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Nonlinear dynamic modeling for control of fusion devices

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***Abstract*— In this paper the issue of modeling for control design with application to fusion devices is discussed. Given the difficulty to provide analytical solutions to the equations describing the system dynamics, the use of numerical tools for evaluating different control architectures is required. Although standard tools for Computer Aided Control System Design (CACSD) can be usefully employed for both analyzing the fusion device dynamics and designing control laws, dedicated tools are required to adequately describe the complex interactions among the different system components. In this paper the use of the nonlinear equilibrium code MAXFEA to support the control design task is described. MAXFEA is a finite element code able to produce quite good approximations of the plasma boundary location and shape, together with internal distributions of current and magnetic fields, and other plasma features. The code provides the simulation of the plasma dynamics, while all the other elements (diagnostics, controller, actuators) in the control loop system can be modeled independently and integrated in the code as external modules, thus making it a candidate tool for Software-in-the-Loop solutions.**

I. INTRODUCTION

One of the most promising and viable approaches to produce energy from nuclear fusion reactions, on Earth and in a controlled way, is to resort to the magnetically confined plasmas, which yield a relevant rate of nuclear reactions while kept inside a metallic toroidal chamber at extremely low pressures and high temperatures.

The equation governing the equilibrium of a magnetically confined plasma is the Grad-Shafranov equation, a two dimensional nonlinear elliptic partial differential equation [1] that basically states the equilibrium of a plasma column with an external magnetic field, in hypothesis of axisymmetry. The toroidal plasma current density profile $J_p(\psi)$ is given as a function of the poloidal flux ψ by the following

$$J_p(\psi) = \mathcal{L}(\psi) = -\mu_0 r \frac{\mathrm{d}p(\psi)}{\mathrm{d}\psi} - \frac{1}{r} f(\psi) \frac{\mathrm{d}f}{\mathrm{d}\psi}, \qquad (1)$$

with $\mathcal{L}$ being the operator

$$\mathcal{L} = \left[\frac{\partial}{\partial z} \left(\frac{1}{r} \frac{\partial}{\partial z} \right) + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} \right) \right]. \qquad (2)$$

The profile is thus parameterized according to the two functions $p(\psi)$ and $f(\psi)$, respectively the pressure profile and the poloidal current density flux function.

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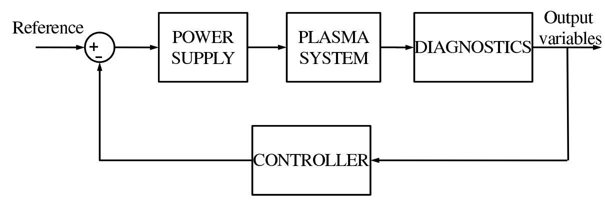

Fig. 1: Control Loop scheme. The open loop is the cascade of power supply, plasma-vessel system, and diagnostics.

Solving the Grad-Shafranov equation means to compute the magnetic flux distribution for a given external coil current configuration, whence the plasma boundary (its flux value, location, and shape) is determined. To compute a boundary for a plasma in equilibrium is known as the *free boundary problem*. Actually, the nonlinear nature of the problem arises from the parametrization chosen for $p(\psi)$ and $f(\psi)$ and the free-boundary condition itself.

Unfortunately, in general this does not offer an analytical solution so that a common way to solve the problem is to employ numerical procedures, as finite element or finite difference methods, implemented in equilibrium codes.

At the same time, the magnetic control of a fusion device is required for the control of the macroscopic characteristics of the plasma mass (shape, position, modes, current) both to drive the plasma during the various phases of the discharge and to counteract disturbances and internal instabilities, and is exerted by acting dynamically on the magnetic field produced by external coils surrounding the vacuum chamber.

The use of equilibrium codes is of fundamental importance not only for the inherent level of detail, but also for the perception of the dynamics of the complete system obtained through the nonlinear simulation. In fact, the complete closed loop system can be simulated in the equilibrium code: The vessel-plasma system, the diagnostics (output measurements), the actuators (input signals). The power supply block and the diagnostics can be easily understood and their accurate modeling poses stringent constraints on system controllability [2]. The vessel-plasma system basically refers to the complex interaction among the effects of the active coil currents, the eddy currents flowing in the machine structure (passive metallic structures, plasma facing components, ...), and the plasma, to produce the output quantities measured by the diagnostic sensors.

