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Magnetic momentum management for a geostationary satellite platform

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Abstract— **The attitude control system of three-axis stabilized geostationary platforms is usually based on a set of reaction wheels, the operation of which requires periodic desaturation maneuvers in order to dump the angular momentum accumulated because of external secular disturbance torques. The need for such maneuvers can be minimized by careful design of the satellite platform. In this paper, the control issues associated with the adoption of a magnetic actuator for (partial) momentum dumping on a geostationary platform are discussed, with specific reference to the robustness issues arising due to the highly uncertain space environment at GEO altitude.**

I. INTRODUCTION

A number of satellites rely for the generation of attitude control torques on the use of (rotating) momentum exchange devices, such as reaction wheels ([1], [2]): wheels are controlled by means of electrical motors in order to absorb the changes in spacecraft momentum due to the action of external torques. Typically, a periodic disturbance torque along one spacecraft axis would result in a cyclic variation in the angular velocity (momentum) of the wheel directed along that axis, while a constant (secular) disturbance would lead to a linear increase in angular velocity (momentum), as the wheel would be accelerated at a constant rate in order to transfer to it the excess angular momentum due to the external disturbance. Clearly, the effect of secular torques can only be managed up to a certain limit, known as wheel saturation, which corresponds to the physical limit for the rotational speed of the device.

In order to prevent this limit from being reached, the so called desaturation of the wheels must be performed, i.e., an extra set of actuators, generating external torques, must be used to dump angular momentum from the spacecraft. The general approach to this problem consists in performing a continuous compensation of the effect of external torques on the wheels' momentum, based on a suitable control scheme. The idea is that the continuous compensation of the effect of (small) secular torques on the reaction wheels should lead to small side effects on the pointing and stability performance of the actual attitude control system.

On board small Low Earth Orbit (LEO) spacecraft the external torques necessary for the control of the wheels' angular momentum are frequently generated by means of magnetic coils. The use of magnetic coils for control purposes has been the subject of extensive study since the early years of satellite missions (see, e.g., [3]). As is well known, the operation of magnetic actuators is based on the interaction with the geomagnetic field ([1], [2]).

Wheel desaturation, however, is not an issue limited to LEO spacecraft, but has to be taken into account in the design of any satellite based on a zero-momentum architecture. To date, the use of magnetic torquers for such purposes has been always limited to LEO satellites, given the rapid decrease of the intensity of the geomagnetic field with altitude. Recently, however, it has been proposed to try and use magnetic desaturation techniques for Geostationary Earth Orbit (GEO) platforms, by relying on suitably designed magnetic torquers capable of providing magnetic dipoles in the order of $3000 \div 4000$ A m^2, which are necessary in order to compensate for the weak intensity of the geomagnetic field at geostationary altitude (see [1]). In addition, while the geomagnetic field at LEO altitude can be accurately modelled using classical representation, such as the IGRF model (see again [1]), at GEO altitude the effect of solar wind causes occasional, rapid and essentially unpredictable variations of the geomagnetic field which make robustness the main requirement in the design of the desaturation loop.

This paper deals with the problem of designing suitable laws for the control of the momentum of a GEO spacecraft's reaction wheels, using magnetic actuators, taking specifically into account the robustness issues associated with the uncertainty of the geomagnetic field and the saturation of the magnetic actuator.

This work relies on the assumption (which is common practice both in the literature and in actual on board implementations, see, e.g., [4], [5]) that the momentum control loop and the attitude control loop can be treated independently. Such an engineering approximation is entirely justified by the wide frequency separation between the attitude control loop (time response of the order of seconds/minutes) and the momentum control loop (time response of the order of minutes, possibly hours).

