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# Passivity based control of a reduced port-controlled hamiltonian model for the shallow water equations

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*Abstract*— **In this paper an extension of an existing reduced port-controlled hamiltonian (PCH) model for the shallow water equations (PDEs) is first proposed. It aims at a new definition for the passive boundary port-variables which allows the application of a passivity-based approach to control the water flows and levels profiles in irrigation channel reaches. Then a control law based on the Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) methodology is developed. It allows to assign desired structure and energy function to the closed loop system. Simulation results made on a micro-channel simulator are presented, showing the effectiveness of the control law.**

## I. INTRODUCTION

The dynamic of fluid flow in an open channel is modelled by a set of hyperbolic PDEs: the Saint-Venant equations also called Shallow water equations. They are derived from conservation laws of mass and kinetic momentum using some assumptions on the flow [1]. A port hamiltonian formulation of the shallow water equations has been proposed in [2] considering elementary volume and momentum density as state variables and the total kinetic and potential energy as the hamiltonian of the system (see [3] and [4] for port hamiltonian formalism). We proposed in [2] a reduced PCH model for these equations which is obtained using a geometric reduction scheme based on mixed finite elements method [5], [6]. This reduction scheme preserves both the interconnection structure and the energetic properties of the actual equations. The obtained model also exhibits some interesting dynamic spectral properties. In [2] the canal reach was discretized using a mixed finite elements method in a finite number of finite-dimensional "cells". With the chosen cells output variables a direct transfer, which does not exist in the shallow water equations, appears between input and output of each of these cells. This direct transfer prevents direct application of a passivity-based output feedback. This situation is not satisfactory even from a physical modelling point of transport phenomena where the transfer between upstream and downstream is characterized by a (variable) time delay. We will thus propose in this paper a new expression of internal reduced variables (efforts) which eliminates these direct transfer terms and preserves the hamiltonian structure of the reduced individual cells.

Many control algorithms for fluid flow through open-air channels have been developed. Most of them are based on reduced models of the shallow water equations. Some works are based on continuous time reduced model obtained by the orthogonal collocation method as in [7] (input-output linearization), in [8] (backstepping) or in [9] (robust optimal control). Other are developed on discrete time models obtained using the Preissmann implicite finite differences scheme [10] as in [11] (predictive control) or in [12] (optimal control). In this paper we intend to use the structured PCH form of the reduced model to design a control law which makes the closed loop system passive with respect to desired storage function. To achieve this result we use the interconnection and damping assignment passivity based control (IDA-PBC) developed in [14] which also allows us to assign prescribed interconnection and damping structures to the closed loop. The regulation problem we address in this paper is to achieve a desired water flow at the downstream of a reach and a water level at the upstream. This is the case when the reach is assumed to provide some defined demand while ensuring a safe operating of the hydraulic works. The paper is organized as follows. In section II, we recall the PCH formulation for the shallow water equations and we present the control objective sought in this work. In section III, a new reduced PCH model for each cell, without upstream-downstream direct transfers, is defined. Then the global reduced PCH model of all interconnected cells is derived. In section IV, it is shown how IDA-PBC control methodology may be easily applied to the new interconnected model to design an output-feedback control law for the open channel regulation problem. In section V, simulation results are presented. The paper ends with a summary of the results and a discussion on their possible extensions.

## II. PORT-HAMILTONIAN FORMULATION FOR THE SHALLOW WATER EQUATIONS

We consider the rectangular open channel of fig. (1) with a single reach of slope $I$, with length $L$ and width $B$. It is delimited by upstream and downstream gates and terminated

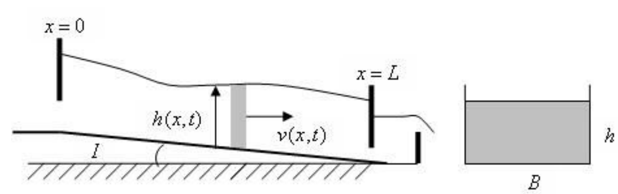


Fig. 1. Longitudinal(left) and lateral (right) sights of an open rectangular hydraulic channel

