# Nonlinear Control Synthesis for Asymptotic Stabilization of the Swing Equation using a Controllable Series Capacitor via Immersion and Invariance

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Abstract—A nonlinear control law is proposed to asymptotically stabilize a single machine infinite bus (SMIB) system based on Immersion and Invariance (I&I) control strategy. The actuator used is a controllable series capacitor (CSC). The SMIB system is described using the nonlinear second order swing equation model, and the CSC is modeled using a first order system. The control objective here is to approximate the complete third order system with a second order dynamics, for which we have an asymptotically stabilizing control law.

## I. INTRODUCTION

Control of power swing oscillations is an important control problem. See [1] for an account of the new issues in power system operations. Recently the application of nonlinear control theory has been investigated for improving the transient stability of a power system. Nonlinear control using turbine control, see [2], and excitation control has been proposed. The excitation control law has been investigated to replace the traditional Automatic Voltage Regulator (AVR) and the Power System Stabilizer (PSS) control structure. In [3], [4], [5], [6], [7] feedback linearization was applied to the nonlinear control problem for single machine as well as multi-machine systems, using output feedback and state observers. However, this method is fragile, as it relies on nonlinearity cancellation, and the issue of robustness remains unanswered. This motivated the investigation of energybased control technique for this control problem. The use of energy function for control application has been given in [8]. The work based on damping injection controllers, also known as  $L_{\rho}V$  controllers, is found in [9], [10], [11], [12]. In [13], [14] a dynamic damping injection controller is presented. It is shown that the domain of attraction becomes larger. In [15], [16] a passivation technique is proposed for power system stabilization. An observer-based controller is given in [17]. Further, in [18] a passivity-based control law is proposed for the excitation control of synchronous generator

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by shaping the total energy function via modification of the energy transfer between the mechanical and electrical components of the system. This control law enlarges the domain of attraction, thus increasing the critical clearing time. An observer-based (adaptive) control is given in [19]. In [20] an output feedback excitation control of synchronous generators is proposed using a nonlinear observer. [21] deals with transient stabilization of a multimachine power system with nontrivial transfer conductances. In [22] energy shaping approach is applied to a power system using direct mechanical damping assignment. Transient stabilization of structure preserving power systems with excitation control using an energy-shaping technique is given in [23]. Recently, in [24] interconnection and damping assignment passivitybased control (IDA-PBC) strategy is used for transient stabilization of a synchronous generator using a CSC.

An important factor, which decides the capacity of a transmission line to transfer the electrical power across the network, the stability margin of the power system, is the reactance of the transmission line. Many power electronic devices have been invented for increasing the capacity and stability margin of the power systems. The concept of Flexible AC Transmission System (FACTS) relies on the use of such power electronic devices, and offers greater control of power flow, secure loading and damping of power system oscillations see, e.g., [25]. These devices can be classified into two categories, one is shunt devices (the injected currents are controlled), and the other is series devices (the inserted voltages are controlled). Static VAR compensator is an example of shunt devices, while series devices include Unified Power Flow Controller (UPFC), Controllable Series Capacitor (CSC) and Quadrature Boosting transformer (QBT). These series devices are known as Controllable Series Devices (CSDs). See [26], [25] for use of CSDs in power system stabilization.

In this paper we address the problem of transient stabilization of the SMIB system using a CSC. The control system consists of two subsystems- the second order nonlinear swing equation of the SMIB system, and a first order system representing the CSC. We employ the I&I control strategy [27] to achieve the control objective. The control synthesis is based on two important nonlinear tools-(system) immersion and (manifold) invariance. This control design involves the following important steps- The complete third order system is immersed in a reduced order target dynamics of order two. The choice of the target system is such that we have an asymptotically stabilizing control law for the system. An invariant manifold is constructed such that the restriction of the full order system dynamics coincides with the target dynamics. We then design a control law that renders the manifold attractive and ensures all signals are bounded.

The paper is organized as follows: In Section II we describe the control system and state the control objective. In Section III we briefly introduce the control synthesis methodology. In Section IV we give the main result of this paper. The simulation plots are given in Section V. And finally Section VI concludes the paper.