II. PROBLEM STATEMENT

A. System description

The considered system is a High Resolution mission platform intended for operation in a GEO orbit. An example of potential satellite architecture is depicted in Figure 1 and described in [6]. From a control perspective, the main features of the current design are the choice of a three-axis stabilised architecture and the presence of a solar array (SA) which can rotate independently from the satellite. The

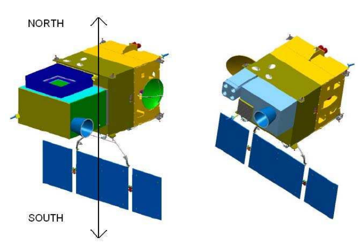

Fig. 1. The considered High Resolution mission platform.

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attitude control system is based on four reaction wheels set up in a pyramidal configuration and three out of four wheels are used in nominal configuration (the fourth wheel is used for failure occurrence purpose). The use of reaction wheels has the advantage of providing a "smooth" and continuous control. As can be seen from Figure 1, the spacecraft is characterised by a significant asymmetry in the North/South (N/S) direction, given the presence of a single solar array (SA) instead of a symmetric couple. This is bound to cause a rapid, significant accumulation of angular momentum in the orbit plane, due to the secular component of the solar radiation pressure which will be captured by the SA.

The maximum capability for angular momentum storage of the considered reaction wheels set is of 32.5 Nms, which is expected to be reached only in a few days.

The "classical" solution, i.e., the use of chemical thrusters for momentum management, requires, instead, the interruption of the mission; in this case, this would occur every 2.75 days. This time period is very short and so it would be more convenient to use the magnetic torque. As a possible solution to the problem, it is proposed to entirely or partially coil the SA in order to obtain an equivalent magnetic torquer capable of producing a dipole (normal to the SA surface) of up to $\simeq 3500$ Am^2 with the considered sizing, which would be sufficient to implement a continuous momentum unloading policy for the in-plane components of the spacecraft angular momentum. The magnetic torque is realized by the interaction of a magnetic dipole moment, produced by coiling the SA with a magnetic coil, with the (essentially orbit-normal) earth magnetic field. However, this solution relies on the assumption that the earth's magnetic field in GEO orbit can allow the generation of the necessary magnetic torque; the field, unfortunately is rather weak and very sensitive to solar activity. The web site of the National Geophysical Data Center (see [7]) contains all the data measured by the GOES satellites concerning magnetic field fine measurements in GEO orbit for several years.

In the following, we will refer to the spacecraft-fixed axes illustrated in Figure 2. The XYZ axes represent the orbit reference frame while the $X'Y'Z'$ axes are related to the rotating SA. The SA coordinate system is rotated, around the Y' axis, of the angle α as regards the orbit reference frame.

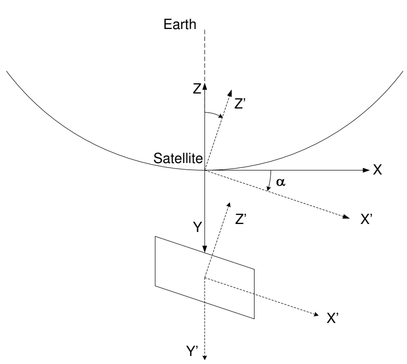

Fig. 2. Definition of the XYZ and $X'Y'Z'$ coordinate frames.

B. Nominal Model for Momentum Management

For the purpose of the present study, a simple model for the angular motion of the spacecraft has been adopted, on the basis of the assumption that the attitude control system can effectively maintain the attitude motion within the linear range with respect to the nominal pointing condition.

The attitude dynamics of a rigid spacecraft can be expressed by the well known Euler's equations ([2], [1]), as

$$I\dot{\omega} = -\omega \times (I\omega + h_w) - \dot{h}_w + T_m + T_{ext}, \tag{1}$$

where $\omega = [\omega_x \quad \omega_y \quad \omega_z]^T \in \mathbb{R}^3$ is the vector of spacecraft angular rates, expressed in body frame, $I \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $h_w = [h_{wx} \quad h_{wy} \quad h_{wz}]^T \in \mathbb{R}^3$ is the vector of the wheels' angular momentum. $T_m \in \mathbb{R}^3$ is the vector of external torques induced by the magnetic torquer, while $T_{ext} \in \mathbb{R}^3$ is the vector of external disturbance torques.