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by an hydraulic outfall. The flow dynamic within the reach is modelled by the well known shallow water equations. Its port-based model has been developed in [2] and is here only briefly recalled. By choosing the elementary volume and kinetic momentum density along the spatial domain $Z = [0, L]$ as energy (state) variables, we can write the port hamiltonian formulation of the shallow water equations as

$$q(x,t) = Bh(x,t)dx, \;\; p(x,t) = \rho v(x,t)dx \tag{1}$$

$$\begin{bmatrix} -\frac{\partial q}{\partial t} \\ -\frac{\partial p}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} \tag{2}$$

$$\begin{array}{rcll} e^0_\partial(t) & = & -\delta_q H_{|x=0} & f^0_\partial(t) = \delta_p H_{|x=0} \\ e^L_\partial(t) & = & -\delta_q H_{|x=L} & f^L_\partial(t) = \delta_p H_{|x=L} \end{array} \tag{3}$$

where $d$ is the exterior derivative which maps $k$-differential forms on $(k+1)$-differential forms and where $H$ denotes the total energy of the fluid. From the kinetic and potential energy balance computed on an "elementary" length of reach, it is easy to obtain

$$H(t) = \frac{1}{2}\int_0^L (\rho g B h^2 - 2\rho B I h g x + \rho B h v^2)dx \tag{4}$$

The effort variables (thermodynamics forces) are derived from the energy expression as the variational derivatives

$$\begin{array}{rcl} e_q(x,t) & = & \delta_q \mathcal{H} = \frac{1}{2}\rho v^2(x,t) + \rho g(h(x,t) - Ix) \\ e_p(x,t) & = & \delta_p \mathcal{H} = Bh(x,t)v(x,t) \end{array} \tag{5}$$

In (2), $G(q,p)$ is the momentum dissipated by friction forces. They are usually modelled by nonlinear and empirical Manning strikler constitutive formula:

$$G = \frac{\rho g|v|}{K^2 Bh(\frac{Bh}{B+2h})^{\frac{4}{3}}}dx \tag{6}$$

The dynamical system (2) admits an infinity of uniform (constant) water flow equilibrium profiles and spatially varying equilibrium water levels profiles. For a constant equilibrium water flow we can obtain uniform, accumulation or drying equilibrium profiles. The uniform equilibrium profile is obtained when the friction forces equal the gravity ones.

## III. Reduced port-controlled Hamiltonian model

We will now derive a new reduced PCH model for the shallow water equations. It differs from the one developed in [2] in the definition of the internal reduced effort variables. We start from the distributed power product:

$$P_d = \int_{\mathcal{Z}_{ab}} [e_q(x,t) \wedge f_q(x,t) + e_p(x,t) \wedge f_p(x,t)] \tag{7}$$

where $f_q(x,t) = -\frac{\partial q(x,t)}{\partial t}$ and $f_p(x,t) = -\frac{\partial p(x,t)}{\partial t}$ are the distributed flow variables, and $e_q(x,t) = \delta_q H$ and $e_p(x,t) = \delta_p H$ the distributed effort variables. We choose the following mixed finite element approximations:

$$\begin{array}{rcl} f_q(x,t) & = & f^{ab}_q(t)\;\; w^{ab}_q(x), f_p(x,t) = f^{ab}_p(t)\;\; w^{ab}_p(x) \\ e_q(x,t) & = & e^a_q(t)\;\; w^q_a(x) + e^b_p(t)\;\; w^q_b(x) \\ e_p(x,t) & = & e^a_p(t)\;\; w^p_a(x) + e^b_p(t)\;\; w^p_b(x) \end{array} \tag{8}$$

where the approximations spaces for the 1-forms $q$ and $p$ are spanned by the bases $\{w^q_{ab}(x)\}$ and $\{w^p_{ab}(x)\}$ and the approximations spaces for the 0-forms $e_q$ and $e_p$ are spanned by the bases $\{w^q_a(x), w^q_b(x)\}$ and $\{w^p_a(x), w^p_a(x)\}$. Moreover we force the following normalizing conditions on the approximation bases:

$$\begin{array}{l} w^i_a(a) = 1; w^i_b(a) = 0; w^i_a(b) = 0; w^i_b(b) = 1; \\ \int_{\mathcal{Z}_{ab}} w^{ab}_i(x) = 1 \text{ with } i \in \{p,q\} \end{array} \tag{9}$$

