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Extremum seeking without external dithering and its application to plasma RF heating on FTU

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Abstract— **We propose a new global extremum seeking algorithm to optimize the coupling between the emitting Lower Hybrid (LH) antennas and the plasma scrape off layer in the radiofrequency (RF) heating system of tokamak plasmas. The new algorithm, where the disturbances affecting the system are used as probing signals, requires less stringent properties than the previous algorithms, and is more robust. Simulation results are presented illustrating the effectiveness of the algorithm on the Lower Hybrid RF heating system of the Frascati Tokamak Upgrade (FTU).**

I. INTRODUCTION

In many control applications certain outputs of a controlled dynamical system have to be driven to values that minimize/maximize an unknown function. Already since the early 1950s (see [1] and [2]), a number of researchers contributed to find the control law that solves this problem, called "extremum control" or "self-optimizing control". Extremum control has been defined in [3] as one of the most promising areas of adaptive control, and indeed, it still is the subject of a number of recent works. In [4], for the first time, local stability properties of an extremum seeking feedback scheme for general nonlinear systems has been formally proved, motivating further interesting results (see [5], [6]). Recently, in [7] an extremum controller slightly different from the one in [4] has been shown, under slightly stronger conditions, to formally guarantee non-local (semiglobal practical) stability properties. In both these approaches, convexity of the unknown function to be minimized is assumed. An application that recently benefited from the use of extremum seeking techniques is that of control of Tokamak plasmas. Tokamak plasmas correspond to experimental facilities wherein controlled nuclear fusion reactions are generated and maintained during pulsed experiments (often called "shots" due to their typical short duration) – an excellent of the current and future control aspects in control of Tokamak plasmas can be found in the two recent special issues [8], [9].

While nuclear fusion energy is extremely desirable from many viewpoints (see, [10]), there are several complications arising from controlling it on earth. Among them, heating is a major one because the high pressures of the sun (naturally caused by gravity therein) are not easily implementable on earth, so that the fusion temperatures have to be even higher here than those in the sun (reasonably, some hundreds of million Kelvin are needed on earth). Most of the heating of the current Tokamak experiments is obtained by Joule effect by way of a high current flowing in the plasma. However, since the plasma resistivity drops down as the temperature increases, alternative heating methods are necessary, especially in future experiments such as ITER [11], which is the ultimate worldwide international experiment jointly built by the international community in Cadarache (France). Among these alternative methods, radiofrequency heating seems to be the most promising and definitely the most widely experimented one. In this method, high frequency waves are delivered to the plasma via suitable antennas and the corresponding energy is absorbed by the plasma and transformed into heat via the same phenomenon that happens in microwave ovens: resonant modes of ions and electrons (or even hybrid resonances). The reason why extremum seeking is needed in these applications is that there's an important coupling between the antenna and the plasma Scrape Off Layer (SOL), namely the outer surface of the plasma within the Tokamak vacuum vessel. If the coupling is poor, then reflected waves can damage the antenna and typically cause undesired safety shutdowns. Moreover, the effectiveness of the radiofrequency heating is evidently proportional to the coupling between antenna and plasma, because optimized coupling causes maximum absorbed power, therefore temperature increase. Another research area within Tokamak plasmas control where extremum seeking has been employed is discussed in [12], but that context is not addressed in this paper.

In [13], some experimental results on the Frascati Tokamak Upgrade (FTU) [14], an experiment owned by ENEA in Frascati (Rome, Italy), showed that when using the Lower Hybrid (LH) antennas of FTU, naive solutions to problem of minimizing the reflected power already gave desirable performance improvements. The problem with these early solutions was slow convergence and lack of any guarantee. Later experiments employed a modified extremum seeking technique to solve the same problem [15], which showed increased performance and robustness in experiments, as compared to the previous solution of [13], even though from the experimental results it was evident that the algorithm employed had some space for improvement. Finally, implementation issues arising from the use of multiple antennas were reported in [16].

