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Generalized Prandtl-Ishlinskii Hysteresis Model: Hysteresis Modeling and Its Inverse for Compensation in Smart Actuators

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Abstract— **Smart actuators such as magnetorestrictive actuators, shape memory alloy (SMA) actuators, and piezoceramic actuators exhibit different hysteresis loops. In this paper, a generalized Prandtl-Ishlinskii model is utilized for modeling and compensation of hysteresis nonlinearities in smart actuators. In the formulated model, a generalized play operator together with a density is integrated to form the generalized Prandtl-Ishlinskii model. The capability of the formulated model to characterize hysteresis in smart actuators is demonstrated by comparing its outputs with experimental results obtained from different smart actuators. As an example, hysteresis nonlinearities of the magnetostrictive and SMA actuators are characterized by the generalized Prandtl-Ishlinskii model. Furthermore, an analytical inverse of the generalized Prandtl-Ishlinskii model is derived for compensations in different smart actuators. In other words, exact inverse of the generalized Prandtl-Ishlinskii model is achievable and it can be implemented as a feedforward compensator to migrate the effects of the hysteresis in different types of smart actuators. Such compensation is experimentally illustrated by piezoceramic actuator.**

I. Introduction

Hysteresis is a nonlinear phenomenon that appears in various systems, including smart actuators and ferromagnetic materials [1-3]. Smart material-based actuators which are widely used for industrial applications exhibit strong hysteresis property under increasing and decreasing inputs. The hysteresis properties of smart actuators are known to cause inaccuracies and oscillations in the system responses that may even lead to instability of the closed loop system [4]. Significant efforts have been made to model the hysteresis properties of smart actuators for effective controller designs.

To describe hysteresis phenomenon in smart actuators, a number of hysteresis models have been utilized, which can be roughly classified into physics based models [19] and phenomenological models [7-12]. Preisach hysteresis model, which is the most well-known phenomological based-operator model, has been formulated to characterize hysteresis phenomenon in smart actuators. In this model, hysteresis is modelled as a cumulative effect (density function) of all possible delayed relay elements (Preisach operator), which are parameterized by a pair of threshold variables [1].

Another popular phenomenological hysteresis model is the Prandtl–Ishlinskii model [2]. This model is a superposition of elementary play or stop operators, which are parameterized by a single threshold variable. It should be noted that PI model [2] is a subset of the Preisach model and is defined in terms of an integral of play operator or stop operator with a density function determining the shape of the hysteresis. However, the Prandtl-Ishlinskii model cannot exhibit neither asymmetric hysteresis loops nor saturated hysteresis output.

There are hysteresis phenomena in smart actuators, which show saturated outputs at maximum and/or minimum input. such as magnetostrictive and shape memory alloy (SMA) actuators. Since one of the main advantages of the PI model is not only the density function can be modified, but also the play operator or stop operator is allowed to be re-defined to enhance the ability to describe hysteresis, to fully exploit such a advantage, in this paper a generalized play hysteresis operator is applied to Prandtl-Ishlinskii model in conjunction with density function to characterize saturated hysteresis nonlinearities in smart actuators. The validity of the resulting generalized Prandtl-Ishlinskii model to characterize hysteresis of both magnetostrictive and SMA actuators is demonstrated using experimental results of magnetostrictive and SMA actuators.

Based on the formulated hysteresis models, a general approach to migrate the effects of the hysteresis is to construct an inversion of the hysteresis model [4,9,13-20]. This method has widely been used for control of smart actuators such as piezoceramic actuators, shape memory alloy actuators, and magnetostrictive actuators. Inverse Preisach model has been widely used to control the smart actuators, see, for example, [15,16,18]. Since the Preisach model is not analytically invertible; numerical implementation has been proposed to obtain the approximate inversion of this model.