Injecting approximations (8) with conditions (9) in (2) and forcing these last equations to be satisfied for all times $t$ and spatial coordinates $x$ one gets the compatibility conditions

$$w^{ab}_i(x) = \mathrm{d}w^i_b(x) = -\mathrm{d}w^i_a(x) \text{ with } i \in \{p,q\} \tag{10}$$

for the chosen approximation bases and the reduced constitutive relations

$$f^{ab}_q(t) = -e^a_p(t) + e^b_p(t), f^{ab}_p(t) = -e^a_q(t) + e^b_q(t) \tag{11}$$

between the reduced variables. Using the compatibility conditions (10), one easily gets the following integrals values [2], [5]:

$$\begin{array}{rcl} \int_{\mathcal{Z}_{ab}} w^q_a(x)w^{ab}_q(x) + \int_{\mathcal{Z}_{ab}} w^p_a(x)w^{ab}_p(x) & = & 1 \\ \int_{\mathcal{Z}_{ab}} w^q_b(x)w^{ab}_q(x) + \int_{\mathcal{Z}_{ab}} w^p_b(x)w^{ab}_p(x) & = & 1 \\ \int_{\mathcal{Z}_{ab}} w^q_a(x)w^{ab}_q(x) - \int_{\mathcal{Z}_{ab}} w^p_b(x)w^{ab}_p(x) & = & 0 \\ \int_{\mathcal{Z}_{ab}} w^p_a(x)w^{ab}_p(x) - \int_{\mathcal{Z}_{ab}} w^q_b(x)w^{ab}_q(x) & = & 0 \end{array} \tag{12}$$

We obtain then from (7), using the approximation scheme (8), the following expression for the (reduced) power product

$$P_d = -e^a_q(t)e^a_p(t) + e^b_q(t)e^b_p(t) \tag{13}$$

The reduced internal effort variables must be defined in such a way that the power product within an element may be written :

$$P_d = f^{ab}_q(t)e^{ab}_q(t) + f^{ab}_p(t)e^{ab}_p(t) \tag{14}$$

This can be achieved in many ways. However, among all linear convex combinations of boundary effort port-variables

$$\begin{array}{l} e^{ab}_q(t) = \lambda e^a_q(t) + (1-\lambda)e^b_q(t) \\ e^{ab}_p(t) = (1-\lambda)e^a_p(t) + \lambda e^b_p(t) \end{array} \tag{15}$$

which satisfy this power expression, there are two special cases where no direct transfer appears in the reduced PCH model of a single cell. We will choose these special cases since they allow to write explicitly the entire model of interconnected cells and they lead to define the natural port variables as a passive outputs for the system. The first of these two cases ($\lambda = 1$) is the choice $e^{ab}_q(t) = e^a_q(t)$ (upstream hydrodynamic pressure) and $e^{ab}_p(t) = e^b_p(t)$ (downstream water flow). The input port variables are then the conjugated ones, i.e. $u_1(t) = e^b_q(t)$ (down stream hydrodynamic pressure) and $u_2(t) = e^a_p(t)$ (upstream water flow). The second choice is $\lambda = 0$, leading to $e_q^{ab}(t) = e_q^b(t)$ and $e_p^{ab}(t) = e_p^a(t)$. The input variables are then $u_1(t) = e_q^a(t)$ (upstream hydrodynamic pressure) and $u_2(t) = e_p^b(t)$ (downstream water flow). The other possible ways to rewrite the reduced power product introduce the reduced efforts as balanced expression of the boundary ones and thus a direct transfert between the inputs and outputs.

Since our control problem is the regulation of the upstream water level (related to the upstream hydrodynamic pressure) and downstream water flow, it is convenient to choose the upstream water flow and downstream hydrodynamic pressure as inputs variables and their conjugate port variables (upstream hydrodynamic pressure and downstream water flow) as outputs. Finally we obtain the following reduced port-controlled hamiltonian model of an elementary cell defined on the elementary spatial domain $x \in [a, b]$) (see [2] for more details on the reduction procedure):

