Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Decentralized Adaptive Approximation Based Control With Safety Scheme Outside the Approximation Region

Panagiotis Panagi and Marios M. Polycarpou

Abstract— **This paper presents a decentralized adaptive approximation based control scheme for a class of interconnected nonlinear systems. The feedback control law consists of two schemes, an adaptive approximation controller operating inside a chosen approximation region and a decentralized safety scheme for outside the approximation region. Within the approximation region, linearly parameterized neural networks with a dead-zone modification are used to adaptively approximate the unknown dynamics of each subsystem, as well as the unknown interconnections. Outside the approximation region, the decentralized safety control scheme is designed to steer back the trajectory by using an adaptive bounding approach. A rigorous stability analysis is presented and a simple simulation example is used to illustrate the decentralized adaptive control methodology.**

I. INTRODUCTION

Decentralized control of large-scale interconnected systems has attracted significant attention during the last two decades. The problem of decentralized adaptive linear control was introduced by Ioannou [5], where weakly interconnected subsystems with relative degree one or two were studied. In [4] and [9] it was shown that stability of the decentralized system is ensured if there exists a positive definite M-matrix, which is related to the bound of the interconnections. Most of these approaches were focused on linear subsystems with possibly nonlinear interconnections. An alternative decentralized adaptive control method using the high gain approach was developed in [2], where a standard strict matching condition is assumed on the disturbances. A methodology for handling higher-order interconnections in a decentralized adaptive control framework was developed in [11].

One of the key challenges in decentralized control is the issue of dealing with uncertainty, both in the nonlinearities of the local subsystems as well as in the interconnections. A recent approach for dealing with uncertainty is based on the use of neural networks to approximate the unknown interconnections. In [13], [12], the authors developed a decentralized control design scheme for systems with interconnections that are bounded by first-order polynomials. In [3], the authors employ a composite Lyapunov function for handling both unknown nonlinear model dynamics and interconnections. The interconnections are assumed to be bounded by unknown smooth functions, which are indirectly approximated by neural networks. In [8], [7] and [6] it is assumed that the decentralized controllers share prior information about their reference models. Based on this assumption, it is then shown that the subsystems are able to asymptotically track their desired outputs.

In this paper, we consider a system composed of nonlinear subsystems coupled by unknown nonlinear interconnections. We develop a decentralized adaptive approximation based control system [1] and derive stability results for the closed-loop system under certain assumptions. We consider both the case where the trajectory is inside the approximation region as well as the case where the trajectory leaves the approximation region. In the latter case, we develop a decentralized safety control scheme based on the sliding mode control approach with adaptive bounding. The presented adaptive approximation based control scheme follows the general approach for decentralized systems developed in [12], [3] and [10]. The main contribution of this work is the synthesis and analysis of the dead-zone modification and the design of a stable decentralized safety control scheme for addressing the problem of the trajectory exiting the approximation region during the transient stage.

The paper is organized as follows. In Section II, we design a decentralized feedback control law with a dead-zone modification in the adaptive laws to account for the residual approximation errors. Section III presents a decentralized safety control scheme for the case where the trajectories go outside the approximation region, while in Section IV a simulation example is used to illustrate the overall control methodology. Finally, Section V contains some concluding remarks.

II. DECENTRALIZED ADAPTIVE CONTROL

We consider a system comprised of n interconnected subsystems. The i-th subsystem, where $i = 1, 2, \ldots, n$, is described by

$$\begin{aligned} \dot{x}_{ij} &= x_{i(j+1)}, \qquad j = 1, 2, \ldots, \rho_i - 1 \\ \dot{x}_{i\rho_i} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, x_2, ..., x_n) \\ y_i &= x_{i1}, \end{aligned}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{i\rho_i}]^\top \in \Re^{\rho_i}$ is the state vector of the i-th subsystem, $f_i : \Re^{\rho_i} \mapsto \Re$ and $g_i : \Re^{\rho_i} \mapsto \Re$ are unknown smooth functions, $\Delta_i : \Re^{\rho} \mapsto \Re$ (where $\rho = \sum_{i=1}^{n} \rho_i$) represents the interconnection effect between subsystems, $u_i \in \Re$ is the input and $y_i \in \Re$ is the output of the i-th subsystem. Our objective is to synthesize decentralized adaptive approximation based control laws u_i such that each y_i tracks a smooth bounded reference trajectory y_{d_i} in the presence of the unknown interconnections Δ_i, using only local measurements.

P. Panagi and M. M. Polycarpou are with the Department of Electrical and Computer Engineering, University of Cyprus, Nicosia 1678, Cyprus (Email: ppanagi@ucy.ac.cy, mpolycar@ucy.ac.cy). This work is supported by the Research Promotion Foundation of Cyprus.

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

It is assumed that each input gain function, $g_i(x_i)$, is bounded from below by $0 < g_{i0} \leq g_i(x_i)$, where g_{i0} is a known constant. This assumption is required in order to guarantee the controllability of the feedback control scheme [1]. In general, each $g_i(x_i)$ is required to be either positive or negative for all x_i in a domain of interest $\mathcal{D}_i \subset \Re^{\rho_i}$. For notational simplicity and without any loss of generality, here we assume that all $g_i(x_i)$ are positive. Furthermore, the desired trajectory vector $Y_{d_i} = [y_{d_i}, \dot{y}_{d_i}, \ldots, y_{d_i}^{(\rho_i)}]^\top$ of the i-th subsystem is assumed to be available and bounded.

