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Parameters identification of a hybrid model for dry friction

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***Abstract*— An original methodology is proposed to tune the parameters of the model of a mechanical system with dry friction which is modeled with a new simple approach recently proposed. This approach is based on a hybrid (two-modes) model approach: one mode models the system motion and the other the motionless case. In this paper, the hybrid modeling of dry friction is first recalled, then the proposed identification methodology is presented. The inertia, the viscous friction and the Coulomb's effect are first identified using the power equation of the system. Secondly, a simple algorithm is proposed to find the system Stribeck effect parameter. The other parameters specific to the used hybrid model are finally tuned to match the real observed data. The proposed parameters identification methodology is compared in simulation with other methods and is validated on a real experimental system.**

I. Introduction

In many mechanical systems dry friction is present. It leads to lower precision and limit cycles may appear. It is well known that the main behavior of systems submitted to dry friction is modeled when Coulomb's force and Stribeck effect are taken into account [1]. Sophisticated models as Dahl's or LuGre models [2] have been developed to account also for other frictions phenomena as presliding displacement or stick-slip motion. Unfortunately, such models need a lot of parameters that are not easily obtained experimentally. Moreover, in many industrial applications the used position sensor doesn't permit to detect the presliding displacement. Then, it is of a real interest to have a simple model easily identifiable whose precision can be improved according to the application. On the other hand, to deal with dry friction parameters that may change with time, it is necessary to design control laws that are robust with respect to uncertainties of the model [3], [4]. The main conclusion is the need of a good estimation of the system state, i.e the system motion or motionless [5]. Recently in [6], a polytopic approach is used to derive a new simple hybrid model for systems with dry friction view as hybrid systems with two operating modes: motion or motionless. The principal characteristics of the proposed approach are that it is easily comprehensible, has few parameters, allows the adjustment of the model complexity to the treated case, models the stick-slip phenomena, and has low simulation time.

A lot of works have been done on the identification of dry friction [7], [8], [9]. In this paper, the experimental identification of the hybrid model for dry friction is addressed. The proposed identification methodology is composed of three steps: the viscous friction and the Coulomb's effect are first identified using a recursive least square method based on the power equation of the system; secondly, a simple algorithm is proposed to find the system Stribeck effect parameter; finally, the other parameters of the system are tuned to match the model behavior. The main contribution of the paper is that the proposed identification methodology of the model parameters does not require difficult experimental tuning of identification algorithms such as threshold on used experimental data or the use of ad'hoc excitation signals. The proposed method is compared in simulation with other methods and is validated with a real experimental system.

The paper is organized as follows. Section II presents the multi-model (hybrid model) for systems with dry friction. Section III introduces the studied experimental system. The identification methodology of the model parameters is presented in section IV. Section V presents the comparison of the results obtained with the experimental system and the tuned model for various cases. Finally, section VI concludes the paper.

II. A multi-model of dry friction

Let consider a single-mass system:

$$\begin{array}{c} \dot{x}\left(t\right)=v\left(t\right) \\ m\dot{v}\left(t\right)=u\left(t\right)-f\left(t,v,x,...\right) \end{array} \tag{1}$$

where $x(t)$ is the position, $v(t)$ is the speed, $u\left(t\right)$ is the input force and the force f represents frictions which may depend on speed, position,... and parameters as wear, Coulomb's friction level, Stribeck parameter... [2].

