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Distributed Control for Radial Loss Network Systems via the Nash Certainty Equivalence (Mean Field) Principle

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Abstract— **The computational intractability of the dynamic programming (DP) equations associated with optimal admission and routing in stochastic loss networks of any non-trivial size (Ma et al, 2006, 2008) leads to the consideration of suboptimal distributed game theoretic formulations of the problem. This work presents a formulation of loss network admission control problems in terms of a class of systems composed of a large population of weakly coupled competitive individual agent networks. The resulting distributed dynamic stochastic game problem is solved and analyzed by application of the so-called Point Process Nash Certainty Equivalence (PPNCE) principle; this is an extension to the network point process context of the NCE Principle originally formulated in the LQG framework by M. Huang et al, (2006, 2007). This methodology has close connections with the mean field models studied by Lasry and Lions (2006, 2007) and the notion of oblivious equilibrium proposed by Weintraub, Benkard, and Van Roy (2005, 2007) via a mean field approximation.**

I. INTRODUCTION

Loss networks can be viewed as systems of multi-server queues with zero internal buffering capacity whereby one customer can simultaneously occupy or release several servers or links along a given route. More precisely, a link of capacity c is equivalent to c parallel servers with zero waiting room each and, as a result, a call request which cannot be admitted instantaneously upon arrival and placed on a route, is immediately lost. The associated optimal admission and routing problems are strongly tied with problems of optimal control of queues with the specific difference that in classical queuing problems servers cannot be chosen simultaneously by the controller. In recent work [10], [11], we have developed a state space representation and the relevant dynamic programming equations for multi-class, general call request arrivals and general connection durations loss networks. They correspond to systems of coupled partial differential equations which reduce to the piecewise linear algebraic equations of Markov decision problems when arrivals are Poisson and connection durations are exponential. See [2], [1], [4], [15] for related dynamic programming papers in a queuing context.

While the availability of a formal system of equations characterizing optimal admission and routing decisions in loss networks under fairly general conditions is noteworthy, one is inescapably faced with their computational intractability for all but the most simple networks, even under the most favorable assumptions of Poisson call request arrivals and exponential connection durations [10], [11], [14]. In this paper, following the work in [13].we employ non-cooperative dynamic game theory for the analysis of networks of large populations of weakly coupled players; this permits the derivation of scalable distributed, suboptimal control laws which have a strong potential for real time implementation. In this formulation, the original large network is split into a collection of small, independent, self-optimizing components which are called *(agent) networks*. Moreover, in its initial form, in this paper, a radial network topology is assumed for the core network (via which the individual agents are interconnected) thus in effect limiting the analysis to admission issues.

The analysis here is motivated by "the individual versus the mass" correspondence which is fundamental to the so-called Nash certainty equivalence (NCE)(or Mean Field) methodology developed by Huang et. al., see e.g. [5], [7], [6]; this has been applied to the construction of explicit distributed control laws in large scale linear quadratic regulator games and to the analysis of large classes of multi-agent nonlinear stochastic dynamic games. A closely related approach has recently been independently developed by Lasry and Lions [9], while for models of many firm industry dynamics, Weintraub, Benkard, and Van Roy proposed the notion of oblivious equilibrium by use of a mean field approximation [16].

First we specify some general classes of coupled agent networks with local state transition equations. In this formulation two distant agent (networks) coordinate their acceptance of a long distance call request, and thus share a long distance connection which is born at both agents at a common instant t and which dies at those agents at a common instant after t (thus correlating the two agents at the birth of a long distance call request and also at the death of the resulting established connection, if such a connection had been established).

Next we introduce the notion of *network decentralized state (NDS)*. We consider a class of finite radial networks such that subject to uniformity and independence hypotheses on the call request characteristics, fixed point theorems yield the existence of a class of feedback induced NDSs in the limit as the number of agents increases to infinity. This is in the sense that a unique NDS is associated with (i) any given common local admission (feedback control) rule and (ii) any given network initial state. An NDS is characterized by

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boundary conditions common to all agents, where by boundary conditions we mean the specification of (i) input Poisson streams of aggregate call requests released (i.e. pre approved) by some distant network, with a deterministic intensity at any agent's input port which are furthermore independent from one agent to the other, and (ii) a fixed common distant network probability of acceptance for locally originating call requests which are destined for the outside. Such boundary conditions have the property that they characterize exogenous inputs for each agent which will be replicated by the (infinite) mass of agents under the common local feedback law (this is in response to both these exogenous inputs and their own internal random call request processes). Furthermore, this implies (in the asymptotic population limit) that the set of state processes of the agents' networks are identically and mutually independently distributed.

It is to be noted that elsewhere, e.g. [3], [14], [8], quite different approaches have been proposed to mitigate the computational complexity of search procedures for optimal admission and routing decisions in loss networks. More specifically, the analysis in [3] is based upon the assumption of the statistical independence of each network link; an approximation result is obtained in [14] using reinforcement learning techniques; and in [8] a game theoretic analysis is employed which uses the notion of shadow prices from decentralized optimization.