Following the universal approximation results of neural networks [1], given any continuous function $f(x)$ where $f : \Re^q \mapsto \Re$ is defined on a compact set $\mathcal{D} \subset \Re^q$, and an arbitrary $\varepsilon^* > 0$, there exists a set of bounded constant weights $\theta_f \in \Re^p$ and a set of basis functions $\phi_f(x)$, where $\phi_f : \Re^q \mapsto \Re^p$ is such that $\forall x \in \mathcal{D}$:

$$f(x) = \phi_f(x)^\top \theta_f + \varepsilon(x), \quad \|\varepsilon(x)\|_{\mathcal{D}} < \varepsilon^*. \tag{1}$$

In the above representation, $\varepsilon(x)$ denotes the Minimum Functional Approximation Error (MFAE) which is the minimum possible deviation between the unknown function $f(x)$ and its approximation, $\phi_f(x)^\top\theta_f$, in the ∞-norm sense over the compact set $\mathcal{D}$.

To design the decentralized controller we consider the tracking error dynamics, $\tilde{x}_{ij} = x_{ij} - y_{d_i}^{(j-1)}$, of the i-th subsystem, which satisfy:

$$\begin{aligned} \dot{\tilde{x}}_{ij} &= \tilde{x}_{i(j+1)}, \qquad j = 1, 2, \ldots, \rho_i - 1 \\ \dot{\tilde{x}}_{i\rho_i} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, x_2, \ldots, x_n) - y_{di}^{(n)}, \end{aligned}$$

The tracking error dynamics can be written in matrix state-space form as

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B(f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, x_2, \ldots, x_n) - y_{d_i}^{(n)}), \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

In this section, we consider the design and analysis of a decentralized control scheme which is valid within a certain compact approximation region $A_D \subset \Re^\rho$. In other words, it is assumed that the state vector $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$ remains within A_D for all $t \geq 0$ (i.e., $x(t) \in A_D$). In the next section, we will consider the design of a control scheme which goes into effect if $x(t)$ leaves the approximation region A_D. For notational purposes, we denote by u_i the adaptive approximation based control law valid for $x(t) \in A_D$. The control law outside the approximation region will be referred to as *safety control scheme* and will be denoted by u_{s_i}.

The decentralized approximation based control law can be broken up as $u_i = u_i^* + u_{l_i}$, where the term u_i^* represents the nominal control that depends on the local nonlinear functions, while the term u_{l_i} represents an augmented control component related to the interconnections, Δ_i between the subsystems. According to (1), the unknown functions f_i and g_i can be represented as follows:

$$\begin{aligned} f_i(x_i) &= \phi_{f_i}(x_i)^\top \theta_{f_i} + \mu_{f_i}(x_i) \\ g_i(x_i) &= \phi_{g_i}(x_i)^\top \theta_{g_i} + \mu_{g_i}(x_i), \end{aligned}$$

where μ_{f_i} and μ_{g_i} are the MFAEs of the approximations of $f_i(x_i)$ and $g_i(x_i)$ respectively. Let $\hat{\theta}_{f_i}$ and $\hat{\theta}_{g_i}$ be the estimated weights of the approximators of f_i and g_i respectively, and define $\tilde{\theta}_{f_i} = \hat{\theta}_{f_i} - \theta_{f_i}$, $\tilde{\theta}_{g_i} = \hat{\theta}_{g_i} - \theta_{g_i}$ as the corresponding parameter estimation errors. The feedback linearizing approximation based control law component u_i^* of the i-th subsystem is defined as

$$u_i^* = \frac{-K_i^\top \tilde{x}_i + y_{d_i}^{(n)} - \phi_{f_i}(x_i)^\top \hat{\theta}_{f_i}}{\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}}, \tag{3}$$

where the vector $K_i = [k_{i1}, k_{i2}, \cdots, k_{i\rho_i}]^\top \in \Re^{\rho_i}$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix. Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists a positive definite matrix P_i satisfying the Lyapunov equation

$$P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i.$$

Define the scalar training error $e_i = B^\top P_i \tilde{x}_i$. We impose the following assumption on the interconnection terms Δ_i.

Assumption 1: The interconnections Δ_i are bounded by

$$|\Delta_i(x_1, x_2, ..., x_n)| \leq \sum_{j=1}^{n} \gamma_{ij}(|e_j|),$$

where $\gamma_{ij} : \Re \mapsto \Re^+$ are unknown analytic functions.

According to Assumption 1, the magnitude of the interconnections is allowed to be significantly large and also unknown. As we will see later on, a surrogate (denoted by $d_i(e_i)$) of the unknown bounding functions γ_{ij} will be adaptively approximated for use in the feedback control law. The above assumption is similar to the corresponding assumption used in [3]. According to (1), $d_i(e_i)$ can be represented as

$$d_i(e_i) = \phi_{d_i}(e_i)^\top \theta_{d_i} + \mu_{d_i}(e_i), \tag{4}$$

where μ_{d_i} is the MFAE of the approximation of d_i. The augmented control law u_{l_i} of the i-th subsystem is defined as:

$$u_{l_i} = -\frac{\phi_{d_i}(e_i)^\top \hat{\theta}_{d_i}}{\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}}. \tag{5}$$

