Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

ThB03.1

Set-membership Identification of LPV models
with uncertain measurements of the time-varying parameter

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Abstract— In this paper the identification of SISO Linear Parameter Varying (LPV) models when both the output and the time-varying parameter measurements are affected by bounded noise is considered. First, the problem is formulated in terms of Error-in-variables (EIV) identification in presence of bounded noise. Then, a polytopic outer approximation of the feasible parameter set (FPS) and the parameters uncertainty intervals (PUI) of the LPV model are computed solving the EIV problem.

Index Terms— Linear Parameter Varying, bounded uncertainty, errors-in-variable, parameter bounding, linear programming.

I. INTRODUCTION

Most physical systems are inherently nonlinear, and, though in some cases they can be represented by linear models over a restricted operating range, nonlinear representations are often required for their proper modeling. In recent years, among the many different available techniques to derive mathematical description of nonlinear systems, Linear Parameter Varying (LPV) modeling approach received a major attention from the identification and control research community, mainly due to the strong connection between LPV models and Gain Scheduling Control which, thanks to the relevant research efforts devoted to the subject in the last two decades, is now recognized as an effective technique for the control of a large class of nonlinear systems (see the survey paper [1] for a thorough review of the literature on the subject).

LPV models belong to the more general class of linear time-varying models and, roughly speaking, they can be defined as linear systems where either the matrices of the state equations or the coefficients of the difference equation relating the input and the output signals depend on one or more time varying parameters whose real-time measurement are assumed to be available. According to [1], a linear parameter varying model of a nonlinear plant can be obtained by means of two different approaches. The first one, called *linearization scheduling*, is based on Jacobian linearization of the nonlinear system around a family of equilibrium points parameterized by a set of measurable parameters (the so called scheduling variables). Such a linearization leads to a family of linear models parameterized by the scheduling variables which actually represent an LPV model of the nonlinear plant. The second approach is the so called *quasi-LPV scheduling* which is based on the idea

of hiding nonlinearities in the system equations by suitably redefining some measurable state variables as time-varying parameters (see [1] for details). A relatively small number of papers can be found in the literature which address the problem of identifying an LPV model from experimental data, although a rapidly increasing attention has been devoted to the problem in recent years. The paper by Nemani, Ravikanth and Bamieh [2] seems to be the first contribution addressing the problem. They consider linear parameter varying models with a single time-varying parameter and assume that the measurement of all the state variables are available. Both the cases of exact and noisy measurement are considered and the identification of the parameter of the LPV model is reformulated as a least squares estimation problem. The proposed algorithm is proved to be convergent under some assumptions on the system. In the paper by Lee and Poolla [3] LPV models with multiple time-varying parameters are considered under the assumption that the parameter dependence can be written as a Linear Fractional Transformation (LFT). A parameter estimation scheme based on the minimization of a prediction error cost function by means of nonlinear (nonconvex) optimization is proposed. In order to minimize the effect of the presence of local minima on the estimation accuracy, a suitable boostrapping approach is proposed for the initialization of the estimation procedure for the particular case of LPV models whose parameter dependence is affine. Bamieh and Giarré in [4] consider the identification of LPV input-output models assuming that measurements of inputs, outputs and scheduling parameters are available. They show that the identification problem can be reformulated in terms of a linear regression model estimation and provide compact formulae for Least Mean Square and Recursive Least Square algorithms solving the problem. Besides, persistency of excitation conditions in terms of inputs and scheduling parameters trajectories are derived for the case of LPV models with polynomial time-varying parameters dependence. The proposed technique has been applied to rotating stall and surge control problem in [5]. Subspace identification of MIMO LPV models with affine parameter dependence is considered in the papers [6], [7] and [8]. In [6], Verdult and Verhaegen show that standard subspace algorithm cannot be used in practice to identify LPV models since the dimensions of the data matrices grow exponentially with the order of the system. Then, they propose a suitable modification of the subspace algorithm in order to overcome the curse of dimensionality. Significant improvements of the method proposed in [6] have been presented in [7], exploiting kernel methods, and in [8]

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where an instrumental variable approach is considered and the positive effect of using periodic scheduling sequences is highlighted. Application of LPV subspace identification algorithms to both periodic and nonlinear systems have been discussed in [9] by Verdult, Lovera and Verhaegen. On the basis of the extensive simulation study conducted in the paper, they also show that the proposed approach can be used to provide accurate models of helicopter rotor dynamics. Separable least squares have been exploited by Previdi and Lovera in [10] to derive a novel algorithm for a class of nonlinear parameter-varying models represented in the form of a linear fractional transformation where the forward part is a standard linear regression and the feedback time-varying part is modeled by means of a neural network.