The paper is organized as the follows. In Section 2, we formulate call admission control problems for a class of loss networks. Section 3 then specifies the class of agent subnetworks which are connected into the radial mass network under study. Then network decentralized states (NDSs) for distributed network systems are defined and their existence established. In Section 4, based upon the decentralized model developed in Section 3, control problems for the global networks under consideration are formulated as distributed control problems and the hybrid dynamic programming (DP) equation systems for each of the agent systems (developed in [11], [12]) are then presented. It is then established that there always exist relaxed admissible local feedback rules which induce an NDS of the type introduced above with the Nash equilibrium property with respect to each agent's loss function. Section 5 contains the conclusions and outlines future work.

II. Call Admission Control for Global Loss Networks

A capacitated global *network* of size M, $M \in \mathbb{Z}_2 \triangleq \{2,3,\cdots\}$, is denoted $Net^M(\mathbb{V},\mathbb{L},\mathbb{C})$, each of which consists of a set of vertices $\mathbb{V} = \{v_0, v_1, \cdots, v_M\}$, a set of (bidirectional) links $\mathbb{L} = \{(v_0, v_i);\ i \in \mathcal{M}\}$, $\mathcal{M} = \{1, \cdots, M\}$, where the capacities of the links are denoted $\mathbb{C} = \{c_l = c; l \in \mathbb{L}\}$, where $c \in \mathbb{Z}_1 \triangleq \{1,2,\cdots\}$.

In other words, the (overall) global $Net^M(\mathbb{V},\mathbb{L},\mathbb{C})$ is a radial network composed of M bidirectional links with an identical finite capacity c, see Figure 1.

In this paper, we assume that the call request processes and connection durations for the infinite sequence of

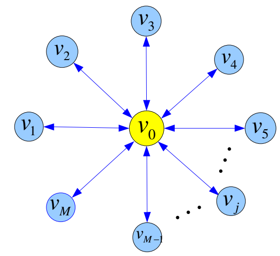

Fig. 1. A global radial network with size M

$Net^M(\mathbb{V},\mathbb{L},\mathbb{C})$, $M \in \mathbb{Z}_2$, satisfy the specifications (S1)^g-(S3)^g given below:

(S1)^g The (bidirectional internal) call request process between a peripheral vertex v_i and the central vertex v_0, denoted $Rq^M_{\{j,0\}}$, with events denoted $e^+_{\{j,0\}}$, is a Poisson process with rate equal to $\lambda_1 < \infty$;
The duration of the m-th established (internal) connection between v_j and v_0, denoted $\eta_m^{\{j,0\}}$, with event denoted $e^-_{\{j,0\}}$, is exponentially distributed with fixed parameter equal to $1/\mu_1 < \infty$. The termination process of $e^-_{\{j,0\}}$ is denoted $Dp^-_{\{j,0\}}$.

(S2)^g The (directional long distance) call request process from any v_j to v_k, denoted $Rq^M_{\langle j,k\rangle}$, with events denoted $e^+_{\langle j,k\rangle}$, is a Poisson process with rate equal to $\frac{1}{M-1}\lambda_2 < \infty$. In other words, the rate of the process $Rq^M_{\langle j,k\rangle}$ is inversely proportional to the M.
The duration of the m-th established (long distance) connection from v_j to v_k, denoted $\eta_m^{\langle j,k\rangle}$ with event $e^-_{\langle j,k\rangle}$, is exponentially distributed with fixed parameter equal to $1/\mu_2 < \infty$. The termination process of $e^-_{\langle j,k\rangle}$ is denoted $Dp^-_{\langle j,k\rangle}$.

(S3)^g The set of the stochastic processes and random variables of any network $Net^M(\mathbb{V},\mathbb{L},\mathbb{C})$, specified in (S1)^g and (S2)^g are mutually independent, and the collections of processes in the resulting family of networks $\{Net^M(\mathbb{V},\mathbb{L},\mathbb{C}); M \in \mathbb{Z}_2\}$ are mutually independent.

The probability space $(\Omega, \mathcal{F}, \mathbb{P})$ carries all the stochastic processes and random variables in this paper.

We further adopt the following global network performance functions (S4)^g and (S4)^g subject to call admission decisions:

(S4)^g The system gains at time t, with $t \in \mathbb{R}_+$, an instantaneous (time discounted negative (to give infimization problems)) reward equal to $e^{-\beta t} b_1$ ($e^{-\beta t} b_2$ respectively), with $\beta \in \mathbb{R}_+$ and $b_1, b_2 \in \mathbb{R}_-$, in case that the call request $e^+_{\{j,0\}}$ ($e^+_{\{j,k\}}$ respectively) is admitted at t on route (v_j, v_0) ((v_j, v_0, v_k) respectively);

(S5)^g The system gains at time t a reward per unit time equal to $e^{-\beta t} g_1$ ($e^{-\beta t} g_2$ respectively), with $g_1, g_2 \in \mathbb{R}_-$, during the duration of each of the active connections on route (v_j, v_0) ((v_j, v_0, v_k) respectively);

III. Distributed Control: Agent Network Systems

We generalize the class of centralized OSC problems developed in [10], [11] to a particular class of distributed OSC problems which in the infinite population game theoretic formulation of this paper leads to a tractable class of problems.