Due to the presence of the MFAE μ_{f_i}, μ_{g_i} and μ_{d_i} of the approximation of the functions f_i, g_i and d_i respectively, we introduce a dead-zone modification in the adaptation laws of the parameters of $\hat{\theta}_{f_i}$, $\hat{\theta}_{g_i}$ and $\hat{\theta}_{d_i}$ to enchance the robustness of the adaptive scheme and to avoid instabilities that may occur due to parameter drift. The parameter estimates are updated according to the following adaptive laws

$$\dot{\hat{\theta}}_{f_i} = \Gamma_{f_i}\phi_{f_i}(x_i)q_i(e_i,\tilde{x}_i,\epsilon_i) \quad (6)$$

$$\dot{\hat{\theta}}_{g_i} = \mathcal{P}_s\{\Gamma_{g_i}\phi_{g_i}(x_i)q_i(e_i,\tilde{x}_i,\epsilon_i)u_i\} \quad (7)$$

$$\dot{\hat{\theta}}_{d_i} = \Gamma_{d_i}\phi_{d_i}(e_i)q_i(e_i,\tilde{x}_i,\epsilon_i), \quad (8)$$

where Γ_{f_i}, Γ_{g_i}, Γ_{d_i} are positive definite matrices characterizing the adaptive gain of the parameter estimates and $\mathcal{P}_s$ is a projection operator [1] that is used to ensure that the term $\phi_{g_i}(x_i)^\top\hat{\theta}_{g_i}$ stays away from zero. The dead-zone function $q_i(e_i,\tilde{x}_i,\epsilon_i)$ is defined as

$$q_i(e_i,\tilde{x}_i,\epsilon_i) = \begin{cases} 0 & \tilde{x}_i^\top P\tilde{x}_i \le \bar{\lambda}_P\epsilon_i^2 \\ e_i & \tilde{x}_i^T P\tilde{x}_i > \bar{\lambda}_P\epsilon_i^2 \end{cases}$$

$$\epsilon_i = 2\|PB\|\delta_{0_i} + \mu_i$$

where $\delta_i = \mu_{f_i}+\mu_{g_i}u_i+\mu_{d_i}$ and δ_{0_i} is an upper bound on δ_i (i.e., $|\delta_i| < \delta_{0_i}$), μ_i is a positive constant and $\bar{\lambda}_P$ and $\underline{\lambda}_P$ are the maximum and minimum eigenvalues of P, respectively. The overall decentralized control law for the i-th subsystem is given by

$$u_i = \frac{-K_i^\top\tilde{x}_i + y_{di}^{(n)} - \phi_{f_i}(x_i)^\top\hat{\theta}_{f_i} - \phi_{d_i}(e_i)^\top\hat{\theta}_{d_i}}{\phi_{g_i}(x_i)^\top\hat{\theta}_{g_i}}. \quad (9)$$

It is important to note that the feedback control law described by (9) is decentralized, since each local control law u_i does not use the states x_j, $j = 1,2,\ldots,n$, $j \neq i$, of the other subsystems.

Theorem 1: Given the tracking error dynamics (2), the decentralized control law (9) with adaptation laws (6), (7) and (8) guarantees that the following hold:

1) $\tilde{x}_i(t)$ is small-in-the-mean-square sense, satisfying
$$\int_t^{t+T}\|\tilde{x}_i^2(\tau)\|^2 d\tau \le 2V(t) + \frac{\bar{\lambda}_P}{\underline{\lambda}_P}\epsilon_i^2 T;$$
2) $\|\tilde{x}(t)\|$ is uniformly ultimately bounded by ϵ; i.e., the total time such that $\tilde{x}_i^\top P\tilde{x}_i > \bar{\lambda}_P\epsilon_i^2$ is finite;
3) In the special case that the approximation errors are all zero (i.e., $\mu_{f_i} = \mu_{g_i} = \mu_{d_i} = 0$, $i = 1,2,\ldots,n$) and the dead-zone size is set to zero ($\epsilon_i = 0$, $i = 1,2,\ldots,n$), the tracking errors $\tilde{x}_{ij}$ converge asymptotically to zero.

The proof of Theorem 1, which follows along the same lines as Lemmas 2 and 3 in [10], is omitted due to space limitations.

III. Decentralized Safety Control Scheme

In the previous analysis, we assumed that the states of each subsystem are restricted within a certain compact approximation region. Within this region, the approximation error δ_i can be arbitrarily reduced by increasing the size of the approximation network. However, outside the approximation region, the size of δ_i is typically significantly large, such that the states of the subsystems may become unbounded. Even in the case that the initial state conditions are inside the approximation region, due to large parameter errors, the states may still leave the approximation region. Therefore, in order to address this problem, in this section we consider the design of a decentralized safety control scheme based on the sliding mode control methodology with adaptive bounding. The state space x_i of each subsystem i is divided into the following subsets,

$$A_{D_i} = \left\{x_i \mid \|x_i - x_{0_i}\|_{p,\beta_i} \le 1\right\}$$

$$A_{\Psi_i} = \left\{x_i \mid \|x_i - x_{0_i}\|_{p,\beta_i} \le 1 + \Psi_i\right\},$$

where Ψ_i is a small positive constant representing the width of the transition region, x_{0_i} is a fixed vector in the state space of subsystem i and $\|x\|_{p,\beta}$ is the weighted p-norm,

$$\|x\|_{p,\beta} = \left[\sum_{j=1}^{k}\left(\frac{|x_j|}{\beta_j}\right)^p\right]^{\frac{1}{p}}.$$

Through the use of the weighted p-norm, for different values of p and β, we are able to specify subsets with arbitrary dimensions in different dimensions (e.g., ellipse, rectangle) and not necessarily of equal dimensions in different coordinates (e.g., circle, square). Fig. 1 illustrates the defined subsets for a second order system, where the weighted 2-norm is used, with $\beta_1 > \beta_2$. Without the use of a transition region, the switching between the approximation based control law, u_i and the sliding mode control, u_{s_i}, may lead to a discontinuity in the control law which in turn may excite unmodelled high-frequency dynamics. We therefore design a transition region, in which the two control schemes (adaptive approximation and sliding mode control) are combined, in a way that we will see later, such that the overall control law is continuous at all times. The sliding manifold of the i-th subsystem is chosen to be the scalar training error $e_i = B^\top P_i\tilde{x}_i = 0$. Let $\Omega_i \supset A_{D_i}$ and $\Omega \equiv \Omega_1 \times \Omega_2 \ \ldots\ \Omega_n \supset A_D$. We impose the following assumptions on the bounds of the unknown functions.