In all the papers mentioned above, the authors assume that the measurement error η_t is statistically described. However, there are many cases where in practice either a priori statistical hypotheses are seldom satisfied or the errors are better characterized in a deterministic way. Some examples are given by systematic and class errors in measurement equipments, rounding and truncation errors in digital devices. A worthwhile alternative to the stochastic description of measurement errors is the bounded-errors or set-membership characterization, where uncertainties are assumed to belong to a given set. In the bounding context, all parameter vectors belonging to the *feasible parameter set* (*FPS*), i.e. parameters consistent with the measurements, the error bounds and the assumed model structure, are feasible solutions of the identification problem. The interested reader can find further details on this approach in a number of survey papers (see, e.g., [11], [12]), in the book edited by Milanese *et al.* [13] and the special issues edited by Norton [14], [15].

To the authors best knowledge, only a couple of contributions can be found which address the identification of LPV models when the measurement errors are supposed to be bounded. In [16] the problem of identification and model validation of LPV systems in presence of bounded noise and a possible nonparametric part is considered by Sznaier and Mazzaro. A solution is proposed recasting the problem in terms of checking the feasibility of a set of linear matrix inequalities. The obtained results are applied to a practical problem arising in the context of active vision showing the effectiveness of the proposed approach. Belforte and Gay [17] consider the identification of discrete-time LPV models with Finite Impulse Response (FIR) structure and output measurement affected by bounded noise. The coefficients of the FIR system are assumed to be nonlinear functions of multiple time-varying parameters. The problem of optimal worst-case experiment design is addressed. More specifically, optimal sequences for both the input signal and the scheduling parameters are derived which guarantee the computation of the lowest worst case parameter uncertainty intervals for LPV-FIR systems.

In this paper the identification of SISO input-output discrete-time Linear Parameter Varying (LPV) models when both the output and the time-varying parameters measurements are affected by bounded noise is considered. To the authors best knowledge this is the first attempt of addressing such a problem. The paper is organized as follows. Section II is devoted to the problem formulation. In Section III the addressed problem is firstly rewritten in terms of Error-In-Variables (EIV) identification, then a polytopic outer-approximation of the feasible parameter set is derived and finally parameter uncertainty intervals are computed. A simulated example is presented in Section IV showing the effectiveness of the proposed identification scheme. Concluding remarks end the paper.

II. PROBLEM FORMULATION

Consider the SISO discrete-time LPV model described by the following equations

$$w_t = \frac{B(q^{-1}, \lambda_t)}{A(q^{-1}, \lambda_t)} u_t = G(q^{-1}, \lambda_t) u_t, \qquad (1)$$

or, equivalently, in terms of a linear difference equation

$$A(q^{-1}, \lambda_t) w_t = B(q^{-1}, \lambda_t) u_t, \qquad (2)$$

where u_t, w_t are the input and the output signals respectively, while $\lambda_t \in \Re^\alpha$, $\lambda_t = [\lambda_1^t \lambda_2^t \dots \lambda_\alpha^t]^{\mathrm{T}}$ is a vector of time-varying parameters which, according to the LPV modeling and control literature (see, e.g., [1]) are assumed to be measurable. The numerator $A(\cdot)$ and the denominator $B(\cdot)$ of the transfer function $G(\cdot)$ are polynomials in the backward shift operator q^{-1}, $(q^{-1} w_t = w_{t-1})$,

$$A(q^{-1}, \lambda_t) = 1 + a_1(\lambda_t) q^{-1} + \dots + a_{na}(\lambda_t) q^{-na}, \quad (3)$$

$$B(q^{-1}, \lambda_t) = b_0(\lambda_t) + b_1(\lambda_t) q^{-1} + \dots + b_{nb}(\lambda_t) q^{-nb} \, (4)$$

whose coefficients a_i and b_j are assumed to be nonlinear memoryless functions of the parameters λ_t respectively described by