In particular, the possibility of performing long time steady state simulations, or even simulations of different phases of the plasma discharge with the relevant transitions, represents

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a major advantage for the control engineer when to design an end-to-end scenario. Lastly, control scientists are facing more and more stringent constraints regarding dedicated time for commissioning, so that the simulation phase assumes a central rôle in proposing new control schemes or modification to already implemented control strategies.

In this paper we describe how all of the above mentioned issues can be successfully dealt with by resorting to the nonlinear equilibrium code MAXFEA, a finite-element code able to produce good approximations of the dynamics of the key quantities that describe the plasma evolution, such as the plasma boundary location and shape, internal distributions of current and magnetic fields, etc.. The code itself provides the simulation of the plasma dynamics, while all the other elements (measurement sensors, controller scheme, actuators) in the control loop system can be modeled independently and integrated in the code as external modules, thus making it a candidate tool for Software-in-the-Loop solutions. In this perspective, the nonlinear code can be seen as an integration platform, *where the emphasis on the term "platform" implies a middleware environment from which integration oriented solutions are derived* [3].

II. NONLINEAR MODELING OF FUSION DEVICES

MAXFEA [4] is a finite element code employing first order elements, able to solve the direct and inverse free boundary problems, and to perform static and dynamic simulations. It provides quite good approximations of the plasma boundary location and shape, together with internal distributions of current and magnetic fields, and other plasma characteristics. Nonetheless, since it requires iterative procedures, MAXFEA is not suitable for the real time application. The use of the code is therefore restricted to the pre-operation phases, in particular during the control system design for the simulation of the proposed algorithms, and then during the tuning and testing phase, for possible software-in-the-loop application with the actual hardware implementation.

The Grad-Shafranov parametrization used in the code for $p(\bar{\psi})$ and $f(\bar{\psi})$ as functions of the normalized flux $\bar{\psi} = (\psi - \psi_a)/(\psi_b - \psi_a)$ expresses the profiles in terms of parameters α and β related respectively to the plasma internal inductance ℓ_i and the poloidal beta β_p. Here, and in the remainder, $(\bullet)_a$ and $(\bullet)_b$ indicate values at respectively the magnetic axis and the boundary. For a tokamak machine, the pressure p and the poloidal field current density f functions result

$$p(\bar{\psi}) = p_a \left[-\frac{\alpha+1}{\alpha}\bar{\psi} + \frac{1}{\alpha}\bar{\psi}^{\alpha+1} + 1 \right] \quad (3)$$

$$f(\bar{\psi}) = f_a \sqrt{\left(1+\frac{1}{\alpha}\right)\left[\frac{\alpha}{\alpha+1} - \bar{\psi} + \frac{\bar{\psi}^{\alpha+1}}{\alpha+1}\right]}. \quad (4)$$

As a note, parameter α appears explicitly in the equations, while β is implicitly absorbed in the axis values p_a and f_a.

In this work we will refer to fusion devices such as JET [5], ITER [6], and RFX [7], different in nature and in the characterizing features. Despite the fact that all the aspects shown in the following sections can be applied to the same device, the choice of presenting issues related to different machines is motivated by the idea of versatility the authors desire to convey, about the use of the nonlinear code.

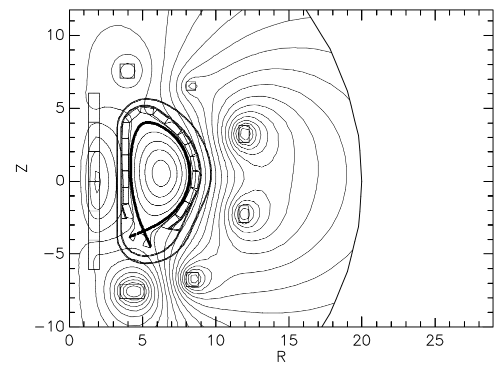

Fig. 2: MAXFEA Flux map. Example of output flux map provided by the code with reference to the ITER machine.