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \left[ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \right] \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{aligned} \tag{16}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \tag{17}$$

where $u_1$ the downstream hydraulic pressure, $u_2$ the upstream water flow, $y_1$ the downstream water flow and $y_2$ the upstream hydraulic pressure are the boundary conjugate port variables through which the system exchanges energy with the environment.The energy stored in the cell is approximated as:

$$H(q,p) = \frac{1}{2}\frac{q(t)^2}{C_{ab}} - \rho g I K_{ab} q(t) + \frac{1}{2}\frac{q(t)p(t)^2}{L_{ab}} \tag{18}$$

with the following reduced elements:

$$C_{ab} = \frac{B}{\rho g}(b-a), L_{ab} = \rho(b-a)^2, K_{ab} = \frac{b+a}{2} \tag{19}$$

The momentum dissipated in the cell is approximated by

$$G(q,p) = g\frac{p}{K^2 q}\left(\frac{B^2(b-a)+2q}{Bq}\right)^{\frac{4}{3}}(b-a) \tag{20}$$

From the reduced energy function we derived the following constitutive equations which give the reduced internal efforts port variables:

$$\frac{\partial H}{\partial q} = \frac{q}{C_{ab}} - \rho g I K_{ab} + \frac{p^2}{2L_{ab}}, \quad \frac{\partial H}{\partial p} = \frac{qp}{L_{ab}} \tag{21}$$

The total channel reach is subdivided into $n$ cells. Each cell $i$ is modelled by the above reduced model with its own boundary port variables $(u_1^i, y_1^i)$ and $(u_2^i, y_2^i)$. Then the interconnection between the $i$ and the $i+1$ cell can be expressed as sequence of series junctions with equations

$$u_1^i = y_2^{i+1} \qquad y_1^i = -u_2^{i+1} \tag{22}$$

We can then write the global reduced PCH model as

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \left[ \begin{bmatrix} 0 & M \\ -M^T & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \right] \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \\ &\quad + g_u \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{aligned} \tag{23}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = {g_u}^T \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \tag{24}$$

where $q = [q_1 \dots q_n]^T$ is the vector of cell volumes and $p = [p_1 \dots p_n]^T$ the vector of cell kinetic momentums. The global interconnection structure sub-matrix $M$ is given by

$$\begin{cases} M(i,i) = -1, \quad M(i+1,i) = 1 \quad i = 1, ..., n \\ \text{and 0 else where} \end{cases} \tag{25}$$

The input matrix $g_u$ is defined as

$$g_u = \begin{bmatrix} 0 & 1 \\ 0_{(2n-2)\times 1} & 0_{(2n-2)\times 1} \\ -1 & 0 \end{bmatrix} \tag{26}$$

The dissipation matrix $G(q,p) \in \Re^{n \times n}$ is given as

$$G(q,p) = Diag\{G_i(q,p)\} \tag{27}$$

with

$$G_i(q,p) = g\frac{p_i}{K^2 q_i}\left(\frac{B^2(b-a)+2q_i}{Bq_i}\right)^{\frac{4}{3}}(b-a) > 0, \forall q_i > 0 \tag{28}$$

The total energy of the system is given as the sum of the individual energies of the cells :

$$H(q,p) = \sum_{j=1}^{n} H_j(q_j, p_j) \tag{29}$$

## IV. Passivity-based control design

The last advances in the structural modelling of physical systems as Port Controlled Hamiltonian (PCH) systems reveal geometric and energetic properties of these systems which stimulate many works on control design taking into account these intrinsic properties. Contrarily to many methodologies where nonlinearities are cancelled using high gain controllers, the PCH formulation leads to a design approach which "truly" accounts for these nonlinearities as well as the structural and energetic properties of the systems, providing less conservative control laws. Among these PCH based control techniques, the Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) is a methodology which gives "three" design degrees of freedom : the closed loop interconnection structure, the closed loop dissipation structure (and values) and the closed loop energy function.