$$a_i(\lambda_t) = \sum_{k=1}^{n_i} a_{i,k} f_{i,k}(\lambda_t) \qquad (5)$$

and

$$b_j(\lambda_t) = \sum_{h=0}^{m_j} b_{j,h} g_{j,h}(\lambda_t) \qquad (6)$$

Let y_t and r_t be the noise-corrupted measurements of w_t and λ_t respectively

$$y_t = w_t + \eta_t. \qquad (7)$$

$$r_t = \lambda_t + \varepsilon_t. \qquad (8)$$

where $\varepsilon_t = [\varepsilon_1^t \varepsilon_2^t \dots \varepsilon_\alpha^t]^{\mathrm{T}}$. Measurements uncertainties η_t and ε_{s_t} are known to range within given bounds $\Delta \eta_t$ and $\Delta \varepsilon_{s_t}$, more precisely

$$|\eta_t| \leq \Delta \eta_t \qquad (9)$$

and

$$\varepsilon_t \in \mathcal{E} = \left\{ \varepsilon_t \in \Re^\alpha : |\varepsilon_s^t| \leq \Delta \varepsilon_s^t, s = 1, 2, \dots, \alpha \right\} \quad (10)$$

The unknown parameter vector $\theta \in R^p$ to be estimated is defined as

$$\theta^{\mathrm{T}} = [a_{1,1} \dots a_{1,n_1} \dots a_{na,1} \dots a_{na,n_{na}} \\ b_{0,1} \dots b_{0,m_1} \dots b_{nb,1} \dots b_{nb,m_{nb}}], \qquad (11)$$

where $p = \sum_{i=1}^{na} n_i + \sum_{j=0}^{nb} m_j$. In the set-membership context, all parameter vectors belonging to the so-called *feasible parameter set* (*FPS*), i.e. parameters consistent with the measurements, the error bounds and the assumed model structure, are feasible solutions of the identification problem. Given N samples of the signals u_t, y_t and r_t, the feasible parameter set for the LPV system described by equations (1) - (10), is defined as

$$\mathcal{D}_\theta = \{\theta \in R^p : A(q^{-1}, r_t - \varepsilon_t)[y_t - \eta_t] = B(q^{-1}, r_t - \varepsilon_t)u_t;$$
$$\mid \eta_t \mid \leq \Delta\eta_t;\ \varepsilon_t \in \mathcal{E};\ t = 1, .., N\}.$$
$$(12)$$

As will be clarified in Section III, the exact feasible parameter region $\mathcal{D}_\theta$ is, in general, a noncovex set described by nonlinear inequalities whose shape may become fairly complex for increasing values of N. Thus, in this paper a polytopic outer approximation $\mathcal{D}'_\theta$ of the exact feasible parameter set $\mathcal{D}_\theta$ will be derived. Such a polytope will be also used to compute Parameter Uncertainty Intervals (PUI) for each component of the parameter vector θ.

III. BOUNDING THE PARAMETERS OF THE LPV MODEL

Combining equations (1) – (6) we get the following-linear-in-parameter regression model:

$$w_t = -\sum_{i=1}^{na} w_{t-i} \sum_{k=1}^{n_i} a_{i,k} f_{i,k}(\lambda_t)$$
$$+ \sum_{j=0}^{nb} u_{t-j} \sum_{h=1}^{m_j} b_{j,h} g_{j,h}(\lambda_t)$$
$$(13)$$

which can be rewritten in the following compact form:

$$w_t = \phi_t^T \theta \qquad (14)$$

where:

$$\phi_t^T = [\rho_{1,1}^t \cdots \rho_{1,n_1}^t \cdots \rho_{na,1}^t \cdots \rho_{na,n_{na}}^t$$
$$\gamma_{1,1}^t \cdots \gamma_{1,m_1}^t \cdots \gamma_{nb,1}^t \cdots \gamma_{nb,m_{nb}}^t] \qquad (15)$$

$$\rho_{i,k}^t = -w_{t-i} f_{i,k}(\lambda_t) \qquad (16)$$

$$\gamma_{j,h}^t = u_{t-j} g_{j,h}(\lambda_t). \qquad (17)$$

According to equations (7) – (10), w_{t-i} and $\lambda_t = [\lambda_1^t \lambda_2^t \dots \lambda_\alpha^t]^T$ are uncertain variables which belong to the following intervals

$$\underline{w}_{t-i} \leq w_{t-i} \leq \overline{w}_{t-i} \qquad (18)$$

$$\underline{\lambda}_s^t \leq \lambda_s^t \leq \overline{\lambda}_s^t, \text{ for } s = 1, \dots, \alpha \qquad (19)$$

where

$$\underline{w}_{t-i} = y_{t-i} - \Delta\eta_{t-i}, \ \overline{w}_{t-i} = y_{t-i} + \Delta\eta_{t-i}$$
$$\underline{\lambda}_s^t = r_s^t - \Delta\varepsilon_s^t, \ \overline{\lambda}_s^t = r_s^t + \Delta\varepsilon_s^t, \text{ for } s = 1, \dots, \alpha. \qquad (20)$$