A. Formulation of Local Control Laws

We first give the required definitions and notions for a generic agent network S^j, $j \in \mathcal{M}$:

1) Local Relaxed Control Law $u_j^r(\mathbf{n}^j, e)$*:* The *set of global events* denoted E, is specified as:

$$E = \{e^0, e^+_{\{j,0\}}, e^+_{\langle j,k\rangle}, e^-_{\{j,0\}}, e^-_{\langle j,k\rangle};\ j,k \in \mathcal{M}\}, \quad (1)$$

with (i) e^0 absence of a call request or a connection ending (event); (ii) $e^+_{\{j,0\}}$ (internal) call request between v_j and v_0 for any $j \in \mathcal{M}$, and $e^+_{\langle j,k\rangle}$ (long distance) call request from v_j to v_k for any $k \in \mathcal{M}_{-j}$; (iii) $e^-_{\{j,0\}}$ end of an (internal) connection between v_j and v_0, and $e^-_{\langle j,k\rangle}$ end of a (long distance) connection from v_j to v_k.

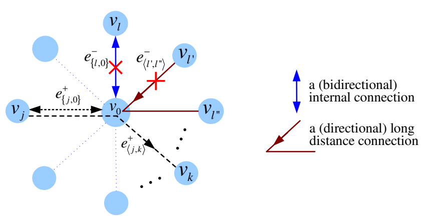

Fig. 2. Examples of Global Events of Global Networks

The *set of (admissible) local connection vectors* of agent network S^j for all $j \in \mathcal{M}$, denoted $\mathcal{N}^j$, is specified as:

$$\mathcal{N}^j = \{\mathbf{n}^j \equiv (n_1^j, n_2^j, n_3^j) \in \mathbb{Z}_+^3;\ \Sigma_{b=1}^3 n_b^j \le c\}, \quad (2)$$

with n_b^j, $b = 1,2,3$, denoting respectively the total number of active connections between v_j and v_0, from v_{-j} to v_j with $v_{-j} \triangleq \{v_k; k \in \mathcal{M}_{-j}\}$, and from v_j to v_{-j}.

See Figure 3 for the illustration of local connection vector of agent network S^j.

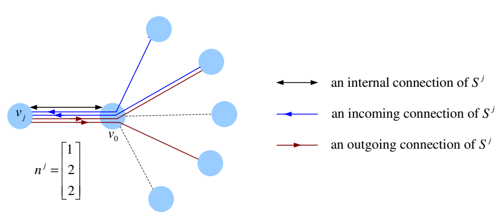

Fig. 3. Example of Admissible Local Connection Vector of Agent S^j

For any agent network S^j, the *(admissible local connection vector and global event dependent) control set* $U^j(\mathbf{n}^j, e)$ given a pair of $(\mathbf{n}^j, e) \in \mathcal{N}^j \times E$, is specified as

$$U^j = \begin{cases} \{\mathbf{0}, a_1\} \\ \{\mathbf{0}, a_2\} \\ \{\mathbf{0}, a_3\} \\ \{-a_1\} \\ \{-a_2\} \\ \{-a_3\} \\ \{\mathbf{0}\} \end{cases} \text{in case} \begin{cases} e = e^+_{\{j,0\}}, \mathbf{n}^j + a_1 \in \mathcal{N}^j \\ e = e^+_{\langle k,j\rangle}, \mathbf{n}^j + a_2 \in \mathcal{N}^j \\ e = e^+_{\langle j,k\rangle}, \mathbf{n}^j + a_3 \in \mathcal{N}^j \\ e = e^-_{\{j,0\}} \\ e = e^-_{\langle k,j\rangle} \\ e = e^-_{\langle j,k\rangle} \\ \text{otherwise} \end{cases} \quad (3)$$

where a_i, $i = 1,2,3$, denotes respectively the i-th unit vector in $\mathbb{R}^3$ and we denote $U_j^+ = \{a_i; i = 1,2,3\}$.

Here, the control actions are specified as follows: (i) $u^j(\mathbf{n}^j, e^+_{\{j,0\}}) = \mathbf{0}$ (or a_1) denotes that the call request $e^+_{\{j,0\}}$ is rejected (or accepted) by S^j; (ii) $u^j(\mathbf{n}^j, e^+_{\langle j,k\rangle}) = \mathbf{0}$ (or a_3) denotes that the call request $e^+_{\langle j,k\rangle}$ is rejected (or released) by S^j; (iii) $u^j(\mathbf{n}^j, e^+_{\langle k,j\rangle}) = \mathbf{0}$ (or a_2) denotes that the call request $e^+_{\langle k,j\rangle}$ is rejected (or accepted) by S^j; and (iv) $u^j(\mathbf{n}^j, e) = -a_b$, with $b = 1,2,3$, denotes that triggered by the end of an active internal, incoming or outgoing connection in S^j respectively, this connection is deleted.