The genesis of the model proposed in [6] for systems with dry friction starts from a simple report that is that a mechanical system has two operating modes: it moves or it is motionless. This can be modeled as a state machine with two states: state 1, the system moves according to (1), and state 0, the system is motionless with model (2) which forces the speed to converge quickly to zero. The pole p_0 has to be chosen by the designer much more faster than the fastest dynamic of the system.

$$\begin{array}{c} \dot{x}=v \\ m\dot{v}=-p_0 v,\, p_0 \longrightarrow +\infty \\ f=u \end{array} \tag{2}$$

It is also necessary to define how this hybrid system commutes between its two states. For that, the force f is first described. Note that this force may depend on system's position [6]. For sake of simplicity, the simpler case of

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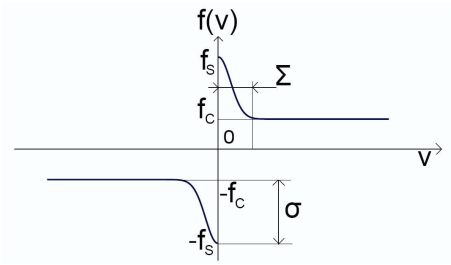

Fig. 1. Dry friction force accounting for Coulomb's and Stribeck effects

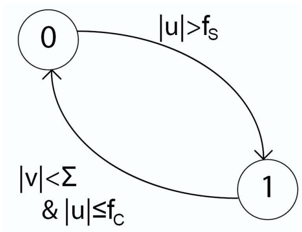

Fig. 2. State machine for dry friction modeling

fig. 1 is considered where the dry friction force accounts for Coulomb's and Stribeck effects. Starting from state 0 (motionless), the unique way to switch to state 1 (motion) is that the absolute value of the input $|u(t)|$ becomes higher than f_s. After switching to state 1, the system stays in this one as long as the absolute value of the speed $|v(t)|$ is not less or equal to Σ or $|u(t)|$ is higher than f_c. Fig. 2 summarizes this state machine. Parameters of force f and $\sigma = f_s - f_c$ are obtained experimentally as it will be shown in section IV. The model parameter Σ is mainly chosen to fit the observed experimental stick-slip phenomena.

The principal characteristics of the proposed approach to model dry friction are that the model is easily comprehensible, has few parameters (Σ, σ), allows the adjustment of the model complexity to the treated case (such as take into account the dry friction dependence to mechanical position), models the stick-slip phenomena, and has low simulation time since the $sign$ function is not used near null speed in mode 0.

III. Model of the studied system

Fig. 3 gives the kinematic diagram of our experimental mechanical system. It is composed of an DC motor and a toothed rack. The displacement of the rack creates dry

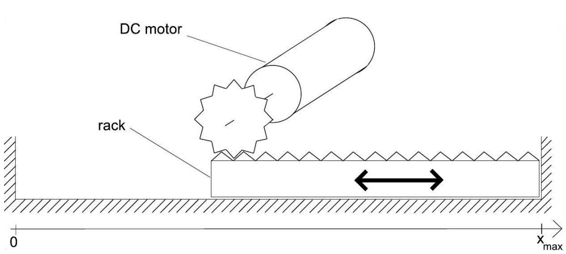

Fig. 3. Kinematic diagram of the studied system

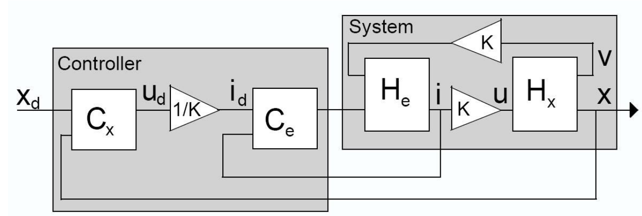

Fig. 4. Nested loops control configuration

friction between the surface of this one and the frame. The input of the system is the torque provided by the motor. The output is the rotor position. Due to the frame, the rack position is bounded, the rotor position varies between 0 and x_{max}.