III. DIAGNOSTICS AND SENSING

In tokamak devices, standing the axisymmetry approximation, it is possible to reduce the study of the toroidal plasma volume to the analysis of the poloidal cross section of the plasma column, therefore a two-dimensional flux map (Fig. 2) is sufficient to describe the magnetic configuration and consequently to locate the plasma shape.

In actual fact, the magnetic field map, generated by the combination of the presence of plasma and the imposed active coil currents, is the main ingredient for the magnetic diagnostic system, made of flux and field sensors; from these measurements, tailored algorithms based on the current moment equations [8], provide measurements of plasma current value and current centroid position.

The current moment method [9] allows measuring some distribution of field sources inside the plasma, distinguishing them from external ones. Given a closed contour l surrounding the plasma of cross section Ω_p and current density j_p (and not comprising any other current source), the current moment q_g is associated to the external magnetic field B (in its tangential and normal components B_T and B_N) as

$$q_g = \int_{\Omega_p} g j_p \cdot \mathrm{d}\Omega = \oint_l (gB_T + rfB_N) \cdot \mathrm{d}l \quad (5)$$

where (g, f) is a couple of scalar functions solving the equation. In particular, the lower order moments provide macroscopic information on the plasma such as total current (q_0), and shape descriptors: Centroid position (q_1, q_2), skew and vertical ellipticity (q_3, q_4), upward and outward triangularity (q_3, q_4). Most importantly, this method lists in order of complexity all the features of the current distribution that can be identified using external magnetic measurements.

This approach to the design of the diagnostic system has two scopes of application. For new devices, the relevant issue is the optimal placement for the sensors, which are the output variables whence to reconstruct the system state. Conversely, for running machines, we are interested in understanding the dimension and the characterization of the plasma shape space achievable with suitable scenarios, in other words, the experimental reachable space.

The discretization of the rhs of Eq. 5 allows reducing the integral over a continuous line to a finite summation:

$$q_g \approx \widehat{q_g} = \sum_{j=0}^{N_T} g_{T,j} B_{T,j} + \sum_{j=0}^{N_N} g_{N,j} B_{N,j} = \sum_{j=0}^{N_s} g_{s,j} s_j \quad (6)$$

where $B_{T,j}$'s ($B_{N,j}$'s) are the measurements of the tangential (normal) field at N_T (N_N) locations and $g_{T,j}$'s ($g_{N,j}$'s) are suitable coefficients: In practice, the calculation of $\widehat{q_g}$ refers to a set of N_s sensors s_j's through coefficients $g_{s,j}$'s.

The macroscopic quantities computed from the sensor signals through Eq. 6 are also given by the equilibrium code as internal variables. Therefore, the design of the diagnostic system (location of the sensors, calculation of discrete coefficients s_j's) can be driven by the nonlinear code, used as a tool for providing data, in this leading to the derivation of an optimization problem of the sort

Problem 1: Diagnostic System Design: Find the combination of sensors and coefficients minimizing the error between the reconstructed moments and the "real" ones, the reality ground truth being given by the nonlinear code.

Similarly to the moment approach, also the location of the boundary at specific gap-points can be derived from magnetic measurements, since the external configuration of the magnetic flux map determines the shape of the plasma. The *gap* is the distance between the plasma boundary and the plasma facing component computed along a virtual line starting from the firstwall: The collection of gaps around the plasma cross section is commonly chosen as a shape descriptor and is used as a feedback variable for shape control [10].

The gap reconstruction algorithm makes use of a wide range of signals, resulting in a linear combination of the active coil currents, the magnetic flux and field measurements signals. Again, the plasma shape is computed within the equilibrium solution by the MAXFEA code which is also able to simulate the sensor probes; therefore, in the nonlinear simulation it is possible to replicate the algorithm procedure and validate the results of the reconstruction.The simulations performed with the nonlinear code show a good agreement between the reconstructed variables and the ones given by the equilibrium code during the evolution (Fig. 3), where the presence of a small offset in all the reconstruction signals can be noticed. Such a difference, larger during the transient and tending to settle as reaching the steady state, may be due to the static characteristic of the algorithm.

