $\phi_c\left(FSS^T\right) = f_c$ is optimal.
ii) $E\left(f_c\right) = \frac{1}{2}\left\|\overline{f} - \underline{f}\right\|_p = r_I = \inf_\phi E\left[\phi\left(FSS^T\right)\right]$.

■

Note that the NSM method can be used either alone or together with any other identification method. Let us suppose that an estimate f_a has been obtained using any desired technique. The NSM method can be applied to identify the residue function $f_\Delta\left(w\right) \doteq f_o\left(w\right) - f_a\left(w\right)$ using the set of values $\Delta y^t = \widetilde{y}^t - f_a\left(\widetilde{w}^t\right)$, $t \in \mathfrak{T}$. Theorem 2 implies that the estimate $f_L\left(w\right) = f_a\left(w\right) + f_c\left(w\right)$, where $f_c\left(w\right)$ is the optimal estimate of $f_\Delta\left(w\right)$, is optimal.

V. APPLICATION TO SEMI-ACTIVE DAMPERS CONTROL

A. Resume of semi-active dampers modeling and control

In this Section, the use of the proposed inversion procedure is shown in the problem of the input computation in controlled semi-active suspension systems (see Fig. 1 for a schematic and a description of a quarter car semi-active suspension).

Such systems, are based on suitable dampers which allow the tuning of the suspension force F^t according to some control requirements through the adjustment of the damping characteristics of the device within its physical limitations. In order to vary the value of F^t several technologies can be employed which range from the case of hydraulic devices where a fluid flows through valves whose opening can be controlled by means of a current to the Magneto-Rheological (MR) dampers where the damping characteristics are modulated through the variation of a magnetic field generated by

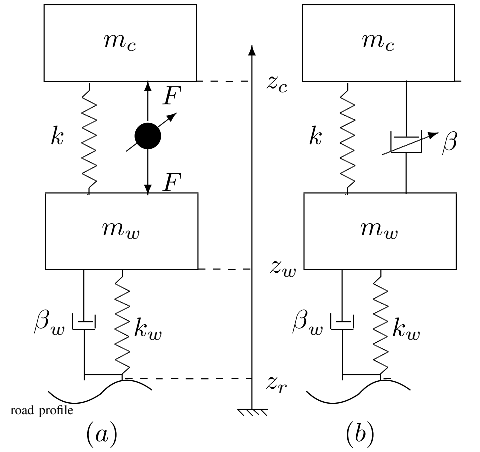

Fig. 1. (a) Quarter car suspension schematic. (b) Quarter car semiactive suspension schematic. z^c sprung mass (1/4 of chassis and load mass) vertical position, z^w unsprung mass (wheel) vertical position z^r road profile

a suitable current. The relationships between the suspension force F^t and the relative speed $v^t_{wc} = \dot{z}^t_w - \dot{z}^t_c$ and the input current i^t of the considered device are described by a "damper map"

$$F^t = f_D(v^t_{wc}, i^t) \quad (14)$$

as reported in Fig. 2 which is obtained by the damper manufacturer by means of static tests and it is usually provided to the costumer. In such a map each curve represents the force behavior for a given constant value of the driving current (for simplicity in Fig. 2 only the minimum and maximum current curves are reported).

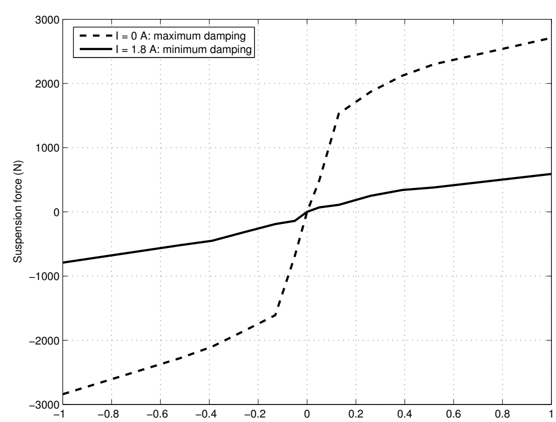

Fig. 2. Employed damper map. Dashed line: maximum damping characteristic obtained for a driving current of 0 A. Solid line: minimum damping characteristic obtained for a driving current of 1.8 A.