### A. IDA-PBC methodology

Hereafter we recall the basic principles of IDA-PBC methodology as developed in [14]. Consider a port hamiltonian system in general form that we want to stabilize around a desired equilibrium point $x_d \in \Re^n$:

$$\dot{x} = (J(x) - R(x))\frac{\partial H}{\partial x} + g_u(x)u \tag{30}$$

If we can find a control law $\beta(x)$, matrices $J_a, R_a$ and an efforts vector $K(x)$ such that:

$$[J+J_a-(R+R_a)]K(x) = -[J_a-R_a]\frac{\partial H}{\partial x}+g_u\beta(x) \quad (31)$$

with:

$$J_d = J+J_a = -J_d^T \quad (32)$$
$$R_d = R+R_a = R_d^T \geq 0 \quad (33)$$
$$\frac{\partial K(x)}{\partial x} = [\frac{\partial K(x)}{\partial x}]^T \quad (34)$$
$$K(x_d) = -\frac{\partial H}{\partial x}(x_d) \quad (35)$$
$$\frac{\partial K}{\partial x}(x_d) > -\frac{\partial^2 H}{\partial x^2}(x_d) \quad (36)$$

Then the closed loop system with feedback $u = \beta(x)$ can be written in the PCH form:

$$\dot{x} = (J_d(x) - R_d(x))\frac{\partial H_d}{\partial x} \quad (37)$$

where $H_d = H(x) + H_a(x)$ is the shaped energy of the closed loop and $\frac{\partial H_a}{\partial x}(x) = K(x)$. Moreover the equilibrium $x_d$ is (locally) stable. It is *asymptotically stable* if, in addition, $x_d$ is an isolated minimum of $H_d$ and if the largest invariant manifold of $\phi = \{x \in \Re^n | [\frac{\partial H_d}{\partial x}(x)]^T R_d(x)\frac{\partial H_d}{\partial x}(x) = 0\}$ is $\{x_d\}$.

## B. Control design

Let us first note that the obtained reduced model (23) is indeed given in the general PCH systems explicit form (30). Using the IDA-PBC approach we will thus develop a control law which conserves this PCH structure. This control law will be derived in three steps :

1) the definition of the controller interconnection and damping structures $J_a$ and $R_a$
2) the controller internal "efforts" $K$
3) the damping parameters design.

The control objective, formulated in the introduction, is to achieve a desired downstream water flow $(\frac{\partial H}{\partial p_n})_d$ and an upstream water level which corresponds to a desired volume $q_{1d}$ of the upstream cell.

*a) Controller interconnection structure $J_a$:* In order to render the system in closed loop passive with a smooth state feedback the system must have relative degree 1 and to be weakly minimum phase [15]. For the reduced PCH model obtained in (23), the internal dynamics of the system $(q_2, \ldots, q_n, p_1, \ldots, p_{n-1})$ have relative degrees greater than 1. This implies that with a static smooth feedback we can act only on the boundary state variables $(q_1, p_n)$. We thus have limited possible choices for the interconnection matrix $J_a$ and the dissipation matrix $R_a$. We can only fix $\{J_a(1, n+1), J_a(n, 2n)\}$ (and thus $\{J_a(n+1, 1), J_a(2n, n)\}$) for the skew-symmetric matrix $J_a$ and only $\{R_a(1, 1), R_a(2n, 2n)\}$ for the nonnegative symmetric matrix $R_a$. As we would like to cancel the conservative part of the "boundary cells" controlled dynamics $(q_1, p_n)$, leaving only the dissipation part for these dynamics, the skew symmetric matrix $J_a \in \Re^{2n\times 2n}$ will be fixed as follows:

$$\begin{cases} J_a(1, n+1) = & J_a(n, 2n) = 1 \\ J_a(n+1, 1) = & J_a(2n, n) = -1 \\ \text{and 0 else where} & \end{cases} \quad (38)$$

Note that $J_a$ may be viewed as a symplectic structure interconnection matrix. Hence we leave all numerical coefficients for the constitutive equations. The symmetric non-negative damping matrix $R_a \in \Re^{2n\times 2n}$ will be fixed as follows:

$$\begin{cases} R_a(1, 1) = \lambda_2 > 0, \quad R_a(2n, 2n) = \lambda_1 > 0 \\ \text{and 0 else where} \end{cases} \quad (39)$$

where the values of parameters $\lambda_1$ and $\lambda_2$ will be discussed further.