Conversely, for existing machines, there may be the need of enhancing or complementing an installed sensor system, as recently in the case of JET. As for the reconstruction capability, it has been fundamental for the sensor definition to understand beforehand how much the enhancement of the diagnostic system will, in principle, increase the measurability of the plasma shape and extend the operating space [11]. Stringent constraints to the design of the refurbishment are put by the existing JET structure and systems.

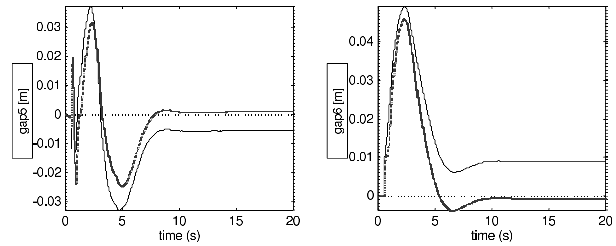

Fig. 3: Example of gap reconstruction. Reconstruction algorithm designed for the ITER machine.

Through the use of the nonlinear code a huge database related to the whole range of plasma configurations has been built. Among the useful output variables, an "ideal observer" has been identified, comprising all magnetic signals S_V (virtual sensors) measured along a closed line l as in Eq. 5. It is also made the reasonable assumption that the ideal observer is related to the real observer (the finite set of real sensors S_R) through a linear relation, $S_V = K_{R2V} S_R$, where each column of S_V (S_R) is related to a specific equilibrium, and the K_{R2V} matrix states the correspondence between the two sets of measurements and how the whole information is distributed among the real sensor sets.

A Singular Value analysis performed on the data matrices S_V and S_R reveals that most of the information is carried by a limited number of "combinations of sensors". This decomposition over the data coming from the virtual sensors shows as expected that the whole space of the available magnetic and consequently plasma configurations (the reachability space) has a limited number of degrees of freedom (less than 60, at JET). As far as the real measurements are concerned this dimension varies according to the choice of the sensors, but in any case adding new measures effectively expands the space of observability, introducing innovation through other orthogonal components. In this sense a refurbished magnetic system would bring in more information.

Therefore, the following problem can be formalized

Problem 2: Diagnostic System Refurbishment: Find the combination of new magnetic signals that augments the observability space w.r.t. the existing magnetic configuration and the machine structural constraints.

This problem is solved through two complementary approaches (see Fig. 4). Firstly, the analysis of the error propagation from the real sensor measurement to the field reconstruction (ideal observer) is carried out for different choices of augmented sensor sets. The reconstruction noise is largely reduced almost everywhere around the plasma; also, the use of all the available magnetic probes can, in principle, improve the reconstruction capability. Instead, this is practically limited by how the enhanced information is exploited. In fact, the sensor data have to be interpreted in the reconstruction code framework: In this sense, the analysis of the boundary shape modification as an error on gaps is obtained resorting to the actual code used during the plasma

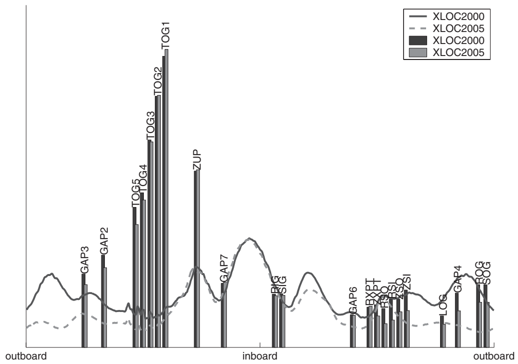

Fig. 4: Magnetic refurbishment. Error propagation on the magnetic field reconstruction and correspondence with the gap reconstruction provided by the real time code.

operation.

IV. LINEARIZATION AND CONTROLLER DESIGN

The purpose of plasma magnetic control is twofold: On the one side there is the need to stabilize the plasma vertical position (naturally unstable for elongated plasmas), on the other there is the need to shape the plasma boundary, for several issues ranging from volume occupancy, to passive structure coupling, to specific energy confinement regime requirements. In general, the design of this controller, both for vertical stabilization and shaping, is carried out by resorting to linear models derived from the linearization of the system stabilized at specific equilibria along the scenario [12].