The region included into the two curves in Fig. 2 represents the set of the all the forces that the damper is able to provide and it is referred to as the "passivity constraints". A common strategy to compute the required amount of the force F^t is the Sky-Hook approach (see e.g. [23], [24]):

$$F^t = \beta_{\text{rel}}(\dot{z}^t_w - \dot{z}^t_c) - \beta_{\text{sky}}\dot{z}^t_c \quad (15)$$

where the parameters β_{rel} and β_{sky} are suitably chosen to meet the control requirements. However, since not every value of the force F^t computed in (15) can be realized, a clipping action (see e.g. [24]) is needed to satisfy the passivity constraints.

In a real suspension, the control algorithm has to provide the value of the input current i^t which realizes the required clipped force. To this end, a typical procedure consists in computing i^t by static inversion (w.r.t. i) of the damper map (14) (see e.g. [6]):

$$i^t = f_D^{-1}(v_{wc}^t, F^t). \tag{16}$$

However, as discussed in [14] and [19]), the use of the map (14) provides a poor description of the damper physical behavior and a more accurate (dynamical) model is needed. In particular, a model of the form

$$y^t = f\left(\mathbf{y}^{t-1}, \mathbf{v}^t, \mathbf{u}^t\right) \tag{17}$$

$$\begin{aligned}\mathbf{y}^{t-1} &= [y^{t-1}; ...; y^{t-n_y}]\\ \mathbf{u}^t &= [u^t; ...; u^{t-n_u}]\\ \mathbf{v}^t &= [v^t; ...; v^{t-n_v}]\end{aligned}$$

where $y^t = F^t$, $u^t = i^t$, $v^t = [v_{wc}^t; z_w^t - z_c^t]$ can be suitably employed to describe the damper dynamics. Models of the form (17) can be obtained from data collected on a test bench in both the cases the damper is mounted or not on the vehicle as described in [14] and [19]. Now, the inversion procedure defined in Section III performed on an identified model of this type can be used to obtain the damper current needed to obtain the force required by the employed control algorithm (e.g. (15)).

B. Simulation results

In order to show the effectiveness of the proposed inversion procedure, the following simulation setting has been considered. In the quarter car model of Fig. 1 described by the physical parameters: $m_c = 432.82$ kg, $m_w = 40$ kg, $k = 20000$ N/m, $k_w = 200000$ N/m, $\beta_w = 0$ Ns/m, the semi-active damper force F^t has been computed according to a clipped Sky-Hook strategy originated from (15) and with the value of the design parameters $\beta_{\text{rel}} = 3000$ Ns/m and $\beta_{\text{sky}} = 1550$ Ns/m which are supposed to be given. The clipping of the force has been performed supposing that the employed damper is represented by the map depicted in Fig. 2. The results obtained by the direct application of such a force define the ideal level of performance to be reached by the controlled suspension. Subsequently, in the considered quarter car model, the model described in [14] has been employed to simulate the damper device. Such a model requires the current i and the relative position $z_w^t - z_c^t$ and speed v_{wc}^t as inputs and produces the force F as output. In particular, the regression structure (17) has been employed, with $n_y = n_u = n_v = 2$. It has to be highlighted that, as reported in [14], such a model has been identified from real data collected on a vehicle equipped with a commercial damper characterized by the map reported in Fig. 2.

At this point, two different kind of simulation tests have been considered:

1) the damper input current is computed according to a straightforward static inversion of the map (14) (IMAP)
2) the damper input current is computed according to the inversion procedure described in Section III, where the approximation $\widehat{f}$ is obtained by means of the NSM identification method of Section IV (INSM)

The simulations have been carried out using "benchmark" road profiles employed in standard industrial tests (see [14]). In particular, the following road profiles have been taken into account:
- English Track: road with irregularly spaced holes and bumps, maximum amplitude of 0.025 m and run at 60 km/h.
- Short Back: impulse road profile, with amplitude of 0.015 m, width of 0.5 m and run at 30 km/h.
- Drain Well: negative impulse road, with amplitude of 0.05 m, width of 0.5 m and run at 30 km/h.