Due to the uncertainty affecting λ_t, also $f_{i,k}(\lambda_t)$ and $g_{j,h}(\lambda_t)$ are uncertain variables which belong to the following intervals:

$$\underline{f}_k(\lambda_t) \leq f_{i,k}(\lambda_t) \leq \overline{f}_k(\lambda_t) \qquad (21)$$

$$\underline{g}_h(\lambda_t) \leq g_{j,h}(\lambda_t) \leq \overline{g}_h(\lambda_t) \qquad (22)$$

where

$$\underline{f}_k(\lambda_t) = \min_{\varepsilon_t \in \mathcal{E}} f_{i,k}(r_t - \varepsilon_t), \ \overline{f}_k(\lambda_t) = \max_{\varepsilon_t \in \mathcal{E}} f_{i,k}(r_t - \varepsilon_t)$$
$$(23)$$

and

$$\underline{g}_h(\lambda_t) = \min_{\varepsilon_t \in \mathcal{E}} g_{j,h}(r_t - \varepsilon_t), \ \overline{g}_h(\lambda_t) = \max_{\varepsilon_t \in \mathcal{E}} g_{j,h}(r_t - \varepsilon_t)$$
$$(24)$$

Remark 1 — The solution of problems (23) and (24) requires the computation of the global maxima and minima of nonlinear functions $f_{i,k}(\cdot)$ and $g_{j,h}(\cdot)$. While such global extremes can be easily computed for the case of convex $f_{i,k}(\cdot)$ and $g_{j,h}(\cdot)$, standard nonlinear optimization methods usually trap in local maxima/minima when $f_{i,k}(\cdot)$ and $g_{j,h}(\cdot)$ are nonconvex functions. In such a case global maxima and minima can be obtained by means of branch and bound optimization method which, however, can be computationally cumbersome when the dimension of the time-varying parameter $\lambda_t \in \Re^\alpha$ is large. Note that a number of significant real world applications involves quite a low number of time-varying parameters.

Once bounds $\underline{f}_k(\lambda_t)$, $\overline{f}_k(\lambda_t)$, $\underline{g}_h(\lambda_t)$, $\overline{g}_h(\lambda_t)$ have been obtained, extreme values of interval variables $\rho_{i,k}^t$ and $\gamma_{j,h}^t$ defined as

$$\underline{\rho}_{i,k}^t = \min_{\varepsilon_t \in \mathcal{E}} \rho_{i,k}^t, \qquad \overline{\rho}_{i,k}^t = \max_{\varepsilon_t \in \mathcal{E}} \rho_{i,k}^t \qquad (25)$$

and

$$\underline{\gamma}_{j,h}^t = \min_{\varepsilon_t \in \mathcal{E}} \gamma_{j,h}^t, \qquad \overline{\gamma}_{j,h}^t = \max_{\varepsilon_t \in \mathcal{E}} \gamma_{j,h}^t \qquad (26)$$

can be easily computed applying basic results on interval arithmetics (see, e.g., [18]) to equations (16) and (17).
A compact description of $\rho_{i,k}$ and $\gamma_{j,h}$ in terms of their central values $c(\rho_{i,k}^t)$, $c(\gamma_{j,h}^t)$ and their perturbations $\delta\rho_{i,k}^t$, $\delta\gamma_{j,h}^t$ are as follows

$$\rho_{i,k}^t = c(\rho_{i,k}^t) + \delta\rho_{i,k}^t \qquad (27)$$

$$\gamma_{j,h}^t = c(\gamma_{j,h}^t) + \delta\gamma_{j,h}^t. \qquad (28)$$

where

$$|\delta\rho_{i,k}^t| \leq \Delta\rho_{i,k}^t, \ |\delta\gamma_{j,h}^t| \leq \Delta\gamma_{j,h}^t \qquad (29)$$

$$\Delta\rho_{i,k}^t = \frac{\overline{\rho}_{i,k}^t - \underline{\rho}_{i,k}^t}{2}, \ c(\rho_{i,k}^t) = \frac{\underline{\rho}_{i,k}^t + \overline{\rho}_{i,k}^t}{2} \qquad (30)$$

$$\Delta\gamma_{j,h}^t = \frac{\overline{\gamma}_{j,h}^t - \underline{\gamma}_{j,h}^t}{2}, \ c(\gamma_{j,h}^t) = \frac{\underline{\gamma}_{j,h}^t + \overline{\gamma}_{j,h}^t}{2}. \qquad (31)$$