*b) Controller internal efforts $K$:* In direct Lyapunov control design, the Lyapunov function is a priori fixed and usually supposed to be quadratic function. However with our modelling PCH approach, a quadratic Lyapunov function may seem a strange choice since it is not homogeneous with the natural energy of the system. More precisely we know the thermodynamical forces in our physical model as the variational derivatives of the energy density and we would like to choose controller effort functions $K(q, p)$ which are compatible (homogeneous) with these physical efforts. These dimensional and thermodynamical considerations lead us to the choice:

$$\begin{aligned} K_1(q, p) &= -(\frac{q_{1d}}{C_{ab}} - \rho g I K_{ab,1} + \frac{{p_1}^2}{2L_{ab}}) \\ &\quad +\gamma(q_1 - q_{1d}) && (40) \\ K_{n+1}(q, p) &= -(\frac{\partial H}{\partial p_1}) = -\frac{q_1 p_1}{L_{ab}} && (41) \\ K_{2n}(q, p) &= -(\frac{\partial H}{\partial p_n})_d = -Q_d \ \text{(constant)} && (42) \\ K_i(q, p) &= 0 && (43) \end{aligned}$$

for $i = 2, \ldots, n, n+2, \ldots, 2n-1$ and where $K_{ab,1}$ is the coordinate of the the first cell middle point. These efforts have been designed to satisfy, besides the homogeneity condition, the desired equilibrium point condition (35) and the integrability condition (34) which ensures the existence of the energy function $H_a$. In order to satisfy also the Lyapunov stability condition (36) $\gamma$ must satisfy the following condition:

$$\gamma > -\frac{1}{C_{ab}} \quad (44)$$

We can now write the control laws that stabilize the system which are implicitly defined in (31) (the first and last rows of this equation may be explicitly solved):

$$\begin{aligned} u_1(t) &= \lambda_1(\frac{\partial H}{\partial p_n} + K_{2n}(q, p)) + \frac{\partial H}{\partial q_n} \\ &\quad + G_n(q, p)K_{2n}(q, p) && (45) \\ u_2(t) &= -\lambda_2(\frac{\partial H}{\partial q_1} + K_1(q, p)) + \frac{\partial H}{\partial p_1} && (46) \end{aligned}$$

We recognize in these control laws the proportional corrections

$$\lambda_1(\frac{\partial H}{\partial p_n}+K_{2n}(q,p)) = \lambda_1(\frac{\partial H}{\partial p_n}-Q_d) \quad (47)$$

$$\lambda_2(\frac{\partial H}{\partial q_1}+K_1(q,p)) = \lambda_2(\gamma+\frac{1}{C_{ab}})(q_1-q_{1d}) \quad (48)$$

and a compensation of the autonomous dynamics $\dot{p}_n=\frac{\partial H}{\partial q_n}$ and $\dot{q}_1=\frac{\partial H}{\partial p_1}$. This controller structure is characteristic of the control laws derived from the IDA-PBC methodology. From the expression of the efforts (40-43) we derive the energy function $H_a(q,p)$:

$$\begin{aligned} H_a(q,p) &= -\frac{q_1 q_{1d}}{C_{ab}}+\rho g I K_{ab,1} q_1-\frac{q_1 {p_1}^2}{2L_{ab}} \\ &\quad -\gamma q_1(\frac{q_1}{2}-q_{1d})-Q_d\ p_n \end{aligned} \quad (49)$$

which shows that only the energy of the boundary cells is "shaped" with the proposed control law. The closed loop system will be passive with respect to the shaped energy function $H_d(q,p)=H(q,p)+H_a(q,p)$.

*c) Damping parameters design:* We can tune the injected damping parameter $\lambda_1$ in order to have local exponential stabilization of the downstream water flow around a small variation of the downstream water level. The variation of the downstream water flow is

$$\begin{aligned} \frac{d}{dt}(\frac{\partial H}{\partial p_n}) &= \frac{d}{dt}(\frac{q_n p_n}{Lab}) \\ &= \frac{p_n}{Lab}\dot{q_n}+\frac{q_n}{Lab}\dot{p_n} \\ &= \frac{p_n}{Lab}(\frac{\partial H}{\partial p_{n-1}}-\frac{\partial H}{\partial p_n})+\frac{q_n}{Lab}(\frac{\partial H}{\partial q_n}-u_1) \end{aligned}$$