In this paper we revisit the application study and the

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experimental results of [15], [16] with the goal of improving the performance and robustness of the scheme. To this aim, we first address analytically the corresponding scenario, by developing a new extremum seeking control architecture. In particular, we use a novel Lyapunov-based proof technique as compared to that in the classical approaches [4], [7], to show that one can exploit the properties of a disturbance already present in the system, without having to inject an external dithering signal (this was the technique adopted in [15], [16] without any formal statement about its effectiveness). The desirable convergence properties of the new control law is achieved while assuming very little (not even convexity or differentiability) on the unknown function to be minimized. Finally, we show that based on this more solid theoretical framework, the effectiveness of the new scheme on the Tokamak plasma control problem in [15], [16] is highly increased, both in terms of performance and robustness, as compared to the previous results of [15]. Only simulations are carried out in this submitted version (using models accurately validated on experimental data) but experiments are scheduled for the next experimental campaign (during March 2009) and experimental results should be included in the next version of the paper, if accepted for publication.

The paper is organized as follows. In Section I-A we give the problem statement and propose the novel extremum seeking solution together with the main theorem, discussing some implementation issues and illustrating the effectiveness of the new schema on a simple case of study. In Section II we illustrate the application to the Tokamak plasma control problem of [15], [16] and show its advantages.

A. The new extremum seeking controller

We address the problem of finding the global minimum of a static unknown map $g(\cdot) : \mathbb{R} \to \mathbb{R}$ whose input is affected by a disturbance $t \mapsto d(t)$ as shown in Figure 1. In particular, we assume that the map $g(\cdot)$ satisfies the following assumption.

Assumption 1: The unknown map $g(\cdot) : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz, locally bounded and there exist a $y^\star \in \mathbb{R}$ and a class $\mathcal{K}$ function[1] $\gamma(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for almost all $s \in \mathbb{R}$:

$$\nabla g(s)(s - y^\star) \geq |s - y^\star|\gamma(|s - y^\star|). \tag{1}$$

□

Condition (1) implies that g is in the incremental[2] sector $(0, \infty)$ around $y^\star$. In our extremum seeking synthesis problem we assume to exploit a probing signal $\theta(\cdot)$ which is constrained to act on the function $g(\cdot)$ through a scalar dynamical system having the following simplified form[3]

$$\varepsilon \dot{y} = -y + \theta, \tag{2}$$

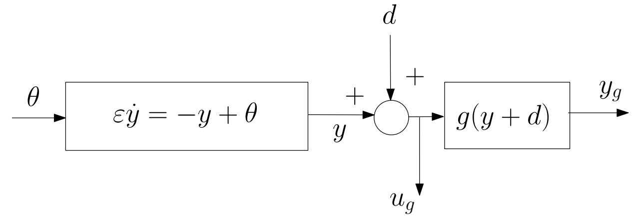

Fig. 1. Block diagram of the cascade connection between the input dynamics and the static map $g(\cdot)$ to be minimized, together with the two disturbances d and d_2.

where ε is a positive scalar. The following two signals are assumed to be available for measurement, as represented in Figure 1, corresponding to the input and the output of the static map $g(\cdot)$:

$$\begin{array}{rcl} u_g(t) &=& y(t) + d(t) \\ y_g(t) &=& g(y(t) + d(t)). \end{array} \tag{3}$$

The schemes proposed next rely on suitable properties of the disturbance d exciting the function $g(\cdot)$. The following assumptions will be necessary to prove the results stated in the next Theorem, the proof of which is omitted due to space constraints (the proof involves Lyapunov techniques and other standard tools for stability analysis of nonlinear systems).

Assumption 2: The disturbance $d(\cdot)$ is bounded and has bounded first and second time derivatives, namely there exist positive numbers $\bar{d}$, $\bar{d}_d$, and $\bar{d}_{dd}$ such that $|d(t)| \leq \bar{d}$, $|\dot{d}(t)| \leq \bar{d}_d$, and $|\ddot{d}(t)| \leq \bar{d}_{dd}$ for all $t \geq 0$. □

Assumption 3: The disturbance $d(\cdot)$ is such that there exist $T > 0$ and $c > 0$ satisfying

$$\int_t^{t+T} |\dot{d}(\tau)| d\tau \geq c \tag{4}$$

for all $t \geq 0$. □

The latter assumption is related to the persistence of excitation property of the disturbance signal $d(t)$. In the following we propose an extremum seeking law assigning θ, assuming that the signals in (3) are available, *i.e.* we assume to know

$$\begin{aligned} z_1(t) &= \dot{u}_g(t) = \dot{y}(t) + \dot{d}(t), \\ z_2(t) &= \dot{y}_g(t) = \frac{\partial g(y(t) + d(t))}{\partial y} \left(\dot{y}(t) + \dot{d}(t)\right). \end{aligned} \tag{5}$$

As detailed in Section II, for actual implementation we will use approximations of the signals in (5). Assume that the signals (5) are available. Then θ is generated by the following dynamics:

$$\dot{\theta} = -k_1 \mathrm{sat}\left(k_2 z_2(t) z_1(t)\right), \tag{6}$$

with $k_1 > 0$, $k_2 > 0$. The block diagram of the corresponding closed-loop (2), (5), (6) is depicted in Figure 2.