The *process of local connection vectors* of agent S^j, denoted $\mathbf{n}^j_\cdot$, and the *global event process* of the global network, denoted $e_\cdot$, are given to be:

$$\mathbf{n}^j : \mathbb{R}_+ \times \Omega \to \mathcal{N}^j \quad \text{and} \quad e : \mathbb{R}_+ \times \Omega \to E. \quad (4)$$

The set of *local relaxed control laws* of S^j, denoted $\mathcal{U}_j^r[0,\infty)$, is given to be

$$\mathcal{U}_j^r[0,\infty) = \big\{u_j^r : \mathbb{R}_+ \times \mathcal{N}^j \times E \times \Omega \to U;\ u_j^r(t) \text{ is } \sigma(\mathbf{n}^j_{t-}, e_t) \times \mathcal{B}_t^j(\Omega) \text{ measurable}\big\}, \quad (5)$$

where $\mathcal{B}_t^j(\Omega)$ is a sigma field on the probability space Ω, such that: (i) Conditioned on the sigma field generated by the set of local states $\{\mathbf{n}^j(t^-); \forall j \in \mathcal{M}\}$, and the global event process e at t, the random control functions $u_j^r(t)$, for all $j \in \mathcal{M}$, are mutually independent and independent of all events in $\{s; s > t\}$ appearing in assumptions (S1)^g and (S2)^g. (ii) For all $(\mathbf{n}^j, e) \in \mathcal{N}^j \times E$ and $u \in U(\mathbf{n}^j, e)$, $\mathbb{P}\big(u_j^r(t) = u \,\big|\, (\mathbf{n}^j, e)\big)$ is right continuous with respect to t, and $\mathbb{P}\big(u_j^r = u \,\big|\, (\mathbf{n}^j, e_k)\big)$ is identical for all $k \in \mathcal{M}_{-j}$.

Definition 3.1: (Global Event Transition Equations)

$$e_t = \begin{cases} e^+_{\{j,0\}} \\ e^+_{\langle j,k\rangle} \\ e^-_{\{j,0\}} \\ e^-_{\langle j,k\rangle} \\ e^0 \end{cases} \text{in case} \begin{cases} Rq^+_{\{j,0\}}(t) = Rq^+_{\{j,0\}}(t^-) + 1 \\ Rq^+_{\langle j,k\rangle}(t) = Rq^+_{\langle j,k\rangle}(t^-) + 1 \\ Dp^-_{\{j,0\}}(t) = Dp^-_{\{j,0\}}(t^-) + 1 \\ Dp^-_{\langle j,k\rangle}(t) = Dp^-_{\langle j,k\rangle}(t^-) + 1 \\ \text{otherwise} \end{cases},$$

with processes Rq_e and Dp_e^- given in (S1)^g and (S2)^g. □

2) (Aggregated Local) State Processes of Agent Networks: In this section we use the standard Boolean summation and multiplication properties of indicator functions.

Subject to local relaxed control law $u_k^r \in \mathcal{U}_k^r[0,\infty)$, we define the point processes $A^M_{\langle -j,j\rangle}$, $A^M_{\langle j,-j\rangle}$: $[0,\infty) \times \Omega \to$

$\{0,1\}$ for any j, such that

$$A^M_{\langle -j,j\rangle}(t,\omega) \triangleq \vee_{k\in\mathcal{M}_{-j}} \mathbf{I}(e_t = e^+_{\langle k,j\rangle}, u^r_k = a_3), \quad (6)$$

$$A^M_{\langle j,-j\rangle}(t,\omega) \triangleq \vee_{k\in\mathcal{M}_{-j}} \mathbf{I}(e_t = e^+_{\langle j,k\rangle}, u^r_k = a_2), \quad (7)$$

where $\mathbf{I}(.)$ is an indicator function.

Definition 3.2: (External Call Request Processes)

For any S^j, $Rq^M_{\langle -j,j\rangle}$ and $Rq^M_{\langle j,-j\rangle} : \mathbb{R}_+ \times \Omega \to \mathbb{Z}_+$ are point processes such that:

$$Rq^M_{\langle -j,j\rangle}(t) = \begin{cases} Rq^M_{\langle -j,j\rangle}(t^-) + 1, & \text{in case } A^M_{\langle -j,j\rangle}(t) = 1 \\ Rq^M_{\langle -j,j\rangle}(t^-), & \text{otherwise} \end{cases}$$

$$Rq^M_{\langle j,-j\rangle}(t) = \begin{cases} Rq^M_{\langle j,-j\rangle}(t^-) + 1, & \text{in case } A^M_{\langle j,-j\rangle}(t) = 1 \\ Rq^M_{\langle j,-j\rangle}(t^-), & \text{otherwise} \end{cases}$$

We call $Rq^M_{\langle -j,j\rangle}$ ($Rq^M_{\langle j,-j\rangle}$ respectively) the (aggregated) incoming (outgoing respectively) call request process of S^j, which is released (accepted respectively) by the mass $S^{\mathcal{M}_{-j}}$. $Rq^M_{\langle -j,j\rangle}$ and $Rq^M_{\langle j,-j\rangle}$ are jointly referred to as the *filtered external call request processes* of S_j. □