The resulting system can be approximated by a lumped parameter system obtained by describing the massive structures via toroidally symmetric elements of finite cross section, writing for each of these elements a circuital equation as:

$$\dot{\Psi}(t) + RI(t) = V(t); \tag{7}$$

this equation describes the relation between the magnetic flux $\Psi(t)$, the current $I(t)$ (in the plasma, in the metallic passive structures, and in the active coils), and the voltage $V(t)$ applied to the active coils, with R being a (diagonal) matrix of resistances. Under commonly accepted assumptions, the linearization of Eq. 7 can be carried out resorting to the equilibrium code, thus yielding

$$L^* \dot{I}(t) + RI(t) = V(t), \tag{8}$$

where the modified inductance matrix L^* takes into account the presence of the plasma. Being n the dimension of the current vector I, the perturbation of the equilibrium for the derivation of the L^* matrix can proceed by perturbing each loop current at the equilibrium $I_{k,0}$, $\{k = 1, \ldots, n\}$ by a small amount $\delta I_{k,0}$ [13]. This approach has been extensively used to derive linear models for control systems design, although attention must be paid to the fact that most free-boundary codes require the plasma to be stabilized while computing the perturbed equilibrium after an external current variation, and the computation of L^* becomes tricky.

Alternatively, under some hypotheses over the machine structure that stand for example for the ITER device (active coils outside the closed vacuum vessel metallic structure separating inside from outside) it is possible to consider flux perturbations, by slightly altering the flux Ψ_0 linking loop currents $I_{k,0}$ and limiting the number of flux perturbations to the flux linking loop currents "facing" the plasma since the other currents are shielded by eddy currents induced in the ideally conducting shell [14]. Remarkably, in this approach, the problem of stabilizing the plasma can be neglected since in the presence of an ideal shell the plasma is always stabilized w.r.t. to the timescale of interest.

In both procedures, the nonlinear code reveals a particularly versatile and agile tool to obtain the equilibrium and perform the whole range of perturbations needed for the linear model derivation. As a matter of fact, a major advantage is in the simplicity of procedure since it eliminates the degrees of freedom related to the plasma fluid displacement making use of free boundary Grad-Shafranov solvers.

Coming back to the structure of the linear model, the plasma current profile is approximated by using three degrees of freedom, namely total plasma current $I_p(t)$, poloidal beta $\beta_p(t)$ (stating the equilibrium between kinetic and magnetic pressures), and internal inductance $\ell_i(t)$ (giving indication of the current profile peaking). Variations of and with respect to their nominal values at a given equilibrium are used to model critical plasma disturbances, so that it is relevant that they are included in the model as external inputs.

From Eq. 8, the system dynamics can be represented in state space form given the positions:

$x(t)$: state vector comprising currents in all structure elements;

$u(t)$: input vector comprising voltages applied to active circuits;

$w(t)$: disturbance vector comprising variations in the $(\beta_p(t), \ell_i(t))$ parameters (basically representing an additional input);

$y(t)$: output vector comprising plasma macroscopic features (plasma current, current centroid position, shape);

leading to the following system equations

$$\dot{x}(t) = Fx(t) + Gu(t) + E\dot{w}(t) \tag{9}$$

$$y(t) = Hx(t) + Fw(t). \tag{10}$$

As is common practice in the field of tokamak control, linearized models are the main ingredient for controller design, since they are easy to analyze, they provide good insight into the system dynamic features, and they well describe the system behavior under normal operational conditions. Furthermore, the use of linearized models greatly simplifies the design of feedback controllers. In this framework, though, the availability of nonlinear codes is crucial after the design phase. Actually, for the idea mentioned before that the nonlinear code acts as an integration platform, the equilibrium code is used to simulate the behavior of the system in closed loop in order to assess the performances of the control system in keeping the magnetic configuration of the plasma in