Remark 1: The selection of θ as in (6) guarantees that $\dot{\theta}(t) \leq k_1$, an appealing property of the control signal θ since it avoids exciting possible high frequency dynamics. Moreover, whenever necessary, the method allows to meet the rate saturation constraints of the physical devices. ∘

[1] A continuous function $\alpha : [0, +\infty) \to [0, +\infty)$ is of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. Moreover, it is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and $\alpha(r) \to +\infty$ as $r \to +\infty$,[17, page 144].

[2] A function $f : \mathbb{R} \to \mathbb{R}$ is in the incremental sector $(0, \infty)$ if ∇f is in the sector $(0, \infty)$.

[3] The theory here developed can be extended to a more general case where system (2) is replaced by a nonlinear dynamical system with suitable properties.

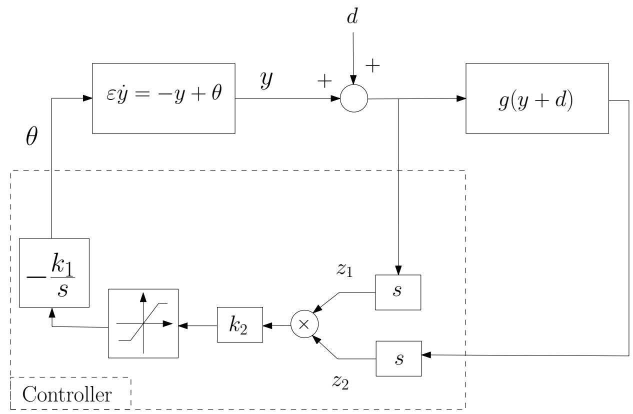

Fig. 2. The dynamic extremum seeking scheme (no measurement of y).

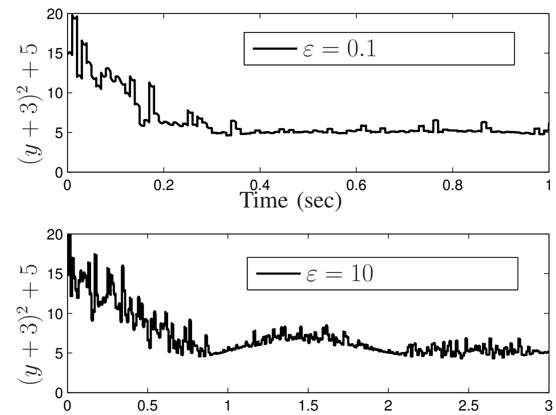

Fig. 3. Extremum seeking of the proposed schema with $\varepsilon\dot{y} = -y + \theta$ and $g(y) = (y+3)^2 + 5$ and with band-limited white noise acting on u_g and g_y.

The next theorem establishes boundedness of the closed-loop signals, as well as invariance and attractivity of the set $\mathcal{A}_\theta = \inf_{a\in\mathcal{A}}\{|\theta - a| \leq 2\varepsilon k_1 + \bar{d}\}$, where ε comes from (2), k_1 comes from (6) and $\bar{d} := \|d(\cdot)\|_\infty$ comes from Assumption 2.