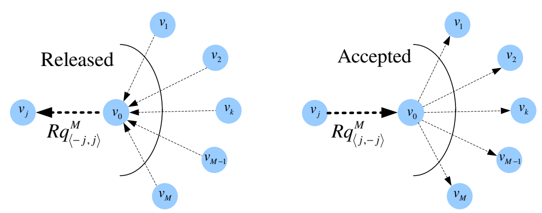

Fig. 4. Filtered Call Request Processes of S^j

Definition 3.3: The *local state process* of agent network S^j, denoted x^j, is defined to be

$$x^j_t \triangleq (\mathbf{n}^j_{t^-}, e^j_t) : [0,\infty) \times \Omega \to X^j, \quad (8)$$

with X^j, the *set of local state values* of S^j, and E^j, the *set of (aggregated) local events* of S^j specified as

$$X^j = \mathcal{N}^j \times E^j = \{(\mathbf{n}^j, e^j); \mathbf{n}^j \in \mathcal{N}^j, e^j \in E^j\}, \quad (9)$$

$$E^j = \{e^0_j, e^+_{\{j,0\}}, e^+_{\langle -j,j\rangle}, e^+_{\langle j,-j\rangle}, e^-_{\{j,0\}}, e^-_{\langle -j,j\rangle}, e^-_{\langle j,-j\rangle}\}, \quad (10)$$

where (i) e^0_j denotes absence of a call request or a connection ending of S^j; (ii) $e^+_1 \equiv e^+_{\{j,0\}}$, $e^+_2 \equiv e^+_{\langle -j,j\rangle}$ and $e^+_3 \equiv e^+_{\langle j,-j\rangle}$ denotes respectively internal, incoming and outgoing call request of S^j; while (iii) $e^-_1 \equiv e^-_{\{j,0\}}$, $e^-_2 \equiv e^-_{\langle -j,j\rangle}$ and $e^-_3 \equiv e^-_{\langle j,-j\rangle}$ denotes respectively the end of an active internal, incoming and outgoing connection of S^j. □

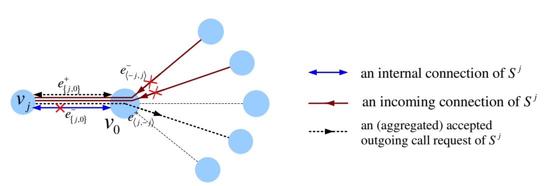

Fig. 5. Illustration of Local Events of Agent Network S^j

Definition 3.4: (Local State Transition Equation)

Subject to any local control law $u^r_k \in \mathcal{U}^r_k[0,\infty)$ for all $k \in \mathcal{M}$, the *local state transition equation* for S^j is specified as follows:

$$e^j_t = \begin{cases} e^+_{\{j,0\}} \\ e^+_{\langle -j,j\rangle} \\ e^+_{\langle j,-j\rangle} \\ e^-_{\{j,0\}} \\ e^-_{\langle -j,j\rangle} \\ e^-_{\langle j,-j\rangle} \\ e^0_j \end{cases} \text{in case} \begin{cases} e_t = e^+_{\{j,0\}} \\ A^M_{\langle -j,j\rangle}(t) = 1 \\ A^M_{\langle j,-j\rangle}(t) = 1 \\ e_t = e^-_{\{j,0\}} \\ \vee_{k\in\mathcal{M}_{-j}} \mathbf{I}(e_t = e^-_{\langle k,j\rangle}) = 1 \\ \vee_{k\in\mathcal{M}_{-j}} \mathbf{I}(e_t = e^-_{\langle j,k\rangle}) = 1 \\ \text{otherwise} \end{cases} \quad (11)$$

$$\mathbf{n}^j_t = \mathbf{n}^j_{t^-} + \widehat{u}^r_j(t, \mathbf{n}^j_{t^-}, e^j_t), \quad (12)$$

with e_t given in Definition 3.1 and $\widehat{u}^r_j(t)$ specified below:

$$\widehat{u}^r_j = \begin{cases} u^r_j, \\ \sum_{k\in\mathcal{M}_{-j}} u^r_j \mathbf{I}(u^r_k = a_3)\mathbf{I}(e_t = e^+_{\langle k,j\rangle}) \\ \sum_{k\in\mathcal{M}_{-j}} u^r_j \mathbf{I}(u^r_k = a_2)\mathbf{I}(e_t = e^+_{\langle j,k\rangle}) \\ -a_b, \ \text{with } b = 1,2,3 \\ \mathbf{0} \end{cases}, \quad (13)$$

in case $e^j_t = e^+_b$, e^-_b and $\emptyset$, with $b = 1,2,3$, respectively. The set of local control laws $\widehat{u}^r_j$ is denoted $\widehat{\mathcal{U}}^r_j[0,\infty)$. □

Remark: An external connection is established in S^j only if both S^j and the associated distant agent accept the arrival of call request.