## II. PROBLEM FORMULATION AND THE CONTROL STRATEGY

Consider the SMIB system with a CSC as shown in Figure 1. In practice, an infinite bus is a large power system with a large center of inertia. Such a system does not exhibit significant oscillations on occurrence of transients and hence we consider it as a reference bus to assess the performance of a synchronous generator connected to it. In Figure 1 the infinite bus is denoted by bus 2 and the generator internal bus 1 is connected to the infinite bus through the transient reactance  $x'_d$ . The controllable series capacitor is represented by the variable capacitor  $-jx_c$ .



Fig. 1. SMIB system with CSC

We use the following notation:  $\delta$  is the rotor angle and  $\omega$  is the rotor angular speed deviation with respect to a synchronously rotating reference for the generator. Further *E* denotes the *q*-axis voltage behind transient reactance of the generator. Let D > 0, M > 0, *P* be the damping constant, moment of inertia constant, and the mechanical power input, respectively. Next we assume that the rotor is round rotor type, and hence neglect the effect of the saliency of the rotor. Since the bus 2 is the infinite bus, *V* is constant. Also  $\theta$  is constant and is assumed to be zero. Let the effective reactance between bus 1 and 2 be denoted by  $x_l$ . Now we make the following assumption.

Assumption 2.1: The region of operation is

$$\mathscr{D} = \left\{ (\delta, \omega, x_l) \mid 0 \le \delta \le \frac{\pi}{2} - d_1, x_l \ge d_2 \right\},$$

where  $d_1 > 0$  and  $d_2 > 0$  are small numbers.

#### A. Swing Equation Model with the First Order Model of CSC

First we describe the SMIB system using the swing equation model given by

$$\dot{\delta} = \omega \dot{\omega} = \frac{1}{M} \left[ P - D\omega - EV \frac{\sin \delta}{x_l} \right].$$
(1)

The actuator dynamics is represented using a first order system of the form

$$\dot{x}_{l} = \frac{1}{T_{dc}} \left[ -x_{l} + x_{l_{\star}} + u \right],$$
(2)

where  $T_{dc}$  is the time constant of the actuator dynamics, and  $x_{l_{\star}}$  is the line reactance at the desired equilibrium point and u is the input to the actuator.

Consider the swing equation model of the SMIB with CSC given by (1) and (2). We define the state variables of the system as  $x_1 = \delta$ ,  $x_2 = \omega$ ,  $x_3 = x_l$  and  $x = [x_1 \ x_2 \ x_3]^T$  as the state vector. Then the open loop operating equilibrium is denoted by  $x_* = (x_{1*}, 0, x_{3*})$ . The complete control system can now be written as

$$\dot{x}_{1} = x_{2} \dot{x}_{2} = \frac{1}{M} \left[ P - Dx_{2} - EV \frac{\sin x_{1}}{x_{3}} \right] \dot{x}_{3} = \frac{1}{T_{dc}} \left[ -x_{3} + x_{3_{\star}} + u \right],$$

$$(3)$$

or equivalently,

$$\dot{x} = f(x) + g(x)u$$

$$= \begin{pmatrix} x_2 \\ \frac{1}{M} \left[ P - Dx_2 - EV \frac{\sin x_1}{x_3} \right] \\ \frac{1}{T_{dc}} \left[ -x_3 + x_{3\star} \right] \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_{dc}} \end{pmatrix} u \quad (4)$$

where  $x_{3_{\star}} = \frac{EV \sin x_{1_{\star}}}{P}$  for a given  $x_{1_{\star}}$ .

## B. Control Objective

As mentioned earlier,  $x_{\star}$  denotes the operating stable equilibrium in  $\mathcal{D}$ . We assume that  $x_{\star}$  is known to us and state the control objective as "to synthesize a control law *u* in order to make the system (4) asymptotically stable at  $x_{\star}$ ."

Next we briefly state a result from [27] to explain the controller design strategy.

### **III. IMMERSION AND INVARIANCE**

The method of I&I for stabilization of nonlinear systems is proposed in [27]. The main result of [27] is now stated.