The dynamics of the reaction wheels is given by

$$\dot{h}_w = -\omega \times h_w - \tau, \tag{2}$$

where $\tau \in \mathbb{R}^3$ is the vector of commanded control torques computed by the attitude control system.

As for the attitude kinematics, a number of possible parameterizations exist (see, e.g., [1]). For the purpose of the present analysis the Euler angles representation is adequate; considering the so-called 1-2-3 sequence of rotations, the kinematic equations relating the derivatives of the Euler angles ϕ, θ and ψ to the components of the body rates are given by

$$\begin{aligned} \dot{\phi} &= (\omega_x \cos(\psi) - \omega_y \sin(\psi))\sec(\theta) \\ \dot{\theta} &= \omega_x \sin(\psi) + \omega_y \cos(\psi) \\ \dot{\psi} &= \omega_z - (\omega_x \cos(\psi) - \omega_y \sin(\psi))\tan(\theta). \end{aligned} \tag{3}$$

The dynamics concerning the angular positions and velocities are significantly faster than momentum dynamics: so the complete model for attitude and momentum dynamics can be reduced to a much simpler one by neglecting the faster dynamics and obtaining a three state model, representing only the angular momentum along the three axis.

As for the magnetic actuator, letting m the magnetic dipole moment of the coil on the SA and b the orbit-normal component of the geomagnetic field, in the rotating frame attached to the SA the generated magnetic torque will be given by mb, directed along the X' axis. Finally, in this study we focus on the disturbance torque due to solar radiation pressure, so we denote by T_d the solar radiation pressure torque acting on the SA, again acting on the X' axis.

By assuming that the attitude is ideally regulated to the nominal Earth pointing reference, linearising the equations for angular momentum around such a reference attitude (taking into account that the nominal angular rate is $\Omega_0 = [0 \; -\omega_\oplus \; 0]^T$) and focusing on the x and z components of the angular momentum we have

$$\dot{h}_{wx} = \omega_\oplus h_{wz} + \cos(\alpha)\,(T_d + mb) \tag{4}$$

$$\dot{h}_{wz} = -\omega_\oplus h_{wx} - \sin(\alpha)\,(T_d + mb), \tag{5}$$

or, equivalently, letting $\tilde{h}_w = [h_{wx} \quad h_{wz}]^T$

$$\dot{\tilde{h}}_w = A\tilde{h}_w + B\,(T_d + mb), \tag{6}$$

where

$$A = \begin{bmatrix} 0 & \omega_\oplus \\ -\omega_\oplus & 0 \end{bmatrix}, \quad B = [\cos(\alpha) \quad -\sin(\alpha)]^T. \tag{7}$$

It is useful to re-write the equations of conservation of angular momentum in a different reference frame, defined by rotating the spacecraft body frame one so that the x axis coincides with the solar array, i.e., the considered rotation is given by

$$T = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}. \tag{8}$$

By applying the corresponding Lyapunov transformation (see [8]) to the (A,B) matrices in (7), we get

$$A' = \dot{T}T^T + TAT^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B' = TB = \begin{bmatrix} 1 & 0 \end{bmatrix}^T \tag{9}$$

recalling that $\dot{\alpha} = \omega_{\oplus}$ (i.e., the SA rotates with an angular rate equal to $\omega_{\oplus}$ to maintain Sun pointing). So equation (6) in the new coordinates $h'_w = Th_w$ is given by

$$\dot{h}'_{wx} = (T_d + mb) \tag{10}$$

$$\dot{h}'_{wz} = 0. \tag{11}$$

It is apparent from the above equations that at each time instant only a one-dimensional subspace of the $X-Z$ plane is controllable using the available magnetic torquer; however, the controllable direction coincides with the one which receives the most significant proportion of the external disturbance torque (actually the entire disturbance under the considered assumptions), so ensuring that the desaturation problem can be effectively solved even though the underlying dynamics is not completely controllable.