Assumption 2: In the region $\Omega_i - A_{D_i}$, the following bounds hold,

$$|f_i(x_i)| \le w_{f_i}^\top M_{f_i}(x_i) \quad (10)$$

$$w_{gl_i}^\top M_{gl_i}(x_i) \le |g_i(x_i)| \le w_{gu_i}^\top M_{gu_i}(x_i) \quad (11)$$

In the region $\Omega - A_{D_1}\times A_{D_2}\times\ldots\times A_{D_n} \equiv \Omega - A_D$,

$$|\Delta_i(x)| \le \sum_{j=1}^{n}|e_j|\sqrt{w_{\Delta_{ij}}}M_{\Delta_{ij}}(x_j) + w_{\Delta_{i0}}, \quad (12)$$

where $M_{f_i}:\Re^{\rho_i}\mapsto\Re$, $M_{gl_i}:\Re^{\rho_i}\mapsto\Re$, $M_{gu_i}:\Re^{\rho_i}\mapsto\Re$, $M_{\Delta_{ij}}:\Re^{\rho_j}\mapsto\Re$ are known functions, and $w_{f_i}, w_{gl_i}, w_{gu_i}$, $w_{\Delta_{ij}}$ $w_{\Delta_{i0}}$ are unknown positive parameters.

It is noted that, in general, we do not need to know the functions $M_{f_i}(x_i)$, $M_{gl_i}(x_i)$, $M_{gu_i}(x_i)$ and $M_{\Delta_{ij}}(x_j)$ since theoretically they can be set to one. However, it is best to incorporate as much prior knowledge as possible into the design to avoid unneccessary large feedback gains. Define the vector $w_{\Delta_i} = [w_{\Delta_{1i}}\ w_{\Delta_{2i}}\ \ldots\ w_{\Delta_{ni}}]^\top$ and let $\hat{w}_{f_i}$,

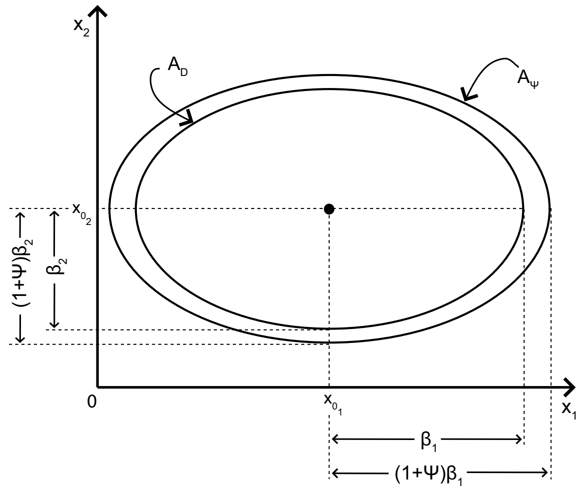

Fig. 1. Illustration of the sets in the state space of a second order system.

$\hat{w}_{gl_i}$, $\hat{w}_{gu_i}$, $\hat{w}_{\Delta_i}$, $\hat{w}_{\Delta_{i0}}$ be the estimates of w_{f_i}, w_{gl_i}, w_{gu_i}, w_{Δ_i} and $w_{\Delta_{i0}}$ respectively. The corresponing parameter estimation errors are defined as $\tilde{w}_{f_i} = \hat{w}_{f_i} - w_{f_i}$, $\tilde{w}_{gl_i} = \hat{w}_{gl_i} - w_{gl_i}$, $\tilde{w}_{gu_i} = \hat{w}_{gu_i} - w_{gu_i}$, $\tilde{w}_{\Delta_i} = \hat{w}_{\Delta_i} - w_{\Delta_i}$ and $\tilde{w}_{\Delta_{i0}} = \hat{w}_{\Delta_{i0}} - w_{\Delta_{i0}}$. The decentralized control law is defined as

$$\bar{u}_i = (1-m_i)u_i + m_i u_{s_i}, \tag{13}$$

where $m_i(t)$ is a modulation function defined by,

$$m_i(t) = \begin{cases} 0 & x_i \in A_{d_i} \\ \frac{\|x_i - x_{0_i}\|_{p,w_i} - 1}{\Psi_i} & x_i \in A_{\Psi_i} - A_{d_i} \\ 1 & x_i \in A^c_{\Psi_i}, \end{cases}$$

and the sliding mode control component is defined by,

$$\begin{aligned} u_{s_i} &= \frac{u_{a_i}}{u_{g_i}} && (14)\\ u_{a_i} &= -K_i^\top \tilde{x}_i + y_{d_i}^{(n)} + \mathrm{sgn}(e_i)\Pi_i && (15)\\ u_{g_i} &= \begin{cases} \hat{w}_{gl_i}^\top M_{gl_i}(x_i) & e_i u_{a_i} \le 0 \\ \hat{w}_{gu_i}^\top M_{gu_i}(x_i) & e_i u_{a_i} > 0 \end{cases} && (16)\\ \Pi_i &= -\frac{n}{2}|e_i| - \hat{w}_{f_i}^\top M_{f_i}(x_i) && (17)\\ &\quad - \frac{|e_i|}{2}\sum_{j=1}^{n} \hat{w}_{\Delta_{ji}} M^2_{\Delta_{ji}}(x_i) - \hat{w}_{\Delta_{i0}}. && (18) \end{aligned}$$