*Theorem 3.1:* Consider the state space model of the system

$$\dot{x} = f(x) + g(x)u \tag{5}$$

where f(x) and g(x) are smooth functions, with state  $x \in \mathbb{R}^n$ and control  $u \in \mathbb{R}^m$ , with an equilibrium point  $x_* \in \mathbb{R}^n$  to be stabilized. Let p < n and assume we can find mappings

$$\begin{split} & \boldsymbol{\alpha}(\cdot): \boldsymbol{R}^p \to \boldsymbol{R}^p, \boldsymbol{\pi}(\cdot): \boldsymbol{R}^p \to \boldsymbol{R}^n, \boldsymbol{c}(\cdot): \boldsymbol{R}^p \to \boldsymbol{R}^m, \\ & \boldsymbol{\phi}(\cdot): \boldsymbol{R}^n \to \boldsymbol{R}^p, \boldsymbol{\psi}(\cdot): \boldsymbol{R}^{n \times (n-p)} \to \boldsymbol{R}^m, \end{split}$$

such that the following hold.

1) (H1) (Target system) The system

$$\xi = \alpha(\xi) \tag{6}$$

with state  $\xi \in \mathbb{R}^p$  has an asymptotically stable equilibrium at  $\xi_* \in \mathbb{R}^p$  and  $x_* = \pi(\xi_*)$ .

2) (H2) (Immersion condition) For all  $\xi \in \mathbb{R}^p$ 

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi} \alpha(\xi).$$
(7)

 (H3) (Implicit manifold) The following set identity holds

$$\{x \in \mathbb{R}^n \mid \phi(x) = 0\}$$
  
=  $\{x \in \mathbb{R}^n \mid x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^p\}.$  (8)

 (H4) (Manifold attractivity and trajectory boundedness) All trajectories of the system

$$\dot{z} = \frac{\partial \phi}{\partial x} [f(x) + g(x)\psi(x,z)]$$
 (9)

$$\dot{x} = f(x) + g(x)\psi(x,z)$$
(10)

are bounded and satisfy

$$\lim_{t \to \infty} z(t) = 0 \tag{11}$$

where  $z = \phi(x)$  and  $u = \psi(x)$ .

Then  $x_{\star}$  is an asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f(x) + g(x)\psi(x,\phi(x)).$$

The above theorem can be interpreted as follows: Given the system (5) and the target dynamical system (6), find if possible, a manifold  $\mathcal{M}$  such that

- restriction of the closed loop system to *M* is the target dynamics
- 2)  $\mathcal{M}$  can be rendered invariant and attractive.

The left hand side of (8) gives an implicit description of  $\mathcal{M}$  while the right hand side is a parametrized description. The control law  $u = c(\pi(\xi))$  renders  $\mathcal{M}$  invariant. A measure of the distance of the system trajectories to  $\mathcal{M}$  is given by *z*, called as off-the-manifold coordinate. Our aim is to design a control law  $u = \psi(x, z)$  that keeps the system trajectories bounded and drives the coordinate *z* to zero.

#### IV. CONTROLLER SYNTHESIS USING I&I

In this section we synthesize a stabilizing controller for the SMIB system with a CSC. The control system is given by (4) and it consists of two subsystems- one is the second order swing equation, a slow system, and the other is the CSC which is a fast dynamics as compared to the swing dynamics.

We use the Immersion and Invariance methodology described earlier to synthesize the controller.

## Target system

Selection of the target dynamics in which the closed loop system is immersed, is a nontrivial task, in general. As discussed in [27] we make a natural choice for the target system as the mechanical subsystem. As a first step in the control synthesis we define a two dimensional dynamical system as follows: Let  $\xi = [\xi_1, \xi_2]^T \in S^1 \times \mathbb{R}$  be the state of the dynamical system.

$$\Sigma_T : \begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -\frac{\partial V(\xi_1)}{\partial \xi_1} - R(\xi)\xi_2 \end{cases}$$
(12)

where  $V(\xi_1)$  denotes the potential energy of the system which is to be chosen, and  $R(\xi_1, \xi_2)$  is a (possibly nonlinear) damping function which is to be chosen. The target system  $\Sigma_T$  is a simple pendulum system with a stable equilibrium  $\xi_* = (\xi_{1_*}, 0)$  with the energy function

$$H(\xi_1,\xi_2) = \frac{1}{2}\xi_2^2 + V(\xi_1).$$
(13)