The main advantage of the PI model over the Preisach model is that that its inverse can be attained analytically, and it can be implemented as a feedforward compensator to control different types of smart actuators in an ideal case [5]. For the formulated the generalized Prandtl-Ishlinskii model, as a further development, a corresponding inverse is also provided in the paper for compensations in different smart actuators. Such compensations are experimentally illustrated by a piezoceramic actuator. We should mention that the main purpose for getting this analytical inverse is to try to provide the stability analysis for the closed-loop systems without assumption on the boundedness of the inverse error. This makes the model attractive for control smart actuators in real

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time systems. The stability analysis will be presented in our future works.

II. Prandtl-Ishlinskii Model

In this paper, the hysteresis nonlinearity is represented by the Prandtl-Ishlinskii model. We shall introduce an essential well known hysteresis operator in order to present the generalized Prandtl-Ishlinskii model.

A. Play hysteresis operator

Play hysteresis operator is a continuous and rate-independent hysteresis operator. A detailed discussion about this operator can be found in [2]. Figure 1 illustrates the play hysteresis operator. Analytically, let $C_m[0,t_E]$ represent the space of piecewise monotone continuous functions. For any input $v(t) \in C_m[0,t_E]$, let $0 = t_0 < t_1 < ..... < t_N = t_E$ be a partition of $[0, t_E]$ such that the function v is monotone on each of the sub-intervals $[t_i,t_{i+1}]$. Then, the play operator is defined by [2]:

$$F_r[v](0) = f_r(v(0),0) = w(0) ,$$
$$F_r[v](t) = f_r(v(t), F_r[v](t_i)); \text{ for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \quad (1)$$

where $f_r(v,w) = \max(v - r, \min(v + r, w))$.

The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to another function. The play operator is characterized by input v and the threshold r. Due to the nature of the play operator, the above definition is based on $v(t) \in C_m[0,t_E]$ of continuous and piecewise monotone functions. This, however, can be extended to space $C[0,t_E]$ of continuous functions [2].

B. Prandtl-Ishlinskii Model

The Prandtl-Ishlinskii model utilizes the play operator $F_r[v](t)$ to describe relationship between output y_P and input v [2,5]:

$$y_P = qv + \int_0^R p(r)F_r[v](t)dr \quad (2)$$

where $p(r)$ is a density function, satisfying $p(r) \ge 0$, which is generally identified from experimental data. q is a positive constant. The Prandtl-Ishlinskii model with the density function maps $C[t_o,\infty)$ into $C[t_o,\infty)$. In other words, Lipschitz continuous inputs will yield Lipschitz continuous outputs [3]. Since the density function *p(r)* vanishes for large values of *r*, the choice of $R = \infty$ as upper limit of integration, widely used in the literature, is just a matter of convenience [2].

The above model was applied to predict and to reduce hysteresis effects of piezoceramic actuators [5]. The reasonably good validity of the model was further demonstrated by comparing the model predictions with the measured data of a piezoceramic actuator.

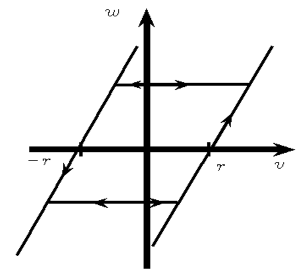


Fig. 1. Play hysteresis operator.

III. A Generalized Prandtl-Ishlinskii Model Based Generalized Play Operator

Owing to the nature of the play operator, the Prandtl-Ishlinskii model predictions were limited to symmetric hysteresis loops of piezoceramic actuators. Moreover, this model cannot show saturation property in hysteresis loops. This property has been widely demonstrated in magnetorestrictive and SMA actuators. Alternatively, the generalized play operator, described in Fig. 2, could be utilized to realize input-output relationships of the smart actuators.

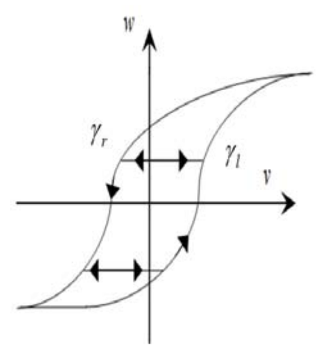


Fig. 2. Input-output relationship of a generalized play operator.