Assuming small spatial variations of water flows between cells $(n-1)$ and $n$:

$$\dot{q}_n=\frac{\partial H}{\partial p_{n-1}}-\frac{\partial H}{\partial p_n}\simeq 0 \quad (50)$$

and the control law $u_1(t)$ defined in (45), we obtain:

$$\begin{aligned} \frac{d}{dt}(\frac{\partial H}{\partial p_n}) &= -\lambda_1\frac{q_n}{Lab}(\frac{\partial H}{\partial p_n}+K_{2n})-G_n(q,p)(\frac{\partial H}{\partial p_n}+K_{2n}) \\ &= -(\lambda_1\frac{q_n}{Lab}+G_n(q,p))(\frac{\partial H}{\partial p_n}-Q_d) \end{aligned}$$

Hence by setting the dissipation parameter $\lambda_1$ as

$$\lambda_1=\lambda_3\frac{Lab}{q_n} \quad ,q_n>0 \quad (51)$$

with $\lambda_3>0$, we guarantee a local exponential stabilization of the downstream water flow with maximum time constant $1/\lambda_3$.

As we can see in (48) that the dynamic of $q_1$ depends on the reduced element $C_{ab}$ which itself depends on the length of the elementary cell. This bring us to define the specific tuning of the injected damping parameter $\lambda_2$ in order to obtain an independent dynamic from the chosen number of cells:

$$\lambda_2=\frac{\lambda_4}{(\gamma+\frac{1}{C_{ab}})}, \quad \lambda_4>0 \quad (52)$$

.

### C. Adding Integral Action

The Manning Strickler friction parameter and the hydraulic gate parameter are poorly known numerical values since they are issued from empirical models of bed friction within the reach and around the gates. A static error may thus appear between the desired and the real equilibrium points. In order to avoid this problem, an integral action is added on each control law [13]. For that purpose, two new states $v_1$ and $v_2$ are introduced whose dynamics are defined by

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = -Kg_u^T(x)\frac{\partial H_d}{\partial x} = -\begin{bmatrix} 0 & -k_I^1 \\ k_I^2 & 0 \end{bmatrix}\begin{bmatrix} \frac{\partial H_d}{\partial q_1} \\ \frac{\partial H_d}{\partial p_n} \end{bmatrix} \quad (53)$$

The augmented system still has a PCH form

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} J_d(x)-R_d(x) & g_u(x)K \\ -Kg_u^T(x) & 0 \end{bmatrix}\begin{bmatrix} \frac{\partial W}{\partial x} \\ \frac{\partial W}{\partial v} \end{bmatrix} \quad (54)$$

with a total desired stored energy $W(x,v)=H_d(x)+\frac{1}{2}v^TK^{-1}v$ where $K$ is the diagonal 2x2 matrix $(K=K^T)$ with $k_I^1>0$ and $k_I^2>0$ on the diagonal. The new control laws of the system are then :

$$\begin{aligned} u_1(t) &= \lambda_3\frac{Lab}{q_n}(\frac{\partial H}{\partial p_n}+K_{2n})+\frac{\partial H}{\partial q_n}+G(q_n,p_n)K_{2n}+v_1 \\ u_2(t) &= -\lambda_2(\frac{\partial H}{\partial q_1}+K_1)+\frac{\partial H}{\partial p_1}+v_2 \end{aligned} \quad (55)$$

Stability is proved using a Lasalle argument [13].

### D. Introduction of the Hydraulic gates

The hydraulic gate is generally modelled as static constitutive relations between the discharge $(Q(t))$ and the difference of fluid levels around the gate. This relation may be written as an invertible constitutive equation between the physical port variables as follows [2]:

$$Q(t)=\frac{\sqrt{2}\alpha B\theta(t)}{\sqrt{\rho}}\sqrt{P_{up}-P_{down}} \quad (56)$$

where $P_{up}$ is the gate upstream pressure, $P_{down}$ the gate downstream pressure, $\alpha$ the gate characteristic and $\theta$ the gate opening.

## V. SIMULATION RESULTS

Simulations presented in this section are obtained with a micro-channel simulator made with the above mentioned reduced port controlled Hamiltonian model. The total length of the channel is subdivided into ten cells. We have used for the parameters values those identified on an experimental micro-channel available at the Laboratory. They are listed in the table I hereafter.