The parameter estimates $\hat{w}_{f_i}, \hat{w}_{gl_i}, \hat{w}_{gu_i}$ and $\hat{w}_{\Delta_{ij}}$ are updated according to the following adaptive laws,

$$\begin{aligned} \dot{\hat{w}}_{f_i} &= \Gamma_{f_i} M_{f_i}(x_i)|e_i| m_i && (19)\\ \dot{\hat{w}}_{gl_i} &= \begin{cases} \frac{\Gamma_{gl_i} e_i u_{a_i} M_{gl_i}(x_i)}{\hat{w}_{gl_i}^\top M_{gl_i}(x_i)} m_i & e_i u_{a_i} \le 0 \\ 0 & e_i u_{a_i} > 0 \end{cases} && (20)\\ \dot{\hat{w}}_{gu_i} &= \begin{cases} 0 & e_i u_{a_i} \le 0 \\ \frac{\Gamma_{gu_i} e_i u_{a_i} M_{gu_i}(x_i)}{\hat{w}_{gu_i}^\top M_{gu_i}(x_i)} m_i & e_i u_{a_i} > 0 \end{cases} && (21)\\ \dot{\hat{w}}_{\Delta_i} &= \Gamma_{\Delta_i} M^{[2]}_{\Delta_i}(x_i) e_i^2 m_i && (22)\\ \dot{\hat{w}}_{\Delta_{i0}} &= \gamma_{\Delta_{i0}} |e_i| m_i, && (23) \end{aligned}$$

where Γ_{f_i}, Γ_{gl_i}, Γ_{gu_i}, Γ_{Δ_i} are positive definite matrices and $\gamma_{\Delta_{i0}}$ is a positive constant corresponding to the adaptive rates of the parameter estimates and $M^{[2]}_{\Delta_i} = \left[M^2_{\Delta_{1i}}(x_i)\ M^2_{\Delta_{2i}}(x_i)\ \ldots\ M^2_{\Delta_{ni}}(x_i)\right]^\top$. The parameter estimates $\hat{\theta}_{f_i}$, $\hat{\theta}_{g_i}$, $\hat{\theta}_{\Delta_i}$ are also processed by the modulation function $m_i(t)$, and are updated according to:

$$\begin{aligned} \dot{\hat{\theta}}_{f_i} &= \Gamma_{f_i}\phi_{f_i}(x_i) q_i(e_i, \tilde{x}_i, \epsilon_i)(1-m_i) && (24)\\ \dot{\hat{\theta}}_{g_i} &= P_s\{\Gamma_{g_i}\phi_{g_i}(x_i) q_i(e_i, \tilde{x}_i, \epsilon_i) u_i (1-m_i)\} && (25)\\ \dot{\hat{\theta}}_{d_i} &= \Gamma_{d_i}\phi_{d_i}(e_i) q_i(e_i, \tilde{x}_i, \epsilon_i)(1-m_i). && (26) \end{aligned}$$

Theorem 2: Given the tracking error dynamics (2), the decentralized control law defined by equations (13), (9) and (14) with adaptation laws (19)-(26) ensures that the trajectories of each subsystem will eventually enter the approximation region. Moreover if $\Omega \equiv \Re^\rho$, this holds for all $\tilde{x}_{ij} \in \Re$.

Proof: Let the Lyapunov function of the i-th subsystem be given by $V_i = V_{i1} + V_{i2}$, where $V_{i1} = \tilde{x}_i^\top P \tilde{x}_i$ and

$$\begin{aligned} V_{i2} &= (1-m_i)\tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1}\tilde{\theta}_{f_i} + (1-m_i)\tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1}\tilde{\theta}_{g_i} \\ &\quad + (1-m_i)\tilde{\theta}_{d_i}^\top \Gamma_{d_i}^{-1}\tilde{\theta}_{d_i} + m_i \tilde{w}_{f_i}^\top \Gamma_{wf_i}^{-1}\tilde{w}_{f_i} \\ &\quad + m_i \tilde{w}_{gl_i}^\top \Gamma_{gl_i}^{-1}\tilde{w}_{gl_i} + m_i \tilde{w}_{gu_i}^\top \Gamma_{gu_i}^{-1}\tilde{w}_{gu_i} \\ &\quad + m_i \tilde{w}_{\Delta_i}^\top \Gamma_{\Delta_i}^{-1}\tilde{w}_{\Delta_i} + m_i \frac{1}{\gamma_{\Delta_{i0}}}\tilde{w}^2_{\Delta_{i0}}. \end{aligned}$$

By substituting the control law (13) in the tracking error dynamics, (2), we obtain,

$$\begin{aligned} \dot{\tilde{x}} &= (1-m_i)\left[A\tilde{x}_i \right.\\ &\quad \left. + B\left(f_i(x_i) + g_i(x_i)u_{n_i} + \Delta_i - y_{di}^{(n)}\right)\right] \\ &\quad + m_i\left[A\tilde{x}_i + B\left(f_i(x_i) + g_i(x_i)u_{s_i} + \Delta_i - y_{di}^{(n)}\right)\right]. \end{aligned}$$

Assume that all $m_i = 1$; i.e., the states x_i are in $A^c_{\Psi_i}$ and that $e_i u_{a_i} \le 0$. Under these conditions using (11), (14) and (16), the time derivative of V_{i1} satisfies

$$\begin{aligned} \dot{V}_{i1} &\le 2\tilde{x}_i^\top P_i\left[A\tilde{x}_i + B\left(f_i(x_i) + u_{a_i}\right.\right. \\ &\quad \left.\left. - \frac{\tilde{w}_{gl_i} M_{gl_i}(x_i)}{\hat{w}_{gl_i} M_{gl_i}(x_i)} + \Delta_i - y_{d_i}^{(n)}\right)\right] \\ &\le -\tilde{x}_i^\top Q_i \tilde{x}_i + 2|e_i|\left(|f_i(x_i)| + \Pi_i + |\Delta_i|\right) \\ &\quad - 2e_i u_{a_i}\frac{\tilde{w}_{gl_i} M_{gl_i}(x_i)}{\hat{w}_{gl_i} M_{gl_i}(x_i)}. \end{aligned}$$