Theorem 1: Assume that Assumptions 1 and 2 hold. Then for any positive constants k_1 and k_2, the closed-loop system (2), (5), (6), is such that both $y(\cdot)$ and $\theta(\cdot)$ are bounded, the set $\mathcal{A}_\theta$ is eventually forward invariant and attractive, moreover

$$g(y(t)) \leq \max_{a\in\mathcal{A}_\theta}\{g(a), g(y(0))\}, \,\forall\, t \geq 0.$$

□

For practical applications, it is unreasonable to assume that the signals in (5) are available, thus z_1 z_2 are estimated by filtering the two measurements u_g and y_g in such a way to eliminate also high frequency noises acting on them. To this aim, we replace the ideal derivator blocks in Figure 2 by two identical band-pass linear filters with transfer function

$$F(s) = \frac{s}{(\tau_1 s + 1)(\tau_2 s + 1)}, \tag{7}$$

where $\tau_1 << 1$ allows to perform an approximated derivative whereas τ_2 should be selected to filter out the measurement noise[4]. In the practical implementation, the following actual measurements from the system will be available, instead of those in (3):

$$\begin{aligned} \bar{u}_g(t) &= n_u(t) + y(t) + d(t) \\ \bar{y}_g(t) &= n_y(t) + g(y(t) + d(t)), \end{aligned} \tag{8}$$

where n_u and n_y represent high frequency measurement noise (possibly biased). Then, the signals z_1 and z_2 are evaluated as (5):

$$\begin{aligned} \bar{z}_1(s) &= F(s)\bar{u}_g(s) \\ \bar{z}_2(s) &= F(s)\bar{y}_g(s), \end{aligned} \tag{9}$$

where $F(s)$ corresponds to (7).

We illustrate the the closed-loop response of the proposed control scheme (2), (7) (8), (9) with (6) on a simple simulation example for two different values of ε (namely, $\varepsilon = 0.1$ and $\varepsilon = 10$), with the disturbance $d(t) = 0.05\sin(50t)$ and measurement noises n_u and n_y corresponding to a band-limited White Gaussian Noise of power 0.001. With these disturbances, we select $\tau_1 = 10^{-4}$ and $\tau_2 = 10^{-2}$ in the filter (7) so that the noise is filtered away from the frequency components of d. The extremum seeking law parameters are selected as $k_1 = 10\varepsilon$ and $k_2 = 1$ (remember that the saturation level is fixed at 1). The simulation results are reported in Figure 3.

[4]We are implicitly assuming that d_1 has some of its spectral components below the "high" frequency components of the measurement noise.

II. Application to plasma RF heating

The Frascati Tokamak Upgrade (FTU) [14] is an experimental facility running at the ENEA research center in Frascati (Rome, Italy) since 1990. Compact, high magnetic field devices are characterized by a large current density value, which makes them suitable for certain specific physics related experiments. The Position and Plasma Current feedback System (PPCS) running at FTU is such that the dominant dynamics of the plasma horizontal position, linearized around the equilibrium point reached at the steady state (when the plasma reaches a stable configuration called *flat-top phase*), can be approximated by the dynamics in (2) with $\varepsilon = 0.004$, where the output y is the plasma horizontal position and the probing signal $d(t)$ encompasses disturbances mainly induced by the power supplies. Note that with such a small value of ε, the dynamics solution is expected to work as good as the static one because the dynamics in (2) is extremely fast.

We rely here on the problem statement of [15], focusing on the lower hybrid (LH) heating antennas. The general setting is only summarized here and the interested reader is referred to [15] for additional details. When LH heating is active, part of the emitted wave is reflected by the plasma scrape off layer (SOL), namely by the outermost plasma surface within the vacuum vessel. The amount of reflected wave is a convex or quasi convex function of the plasma/antenna distance, therefore could be affected by suitably adjusting the plasma horizontal position. [5] The reflected wave, in addition to affecting negatively the heating effectiveness, can damage the emitting antenna and therefore may cause undesired safety shutdowns during the experiment. In terms of the signals involved in the extremum seeking schema of Figure 2, the input of the unknown function $g(\cdot)$ corresponds to the external radius of the plasma cross-section. This radius is estimated by the control system based on the magnetic measurements present on the plant. The input θ acts on the reference signal driving the plasma horizontal position stabilizer. The dynamics induced by the stabilizer+plasma closed-loop is exponentially stable and is represented in a compact way by its dominant time constant in (2). The output of $g(\cdot)$ corresponds to the percentage of reflected power by the plasma and is computed as the ratio between the emitted power and the reflected power measured, and averaged, among the eight central cells of the antenna (the antenna is composed by a grill of 48 cells displaced in four rows of 12 cells each). Note that due to physics related reasons the percentage reflected power cannot drop below a minimum level of $5 \div 8$ percent.