The generalized play operator is a nonlinear play operator, where an increase in input v causes the output w to increase along the curve γ_l or a decrease in input v causes the output w to decrease along the curve γ_r, resulting in asymmetric hysteresis loops about the input or the output. The curves γ_l and γ_r are continuous non decreasing functions with $\gamma_l \le \gamma_r$ [3]. Analytically, the generalized play operator for any input $v(t) \in C_m[0,t_E]$ is defined by [2,17]:

$$F_{lr}^{\gamma}[v](0) = f_{lr}^{\gamma}(v(0),0) = w(0) ;$$
$$F_{lr}^{\gamma}[v](t) = f_{lr}^{\gamma}(v(t), F_{lr}^{\gamma}[v](t_i)); \text{ for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \quad (3)$$

where, $f_{lr}^{\gamma}(v,w) = \max(\gamma_l(v) - r, \min(\gamma_r(v) + r, w))$

A. Symmetric generalized play operator

In this paper, a generalized Prandtl-Ishlinskii model is formulated using symmetric generalized play operator. Analytically, this operator is expressed as:

$$F_r^\gamma[v](0) = f_r^\gamma\big(v(0),0\big) = w(0)\,;$$
$$F_r^\gamma[v](t) = f_r^\gamma\big(v(t), F_r^\gamma[v](t_i)\big);\ \text{for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \quad (4)$$
$$f_r^\gamma(v,w) = \max\big(\gamma(v) - r, \min(\gamma(v) + r, w)\big)$$

where $\gamma : R \to R$ is an envelope function ; strictly increasing, continuous and odd.

Some of mathematical properties of the generalized play operator are:

- Lipschitz-continuity:
  For a given input $v(t)$, Lipschitz-continuity of the generalized play operator can be ensured if the function γ is Lipschitz continuous [2].

- Rate-Independent:
  The generalized play operator $F_r^\gamma[v]$ is rate independent hysteresis operator if:

  $$F_r^\gamma[v] o\, \varphi = F_r^\gamma[v o\, \varphi] \quad (5)$$

  where φ is continuous increasing function $\varphi : [0,T]$ satisfying $\varphi(0) = 0$ and $\varphi(T) = T$.

- Range of the generalized play operator:
  For a given input $v(t) \in C[0,T]$ and $r \ge 0$:

$$\max_{t\in[0,T]} F_r^\gamma[v](t) = f_r\big(\max_{t\in[0,T]} \gamma(v(t)), w(0)\big) \quad (6a)$$
$$\min_{t\in[0,T]} F_r^\gamma[v](t) = f_r\big(\min_{t\in[0,T]} \gamma(v(t)), w(0)\big) \quad (6b)$$

B. Generalized Prandtl-Ishlinskii model

Generalized Prandtl-Ishlinskii model is formulated using symmetric generalized play operator $F_{lr}^\gamma[v](t)$ to yield output $y_{P\gamma}(t)$ as:

$$y_{P\gamma}(t) = q\gamma(v) + \int_0^R p(r) F_r^\gamma[v](t)\,dr \quad (7)$$

Using the generalized play operator, the generalized Prandtl-Ishlinskii model can be used to characterize hysteresis effects of different smart actuators.