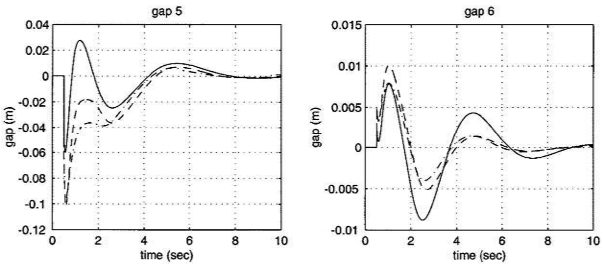

Fig. 5: Linear vs nonlinear simulations: gap behavior after application of a disturbance. Solid line: nonlinear simulation; dashed and dash-dotted lines: two different linear models.

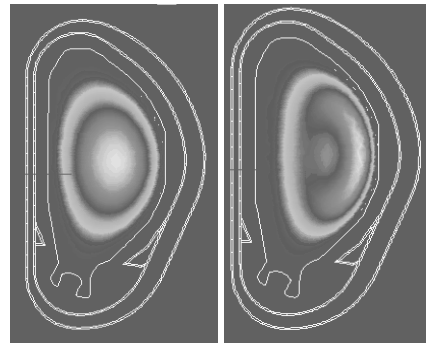

Fig. 6: Plasma current distribution for ITER equilibria. Nominal (left) and reverse shear (right) current profiles.

presence of disturbances. To do so, the designed controllers are discretized so as to fit the digital control scheme of the equilibrium code and also of the real machine system. Then the algorithms implemented in MAXFEA are tested against the whole set of nominal disturbances at different equilibria and the results compared with those provided by the linear models [15] [16] (see example in Fig. 5).

When simulating the complete closed loop system, according to the scheme in Fig.1, the feedback variables are computed from the simulated magnetic diagnostics signals by means of two reconstruction parallel algorithms: One algorithm is devoted to evaluate plasma current and current centroid vertical position, the other one to reconstruct shape in terms of gap displacements or a similar procedure. The control scheme consists of three feedback loops: The two controllers designed according to a frequency separation approach [17], one deputed to vertical position stabilization, and the other to plasma current and shape control, and the Anti-Saturation device, whose strategy consists in modifying the set of gap references, as soon as the current in one or more coils is approaching its saturation limit, in order to avoid the incoming saturation. This action is performed by a nonlinear feedback controller, which takes the active coil currents as input and acts a smooth transition from the nominal reference set points to a previously computed set of emergency gaps. The evaluation of the emergency gaps is carried out off-line by means of a least square optimization on the basis of the amplitude of the disturbances to be rejected, the maximum allowed gap displacements and the current limits.

In addition, it is also possible to reconstruct the pulse reference scenario implementing the complete control scheme, including voltage feedforward components and the capability to switch between different controllers to manage change in the plasma topology. This allows to perform the dynamic simulation of the whole discharge and therefore to assess the real performances of the control system in driving the plasma magnetic configuration along the desired evolution [18]. This is particularly important for scenarios where the physics features of the plasma throughout the entire discharge play a crucial rôle: For example in reverse shear configuration for ITER, the nonlinearities of the MHD dynamics leads to a particular current distribution (see Fig. 6). Although being affected by the specific equilibrium point and the applied control, the internal plasma parameters are not observable (they depend on transport dynamics, not modeled in Eq. 9-10). On the other side, these aspects would lead to complex models whose details are overabundant for the controller design. In summary, the description provided by the linear model discussed so far is simple and suitable for the design of plasma current, position, and shape controllers, but it is mandatory to resort to the nonlinear MHD modeling built in the equilibrium solver to assess the control approach, to tune the controller parameters, and to study the detail of the whole scenario simulation. As an example, two different equilibrium configurations - corresponding to ITER nominal and weak reverse shear scenarios - have been computed with MAXFEA for the current flat-top phase of the pulse (see Fig. 6.) Models of both nominal and reverse shear plasma response are obtained adopting the flux perturbation technique. As a first validation of the two linear models, the inverse of the unstable eigenvalue (yielding vertical instability) is compared to the growth time estimated from the nonlinear simulations, finding a satisfactory agreement. Then, proceeding with the control systems design, a frequency decoupling control scheme is developed on the linear model and implemented in the MAXFEA code.