To ensure the stability at the equilibrium we assume that *Assumption 4.1:* 

1) The potential energy function  $V(\xi_1)$  satisfies

$$\begin{cases} \left. \left. \frac{\partial V(\xi_1)}{\partial \xi_1} \right|_{\xi_1 = \xi_{1\star}} \right|_{\xi_1 = \xi_{1\star}} = 0 \\ \left. \left. \frac{\partial^2 V(\xi_1)}{\partial \xi_1^2} \right|_{\xi_1 = \xi_{1\star}} \right|_{\xi_1 = \xi_{1\star}} > 0 \end{cases}$$

2) The damping function satisfies  $R(\xi_{\star}) \ge 0$ .

#### Immersion Condition

Once we define a desired target dynamics, we define a mapping  $\pi: S^1 \times \mathbb{R} \to \mathbb{R}^3$  as follows:

$$\pi(\xi) \quad : = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi) \end{bmatrix} \tag{14}$$

where  $\pi_3(\xi)$  is to be chosen. Then with this choice of  $\pi(\xi)$  and the target dynamics (12), Equation (7) becomes

$$\begin{bmatrix} \xi_{2} \\ \frac{1}{M} \begin{bmatrix} P - D\xi_{2} - EV \frac{\sin \xi_{1}}{\pi_{3}(\xi)} \end{bmatrix} \\ \frac{1}{T_{dc}} \begin{bmatrix} -\pi_{3}(\xi) + x_{3_{\star}} \end{bmatrix} \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{dc}} \end{bmatrix} c(\pi(\xi))$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial \pi_{3}(\xi)}{\partial \xi_{1}} & \frac{\partial \pi_{3}(\xi)}{\partial \xi_{2}} \end{bmatrix} \begin{bmatrix} \xi_{2} \\ -\frac{\partial V(\xi_{1})}{\partial \xi_{1}} - R(\xi)\xi_{2} \end{bmatrix}.$$
(15)

Next we choose  $\pi_3(\xi)$  and  $c(\pi(\xi))$  to satisfy the above equation as follows: The first row of (15) is already satisfied. From the second row we have

$$\frac{1}{M}\left[P-D\xi_2-EV\frac{\sin\xi_1}{\pi_3(\xi)}\right] = -\frac{\partial V(\xi_1)}{\partial\xi_1}-R(\xi)\xi_2.$$

We choose  $R(\xi) = \frac{D}{M}$  and  $V(\xi_1) = -\beta \cos \tilde{\xi}_1$  for some  $\beta > 0$  (to be chosen). We use  $\tilde{\xi}_1$  to denote  $\xi_1 - \xi_{1_*}$ . Then the above equation becomes

$$\frac{1}{M} \left[ P - EV \frac{\sin \xi_1}{\pi_3(\xi)} \right] = -\beta \sin \tilde{\xi}_1$$

from which we get

$$\pi_3(\xi) = \frac{EV\sin\xi_1}{P+M\beta\sin\tilde{\xi}_1}$$

Notice that  $\pi_3$  is a function of  $\xi_1$  only. Here we make the following assumption:

Assumption 4.2:  $\beta < \frac{P}{M}$ .

This assumption makes  $\pi_3(\xi_1)$  bounded for all  $\xi_1$ . From the third row we have

$$\frac{1}{T_{dc}} \left[ -\pi_3(\xi_1) + x_{3\star} \right] + \frac{1}{T_{dc}} c(\pi(\xi)) = \frac{\partial \pi_3(\xi_1)}{\partial \xi_1} \xi_2.$$

By substituting for  $\pi_3(\xi_1)$  and  $V(\xi_1)$  in the above equation we get

$$c(\pi(\xi)) = T_{dc}EV\xi_2 \left[ \frac{\cos\xi_1}{P + M\beta\sin\tilde{\xi}_1} - \frac{M\beta\sin\xi_1\cos\tilde{\xi}_1}{\left[P + M\beta\sin\tilde{\xi}_1\right]^2} \right] + \frac{EV\sin\xi_1}{\left[P + M\beta\sin\tilde{\xi}_1\right]} - x_{3_{\star}}.$$

Thus we get  $\pi(\xi)$  and  $c(\pi(\xi))$ .