Definition 3.5: (Agent and Mass Network Systems)

For any agent network S^j, $j \in \mathcal{M}$, a family of local state processes x^j induced by (11), (12) and a set (13) of local control laws $\mathcal{U}^r_j[0,\infty)$, is called an *agent (loss) network system* and is denoted S^j, while the collection of agent network systems $S^{\mathcal{M}} = \{S^j; j \in \mathcal{M}\}$ is called a *mass (loss) network system* (with the population of size M) and a *sequence of mass networks systems* $S^\infty = \{S^{\mathcal{M}}; M \in \mathbb{Z}_2\}$ is referred to as an *infinite mass system* S^∞. □

3) Asymptotic Independence of a Set of Processes $\{\mathbf{n}^j; j \in \mathcal{M}\}$: We now specify the following critical independence hypothesis $A1(t)$ for each $t \geq 0$ as the following:

A1(t): The set of local connection vector values $\mathbf{n}^{\mathcal{M}}_t \equiv \{\mathbf{n}^j_t; j \in \mathcal{M}\}$ is asymptotically i.i.d. at $t \geq 0$, as M goes to infinity. □

Theorem 3.1: Assume that the set $\mathbf{n}^{\mathcal{M}}_{t_0}$ is asymptotically independent as M tends to infinity for some $t_0 \geq 0$, i.e. assume that A1(t_0) holds, then asymptotic independence holds for the set of processes $\{\mathbf{n}^j_t; t \geq t_0, j \in \mathcal{M}\}$, in other words $\{A1(t); t \geq t_0\}$ holds. □

B. The Network Decentralized State (NDS)

Definition 3.6: (Network Decentralized State)

Consider the infinite mass system $S^\infty = \{S^{\mathcal{M}}; M \in \mathbb{Z}_2\}$, subject to the local state transition equations (11) and (12). We say the infinite mass system is in an *asymptotic network decentralized state (NDS)* with a pair of parameters $(\lambda_{in}, \lambda_{out})$, if the independent filtered external call request processes $Rq^M_{\langle -j,j\rangle}$ and $Rq^M_{\langle j,-j\rangle}$ form a mutually independent set with respect to j and are Poisson processes with rates respectively equal to $\lambda_{in}(t)$ and $\lambda_{out}(t)$ independent of the value of j, as M goes to infinity. □

In Theorem 3.1, we have shown under the initial assumption $A1(t)$ with $t = 0$ the set of local connection vector processes $\mathbf{n}^{\mathcal{M}}$ is asymptotically mutually independent as M goes to infinity; while in Theorem 3.2 below, we will show that the mass system will converge to an NDS asymptotically, as M goes to infinity. However, we first give in Lemma 3.1, a technical result which is required in the proof of Theorem 3.2. Before giving Lemma 3.1 as below, we first specify a class of (parameterized) isolated single systems as following:

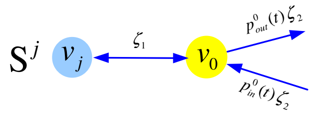

Fig. 6. Local and External Poisson Call Request Processes of An Isolated Single Agent System

Definition 3.7: (Isolated Single Agent System)

We specify a class of *(parameterized) isolated single agent systems* S^j, $j \in \mathcal{M}$, with parameters $(\xi_1^+, \xi_2^+, \xi_1^-, \xi_2^-)$, where the state set is the local connection vector set $\mathcal{N}$ (see (2)) with local, incoming and outgoing independent Poisson (call request) point processes (associated with events e_1^+, e_2^+, e_3^+) denoted Rq_b^+, $b = 1, 2, 3$, respectively.

(1) The rate of the processes Rq_b^+, $b = 1, 2, 3$ at a time t is equal to ξ_1^+, $p_{in}^0(t)\xi_2^+$ and $p_{out}^0(t)\xi_2^+$ respectively, with $p_{in}^0(t)$, $p_{out}^0(t) \in [0, 1]$.
The duration of b-th class of connections with $b = 1, 2, 3$ is exponentially distributed with rate equal to $1/\xi_1^-$, $1/\xi_2^-$ and $1/\xi_2^-$ respectively.

(2) The stochastic dynamics of S^j (see Figure 6) with a relaxed local feedback control law $\widehat{u}^r$ are given by the local state transition equation:

$$e_t^j = \begin{cases} e_b^+, & \text{in case } Rq_b^+(t) = Rq_b^+(t^-) + 1 \\ e_b^-, & \text{in case } Dp_b^j(t) = Dp_b^j(t^-) + 1 \\ 0, & \text{otherwise} \end{cases}, \quad (14)$$

$$\mathbf{n}_t^j = \mathbf{n}_{t^-}^j + \widehat{u}_t^r \quad (15)$$

□

Lemma 3.1: (Existence of NDS Parameters Compatible with Local Control Law)