As an additional aspect, it is worth mentioning the use of the nonlinear code for the design of the control system for the RFX-mod experiment, a Reverse Field Pinch (RFP) machine different in nature from ITER or JET (Fig. 7). Since RFX-mod is the first RFP experiment featuring an active control of the plasma axisymmetric position, it was necessary to develop suitable tools for the control system design and simulation [19]. The two-dimensional nonlinear simulation code, solving the free boundary equilibrium problem, is developed for the RFP plasma configuration. The nonlinear modeling is carried out by expanding the code capabilities to model time varying non monotonic and negative profiles of the toroidal field and subsequently to include the RFP plasma internal profiles [20]. The original Grad-Shafranov parametrization used in the code for pressure p (Eq. 3) and poloidal field current density f (Eq. 4) are substituted by the following expressions (depending on α, β, and the additional

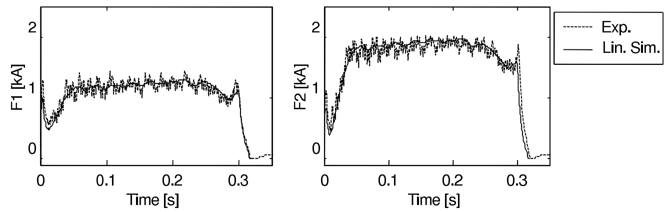

Fig. 7: RFX model. Validation with experimental data of the linear model of RFX derived from the nonlinear model implemented in MAXFEA.

γ parameters):

$$p(\bar{\psi}) \;=\; p_a\left[(\gamma-1)\,\bar{\psi}^{\gamma}-\gamma\bar{\psi}^{\gamma-1}+1\right] \tag{11}$$

$$f(\bar{\psi}) \;=\; f_a\left[1+\nu\left(-\bar{\psi}+\frac{\bar{\psi}^{\alpha+1}}{\alpha+1}\right)\right] \tag{12}$$

where ν is defined as

$$\nu=\left(1-\frac{f_b}{f_a}\right)\left(1+\frac{1}{\alpha}\right), \tag{13}$$

and again β does not appear explicitly but through the axis value terms. The results of the simulations are in good agreement with the experimental measurements: The nonlinear model through the numerical solver provides a correct evolution of the axisymmetric plasma position and reproduces with good precision the whole set of diagnostic signals.

V. OPEN RESEARCH ISSUES AND CONCLUSIONS

Several other aspects can be included in the modeling, covering issues that are positioned at the edge of the state of the art in technology or academic research. As an example, the response of the whole system when a physical module of known dynamics is inserted in the loop can be studied (e.g. blanket modules covering the vessel, and altering the system frequency response).

Also, there can be tested novel approaches to shape modeling, description, and control, that have been inspired by an active vision point of view [10]. Starting from this idea, the study and the derivation of a new control strategy have been developed: The plasma shape is represented no longer as a set of gap distances but as a spline curve modeling the boundary contour. Also, the control points assume the rôle of curve parameters and constitute the feedback quantities for the shape controller.

Remarkably, not only does the control technique based on contours comprise the control of the curve at specific locations, but also it yields the control of integral parameters such as the elongation and the triangularity. This spline controller, again, have been first designed according to the linear system theory, and then implemented in MAXFEA for assessing the control performance with more realistic nonlinear simulations [21].

In summary, the availability of nonlinear equilibrium codes such as MAXFEA is crucial for designing the control architecture of fusion devices and for assessing their performance in a virtual environment. In this sense, the nonlinear modeling provided by the equilibrium code can be interpreted as a software-in-the-loop tool before the actual experiment on the real machine. Currently, many activities are ongoing on these subject in the framework of the task EFDA-ITM (European Fusion Development Agreement - International Tokamak Modelling) aiming at providing the fusion scientists with a set of tools allowing the analysis, the preparation, and the simulation of the perspective experimental ITER operation [22].