C. Uncertainty and saturation

In addition to the above equation (10), a complete formulation of the desaturation problem in this case should take into account two major issues, which usually do not pose any specific problem in the LEO case, but become of primary importance in the GEO one, namely the modelling of the geomagnetic field and the saturation of the magnetic actuator. As far as b is concerned, the difficulty is related to the much higher uncertainty associated with the time behaviour of the geomagnetic field at GEO altitude with respect to the much more predictable LEO environment. Therefore, for all practical purposes, b should be treated as an uncertain, slowly varying parameter. In order to highlight the type of time-variability one can expect from b, in Figure 3 the measured time histories of the $-Y$ component of the geomagnetic field are illustrated, as measured by the GOES spacecraft (see [7]) during three days which provide a representative sample of possible perturbations of b with respect to its nominal value of 100 nT:

- 1 January 2006: a very regular, sinusoidal oscillation of b around its mean value;
- 23 January 2006: a very high level of activity, with changes of almost 50% with respect to the mean value;
- 29 January 2006: a very "quiet" day, with an almost constant value of b.

As a consequence of the limited knowledge of b, it is very likely that saturation of the magnetic actuator will come into play during normal operation of the momentum management loop: indeed, a given request for the desaturation torque might give rise to a very large magnetic dipole because of an unexpected reduction of the local value of b.

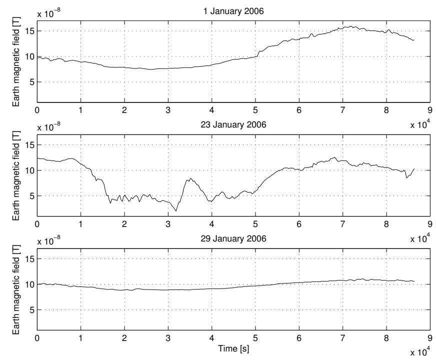

Fig. 3. Time histories of the earth's magnetic field in GEO orbit during three different days of January 2006.

III. ACTUATOR SIZING AND DEFINITION OF THE CONTROL LAW

A. Magnetic Actuator Sizing

A preliminary sizing of the magnetic actuator has been realized starting from a disturbance torque evaluation. The objective is to obtain a maximum magnetic torque larger than the disturbance torque. To achieve this, the magnetic dipole moment feasible from the magnetic coil has to be at most of 3571 Am^2; the mass of the magnetic coil is sizing to be of 38 kg. This first actuator sizing is likely to be conservative since it is based on a magnetic torque requirement by far superior to the actual disturbance torque. After the following considerations on the robustness of the control system, we will able to optimize this actuator sizing obtaining also a reduction of the magnetic coil's mass.

B. Definition of the Control Law

In order to complete the formulation of the analysis problem, the model of the system has to be augmented with a suitable desaturation control law. In this work we consider a classical proportional feedback, i.e., the magnetic dipole is proportional to the angular momentum h'_{wx} to be removed

$$m = -\frac{1}{\tilde{b}} K_{wx} h'_{wx}, \tag{12}$$

where K_{wx} is the control gain and $\tilde{b}$ is a suitable representation of the geomagnetic field to be included in the controller. The corresponding magnetic torque, due to the interaction of the magnetic dipole moment with the earth's magnetic field, turns out to be proportional to the angular momentum, i.e.,

$$T_m = mb = -\frac{b}{\tilde{b}} K_{wx} h'_{wx}, \tag{13}$$

and should, provided that closed loop stability can be guaranteed, ensure that the angular momentum remains bounded. The closed-loop system therefore becomes

$$\begin{aligned} \dot{h}'_{wx} &= mb + T_d, \\ m &= -\frac{1}{\tilde{b}} K_{wx} h'_{wx}. \end{aligned} \tag{14}$$