In [15], experimental results showed that using a generalization of the classical extremum seeking scheme of [4] (based on intuitive reasonings) could reduce significantly the reflected power thus making the heating process more effective. In [15] three most important experiments (also called "shots") were reported, where different gains were used on the extremum seeking loop:

1) in shot number 26725, using the gain $k = 200$, the extremum seeking algorithm performed excellently;
2) in shot number 26722, using the gain $k = 250$, the extremum seeking algorithm performed reasonably well, by converging to the desired minimum, even though the convergence was slow mainly due to a time interval where the algorithm dwelled for some reason on a non optimal value of the function; this showed that there was space for improvement of the closed-loop performance;
3) in shot number 26723, using a larger gain of $k = 350$ the extremum seeking induced violent and undesirable oscillations, thus showing a weak robustness of the algorithm, and possible space for improvement.

The fact that the choice of the parameter k is critical in the approach proposed in [15] is evident by the fact that very similar values of k lead to behaviors that are so different from each other. In particular, it is alarming to see that the performance at item 2 is worse than the performance at item 1, even though the selected gain k was higher. Moreover, it would be desirable to propose an algorithm with a larger stability region in light of the selection of the gain parameters. To suitably analyse the behavior of the extremum seeking law of [15] and validate the improved technique proposed in this paper in view of future experiments, signals stored in the FTU archive and corresponding to past experiments have been processed to set up a simulation tool.

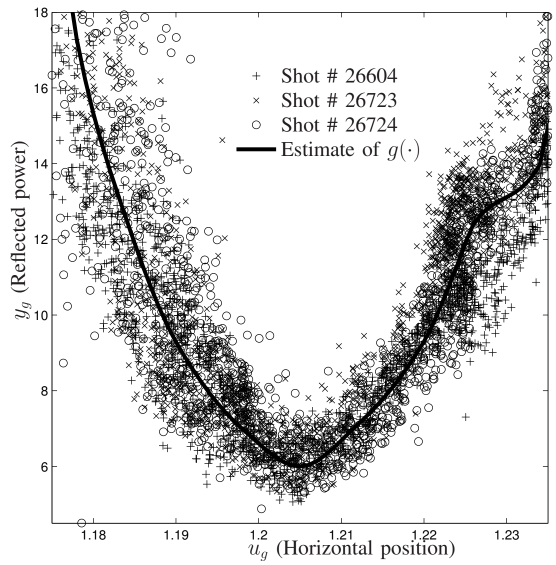

Fig. 4. The estimated graph of $g(\cdot)$ based on the experimental data from shots # 26604, 26723 and 26724.

In Figure 4 we show the experimental data of the measured reflected power $\overline{y}_g$ for three shots where the plasma horizontal position was oscillating. The experimental data are different from shot to shot since the position of the LH antenna is adjusted at each experiment. To suitably reproduce the experimental environment in the numerical simulation, we performed a numerical approximation (by averaging) of the function $g(\cdot)$ for each shot, and then the measurement noise $n_y(t)$ has been evaluated for each shot as well. The approximations of the function $g(\cdot)$ are nonconvex (see Figure 4, where the approximation of $g(\cdot)$ among three shots is shown) but satisfy our Assumption 1, whereas they do not satisfy the convexity assumption usually required in the classical extremum seeking solutions [4], [7].

As an example, Figure 5 depicts the estimates of $d+n_u$ and n_y for shot number 26722. The two disturbances have been estimated as follows: $d+n_u$ corresponds to the difference between the actual plasma horizontal position and the expected plasma horizontal position based on the extremum seeking adaptation signal and on the model (2); n_y corresponds to the difference between the actual reflected power and expected reflected power based on the input of $g(\cdot)$ (corresponding to the signal $d+n_u$ just estimated) and on the estimate of $g(\cdot)$ for that particular shot.

Remark 2: Figure 6 represents the wavelet analysis of the signal $d(t)+n_u(t)$ extracted from shot # 26722 (this signal is represented in the top plot of Figure 5). The analysis has been carried out using the Daubechies wavelets with $N = 8$ (referred to as `db8` in the routine `cwt` of Matlab). In the

[5] As commented in [15], adjusting the antenna position is unviable due to its huge mass and the required speed.