TABLE I

MICRO-CHANNEL PARAMETERS USED FOR THE SIMULATIONS

| | |
|---|---|
| length $L$ | 7 meters |
| width $B$ | 0.1 meter |
| slope $I$ | $1.6\times10^{-3}$ |
| Manning-Strickler coefficient $K$ | 97 |
| upstream gate parameter $\alpha_1$ | 0.66 |
| downstream gate parameter $\alpha_2$ | 0.73 |
| downstream outfall height $(H_{dev})$ | 0.05 meter |

Fig. (2) shows the downstream water flow response signal for an arbitrary water flow reference signal. Good regulation of the downstream flow is obtained as it is shown by the regulation error in the same figure. Fig. (3) shows the water

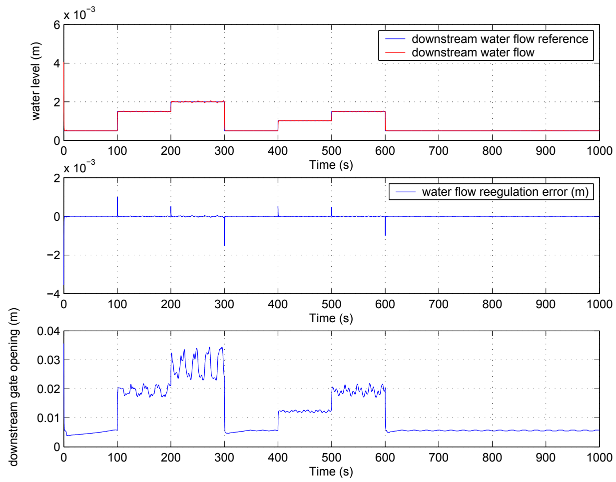


Fig. 2. Regulation of the downstream water flow

level response signal for an arbitrary upstream water level reference signal. Again, the upstream water level is well regulated with an admissible gate opening.

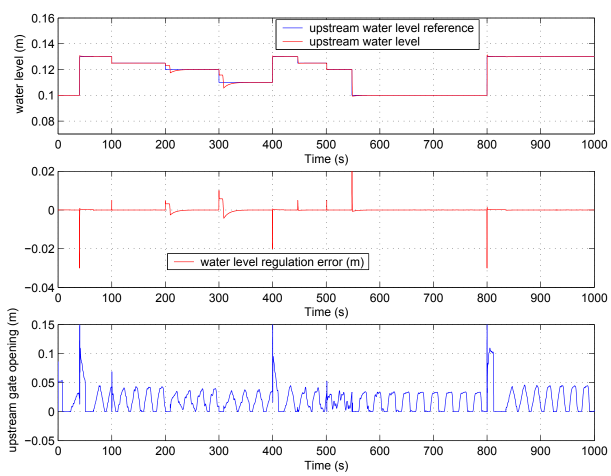


Fig. 3. Regulation of the upstream water level

With both simulation tests one observes oscillations on the opening gate signal. This is due to the wave propagation phenomenon in the channel. Indeed, the walls of the used micro-channel are made out of plexiglass which introduces very low friction forces (the Manning-Strickler parameter K is closed to 100). As a consequence waves may go and come along the channel with a low attenuation. These waves do not affect the downstream water flow and/or upstream water level with the proposed control thanks to the oscillations of the gates.

## VI. CONCLUSIONS AND FUTURE WORKS

In this paper, we first developed a reduced PCH model for the hyperbolic shallow water partial differential equations. This model is derived from the mixed finite elements decomposition of a channel reach in small "cells" PCH models without any direct transmission term between the pairs of power-conjugated input and output variables of the cells. The IDA-PBC control methodology allowed us to design a nonlinear static state feedback based on the choice of the controller interconnection structure, the desired closed shaped energy (with a prescribed minimum state) and the design of damping parameters. The whole approach has been tested in simulation and appears to be very effective.

Among the expected developments of this work are its experimental testing on the Laboratory micro-channel, followed by its real-scale testing. We also intend to develop an external control action on the boundaries which will allow to eliminate the gates oscillations due to the reflecting and superposing wave effects. Finally we must point out that the passivity approach developed here could also be applied directly on the port-based distributed parameter model developed in section II. The development of a passivity based nonlinear boundary control which apply to the whole class of shallow water equations (or to general nonlinear transmission line equations) evidently appears as an expected and significant further development of this work.

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