We have to consider also the fact that the earth's magnetic field intensity is an uncertain time-varying parameter; besides, the control law presents a saturation due to the fact that the magnetic moment has a maximum bound. Considering these two factors equations (14) become

$$\begin{aligned} \dot{h}'_{wx} &= m(b_0 + \delta\Delta b(t)) + T_d, \\ m &= m_{max} sat(-\frac{1}{\tilde{b}} K_{wx} h'_{wx}), \end{aligned} \tag{15}$$

with:

- b_0 : average value of the earth's magnetic field along the Y' axis ($100nT$),
- δ : scale factor ($20 \div 60nT$),
- $\Delta b(t)$: time-varying, norm bounded uncertainty ($|\Delta b(t)| < 1$),
- m_{max} : saturation limit for the magnetic torquer,
- $\tilde{b}$: magnetic field value used in the control law.

Concerning the choice of $\tilde{b}$, we will consider three different possibilities:

1) $\tilde{b} = b$, assuming that we have access to an exact measurement of the earth's magnetic field;
2) $\tilde{b} = b_0 = 100nT$, i.e., we use only the average value of the earth's magnetic field in the controller;
3) $\tilde{b} = b_s = 100 + 40\sin(2\pi/86400 + \pi)nT$, a sine approximation of the earth's magnetic field over a 24h period.

In the first case we could obtain a magnetic torque exactly proportional to the angular momentum, but we would need a magnetometer on board the satellite. In the second case we would not need an additional sensor; moreover it can be expected that the computed magnetic dipole moment has a more regular behaviour. Finally, the third case is considered as a tradeoff between the availability of a measurement and a constant model for the geomagnetic field. In Figures 4-6 the time histories of the angular momentum and the magnetic dipole moment are reported for simulations carried out during the three days of January considered in Figure 3 using each of the proposed choices for $\tilde{b}$, under the effect of a constant disturbance torque. From this preliminary analysis,

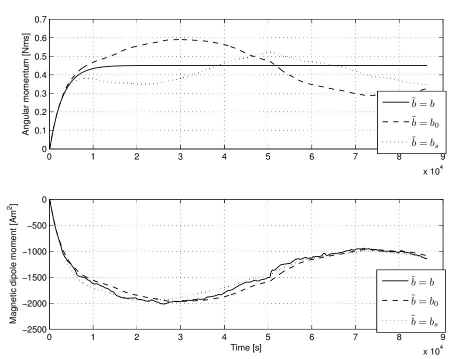

Fig. 4. Angular momentum (top) and magnetic dipole moment (bottom): 1 January 2006.

it was chosen to use the average value of the earth's magnetic field in the control law.

IV. ROBUSTNESS ANALYSIS

As mentioned in Section II, the time-variability of the geomagnetic field and the presence of a saturation in the feedback path represent the main issues associated with the design of the momentum management system. In particular, it is relatively simple to check *separately* the robustness of the closed loop system with respect to uncertainty in the value of the geomagnetic field and to the presence of the saturation (using, respectively, small gain and absolute stability theory). Dealing with both sources of uncertainty in a simultaneous way, however, calls for more advanced methods for robustness analysis. In this work, the analysis problem has been formulated in the Integral Quadratic Constraints (IQC) framework.

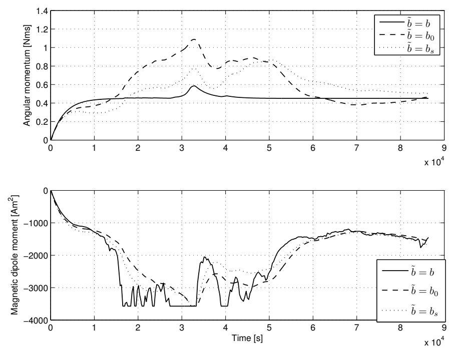

Fig. 5. Angular momentum (top) and magnetic dipole moment (bottom): 23 January 2006.