In this way, the controlled suspensions behavior is tested in different driving and road regularity conditions. The simulations were performed using a sampling time $T = 1/512$ s and a simulation time of about 14 s for each profile type. The accuracy properties of the inversion procedures IMAP and INSM are evaluated using the following RMS errors:

$$E_{\text{IMAP}} = \frac{1}{\sqrt{S}} \sum_{k=0}^{S-1} (\overline{F}^k - F_{\text{IMAP}}^k)^2$$
$$E_{\text{INSM}} = \frac{1}{\sqrt{S}} \sum_{k=0}^{S-1} (\overline{F}^k - F_{\text{INSM}}^k)^2$$

where S is the considered number of samples, $\overline{F}$ is the ideal force computed according to the considered clipped Sky-Hook strategy while F_{IMAP} and F_{INSM} represent the suspension forces obtained using the IMAP and INSM inversion procedures respectively. To better highlight the differences between the IMAP and the INSM approaches the corresponding suspension comfort performance in terms of the RMS value of the sprung mass acceleration:

$$\ddot{z}_{c,\text{RMS}} = \frac{1}{\sqrt{S}} \sum_{k=0}^{S-1} (\ddot{z}_c^k)^2$$

have been evaluated too. In Table I the obtained results in term of inversion accuracy and suspension performance are reported.

TABLE I
ACCURACY AND COMFORT PERFORMANCE RESULTS.

Road Profile	IMAP		INSM		$\overline{F}$
	E_{IMAP}	$\ddot{z}_{c,\text{RMS}}$	E_{INSM}	$\ddot{z}_{c,\text{RMS}}$	$\ddot{z}_{c,\text{RMS}}$
English Track	150.3	1.270	82.7	1.162	1.154
Short Back	50.3	0.399	29.1	0.362	0.348
Drain Well	73.7	0.570	37.1	0.515	0.513

The data in Table I show the improvements on the required force computation obtained using the INSM procedure. The immediate consequence is a significant enhancement of the suspension comfort characteristics in all the considered road profiles. In order to evaluate more directly such results, in Figure 3 the courses of the sprung mass acceleration and on the computed forces are reported for the case of the drain well. Observing Figure 3, it can be noted that the inaccuracies in the force computation in the IMAP case cause a worsening also on the bounce performance of the suspension since the acceleration peaks in the opposite direction of the hole (i.e. negative for short back) are greater than the ones obtained with INSM which are quite close to the case of the direct application of the desired force.

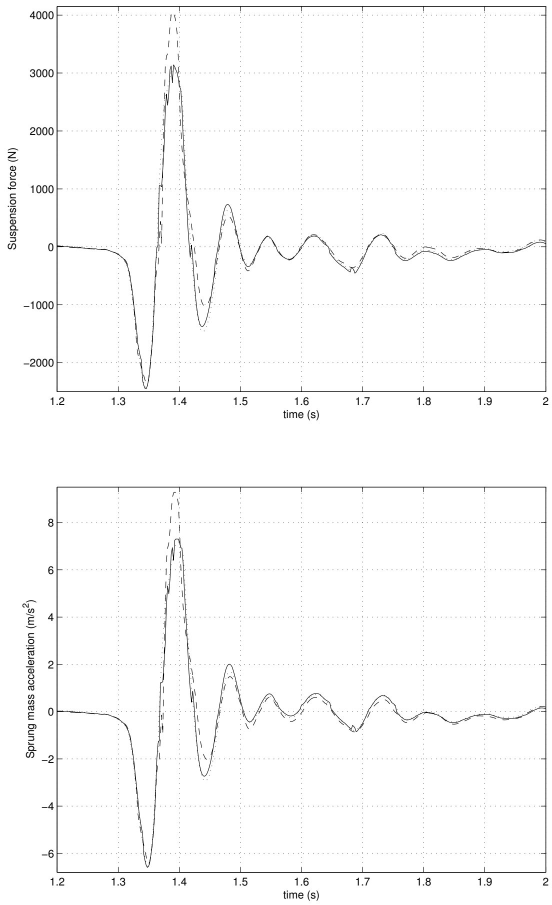

Fig. 3. Drain Well profile. Above: Desired suspension forces (dotted) and IMAP (dashed) and INSM (solid) approximations. Below: Comparison between the desired sprung mass acceleration (dotted) and the ones obtained with limitations (dotted) IMAP (dashed) and INSM (solid) procedures.

VI. Conclusions

In this paper a new approach for obtaining an approximate inverse of fading memory dynamical systems has been introduced. In particular, it has been shown that under some mild assumptions on the considered system, the inversion error is bounded. An example related to the inversion of a semi-active damper has been introduced to show the effectiveness of the proposed approach.

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