Substituting Π_i from (17) and using the bounds of f_i and Δ_i defined in equations (10) and (12) respectively, we obtain

$$\begin{aligned} \dot{V}_{i1} &\le -\tilde{x}_i^\top Q_i \tilde{x}_i + 2|e_i|\left[-\tilde{w}_{f_i}^\top M_{f_i}(x_i) - \frac{n}{2}|e_i|\right. \\ &\quad + \sum_{j=1}^{n}|e_j|\sqrt{w_{\Delta_{ij}}} M_{\Delta_{ij}}(x_j) - \tilde{w}_{\Delta_{i0}} \\ &\quad \left. - \frac{|e_i|}{2}\sum_{j=1}^{n}\hat{w}_{\Delta_{ji}} M^2_{\Delta_{ji}}(x_i)\right] - 2e_i u_{a_i}\frac{\tilde{w}_{gl_i} M_{gl_i}(x_i)}{\hat{w}_{gl_i} M_{gl_i}(x_i)}. \end{aligned}$$

Therefore, using the inequality $2\alpha\beta \leq \alpha^2+\beta^2$ for $\alpha,\beta \in \Re$,

$$\begin{aligned}\dot{V}_{i1} \leq\ & -\tilde{x}_i^\top Q_i\tilde{x}_i - 2|e_i|\tilde{w}_{f_i}^\top M_{f_i}(x_i) \\ & + \sum_{j=1}^{n} e_j^2 w_{\Delta_{ij}} M_{\Delta_{ij}}^2(x_j) - e_i^2 \sum_{j=1}^{n} \hat{w}_{\Delta_{ji}} M_{\Delta_{ji}}^2(x_i) \\ & - 2\,|e_i|\,\tilde{w}_{\Delta_{i0}} - 2e_i u_{a_i}\frac{\tilde{w}_{gl_i}M_{gl_i}(x_i)}{\hat{w}_{gl_i}M_{gl_i}(x_i)}.\end{aligned}$$

After some reordering of terms,

$$\begin{aligned}\sum_{i=1}^{n}\dot{V}_{i1} \leq\ & \sum_{i=1}^{n}\Big[-\tilde{x}_i^\top Q_i\tilde{x}_i - 2|e_i|\tilde{w}_{f_i}^\top M_{f_i}(x_i) \\ & - e_i^2\sum_{j=1}^{n}\tilde{w}_{\Delta_{ji}}M_{\Delta_{ji}}^2(x_i) \\ & -2\,|e_i|\,\tilde{w}_{\Delta_{i0}} - 2e_i u_{a_i}\frac{\tilde{w}_{gl_i}M_{gl_i}(x_i)}{\hat{w}_{gl_i}M_{gl_i}(x_i)}\Big].\end{aligned}$$

The time derivative of the Lyapunov function of the overall system is given by

$$\begin{aligned}\dot{V} =\ & \sum_{i=1}^{n}\dot{V}_{i1}+\dot{V}_{i2} \\ \leq\ & \sum_{i=1}^{n}\Big[-\tilde{x}_i^\top Q_i\tilde{x}_i - 2|e_i|\tilde{w}_{f_i}^\top M_{f_i}(x_i) - 2\,|e_i|\,\tilde{w}_{\Delta_{i0}} \\ & - e_i^2\sum_{j=1}^{n}\tilde{w}_{\Delta_{ji}}M_{\Delta_{ji}}^2(x_i) - 2e_i u_{ai}\frac{\tilde{w}_{gl_i}M_{gl_i}(x_i)}{\hat{w}_{gl_i}M_{gl_i}(x_i)}\Big] \\ & + \sum_{i=1}^{n}\Big[2\tilde{w}_{f_i}^\top\Gamma_{f_i}^{-1}\dot{\tilde{w}}_{f_i} + 2\tilde{w}_{gl_i}^\top\Gamma_{gl_i}^{-1}\dot{\tilde{w}}_{gl_i} \\ & + 2\tilde{w}_{\Delta_i}^\top\Gamma_{\Delta_i}^{-1}\dot{\tilde{w}}_{\Delta_i} + \frac{2}{\gamma_{\Delta_{i0}}}\tilde{w}_{\Delta_{i0}}\dot{\tilde{w}}_{\Delta_{i0}}\Big].\end{aligned}$$

By grouping terms we obtain

$$\begin{aligned}\dot{V} \leq\ & \sum_{i=1}^{n}\Big[-\tilde{x}_i^\top Q_i\tilde{x}_i + 2\tilde{w}_{f_i}^\top\Gamma_{f_i}^{-1}\left(\dot{\tilde{w}}_{f_i} - \Gamma_{f_i}|e_i|M_{f_i}(x_i)\right) \\ & + 2\tilde{w}_{gl_i}^\top\Gamma_{gl_i}^{-1}\left(\dot{\tilde{w}}_{gl_i} - \frac{\Gamma_{gl_i}e_i u_{a_i}M_{gl_i}(x_i)}{\hat{w}_{gl_i}M_{gl_i}(x_i)}\right) \\ & + 2\tilde{w}_{\Delta_i}^\top\Gamma_{\Delta_i}^{-1}\left(\dot{\tilde{w}}_{\Delta_i} - \Gamma_{\Delta_i}M_{\Delta_i}^{[2]}(x_i)e_i^2\right) \\ & + 2\gamma_{\Delta_{i0}}^{-1}\tilde{w}_{\Delta_{i0}}\left(\dot{\tilde{w}}_{\Delta_{i0}} - \gamma_{\Delta_{i0}}|e_i|\right)\Big].\end{aligned}$$