The system is controlled by two known controllers. The controller C_e drives the electrical dynamics H_e of the motor and the controller C_x drives the position of the rotor with mechanical dynamics H_x. The global nested loops synopsis is given in fig. 4 where the desired position is noted x_d, and K is the motor torque constant. The electrical dynamics have been identified with a closed loop identification method [10]. For simulation, the model takes into account electrical and mechanical dynamics but for mechanical parameters identification, the electrical dynamics, supposed much more faster than the mechanical one, are neglected. Moreover the dynamic of the toothed rack is also neglected in front of these of the motor. The system is simulated with (3) and (4):

For state 1:

$$\begin{aligned} Ja(t) &= -f_v v(t) - f_d(v(t)) + u(t) \\ f_d(v(t)) &= \left(f_c + (f_s - f_c)\, e^{-\left(\frac{v(t)}{v_s}\right)^2} \right) sign(v) \end{aligned} \tag{3}$$

For state 0:

$$Ja(t) = -p_0 v(t) \tag{4}$$

where $a(t)$, $v(t)$ and $u(t)$ are respectively the acceleration, the speed and motor torque. The frictions of the system are represented by resistant torques: f_v is the parameter of the viscous friction and $f_d(v(t))$ is the function modeling the dry friction. J is the inertia of the motor. The parameters Σ, p_0 and v_s are specific to the model.

IV. Parameters identification

As said in the introduction, the proposed identification methodology is cut into 3 steps. First, inertia J, viscous friction parameter f_v and Coulomb's force f_c are identified using a least square identification method with weighted data. Secondly, a simple algorithm is proposed to find the Stribeck parameter f_s. Thirdly, the last parameters of the model p_0, v_s and Σ are tuned to better fit the model behavior with the real one.

The first step (the main step) of the proposed methodology will be compared in simulation with other methods with parameters of table I.

TABLE I

SIMULATION PARAMETERS VALUES

J	$7e-6$kg.m	p_0	$1e4$N.m.s.rad^{-1}
f_v	$1.6e-4$N.m.s.rad^{-1}	v_s	0.01rad.s^{-1}
f_c	0.01N.m	Σ	0.01rad.s^{-1}
f_s	0.05N.m		

A. Step 1: Identification with the power equation

First, for the identification of parameters J, f_v and f_c, the multi-model of dry friction is replaced by the function $f_c sign(v)$, so the system is described by equation (5) using a simple Coulomb model for dry friction.

$$Ja(t) = -f_v v(t) - f_c sign(v(t)) + u(t) \qquad (5)$$

The only measurable output of the system is the position $x(t)$, so system speed and acceleration are obtained using respective filters F_v and F_a in (6). The cut-off pulsation p_n is chosen to cut the high-frequency noise and has to be higher than the system dynamic; in the continuation, it is set to 200rad.s^{-1}. The input u is also filtered by F_u in (6) to keep the same phase between the different signals.

$$F_v = \frac{p}{\left(\frac{p}{p_n}+1\right)^2}, \qquad F_a = \frac{p^2}{\left(\frac{p}{p_n}+1\right)^2}, \qquad F_u = \frac{1}{\left(\frac{p}{p_n}+1\right)^2} \qquad (6)$$

At each time $t = kT_e$ ($k \in N$), where $T_e = 1ms$ is the sampling time, $u(kT_e)$, $v(kT_e)$ and $a(kT_e)$ are used by the identification algorithm and are noted u_k , v_k and a_k.

The parameters are identified using the least square identification method with weighted data and with a forgetting variable factor since we have a stationary system [10]. The criterion to be minimized is:

$$I = \sum_{k=1}^{n}\left(\prod_{j=1}^{k-1}\lambda_1(k-j)\right)\left(u_k - \hat{\theta}_k^T\varphi_k\right)\eta_k\left(u_k - \hat{\theta}_k^T\varphi_k\right) \qquad (7)$$

$$\text{where } \lambda_1(i) = \lambda_0\lambda_1(i-1) + 1 - \lambda_0$$

and $\lambda_1(0) = 0.9$, $\lambda_0 = 0.99$. η_k is a weighting function, $\hat{\theta}_k = \begin{pmatrix} \hat{J}_k & \hat{f}_{v_k} & \hat{f}_{c_k} \end{pmatrix}^T$ is the estimated parameters vector, and $\varphi_k = \begin{pmatrix} a_k & v_k & sign(v_k) \end{pmatrix}^T$ is the measurements vector.