REFERENCES

[1] J.P.Freidberg, *Ideal Magnetohydrodynamics*. New York, USA: Plenum Press, 1987.
[2] M.L.Walker, D.A.Humphreys, and E.Schuster, “Some nonlinear controls for nonlinear processes in the DIII-D tokamak,” in *Proc. IEEE Conf. on Dec. and Contr., HI (USA)*, 2003, pp. 6565–6571.
[3] Wikipedia, “Universal integration platform,” http://en.wikipedia.org/wiki/Universal integration platform, Online [March 2008].
[4] P.Barabaschi, “The MAXFEA code,” Plasma Control Technical meeting, Naka (Japan), 1993.
[5] “Jet website,” http://www.jet.efda.org/, Online [March 2008].
[6] “Iter website,” http://www.iter.org/, Online [March 2008].
[7] “Rfx website,” http://www.igi.cnr.it/, Online [March 2008].
[8] B.Ph.VanMilligen, “Exact relations between multipole moments of the flux and moments of the toroidal current density in tokamaks,” *Nuc. Fus.*, vol. 30, no. 1, pp. 157–160, 1990.
[9] L. E. Zacharov and V. D. Shafranov, “Equilibrium of a toroidal plasma with noncircular cross section,” *Sov. Phys. Tech. Phys.*, vol. 18, no. 2, pp. 151–156, 1973.
[10] A.Beghi and A.Cenedese, “Advances in real-time plasma boundary reconstruction: from gaps to snakes,” *IEEE Contr. Sys. Mag.*, vol. 25, no. 5, pp. 44–64, 2005.
[11] A.Cenedese, R.Albanese, G.Artaserse, M.Mattei, and F.Sartori, “Reconstruction capability of JET magnetic sensors,” *Fus. Eng. and Des.*, vol. 74, no. 1-4, pp. 825–830, 2005.
[12] G.Ambrosino, M.Ariola, A.Pironti, and F.Sartori, “Design and implementation of an output regulation controller for the JET tokamak,” *IEEE Trans. Contr. Sys. Tech.*, vol. to appear.
[13] R.Albanese, E.Coccorese, and G.Rubinacci, “Plasma modelling for the control of vertical instabilities,” *Nuc. Fus.*, vol. 29, no. 6, pp. 1013–1023, 1989.
[14] M.Cavinato, “2d and 3d modeling for plasma control in nuclear fusion reactors,” Ph.D. dissertation, Dept. of Electric Engineering, University of Padova, Padova, Italy, 2005.
[15] F.Villone, P.Vyas, J.B.Lister, and R.Albanese, “Comparison of the CREATE-L plasma response model with TCV limited discharges,” *Nuc. Fus.*, vol. 37, no. 10, pp. 1395–1410, 1997.
[16] A. Beghi, “An application of selective modal analysis to tokamak modeling and control,” *IEEE Trans. Contr. Sys. Tech.*, vol. 9, no. 4, pp. 574–589, 2001.
[17] A. Portone, “Design of the ITER-FEAT plasma control system by a frequency separation approach,” *Fus. Eng. and Des.*, vol. 56-57, pp. 731–734, 2001.
[18] M.Cavinato, A.Kavin, V.E.Lukash, and R.R.Khayrutdinov, “Nonlinear simulations by numerical magnetohydrodynamics equilibrium codes in ITER-FEAT,” in *Proc. IEEE Conf. on Contr. App.*, 2000, pp. 406–411.
[19] M.Cavinato and G.Marchiori, “Design of the new RFX equilibrium active control system,” *Fus. Eng. and Des.*, vol. 74, no. 1-4, pp. 567–572, 2005.
[20] P.Bettini, M.Cavinato, and G.Marchiori, “Two-dimensional non-linear model of reversed field pinch plasma evolution,” *Nuc. Fus.*, vol. 43, pp. 119–129, 2003.
[21] A.Cenedese and A.Beghi, “Optimal approach to shape parameter control,” in *Proc. 6th As. Contr. Conf., Bali, Indonesia*, 2006, pp. 556–563.
[22] “Efda integrated tokamak modelling website,” http://www.efda-taskforce-itm.org/, Online [March 2008].