Thanks to equations (27) and (28), the regressor ϕ_t of equation (15) can be rewritten as

$$\phi_t = \varphi_t + \delta\varphi_t \qquad (32)$$

where

$$\varphi_t^T = [\varphi_1^t \dots \varphi_p^t]$$
$$= [c(\rho_{1,1}^t) \dots c(\rho_{1,n_1}^t) \dots c(\rho_{na,1}^t) \dots c(\rho_{na,n_{na}}^t) \qquad (33)$$
$$c(\gamma_{1,1}^t) \dots c(\gamma_{1,m_1}^t) \dots c(\gamma_{nb,1}^t) \dots c(\gamma_{nb,m_{nb}}^t)]$$

$$\delta\varphi_t^{\mathrm{T}} = [\delta\varphi_1^t \ldots \delta\varphi_p^t]$$
$$= [\delta\rho_{1,1}^t \ldots \delta\rho_{1,n_1}^t \ldots \delta\rho_{na,1}^t \ldots \delta\rho_{na,n_{na}}^t \qquad (34)$$
$$\delta\gamma_{1,1}^t \ldots \delta\gamma_{1,m_1}^t \ldots \delta\gamma_{nb,1}^t \ldots \delta\gamma_{nb,m_{nb}}^t]$$

Reduction of equations (7), (14) and (32) leads to the following regression model

$$y_t = (\varphi_t + \delta\varphi_t)^{\mathrm{T}}\theta + \eta_t. \qquad (35)$$

which fits in the framework of the Errors-In-Variables (EIV) identification, i.e. a parameter estimation problem in a linear-in-parameter model where the output and some or all the explanatory variables are uncertain. From equation (35) it can be seen that consecutive regressions are related deterministically by uncertain measurement samples (dynamic EIV) giving rise to an exact feasible parameter set $\mathcal{D}_\theta$, described by a set of nonlinear nonconvex inequalities, whose shape may become fairly complex for increasing values of N. As a consequence, tight parameters uncertainty intervals could not be easily and exactly computed on the basis of $\mathcal{D}_\theta$ [19]. On the other end, in the static EIV case exact parameter bounds are piecewise linear and, although generally non convex, the feasible parameter region is the union of at most 2^p polytopes: each being the intersection of the FPS with a single orthant of the p-dimensional parameter space (a detailed discussion on the geometrical and topological structure of the feasible parameter region for static EIV problems can be found in [20]). Thus, as shown in [19], the FPS of static EIV can be more conveniently handled than the FPS of dynamic EIV. That motivates the use, in this paper, of results from the static EIV [20] which, however, will lead to outer approximations of the exact FPS. More precisely, assuming that the uncertain components $\delta\varphi_t$ of the regressor ϕ_t are permitted to vary independently, the following outer approximation $\mathcal{D}_\theta'$ of the exact FPS $\mathcal{D}_\theta$, i.e. $\mathcal{D}_\theta' \supset \mathcal{D}_\theta$, is obtained:

$$\mathcal{D}_\theta' = \{\theta \in R^p : y_t = (\varphi_t + \delta\varphi_t)^{\mathrm{T}}\theta + \eta_t; \mid \eta_t \mid \le \Delta\eta_t;$$
$$\mid \delta\varphi_1^t \mid \le \Delta\varphi_1^t; \ldots \mid \delta\varphi_p^t \mid \le \Delta\varphi_p^t; t = 1, \ldots, N\}. \qquad (36)$$

When we apply results from [20] to our problem we get the following description of the set $\mathcal{D}_\theta'$ at the single time t

$$(\varphi_t - \Delta\varphi_t)^{\mathrm{T}}\theta \le y_t + \Delta\eta_t \qquad (37)$$
$$(\varphi_t + \Delta\varphi_t)^{\mathrm{T}}\theta \ge y_t - \Delta\eta_t \qquad (38)$$

where

$$\Delta\varphi_t^{\mathrm{T}} = [\Delta\rho_{1,1}^t \mathrm{sgn}(a_{1,1}) \ldots \Delta\rho_{1,n_1}^t \mathrm{sgn}(a_{1,n_1})$$
$$\Delta\rho_{na,1}^t \mathrm{sgn}(a_{na,1}) \ldots \Delta\rho_{na,n_{na}}^t \mathrm{sgn}(a_{na,n_{na}})$$
$$\Delta\gamma_{0,1}^t \mathrm{sgn}(b_{0,1}) \ldots \Delta\gamma_{0,m_1}^t \mathrm{sgn}(b_{0,m_1}) \qquad (39)$$
$$\Delta\gamma_{nb,1}^t \mathrm{sgn}(b_{nb,1}) \ldots \Delta\gamma_{nb,m_{nb}}^t \mathrm{sgn}(b_{nb,m_{nb}})].$$