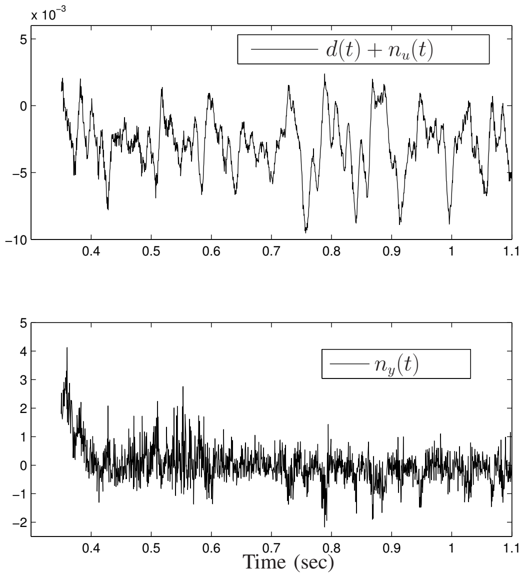

Fig. 5. The reconstructed signals $d(t) + n_u(t)$ (top) and $n_y(t)$ (bottom) for shot ♯26722.

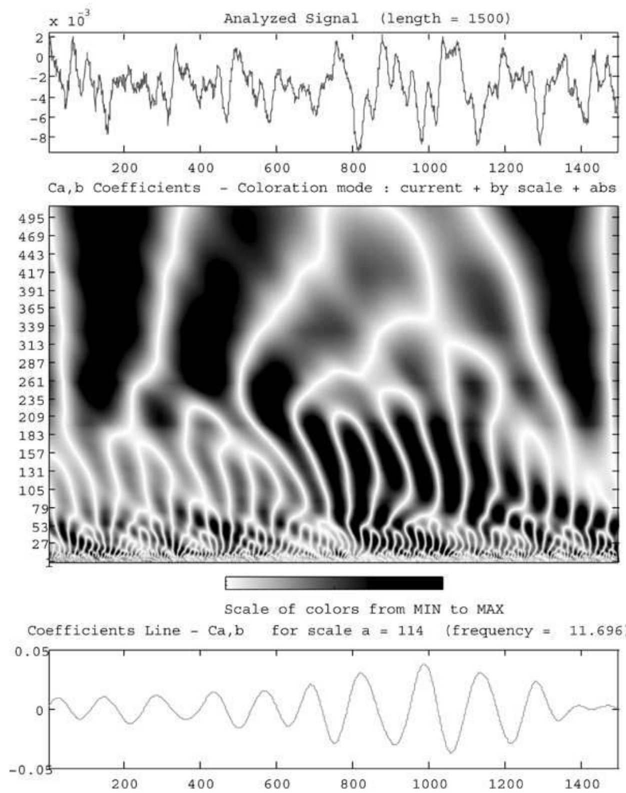

Fig. 6. Wavelet analysis of the disturbance $d + n_u$ extracted from shot # 26722.

middle graph of Figure 6 the colors represent different values of the wavelets coefficients associated with the wavelets scales; these scales, in turns, are shown on the vertical axis and they range from 2 up to 500. A pseudo-frequency from 660Hz up to 2.6Hz, respectively, can be associated with this range. An example of the wavelet associated with the scale value 114 is shown in the bottom plot of Figure 6. This analysis reveals that d contains different time varying frequencies (the wavelets with larger magnitude are associated with darker colors in the central plot). This does not allow, at least formally, to use the results in [4] nor [7], where the sinusoidal dithering signal has a suitable constant frequency. Conversely, the approach proposed here is able to exploit any type of excitation coming from d. ◦

With the disturbances identified above and shown in Figure 5, and also based on the wavelet analysis of d commented above and shown in Figure 7, we select the two time constants in the filter (7) as $\tau_1 = 10^{-4}$ and $\tau_2 = 10^{-2}$. Moreover, to induce a reasonably mild large signal aggressiveness, we select $k_1 = 0.5$, and to induce desirable small signal transients we select $k_2 = 0.03$.

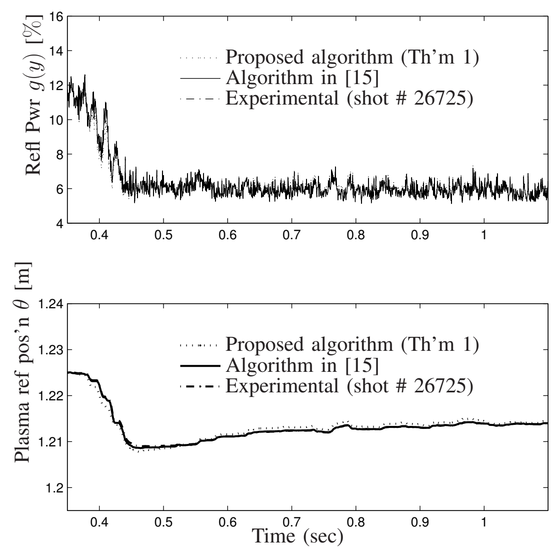

Fig. 7. Shot # 26725: Experimental and simulated data.