With traditional least square identification method, weighting function η_k is set to 1. In this case, the Coulomb's force is not well estimated because the Stribeck effect is not modeled by (5). The same problem is reported in [8]. To remedy this problem, [11] proposes to only use data when the speed v_k is higher than a threshold $v_{\min}$. This is done by setting in (7), $\eta_k = 0$ when the absolute value of the speed v_k is lower than the associated threshold and $\eta_k = 1$ in the other case. So to obtain available data, the input u needs to be chosen to impose a speed v higher than $v_{\min}$. But in many mechanical systems, as with our system, the position is bounded and so the boundaries are reached before the system parameters have been identified. So the test needs to be made around a fixed position. The difficulty with this method is then to choose a threshold that is high enough to remove the Stribeck

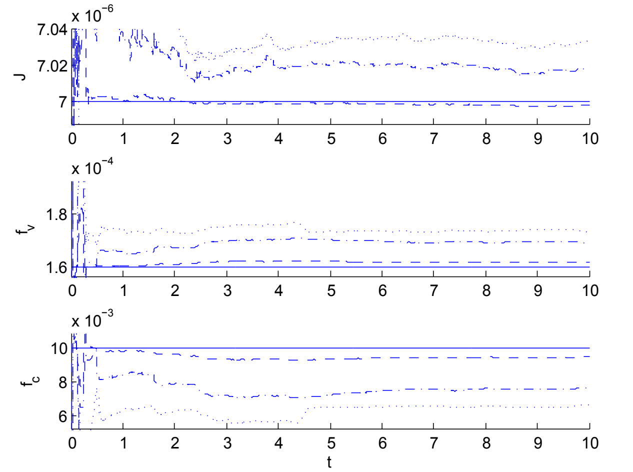

Fig. 5. Convergence of the different algorithms in simulation: real value (solid), $\eta_k = 1$ (dotted), $v_{\min} = 20$rad.s^{-1} (dash-dotted), $\eta_k = v_k^2$ (dashed)

effect, and small enough to keep a good sensitivity to the Coulomb parameter f_c.

We propose to use the following weighting function: $\eta_k = v_k^2$. With this choice, the smaller the speed is, the less the data influence criterion (7). It is easy to show that the proposed method is based on the system power equation. Indeed, there is four powers: the useful power P_u, the provided power P_p and the dissipated powers by the viscous friction (P_v) and by the dry friction (P_d). The expression of each are given by:

$$\begin{array}{r} P_u = Ja(t)v(t) \\ P_p = u(t)v(t) \\ P_v = f_v v(t)^2 \\ P_d = f_c|v(t)| \end{array} \qquad (8)$$

The relationship between these powers is:

$$P_p = P_u + P_v + P_d \qquad (9)$$

$$u(t)v(t) = Ja(t)v(t) + f_v v(t)^2 + f_c|v(t)| \qquad (10)$$

$$u(t)v(t) = \begin{pmatrix} J & f_v & f_c \end{pmatrix}\begin{pmatrix} a(t)v(t) \\ v(t)^2 \\ |v(t)| \end{pmatrix} = \theta^T v(t)\varphi$$

where (10) is (5) multiplied both sides by the speed $v(t)$.

The three different choices presented above for weighting function η_k will be know compared in simulation. For that, data is obtained in simulation by applying a pseudo random binary sequence to the reference input of the controlled system. The data are filtered respectively with filters (6) and used to test the different algorithms.