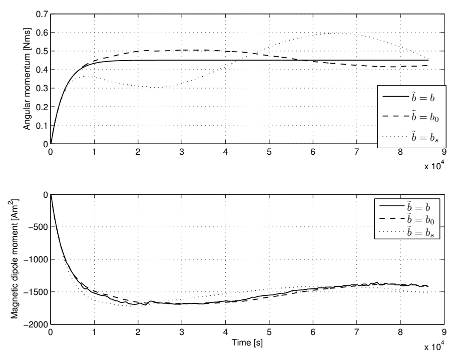

Fig. 6. Angular momentum (top) and magnetic dipole moment (bottom): 29 January 2006.

1) Parametric Stability Analysis: Considering only the uncertainty due to the value of the earth's magnetic field (treated as an uncertain parameter) and neglecting the non-linearity due to the saturation of the control variable, the model can be written as

$$\begin{aligned} \dot{h}'_{wx} &= m(b_0 + \delta\Delta b) + T_d, \\ m &= (-\frac{1}{b_0} K_{wx}) h'_{wx} = K_x h'_{wx}, \end{aligned} \tag{16}$$

where δ is a scale factor such that $\|\Delta b\|_\infty < 1$ and the controller gain K_x is negative. Extracting the uncertain parameter from the state equation we get

$$\begin{aligned} \dot{h}'_{wx} &= b_0 K_{wx} h'_{wx} + K_{wx} w, \\ z &= \delta h'_{wx}, \\ w &= \Delta b z. \end{aligned} \tag{17}$$

so the system is robustly stable if, letting $G(s) = \frac{\delta K_{wx}}{s - b_0 K_{wx}}$, we have $\|G(s)\|_\infty < 1$. Considering the nominal values

- $\delta = 40 \cdot 10^{-9}$ T
- $b_0 = 100 \cdot 10^{-9}$ T

we can easily compute $\|G(s)\|_\infty = \delta/b_0 = 0.4 < 1$, so, as expected, robust stability in the face of an uncertain value of the geomagnetic field is not an issue, provided that the closed-loop system operates linearly. Note, in passing, that the condition $\|G(s)\|_\infty < 1$ holds independently of the value of the control gain K_x.

2) Saturation stability analysis: The second robustness analysis focused on considering only to the saturation of the control variable and assuming that the earth's magnetic field coincides with its average value. Under these assumptions, the model is given by

$$\begin{aligned} \dot{h}'_{wx} &= mb_0 + T_d, \\ m &= m_{max} sat(-\frac{1}{b_0} K_{wx} h'_{wx}), \end{aligned} \tag{18}$$

where m_{max} is the saturation limit for the magnetic torquer. The stability analysis problem can be solved very simply using Lyapunov theory: for $T_d = 0$, the system (18) has $h'_{wx} = 0$ as a globally asymptotically stable equilibrium for all positive K_{wx}. It is interesting to point out, however, that since the linear part of the model is not asymptotically stable, an absolute stability approach to the analysis would not be viable. However, the marginally stable dynamics in (18) is only the result of some modelling simplifications, as (see, e.g., [1]) the dynamics of the actual reaction wheels will be asymptotically stable due to (small but nonzero) friction effects. Therefore, for analysis purposes it would be even more appropriate to modify (18) by including a friction term as in

$$\begin{aligned} \dot{h}'_{wx} &= -\xi h'_{wx} + mb_0 + T_d, \\ m &= m_{max} sat(-\frac{1}{b_0} K_{wx} h'_{wx}), \end{aligned} \tag{19}$$

where $\xi > 0$ is a suitable friction coefficient. At this point we are able to apply conditions such as the circle and Popov criteria (see, e.g., [9]) in order to prove absolute stability of (19). In particular, since we are dealing with a SISO system it is straightforward to verify that the graphical condition for absolute stability associated with the circle criterion is trivially satisfied.