By substituting the adaptive laws (19)-(23), the Lyapunov function derivative satisfies

$$\dot{V} \leq -\sum_{i=1}^{n}\tilde{x}_i^\top Q_i\tilde{x}_i,$$

which shows that the state vector x_i of each subsystem enter A_{Ψ_i} in finite time. Within the transition region, $A_{\Psi_i} - A_{D_i}$, the modulation function satisfies $0 < m_i(t) < 1$. Using the previous results and that of the proof of Theorem 1, the time derivative of the Lyapunov function in $A_{\Psi_i} - A_{D_i}$ satisfies

$$\begin{aligned}\dot{V} \leq\ & \sum_{i=1}^{n}\left[-(1-m_i)\left(\tilde{x}_i^\top Q_i\tilde{x}_i + 2e_i\delta_i\right) - m_i\tilde{x}_i^\top Q_i\tilde{x}_i\right] \\ \leq\ & -\sum_{i=1}^{n}\left[\tilde{x}_i^\top Q_i\tilde{x}_i + 2(1-m_i)e_i\delta_i\right].\end{aligned}$$

Thus, using Theorem 1, it is simple to deduce that the state vector x_i of each subsystem enters A_{d_i} in finite time. It is also straightforward to obtain the same result when $e_i u_{a_i} > 0$. ■

IV. Simulation Example

In this section, we use a simple simulation example to illustrate the previous theoretical results. Consider the following interconnected uncertain system:

$$\begin{aligned}\Sigma_1 :\ \dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= 0.7\cos\left(\frac{\pi}{1.2}R_1\right) + \Delta_1(x_1,x_2) \\ &\quad + \left(2 + (x_{11}+x_{12})^2 + 2e^{-R_1}\right)u_1 \\ \Sigma_2 :\ \dot{x}_{21} &= x_{22} \\ \dot{x}_{22} &= 0.5\cos\left(\frac{\pi}{1.2}R_2\right) + \Delta_2(x_1,x_2) \\ &\quad + \left(2 + (x_{21}+x_{22})^2 + 2e^{-R_2}\right)u_2,\end{aligned}$$

where $R_i(x_{i1},x_{i2}) = x_{i1}^2 + x_{i2}^2$, $\Delta_1(x_1,x_2) = (x_{21}+0.6x_{22})^2$ and $\Delta_2(x_1,x_2) = (x_{11}+0.5x_{12})^2$. The matrix P satisfying the Lyapunov equation is given by

$$P_i = \begin{bmatrix}1.5 & 0.5\\ 0.5 & 1\end{bmatrix}, \quad i = 1,2$$

where $K_1 = K_2 = \begin{bmatrix}1 & 1\end{bmatrix}^\top$. The desired trajectory vector $Y_{d_i} = [y_{d_i}, \dot{y}_{d_i}]^\top$ and the signal $\ddot{y}_{d_i}$ are generated using a third-order filter with a bandwidth of 5 (rad/sec) and unity gain below this frequency. Therefore, y_{d_i} is close to the input of the filter, for any input that has bandwidth below 5 rad/sec. The filter input of the first subsystem is chosen as a square wave of zero mean, 1.1 amplitude and a frequency of 0.5 Hz, while that of the second subsystem is chosen as a square wave of zero mean, 1.3 amplitude and frequency 0.4 Hz. The approximation regions A_{d_i} $i = 1,\ 2$ are chosen as

$$A_{D_i} = \max\left\{\frac{|x_{i1}|}{2},\ \frac{|x_{i2}|}{7}\right\} \leq 1, \quad i = 1,\ 2.$$

Within this region (i.e., $(x_{i1},x_{i2}) = [-2,2]\times[-7,7]$) a lattice of equally spaced radial basis functions are designed for the approximation of the unknown functions f_i, g_i and d_i. The width of the transition region, $A_{\Psi_i} - A_{D_i}$, is chosen as $\Psi_i = 0.05$ $i = 1,\ 2$, such that the region A_{Ψ_i} is defined by,

$$A_{\Psi_i} = \max\left\{\frac{|x_{i1}|}{2},\ \frac{|x_{i2}|}{7}\right\} \leq 1.05, \quad i = 1,\ 2.$$

The dead-zone parameters ϵ_i are set to $\epsilon_1 = \epsilon_2 = 0.4$. The initial conditions are assumed to be: $x_{11}(0) = 4.5$, $x_{12}(0) = 5.5$, $x_{21}(0) = -4$ and $x_{22}(0) = -6$. We assume that

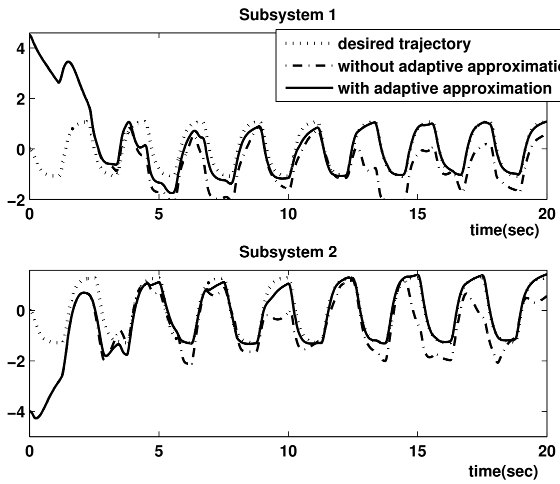

Fig. 2. Tracking performance with and without adaptive approximation.