For each component θ_l of the parameter vector θ, lower and upper bounds θ_l^{min} and θ_l^{max} can be computed solving the following two linear programming problems:

$$\theta_l^{min} = \min_{\theta \in \mathcal{D}_\theta'} \theta_l, \quad \theta_l^{max} = \max_{\theta \in \mathcal{D}_\theta'} \theta_l. \qquad (40)$$

The computed bounds, which implicitly define the so called parameter uncertainty interval $PUI_{\theta_l} = [\theta_l^{min}\ \theta_l^{max}]$, can be used to computed the central estimate θ^c of the parameter vector θ defined as:

$$\theta^c = [\theta_1^c \ldots \theta_p^c]^{\mathrm{T}} \qquad (41)$$

where

$$\theta_l^c = \frac{\theta_l^{min} + \theta_l^{max}}{2}, \ l = 1 \ldots p. \qquad (42)$$

The parameter vector θ^c is the Chebishev centers in the ℓ_∞ norm of $\mathcal{D}_\theta'$ and enjoys peculiar optimality properties (see [21] for details).

Remark 2 — In principle, the computation of θ_l^{min} and θ_l^{max}, $l = 1, \ldots, p$ requires the solution of $2p2^p$ LP problems (the coefficient $2p$ accounts for p minimization problems and p maximization problems while 2^p is the number of orthant in the p-dimensional parameter space in which the above $2p$ optimization problems must be carried out) with $2N + p$ constraints ($2N$ constraints derive from equations (37) and (38) with $t = 1, \ldots, N$; p is the number of constraints defining the orthant in the parameter space). In practice, however, the computational load can be significantly reduced if the signs of θ_l, $l = 1, \ldots, p$ are a-priori known. Indeed, in that case the number of LP problems dramatically decreases to $2p$. If not available, information about the signs of θ_l can be achieved through a point estimate (using least squares estimates, for example) which will indicate the orthant where the optimization should be carried out. If some of θ_l^{min} (θ_l^{max}) obtained working on such an orthant are zero, then the optimization problems should be also solved in the orthants characterized by $\theta_l < 0$ ($\theta_l > 0$).

IV. A SIMULATED EXAMPLE

In this section we illustrate the proposed parameter bounding procedure through a numerical example. The system considered here is characterized by (2) with: $A(q^{-1}) = 1 + (0.5\lambda_t)q^{-1} + (-0.2 + 0.3\lambda_t^2)q^{-2}$ and $B(q^{-1}) = 0.1q^{-1} + (1\lambda_t + 0.2\lambda_t^2)q^{-2}$. Thus, in this example, the system coefficients depend on a single time-varying parameter λ_t and the function $f_{i,k}$ and $g_{j,h}$ of equations (5) and (6) are the following polynomial basis functions $f_{1,1} = \lambda_t$, $f_{2,1} = 1$, $f_{2,2} = \lambda_t^2$, $g_{1,1} = 1$, $g_{2,1} = \lambda_t$, $g_{2,2} = \lambda_t^2$. The true parameter vector is $\theta = [a_{1,1}\ a_{2,1}\ a_{2,2}\ b_{1,1}\ b_{2,1}\ b_{2,2}]^{\mathrm{T}} = [0.5\ -0.2\ 0.3\ 0.1\ 1\ 0.2]^{\mathrm{T}}$. From the simulated transient sequences $\{w_t, \eta_t\}$ and $\{\lambda_t, \varepsilon_t\}$, the signal to noise ratios SNR are evaluated through

$$SNR_w = 10 \log \left\{ \sum_{t=1}^N w_t^2 \bigg/ \sum_{t=1}^N \eta_t^2 \right\} \qquad (43)$$

$$SNR_\lambda = 10 \log \left\{ \sum_{t=1}^N \lambda_t^2 \bigg/ \sum_{t=1}^N \varepsilon_t^2 \right\}. \qquad (44)$$