Implicit Manifold

The manifold  $\mathcal{M}$  is implicitly described by

$$\mathcal{M} = \{x \in S^1 \times \mathbb{R}^2 | \phi(x) = 0\}$$

with

$$\phi(x) = x_3 - \pi_3(x_1)$$
  
=  $x_3 - \frac{EV \sin x_1}{P + M\beta \sin \tilde{x}_1}$ 

where  $\tilde{x}_1$  denotes  $x_1 - x_{1_{\star}}$ .

#### Manifold attractivity and trajectory boundedness

Here the off-the-manifold coordinate is  $z = \phi(x)$  and we have that

$$\begin{aligned} \dot{z} &= \dot{x_3} - \dot{\pi}_3(x_1) \\ &= \dot{x_3} - \frac{\partial \pi_3(x_1)}{\partial x_1} \dot{x_1} \\ &= \frac{1}{T_{dc}} \left[ -x_3 + x_{3\star} + \psi(x,z) \right] - \frac{\partial \pi_3(x_1)}{\partial x_1} x_2 \\ &= \frac{\psi(x,z)}{T_{dc}} + \left[ \frac{-x_3 + x_{3\star}}{T_{dc}} - \frac{\partial \pi_3(x_1)}{\partial x_1} x_2 \right]. \end{aligned}$$

To ensure the boundedness of the trajectories of the offthe-manifold coordinate z and also that  $\lim_{t\to\infty} z(t) = 0$  we take

$$\dot{z} = -\gamma z, \quad \gamma > 0 \tag{16}$$

and then we have

$$\Psi(x,z) = T_{dc} \left[ -\gamma z + \frac{x_3 - x_{3\star}}{T_{dc}} + \frac{\partial \pi_3(x_1)}{\partial x_1} x_2 \right].$$

The control law u(x)

Next we calculate the control law as

$$u(x) = \Psi(x, \phi(x))$$

$$= T_{dc} \left[ -\gamma \phi(x) + \frac{x_3 - x_{3_{\star}}}{T_{dc}} + \frac{\partial \pi_3(x_1)}{\partial x_1} x_2 \right]$$

$$= (x_3 - x_{3_{\star}}) - T_{dc} \gamma \left[ x_3 - \frac{EV \sin x_1}{P + M\beta \sin \tilde{x}_1} \right]$$

$$- T_{dc} EV x_2 \left[ \frac{M\beta \sin x_1 \cos \tilde{x}_1}{[P + M\beta \sin \tilde{x}_1]^2} \right]$$

$$+ T_{dc} EV x_2 \left[ \frac{\cos x_1}{P + M\beta \sin \tilde{x}_1} \right]. \quad (17)$$

Finally, we establish boundedness of the trajectories of the closed-loop system (4) with the control law (17) and the off-the-manifold coordinate z

$$\begin{array}{l} \dot{x}_{1} = x_{2} \\ \dot{x}_{2} = \frac{1}{M} \left[ P - Dx_{2} - EV \frac{\sin x_{1}}{x_{3}} \right] \\ \dot{x}_{3} = \frac{1}{T_{dc}} \left[ -x_{3} + x_{3_{\star}} + u \right], \\ \dot{z} = -\gamma z. \end{array}$$

$$(18)$$

Here  $x \in S^1 \times \mathbb{R}^2$  and  $z \in \mathbb{R}$ . This implies  $x_1 \in \mathscr{L}_{\infty}$  where  $\mathscr{L}_{\infty}$  denotes the space of bounded functions. Now,

$$\dot{x}_{2} = \frac{1}{M} \left[ P - Dx_{2} - EV \frac{\sin x_{1}}{x_{3}} \right] \\ = -\frac{D}{M} x_{2} + \Delta(x_{1}, x_{3})$$
(19)

where  $\Delta(x_1, x_3) = \frac{1}{M} \left[ P - EV \frac{\sin x_1}{x_3} \right]$ . From Assumption 2.1 we have  $x_3 \ge d_2 > 0$  and also  $x_1$  is bounded as stated earlier. This implies  $\Delta(x_1, x_3)$  is bounded. As we have D > 0 and M > 0, (19) is an asymptotically stable linear system in  $x_2$  with a bounded driving function  $\Delta(x_1, x_3)$ . This implies  $x_2 \in \mathscr{L}_{\infty}$ .