3) IQC analysis: The third analysis has been realized taking into account both the uncertainty due to the earth's magnetic field's variability and the non-linearity due to the saturation of the control variable. This analysis has been performed using integral quadratic constraints (IQCs) to model the uncertain components. The IQC approach to robust stability analysis can be shortly described (see [10] for a complete treatment) with reference to the system

$$\begin{aligned} v &= Gw + e, \\ w &= \Delta(v), \end{aligned} \tag{20}$$

(see also the block diagram in Figure 7), where $G(s)$ is the transfer function of a linear time-invariant system without poles in the closed right half plane and Δ is a bounded operator taking into account the uncertain, nonlinear and time-varying components of the system to be dealt with in the analysis. Given a bounded and self-adjoint operator Π, Δ satisfies the IQC defined by Π if

$$\sigma_\Pi(v, \Delta(v)) = \int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix} d\omega \geq 0, \tag{21}$$

where $\hat{v}(j\omega)$ and $\hat{w}(j\omega)$ represent the Fourier transforms of the signals v and w. At this point, after having characterized

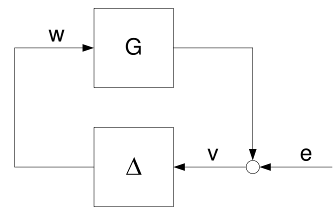

Fig. 7. Model of a system for IQC robust stability analysis.

the uncertainty by means of an IQC, this can be transformed, through the Kalman-Yakubovich-Popov Lemma, into a LMI, simpler to solve by a computational point of view.

For the purpose of IQC robustness analysis, the system under study must be described as (see also the block diagram in Figure 8)

$$\begin{aligned} \dot{h}'_{wx} &= -\xi h'_{wx} + mb + T_d, \\ b &= b_0 + \delta\Delta b(t), \\ m &= m_{max}\tilde{m}, \\ \tilde{m} &= sat(u), \\ u &= K_x h'_{wx}. \end{aligned} \tag{22}$$

By "pulling out" the uncertain elements and defining inter-

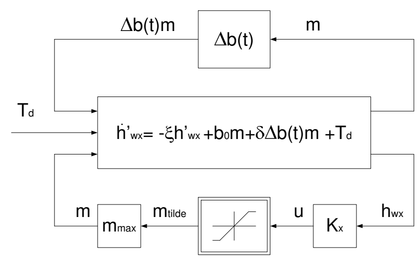

Fig. 8. Block diagram associated with (22).

face variables, (22) can be equivalently written as

$$\begin{aligned} \dot{h}'_{wx} &= -\xi h'_{wx} + m_{max}[w_1 b_0 + w_2] + T_d, \\ w_1 &= sat(v_1), \\ v_1 &= K_x h'_{wx}, \\ w_2 &= \Delta b(t) v_2, \\ v_2 &= w_1 \delta, \end{aligned} \tag{23}$$

which is a form suitable for analysis purposes.

As far as the present study is concerned, IQCs for time-varying uncertain scalars and for the saturation function are needed. Consider first a time-varying uncertain coefficient given by

$$w_2 = \Delta b(t) v_2, \tag{24}$$

where $\Delta b(t)$ is a norm-bounded scalar function, i.e., $|\Delta b(t)| < 1$. Then (see, again, [10]), it can be shown that the uncertainty $\Delta b(t)$ satisfies an IQC defined by the operator

$$\Pi(j\omega) = \begin{bmatrix} X(j\omega) & Y(j\omega) \\ Y^*(j\omega) & -X(j\omega) \end{bmatrix}$$

where $X = X^*(j\omega) \geq 0$ and $Y^*(j\omega) = -Y(j\omega)$ are square, real-valued matrix functions.