For any (parameterized) isolated single agent network, given any initial state distribution P_0 and any control law, there exists a unique two component vector of probabilities $\mathbf{p}_t^0 = (p_{in}^0(t), p_{out}^0(t))$, independent of j, such that there exist incoming and outgoing Poisson processes with rates equal respectively to $p_{in}^0(t)\xi_2^+$ and $p_{out}^0(t)\xi_2^+$ such that the fixed point equations

$$p_{in}^0(t) = \mathbb{P}_{\mathbf{p}_t^0}(\widehat{u}_t^r = a_3 \,|\, e_3^+), \quad (16)$$

$$p_{out}^0(t) = \mathbb{P}_{\mathbf{p}_t^0}(\widehat{u}_t^r = a_2 \,|\, e_2^+), \quad (17)$$

hold where $\mathbb{P}_{\mathbf{p}_t^0}(.|.)$ displays the (parametric) dependence of $\mathbb{P}(.|.)$ on $\mathbf{p}_t^0$ and where a_2 and a_3 are defined in (3). □

Remark: The RHS of (16) captures the statistical behavior of the global mass system in terms of rate of release of external call requests (by taking action a_3) to agent S^j; while the RHS of (17) is its counterpart in terms of rate of acceptance by the mass system (by taking action a_2) of external call requests from S^j. The fact that the RHSs of (16) and (17) depend upon the vector $\mathbf{p}_t^0$ (defined for S^j) corresponds to the mass-individual symmetry of the global system's behaviour under the assumptions of (i) radial network symmetry and (ii) uniform control laws for all agents.

Theorem 3.2: (Existence of Local Feedback Control Compatible NDS)

Subject to a uniform local control law $\widehat{u}_t^r$ for all agents and under hypothesis $A1(t)$ with $t = 0$, the infinite mass system S^∞ is in an asymptotic NDS with the parameter $\mathbf{p}_t^0\lambda_2$ with $\mathbf{p}_t^0$ given in (16) and (17). □

IV. Distributed Control for Global Networks and the PPNCE Principle

In this section we establish the existence of network decentralized equilibria (NDE) subject to uniform local optimal control laws for given class of local cost functions, (see Definition 4.1).

Consider the global network performance functions given in $(S4)^g$ and $(S5)^g$, subject to any u^r for all agents, and assume following local performance specifications:

$(S4)^l$ With acceptance of $e_{\langle -j,j\rangle}^+$ and $e_{\langle j,-j\rangle}^+$, S^j yields a negative valued instant cost equal to αb_2 and $(1-\alpha)b_2$ respectively with $\alpha \in [0, 1]$.
During the duration of each of active incoming and outgoing connection, S^j yields a negative valued cost per unit time equal to αg_2 and $(1-\alpha)g_2$ respectively.

$(S5)^l$ With acceptance of a call request e at t, S^j gains a reward equal to $\varepsilon\mathbb{P}_{|e}(\widehat{u}_t^r \in U^+)$, with $\varepsilon \in [0, \infty)$.

Remark: (i) Given any uniform control u_t^r, the aggregated control law $\widehat{u}_t^r \equiv \widehat{u}_t^r(u_t^r)$ is specified in (13); (ii) The reward $\varepsilon\mathbb{P}(\widehat{u}_t^r \in U^+)$ in $(S5)^l$ is a mathematical smoothing device.

A. Optimal Controls of Isolated Single Agent Networks

Consider the local assumptions $(S4)^l$ and $(S5)^l$, with parameters (α, ε) and subject to any uniform local control law $\widehat{u}^r$. Then the cost function of the *local OSC problem* for an isolated agent network S^j, with (vector valued) parameter $\lambda(t, \mathbf{n}) \equiv (\lambda_b^+(t), \lambda_b^-(\mathbf{n});\ b = 1, 2, 3) \in \mathbb{R}_+^6$, such that $\lambda_b^+(t) = \xi_1^+$, $p_{in}^0(t)\xi_2^+$ and $p_{out}^0(t)\xi_2^+$ with $b = 1, 2, 3$, respectively, with $p_{in}^0(t)$, $p_{out}^0(t) \in [0, 1]$, and $\lambda_b^-(\mathbf{n}) = n_1/\xi_1^-$, n_2/ξ_2^- and n_3/ξ_2^- with $b = 1, 2, 3$ respectively, is specified to be:

$$J_{(\lambda,\alpha,\varepsilon)}^j(s, \mathbf{n}; \widehat{u}^r) = \mathbb{E}_{|(s,\mathbf{n})}\Big\{\int_s^\infty e^{-\beta t} G^j(\mathbf{n}_t^j)dt \quad (18)$$
$$+ \Sigma_{k=1}^\infty e^{-\beta t_k}\big(B^j(e_{t_k}^j) + \varepsilon\mathbb{P}(\widehat{u}_{t_k}^r \in U^+)\big)\mathbf{I}(\widehat{u}_{t_k}^r \in U^+)\Big\},$$

where by assumptions $(S4^g)$, $(S5^g)$, $(S4^l)$ and $(S5^l)$: $G^j(\mathbf{n}) = g_1 n_1 + \alpha g_2 n_2 + (1-\alpha) g_2 n_3$, $B^j(e_{\{j,0\}}^+) = b_1$, $B^j(e_{\langle -j,j\rangle}^+) = \alpha b_2$, and $B^j(e_{\langle j,-j\rangle}^+) = (1-\alpha)b_2$.