Fig. 5 presents the results obtained for the three cases of the weighting function η_k: without threshold, with a threshold $v_{\min} = 20$rad.s^{-1} and with $\eta_k = v_k^2$. Table II gives the error between the different identified values and the real values. The use of $\eta_k = v_k^2$ in criterion (7) gives better results: the error is of the same order of magnitude for parameter J, but more than 4 times lower than others for the two friction parameters. This is due to the fact that

TABLE II

ESTIMATED PARAMETERS VALUES WITH SIMULATED DATA

	$\eta_k = 1$	$v_{\min} = 20\text{rad.s}^{-1}$	$\eta_k = v_k^2$
J	0.49%	0.28%	0.026%
f_v	8.42%	5.68%	1.04%
f_c	33.97%	23.45%	5.05%

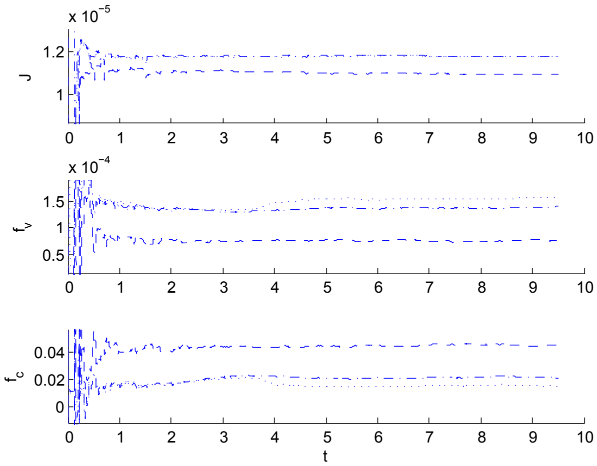

Fig. 6. Convergence of the different algorithms with real data: $\eta_k = 1$ (dotted), $v_{\min} = 20\text{rad.s}^{-1}$ (dash-dotted), $\eta_k = v_k^2$ (dashed)

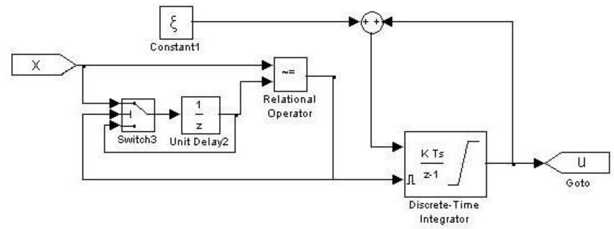

Fig. 7. Identification algorithm of the Stribeck force f_s

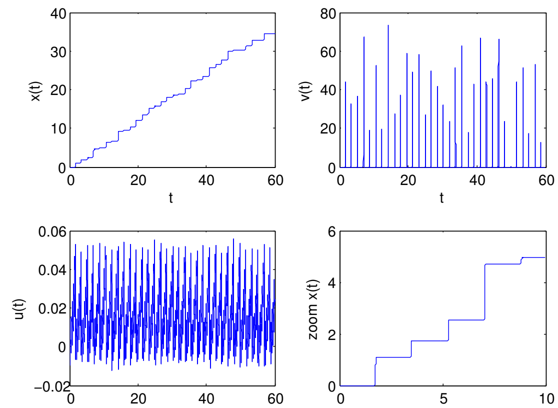

Fig. 8. Simulation results during Stribeck parameter identification

the non-linearity around the null speed is removed by the multiplication of the speed and that no other discontinuity is added in the system as with the threshold method.

The different methods are now tested with data obtained with the real system. The filters cut-off frequency p_n and the threshold $v_{\min}$ are the same as in the simulation part. The system is excited with the same desired position vector as in simulation case. The parameters estimation behaviors obtained with the three methods are shown in fig. 6 and the obtained parameters are presented in table III. The methods with no threshold and with a threshold of 20rad.s^{-1} give similar results, whereas the parameters given by the speed weighting method are rather different (especially for f_v and f_c). This observation was the same for the tests in simulation. Finally the model is tuned with the parameters J, f_v and f_c obtained with the proposed speed weighting method.