Next, we have  $x_3 = z + \pi_3(x_1)$ . We have from (16) that *z* is bounded. Also, from Assumption 4.2 we have that  $\pi_3(x_1)$  is bounded for all  $x_1$ , and hence we can conclude boundedness of  $x_3$ .

The above discussion on the control synthesis can be summarized in the following proposition which is the main result of this paper.

*Proposition 4.1:* The closed-loop system (4) with the control law (17) is asymptotically stable at  $x_{\star}$ .

Proof:

Based on the arguments given above.

#### V. SIMULATION RESULTS

In this section we give simulation results for the control law given by (17). The simulation parameters are [26]:  $M = \frac{8}{100\pi}$ ,  $D = \frac{2}{100\pi}$ , E = 1.075 (p. u.), V = 1 (p. u.),  $x'_d = 0.85$ , P =



Fig. 2. Open loop performance



Fig. 3. Closed-loop performance with  $\gamma = 20$  and  $\beta = 1$ 

1.1(p. u.) and we assume  $T_{dc} = 0.1$ . The tuning parameters are  $\gamma$  and  $\beta$ . The performance of the controller was assessed for the following two different transients:

- 1) A short circuit fault occurs on the far end of the transmission line at time t = 20 s for a duration of about 0.1 s.
- 2) An open circuit fault occurs on one of the two parallel transmission lines resulting in change in  $x'_d$  at time t = 60 s for a duration of about 0.1 s.

The open loop performance as well as closed-loop performance is presented in the plots. Figure 2 shows the open loop response of the system to both the transients. The open loop response shows heavy oscillations in the swing angle and the angular velocity. For the first transient the magnitude of the oscillations is greater as compared to the second transient.

To assess the closed-loop performance we choose different values for  $\beta$  and  $\gamma$ . The tuning parameter  $\beta$  decides the shape of the energy function for the closed-loop system, and  $\gamma$  decides the rate at which the closed-loop system trajectories come closer to the desired trajectories.

Figure 3 shows simulation plots for  $\gamma = 20$  and  $\beta = 1$ . In this case the oscillations in the swing angle are significantly slow and have smaller amplitude as compared to the open loop response. Also, the response of the angular velocity shows very small amplitude oscillations.

In the second case we keep  $\gamma = 20$  and change  $\beta = 1$  to  $\beta = 5$ . The simulation results are shown in Figure 4.



Fig. 4. Closed-loop performance with  $\gamma = 20$  and  $\beta = 5$ 



Fig. 5. Closed-loop performance with  $\gamma = 100$  and  $\beta = 5$ 

In this case the closed-loop response improves slightly in magnitude, however, it is oscillatory in nature. The angular velocity experiences rapid oscillations as compared to the first case. This further reflects in the response of the swing angle. This shows the effect of increase in  $\beta$ , that is, an increase in  $\beta$  introduces oscillations in the load angle and the angular velocity.

In the third case we keep  $\beta = 5$  and increase  $\gamma = 20$  to  $\gamma = 100$ . Figure 5 shows simulation plots. In this case the oscillatory form of the closed-loop response is the same as that of the second case, since  $\beta$  is kept unchanged. However, oscillations decay fast as compared to the first two cases, which is the effect of an increase in  $\gamma$ .

Further, for a short circuit fault on the far end of the transmission line at time t = 20 s and duration of about 0.5 s the open loop system was found to be unstable. The closed-loop response is shown in Figure 6 for  $\gamma = 100$  and  $\beta = 40$ .

## VI. CONCLUSION

In this paper we presented a nonlinear control law based on Immersion and Invariance methodology to asymptotically stabilize the SMIB system with a CSC at an equilibrium. The SMIB was described by the swing equation model and the actuator by a first order model. A simple pendulum system with a suitable energy function was chosen as the target dynamics. We have chosen a manifold such that the closed loop system restricted to the manifold is the target dynamics.



Fig. 6. Closed-loop performance with  $\gamma = 100$  and  $\beta = 40$ 

The control law has been synthesized in order to render the manifold invariant and attractive. A few simulation results have been provided to show the controller performance.

## VII. ACKNOWLEDGMENTS

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