A family of *local OSC problems* (for an isolated single agent network S^j with parameter λ), with local cost function as in (18), is given by the infimization:

$$V^j_{(\lambda,\alpha,\varepsilon)}(s,\mathbf{n}^j_s) = \inf_{\widehat{u}^r \in \widehat{\mathcal{U}}^r[s,\infty)} J^j_{(\lambda,\alpha,\varepsilon)}(s,\mathbf{n}^j_s;\widehat{u}^r), \qquad (19)$$

where the function $V^j_{(\lambda,\alpha,\varepsilon)} : [s,\infty)\times\mathcal{N} \to \mathbb{R}$ with $\mathcal{N}$ defined in (2), is called the *value function* (of the family of local OSC problems for isolated agent network S^j). In the case an infimizing function $u^{r,*} \in \mathcal{U}^r[s,\infty)$ exists, $u^{r,*}$ shall be called an *optimal control law* for local OSC problem.

Theorem 4.1: [12] (The Hybrid HJB Equation for Stationary Isolated OSC Problems)

The HJB equation for a family of isolated local OSC problems with time-homogeneous call request processes, is a collection of coupled piecewise linear equations:

$$\begin{aligned}\beta V_{\mathbf{n}} = G^j(\mathbf{n}) &+ \Sigma_{e_b^- \in E^-}\lambda_b^-(\mathbf{n})\big(V_{\mathbf{n}-a_b} - V_{\mathbf{n}}\big) \qquad (20)\\ &+ \Sigma_{e_b^+\in E^+}\lambda_b^+ \inf_{\widehat{u}^r\in\widehat{\mathcal{U}}^r}\Big\{\varepsilon\big(\mathbb{P}(\widehat{u}^r = a_b)\big)^2 \\ &\quad + \big(B^j(e_b^+) + V_{\mathbf{n}+a_b} - V_{\mathbf{n}}\big)\mathbb{P}(\widehat{u}^r = a_b)\Big\},\end{aligned}$$

for all $\mathbf{n} \in \mathcal{N}$, with constant $\lambda_b^-(\mathbf{n})$ and λ_b^+, given in Definition 3.7 and $V \equiv V^j_{(\lambda,\alpha,\varepsilon)}$. □

B. The Network Decentralized (Nash) Equilibrium (NDE)

Definition 4.1: Consider the infinite mass system $S^\infty = \{S^{\mathcal{M}}; M \in \mathbb{Z}_2\}$, with the local state transition equation (11) and (12) and local cost function (18) subject to uniform local relaxed control law $\widehat{u}^r$. We say the infinite mass system is in an *asymptotic network decentralized (Nash) equilibrium (NDE)* with a pair of constant parameters $(\lambda^0_{in},\lambda^0_{out})$ if: (i) The infinite mass system is in an asymptotic NDS with parameters $(\lambda^0_{in},\lambda^0_{out})$; and (ii) $\widehat{u}^r$ is a uniform local optimal control law for the local OSC problems of all agent networks with cost function (18) and parameters $(\lambda^0_{in},\lambda^0_{out})$. □

Remark: An NDE is an NDS property with the usual reciprocity requirements, together with the added robustness that comes from the assumed individual optimality properties but with deterrence from unilateral deviations properties which flow from optimality (see (ii) in Definition 4.1).

Corollary 4.1: (Existence of Stationary NDE)

Under initial hypothesis $A1(0)$ as M tends to infinity the mass system with the local OSC problems (18) is asymptotically in a stationary NDE with the constant parameters $(p^0_{in}\lambda_2, p^0_{out}\lambda_2)$ with p^0_{in} and p^0_{out} specified as follows:

$$p^0_{in} = \mathbb{P}_{\lambda^0}(\widehat{u}^{r,*}_{(\lambda^0,\alpha,\varepsilon)}(\mathbf{n}^j_{t^-}, e^j_t) = a_3 \,|\, e_3^+), \qquad (21)$$

$$p^0_{out} = \mathbb{P}_{\lambda^0}(\widehat{u}^{r,*}_{(\lambda^0,\alpha,\varepsilon)}(\mathbf{n}^j_{t^-}, e^j_t) = a_2 \,|\, e_2^+). \qquad (22)$$

□

V. Conclusion and Future Work

This paper presents an analysis of distributed call admission control problems for a class of global loss networks each of which is composed of a group of weakly coupled individual systems. Asymptotically, under initial independence of states hypothesis, and for uniform local control laws, agent network state processes and their boundary conditions remain mutually independent. Furthermore, moving from centralized OSC problems to a distributed OSC paradigm whereby agents apply local control laws to optimize their individual costs, it is shown that there exists boundary conditions and uniform relaxed local control law pairs such that the local control laws are optimal with respect to the very boundary conditions they collectively induce. In other words, the NCE (or mean field) property holds and in this context we call it the Point Process NCE (PPNCE) principle. Future work will include the study of the simultaneous solution of the problems of call admission and routing control within the NCE framework; a solution to this problem already having been given in the centralized control case in [10], [11].

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