Bounded absolute errors have been considered when simulating the collection of transient data. More precisely, we assumed $|\eta_t| \le \Delta\eta_t$, $|\varepsilon_t| \le \Delta\varepsilon_t$ where $\{\eta_t\}$ and $\{\varepsilon_t\}$ are random sequences belonging to the uniform distributions

$U[-\Delta\eta_t, +\Delta\eta_t]$ and $U[-\Delta\varepsilon_t, +\Delta\varepsilon_t]$ respectively. Bounds on output and varying parameter measurement errors were supposed to have the same value, i.e., $\Delta\eta_t = \Delta\varepsilon_t = \Delta E$. Four different values of uncertainty bounds ΔE were considered corresponding to four different signal-to-noise ratios. For each given ΔE, we consider two different data lengths, $N = 100$ and $N = 1000$. The input sequence $\{u_t\}$ is a random uniform distributed signal which takes values in the interval $[-1, 1]$ while two different cases are considered for the trajectory of the time-varying parameter λ_t. In the first case λ_t is assumed to be a random uniform distributed signal which takes values in the interval $[-1, 1]$, while in the second one $\lambda_t = \sin(0.1t)$ is assumed.

The obtained results are reported in Tables I — IV. For low noise level ($SNR_w, SNR_\lambda \approx 60$ dB) and for all N, the central estimates are consistent with the true parameters. For higher noise level ($SNR_w, SNR_\lambda \leq 40$dB), θ^c gives satisfactory estimate of the true parameters. As the number of observations increases (from $N = 100$ to $N = 1000$), parameter uncertainty bounds $\Delta\theta_l = |\theta_l^{max} - \theta_l^{min}|/2$ decreases unsurprisingly.

V. CONCLUSION

A parameter bounding procedure for SISO discrete-time LPV models when both the output and the time-varying parameter measurements are affected by bounded noise has been outlined. First, the problem is formulated in terms of Error-in-variables (EIV) identification in presence of bounded noise. Then, a polytopic outer approximation of the feasible parameter set (FPS) and the parameters uncertainty intervals (PUI) of the LPV model are computed solving the EIV problem. The effectiveness of the proposed approach has been demonstrated by means of a simulated example.

ACKNOWLEDGMENT

Table I: Parameter central estimates (θ_l^c) and parameter uncertainty bounds ($\Delta\theta_l^c$) against signal-to-noise ratios (SNR_w and SNR_λ) when λ_t is a uniform random sequence and $N = 100$

SNR_w (dB)	SNR_λ (dB)	θ_l	True Value	θ_l^c	$\Delta\theta_l$
58.9	61.3	θ_1	0.5	0.500	1.6e-3
		θ_2	-0.2	-0.200	2.7e-3
		θ_3	0.3	0.299	3.8e-3
		θ_4	0.1	0.100	9.4e-4
		θ_5	1.0	1.000	2.5e-3
		θ_6	0.2	0.201	2.8e-3
44.2	48.1	θ_1	0.5	0.501	8.9e-3
		θ_2	-0.2	-0.197	1.5e-2
		θ_3	0.3	0.295	2.0e-2
		θ_4	0.1	0.102	6.4e-3
		θ_5	1.0	0.999	1.1e-2
		θ_6	0.2	0.198	1.3e-2
38.6	40.8	θ_1	0.5	0.501	2.2e-2
		θ_2	-0.2	-0.185	3.5e-2
		θ_3	0.3	0.278	4.8e-2
		θ_4	0.1	0.100	1.1e-2
		θ_5	1.0	1.001	2.8e-2
		θ_6	0.2	0.202	3.3e-2
24.5	27.5	θ_1	0.5	0.513	8.1e-2
		θ_2	-0.2	-0.188	1.5e-1
		θ_3	0.3	0.268	1.8e-1
		θ_4	0.1	0.106	5.0e-2
		θ_5	1.0	1.020	1.1e-1
		θ_6	0.2	0.178	1.1e-1

Table II: Parameter central estimates (θ_l^c) and parameter uncertainty bounds ($\Delta\theta_l^c$) against signal-to-noise ratios (SNR_w and SNR_λ) when λ_t is a uniform random sequence and $N = 1000$