B. Step 2: Identification of the Stribeck effect

The Coulomb's effect is well modeled with model (5) at high speeds. But at very low speed stick-slip motion may appear and this model is no more suited to describe the system behavior: the Stribeck effect has to be taken

TABLE III

ESTIMATED PARAMETERS VALUES WITH EXPERIMENTAL DATA

	$\eta_k = 1$	$v_{\min} = 20\text{rad.s}^{-1}$	$\eta_k = v_k^2$
$J(kg.m)$	1.1745e-005	1.1753e-005	1.0953e-005
$f_v(N.s.rad^{-1})$	1.5666e-004	1.3912e-004	7.5870e-005
$f_c(N.m)$	0.0148	0.0212	0.0456

into account. The model parameter f_s can be obtained experimentally with simple experiments with the following algorithm: starting with the system at standstill and position $x = 0$, the input motor torque is slowly increased from 0, when a rotor movement is detected, this means that the input torque is higher than the f_s value at the actual position, the value of the torque is memorized. After, the torque input is reset to 0 and the loop can restart again. When the maximal position is reached $x = x_{max}$, the loop is done again but with negative decreasing input torque values. An implementation example, for positive input torques, of this algorithm in the Simulink workspace is presented in fig. 7. Depending on the desired precision for f_s, one has to use sufficiently small input torque increments. Moreover, the delay between each increment should be sufficiently high to allow the electrical and mechanical motor dynamics reach steady-state.

The method has been tested in simulation with parameters of table II and the full hybrid model (1)-(2). The input torque increment ξ is fixed to $0.005N.m$. Fig. 8 shows the obtained simulation results. The slip-stick motion is well represented and we can note that the crest values of $u(t)$ are very close to the value of parameter $f_s = 0.05N.m$ used in simulation. In order to test the robustness of the method, a gaussian noise with a variance of 0.01N.m, which is coherent with the real system, has been added to the measured torque (given the explication of small negative values of $u(t)$). The obtained estimated f_s values are shown in fig. 9. The relative error is 6% with a mean value of 0.047N.m. The agreement between estimated and real values is again satisfactory. Experimentally, the input torque increment ξ has

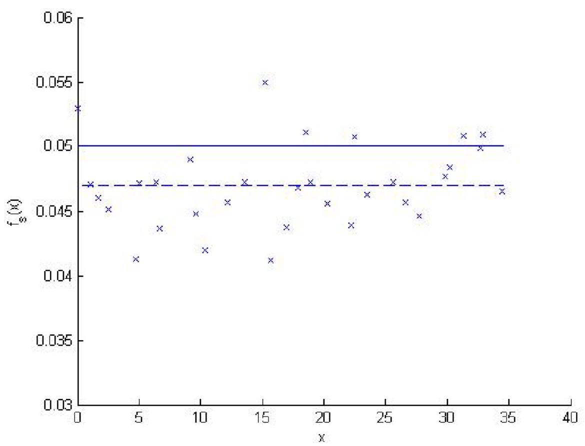

Fig. 9. Simulated Stribeck parameter identification with measurement noise: real value (solid), identified value (dashed) and used data (crosses)

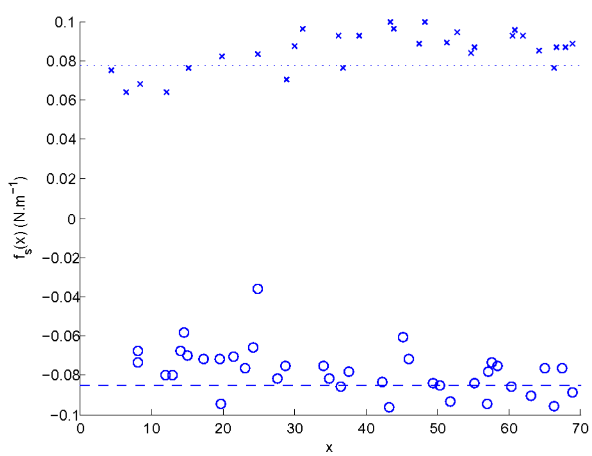

Fig. 10. Experimental Stribeck parameter identification: for negative torque (identified value (dashed), used data (circles)) and for positive torque (identified value (dotted), used data (crosses))