Similarly, the uncertainty associated with the saturation of the magnetic torquer is given by

$$w_1 = sat(v_1), \tag{25}$$

with obvious definition of the $sat(\cdot)$ function. This uncertainty can be embedded in IQC form using the matrix

$$\Pi(j\omega) = \begin{bmatrix} 0 & 1+H(j\omega) \\ 1+H(-j\omega) & -(2+2ReH(j\omega)) \end{bmatrix}$$

where $H(s)$ is a proper rational transfer function, such that for the associated impulse response h the constraint

$$||h||_1 = \int_{-\infty}^{\infty} |h(t)|dt \leq 1$$

holds.

The numerical stability analysis has been performed using the IQC-β toolbox (see [11]), which has allowed to verify that the $\mathbf{L}_2$ gain from the external disturbance torque T_d to the angular momentum h'_{wx} remains finite, even for wide perturbations of the system's parameters, such as:

- perturbing the controller gain $K_x = -3330$ in a range from 50% to 200%;
- changing the scale factor δ from 20nT to 60nT;
- reducing the maximum magnetic dipole moment from 3571 Am^2 to 2500 Am^2.

V. SIMULATION STUDY

The simulation results presented in this section have been obtained by making use of the Spacecraft Modelling Library (see [12]), which is currently being developed using the Modelica language and the Dymola simulation environment.

The control law has been implemented in the Modelica simulator for the considered platform. Through simulations executed in different periods of the year 2006 the following results have obtained. The following Figures are related to ten days of April (5-14 April 2006). Figure 9 shows the time histories of the angular momentum and of the magnetic dipole moment for different values of the maximum magnetic dipole in the considered ten days of April (5-14 April 2006). As expected, the behaviour of the closed-loop system remains essentially unchanged.

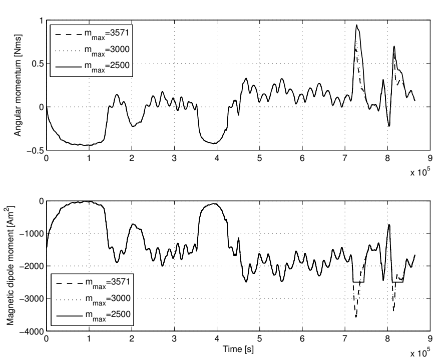

Fig. 9. Angular momentum (top) and magnetic dipole moment (bottom): 5-14 April 2006.

Similarly, in Figure 10 the response of the angular momentum and of the magnetic dipole moment are shown, for different values of the control gain K_x. Here, the tradeoff inherent in the choice of the control gain becomes apparent. A larger value of K_x leads to smaller oscillations in the angular momentum but causes the magnetic dipole to become more irregular: in view of the significant currents involved in the operation of this device, this might not be acceptable from a practical point of view.

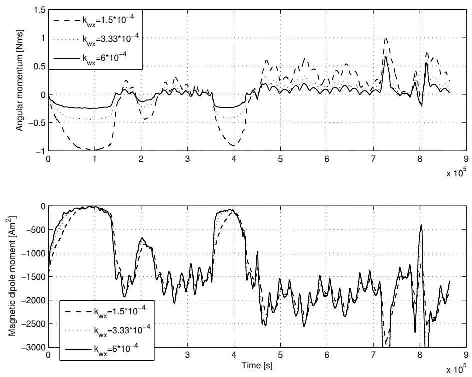

Fig. 10. Angular momentum (top) and magnetic dipole moment (bottom): 5-14 April 2006.

VI. CONCLUSIONS

The problem of reaction wheel desaturation for a geostationary platform has been considered and the feasibility - at least from the control engineering perspective - of a solution based on a magnetic torquer has been demonstrated. A detailed robusteness analysis has been performed, taking into account both actuator saturation and the time-variability of the geomagnetic field. Simulation results show that the closed-loop performance is compatible with the operation of the platform.

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