The first simulation test that we carry out is aimed at reproducing the experimental results of [15], so that the simulation tests with the new algorithm are reasonably reliable. The simulation test related to shot # 26725 is reported in Figure 7 (solid curves), where it is compared to the experimental data (dash-dotted curves). Note that these simulation are not open loop ones, but closed-loop simulations with the identified $g(\cdot)$ function and disturbances $d + n_u$, n_y acting on the closed-loop. It is therefore reasonable to expect some differences due to the fact that the dynamics (2) is a very rough first order linear approximation of very much more complex dynamics underlying the plasma response. Nevertheless, the two experimental and simulated responses are almost coincident, which indicates the suitability of our simulation setup to test the new control law in (6). A similar test is reported in Figure 8, which refers to shot # 26722.

Also in this second case, the simulated response (bold) is almost coincident with the experimental data (dash-dotted). Note that in all these simulations the disturbances $d + n_u$, n_y are different from shot to shot (Figure 5 shows, as an example, the noises used for shot # 26722) while the $g(\cdot)$ function is always the same and corresponds to the solid curve in Figure 4.

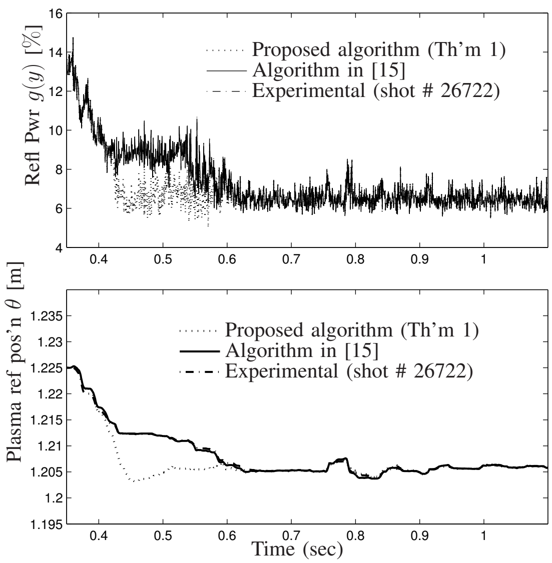

Fig. 8. Shot # 26722: Experimental and simulated data.

The advantages arising from using the dynamic technique (6) proposed in Section I-A is almost absent in the first simulation of Figure 7 since for this experiment there is not much space for improvement and the behavior of the new extremum seeking algorithm confirms its effectiveness, similar to the previous algorithm. Conversely, the simulation reported by dashed curves in Figure 8 shows that the new algorithm is effective at guaranteeing the performance improvement envisioned at item 2: indeed, differently from the algorithm of [15], the new algorithm does not dwell on a suboptimal value but rather converges monotonically to the optimal value, producing a very similar response to the desirable one of Figure 7.

As a final remark, it should be noted that, based on the discussion in Remark 1, the problems created by the algorithm in [15] corresponding to the selection $k = 350$, and commented at item 3 above, cannot occur with the new dynamic extremum seeking controller (6), as a matter of fact, the derivative of θ required to generate the experimental responses of shot # 26723 (which is not reported here due to space constraints) is impossible to generate with the selection $k_1 = 0.5$ used above. This property is a key fact for increased robustness of the extremum seeking scheme to be implemented on FTU.

III. Conclusions

In this paper we proposed a global extremum seeking controller without external dithering, exploiting disturbances already present in the control system. The proposed scheme also relaxes the convexity condition on the unknown function, typically required in classical extremum seeking schemes. The proposed extremum seeking controller has been shown to induce very desirable performance and robustness on plasma radiofrequency heating via Lower Hybrid antennas at the Frascati Tokamak Upgrade. Only simulation results (validated using past experimental data) are reported here but experiments are scheduled for the next experimental campaign.

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