to be chosen depending on the measurement noise. It should be close to the variance of the observed noise. Using this method, the stribeck parameter of the experimental system is then identified. In order to decrease the effect of noise on the measured variables, several go-and-returns are done to obtain a better approximation of the estimated parameter f_s. Results shown in fig. 10 are multiplied by the sign of the speed. The mean values are -0.0777N.m for negative torque and 0.0853N.m for positive torque. Theoretically, due to the symmetric structure of our experimental system, absolute value of f_s should not change with the sign of the input torque. It is chosen to take the mean value, 0.0815N.m, of the absolute values to calibrate our model.

C. Step 3: Final tuning of the system

Three parameters haven't yet been tuned: p_0, Σ and v_s. The first corresponds to the time that takes the system to become motionless when the used hybrid model for dry friction enters in operating state 0 (motionless). This time

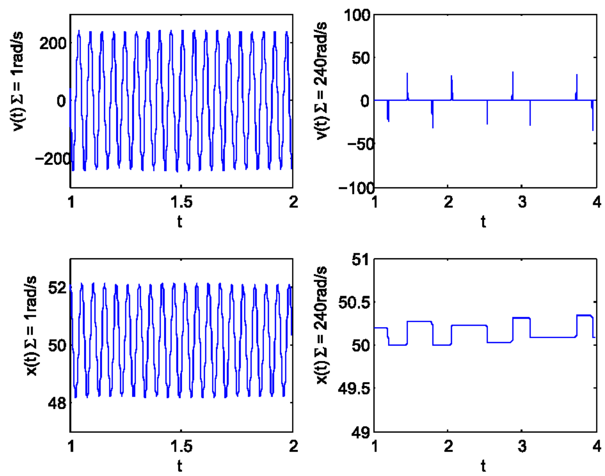

Fig. 11. Tuning of the parameter Σ

should be smaller than the sampling time T_e of the controller. On the other hand, to prevent from too large simulation time, this dynamic needs to be close to the others of the system. One can take $p_0 = 3/T_e$.

A simple test is needed to tune parameter Σ. First, Σ is set to an arbitrary small value, for example 1rad.s^{-1}. Secondly, a constant position reference is applied to the controller. If Σ is too small, a speed oscillation around zero appears as in fig. 11. Then, in order to obtain accurate slip-stick motion, Σ should be chosen higher than the maximal amplitude of this observed speed.

The last parameter v_s has a small influence in front of the others. A lot of advanced methods exist to find it with slightly disturbed data [11],[7]. In most of time, this value is very small in front of the minimal detectable speed and it is very difficult to find it in real industrial situation. So due to low resolution of the experimental position sensor, an approximate value is sufficient to have a good behavior of the system in simulation. To finish the tuning of the model, v_s is chosen equal to 0.01rad.s^{-1}. In the next part, it is showed that this approximation gives good results but if it was not the case, a dichotomy search method may give acceptable results more quickly than with advanced methods.

V. COMPARISON BETWEEN THE TUNED MODEL AND THE EXPERIMENTAL SYSTEM

To evaluate the performance of the proposed hybrid model for dry friction and the parameters identification methodology, the position test vector of fig. 12 is applied to the experimental and the simulated systems. This test vector contains ramps and steps to evaluate the model during stick-slip motions, constant position and step responses. Globally, the results are very satisfactory. Zoom 1 of fig. 12 shows clearly that the stick-slip motion is very well simulated. The second zoom shows the responses for a step input: experimental and simulated results are very close one of the other. This shows that the dynamic parameters J and f_v have been well estimated. The simulated input torque behavior is really near to the one of the real input torque for ramp