SNR_w (dB)	SNR_λ (dB)	θ_l	True Value	θ_l^c	$\Delta\theta_l$
56.0	60.0	θ_1	0.5	0.500	1.1e-3
		θ_2	-0.2	-0.200	6.6e-4
		θ_3	0.3	0.300	1.3e-3
		θ_4	0.1	0.100	4.3e-4
		θ_5	1.0	1.000	1.2e-3
		θ_6	0.2	0.200	1.2e-3
41.9	46.2	θ_1	0.5	0.502	8.5e-3
		θ_2	-0.2	-0.201	4.5e-3
		θ_3	0.3	0.301	9.2e-3
		θ_4	0.1	0.100	2.5e-3
		θ_5	1.0	0.999	6.7e-3
		θ_6	0.2	0.199	6.7e-3
36.0	40.2	θ_1	0.5	0.498	1.2e-2
		θ_2	-0.2	-0.202	6.5e-3
		θ_3	0.3	0.306	1.1e-2
		θ_4	0.1	0.098	2.6e-3
		θ_5	1.0	0.999	1.1e-2
		θ_6	0.2	0.203	1.1e-2
22.1	26.1	θ_1	0.5	0.521	7.6e-2
		θ_2	-0.2	-0.192	3.1e-2
		θ_3	0.3	0.281	7.7e-2
		θ_4	0.1	0.093	2.4e-2
		θ_5	1.0	0.996	7.9e-2
		θ_6	0.2	0.218	8.7e-2

Table III: Parameter central estimates (θ_l^c) and parameter uncertainty bounds ($\Delta\theta_l^c$) against signal-to-noise ratios (SNR_w and SNR_λ) when $\lambda_t = \sin(0.1t)$ and $N = 100$

SNR_w (dB)	SNR_λ (dB)	θ_l	True Value	θ_l^c	$\Delta\theta_l$
57.8	61.3	θ_1	0.5	0.499	1.9e-3
		θ_2	-0.2	-0.200	3.4e-3
		θ_3	0.3	0.299	5.0e-3
		θ_4	0.1	0.100	1.3e-3
		θ_5	1.0	1.000	2.2e-3
		θ_6	0.2	0.200	2.4e-3
43.1	48.1	θ_1	0.5	0.499	1.2e-2
		θ_2	-0.2	-0.204	1.7e-2
		θ_3	0.3	0.304	2.6e-2
		θ_4	0.1	0.101	4.3e-3
		θ_5	1.0	1.001	1.4e-2
		θ_6	0.2	0.197	1.6e-2
37.5	40.8	θ_1	0.5	0.498	2.5e-2
		θ_2	-0.2	-0.199	3.5e-2
		θ_3	0.3	0.292	5.8e-2
		θ_4	0.1	0.101	1.0e-2
		θ_5	1.0	1.003	2.8e-2
		θ_6	0.2	0.198	3.2e-2
23.3	27.5	θ_1	0.5	0.504	1.1e-1
		θ_2	-0.2	-0.186	1.4e-1
		θ_3	0.3	0.244	2.0e-1
		θ_4	0.1	0.096	5.5e-2
		θ_5	1.0	1.003	1.1e-1
		θ_6	0.2	0.229	1.1e-1

Table IV: Parameter central estimates (θ_l^c) and parameter uncertainty bounds ($\Delta\theta_l^c$) against signal-to-noise ratios (SNR_w and SNR_λ) when $\lambda_t = \sin(0.1t)$ and $N = 1000$

SNR_w (dB)	SNR_λ (dB)	θ_l	True Value	θ_l^c	$\Delta\theta_l$
57.4	61.9	θ_1	0.5	0.500	2.0e-4
		θ_2	-0.2	-0.200	2.9e-4
		θ_3	0.3	0.301	3.1e-4
		θ_4	0.1	0.100	7.0e-5
		θ_5	1.0	1.000	3.4e-4
		θ_6	0.2	0.200	3.6e-4
43.5	48.0	θ_1	0.5	0.503	7.5e-4
		θ_2	-0.2	-0.201	6.0e-4
		θ_3	0.3	0.300	7.6e-4
		θ_4	0.1	0.099	1.0e-4
		θ_5	1.0	1.000	1.9e-3
		θ_6	0.2	0.201	7.8e-4
37.7	41.8	θ_1	0.5	0.496	6.5e-3
		θ_2	-0.2	-0.199	1.3e-2
		θ_3	0.3	0.296	1.4e-2
		θ_4	0.1	0.101	5.3e-3
		θ_5	1.0	0.997	9.1e-3
		θ_6	0.2	0.202	1.2e-2
23.5	28.0	θ_1	0.5	0.492	3.0e-2
		θ_2	-0.2	-0.159	4.9e-2
		θ_3	0.3	0.247	5.3e-2
		θ_4	0.1	0.114	1.4e-2
		θ_5	1.0	1.002	5.0e-2
		θ_6	0.2	0.209	5.6e-2

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