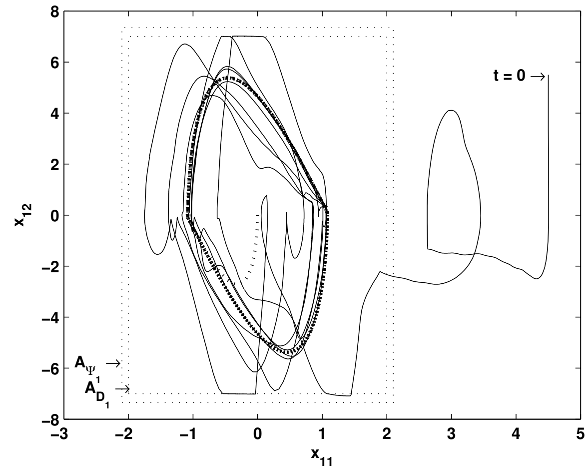

Fig. 3. Phase plane plot of x_{11} versus x_{12}.

outside the approximation region A_{d_i}, the unknown functions are bounded by,

$$\begin{aligned} |f_i(x_i)| &\le w_{f_i}\left[2-\cos\left(x_{i1}+x_{i2}\right)\right] \quad i=1,\ 2\\ 3w_{gl_i} \le |g_i(x_i)| &\le w_{gu_i}\left[3+\left(x_{i1}+x_{i2}\right)^2\right] \quad i=1,\ 2\\ |\Delta_1(x_1,x_2)| &\le w_{\Delta_{12}}\,|e_2|\,|x_{21}+x_{22}|+w_{\Delta_{10}}\\ |\Delta_2(x_1,x_2)| &\le w_{\Delta_{21}}\,|e_1|\,|x_{11}+x_{12}|+w_{\Delta_{20}}. \end{aligned}$$

In Fig. 2 we plot the tracking performance of each subsystem with and without adaptive approximation of the function d_i. In the case that no adaptive approximation is used, the radial basis function neural networks are turned off even when the trajectory is inside the approximation regions. As illustrated by the plot, although the use of sliding mode control guarantees boundedness of all tracking errors, the use of adaptive approximation results in a significantly better tracking performance. In fact, in the case of adaptive approximation, the tracking performance continues to improve after the time period shown in the plot. However, the rate of improvement is reduced as the subsystems spend more time in the dead-zone, until approximately the time $t = 60$ sec, when the scalar errors e_i stay within the dead-zone thereafter.

Fig. 3 shows the phase plane plot of the states of subsystem 1. The desired trajectory is shown as a thick dotted line inside the approximation region. The use of decentralized sliding mode control with adaptive bounds is able to steer the states of the subsystems back to the approximation region. Due to large approximation errors at the initial stages of the simulation, the trajectories of the subsystems leave the approximation region several times, but the decentralized safety control scheme succeeds in steering the trajectories back to the approximation region, in most cases before they cross the sets A_{Ψ_i}.

V. Concluding Remarks

In this paper, we have presented a decentralized, adaptive approximation based controller with a safety control scheme outside the approximation region. Inside the approximation region, we use a dead-zone modification to the adaptive laws for enhanced robustness in the presence of approximation errors. A decentralized sliding mode control scheme is used outside the approximation region to steer the trajectories back to the desired region, which increases the region of attraction for the closed-loop system. In the case that the assumed bounding inequalities hold for all $x \in \Re^\rho - A_D$, then the stability results become global.

References

[1] J.A. Farrell and M.M. Polycarpou. *Adaptive Approximation Based Control*. John Wiley & Sons, Hoboken, NJ, USA, 2006.

[2] D.T. Gavel and D.D. Siljak. Decentralized adaptive control: structural conditions for stability. *IEEE Transactions on Automatic Control*, 34(4):413–426, 1989.

[3] S. Huang, K.K. Tan, and T.H. Lee. Decentralized control design for large-scale systems with strong interconnections using neural networks. *IEEE Transactions on Automatic Control*, 48(5):805–810, 2003.

[4] P. Ioannou and P. Kokotovic. Decentralized adaptive control of interconnected systems with reduced-order models. *Automatica*, 21(4):401–412, 1985.

[5] P.A. Ioannou. Decentralized adaptive control of interconnected systems. *IEEE Transactions on Automatic Control*, 31:291–298, 1986.

[6] B.M. Mirkin and P.O. Gutman. Decentralized Output-Feedback MRAC of Linear State Delay Systems. *IEEE Transactions on Automatic Control*, 48(9):1613, 2003.

[7] B.-J. Yang N. Hovakimyan, E. Lavretsky and A.J. Calise. Coordinated decentralized adaptive output feedback control of interconnected systems. *IEEE Transactions on Neural Networks*, 16:1045–9227, 2005.

[8] K.S. Narendra and N.O. Oleng. Exact output tracking in decentralized adaptive control systems. *IEEE Transactions on Automatic Control*, 47(2):390–395, 2002.

[9] K.A. Ossman. Indirect adaptive control for interconnected systems. *IEEE Transactions on Automatic Control*, 34(8):908–911, 1989.

[10] P. Panagi and M.M. Polycarpou. Decentralized adaptive approximation based control of a class of large-scale systems. *Proc. 2008 American Control Conference*, pages 4191–4196, 2008.

[11] L. Shi and SK Singh. Decentralized adaptive controller design for large-scale systemswith higher order interconnections. *IEEE Transactions on Automatic Control*, 37(8):1106–1118, 1992.

[12] J.T. Spooner and K.M.Passino. Decentralized adaptive control of nonlinear systems using radialbasis neural networks. *IEEE Transactions on Automatic Control*, 44(11):2050–2057, 1999.

[13] J.T. Spooner and K.M. Passino. Adaptive control of a class of decentralized nonlinear systems. *IEEE Transactions on Automatic Control*, 41(2):280–284, 1996.