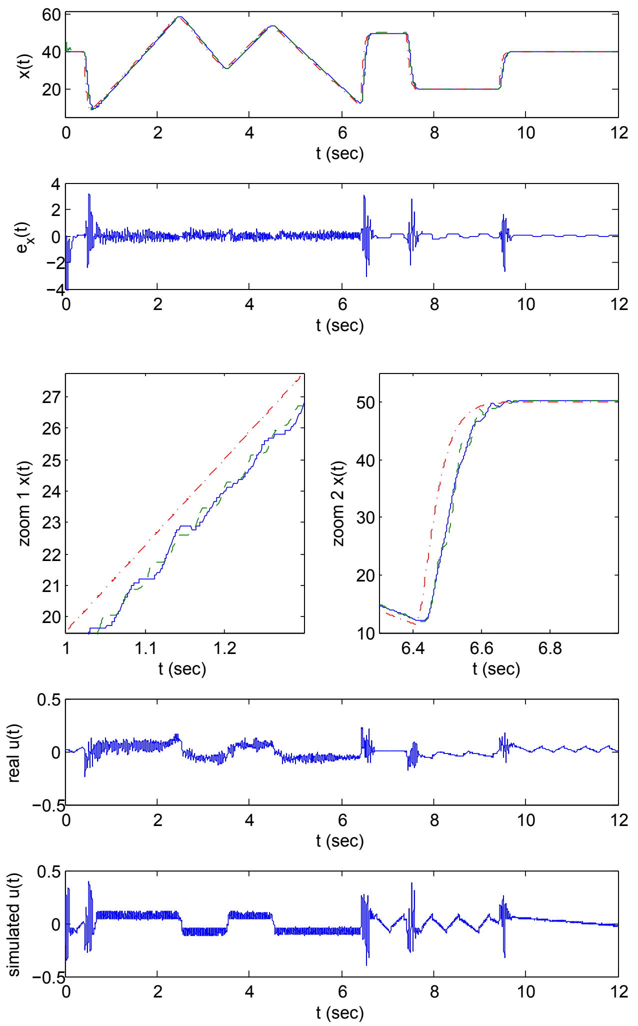

Fig. 12. Comparison between experimental and simulated data: reference position $x_d(t)$ (dash-dotted), real position $x(t)$ (solid), simulated position $x_s(t)$ (dashed), error $e_x(t)$ between $x(t)$ and $x_s(t)$, real input torque and simulated input torque (solid)

and step references. But when system is stabilized around a fixed position, they are different (chaos phenomena appear). For example between 10 to 12 seconds, the real torque is a triangle signal whereas the simulated one is a soft slope signal. But between 6.6 to 7.4 seconds it is the contrary and between the 8 to 9.5 seconds the signals have the same behavior. So the model is able to produce all situations of a system submitted to dry friction.

VI. Conclusion and Future Works

This paper has presented a complete methodology to experimentally identify the parameters of a recent new hybrid model for dry friction. The main advantages of the proposed modeling have been used to propose the simple but complete parameters identification methodology which has been explained and validated in simulation and with a real experimental system. This method can be applied to systems having a bounded position which it is not easy with other methods. Moreover, the proposed identification methodology may be applied in a quasi-automatic procedure since no particular tuning of the identification procedure is needed. This is of major importance from an engineering point of view. Only the presliding displacement observed with systems subject to dry friction is not modeled but most of the time, this behavior is hidden by the resolution of the position sensors and so it is not necessary to model it. Work is in progress to develop a control with dry friction compensation based on the presented model.

With the used switching conditions of the state machine, the model does not switch always accurately from mode 1 to mode 0 when there are large amplitude variations, with change of sign, of the input force (or torque). This bad-behavior explains the large value needed for parameter Σ, which is used to fit the observed experimental stick-slip phenomena and the simulated one. In order to give this parameter a physical meaning and determine it more easily, the state machine switching condition from mode 1 to mode 0 will be improved and will be part of a future paper.

VII. Acknowledgement

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