C. Discussions

The parameters of the generalized play hysteresis operator and density function need to be defined on the basis of known characteristics of smart actuators. In this study, the experimental data obtained for a magentostrictive actuator [22] and a SMA actuator [7] are used to identify the model parameters. The experimental data for the magnetostrictive and the SMA actuators were acquired under input currents between -1 to 1 A and 0.75 to 0.75 A, respectively. For the SMA actuator, the input temperatures corresponding to the input currents are between -175 to 175°C. The generalized play hysteresis operator is analyzed using a nonlinear envelop function. The proposed envelop function of generalized play operator is hyperbolic tangent function (tanh). This function, which exhibits saturated output beyond certain input, is expressed as:

$$\gamma = c_o \tanh(c_1 v + c_2) + c_3 \quad (8)$$

The following density function is employed:

$$p(r) = \rho e^{-\tau r} \quad (9)$$

where c_o, c_1, c_2, c_3 , ρ, and τ are constants identified from experimental data. It should be mentioned that the proposed density function and the proposed envelop function of the generalized play operator are not unique; they depend upon the nature of hysteresis of particular material or actuator. In this study, the experimental data obtained for SMA and magnetostrictive actuators are used to identify the generalized Prandtl-Ishlinskii model parameters.

The model parameters were identified through minimization of an error sum-squared function given by:

$$J = \sum_{i=1}^{n} \big(y_{P_\gamma}(i) - y_m(i)\big)^2 \quad (10)$$

where $y_{P\gamma}$ is the displacement response of the generalized Prandtl-Ishlinskii model corresponding to a particular actuator, and y_m is the measured displacement. The error function is constructed through summation of squared errors, denoted by i (i=1…n). The index i refer to the number of data points considered to compute the error function J for major and minor hysteresis loops. The error minimization problem was solved using the MATLAB optimization toolbox.

The validity of the generalized Prandtl-Ishlinskii model that is constructed by generalized play operator of hyperbolic-tangent envelop function is examined by comparing the model responses with the measured data of magnetostrictive and SMA actuators. Figures 3 and 4 illustrate comparisons of the displacement responses of the generalized model with the measured data. The results clearly suggest that the model can predict the hysteresis properties of the SMA and magnetostrictive actuators.

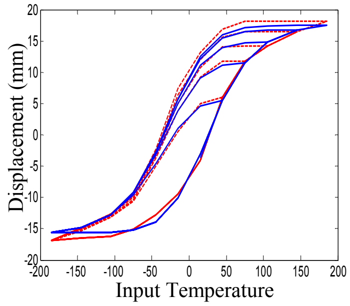


Fig.3. Comparisons of displacement responses of the generalized model with the measured responses of SMA actuator (——, model; - - -, measured).

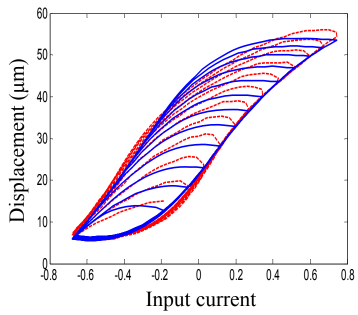


Fig.4. Comparisons of displacement responses of the generalized model with the measured responses of magnetostrictive actuator (——, model; - - - -, measured).

IV. Inverse Generalized Prandtl-Ishlinksii Model

As shown in the previous section, hysteresis effects of SMA and magnetostrictive actuators could be successfully characterized by the generalized Prandtl-Ishlinskii model. In this paper, inversion of the Generalized Prandtl-Ishlinskii model is presented for the purpose of reducing the hysteresis effects in control systems. The inverse of generalized Prandtl-Ishlinskii model is used as a feedforward compensator to compensate hysteresis effects. It should be mentioned that this inverse is computed analytically. In other words, exact inverse of this model is reachable, consequently making it more attractive for real-time applications of smart actuators.

Theorem: For the generalized Prandtl-Ishlinskii model in equation (7), and the generalized play operator used in equation (4)**, if** the inverse of the envelope function $\gamma^{-1}: R \rightarrow R$ exists; the inverse of the generalized Prandtl-Ishlinskii model that is presented in (7) can be analytically expressed as:

$$y_{P\gamma}^{-1} = \gamma^{-1}\left(y_P^{-1}\right) \quad (11)$$

Proof:

The output of the Generalized Prandtl-Ishlinskii model is presented as:

$$z = y_{P\gamma}[v] \quad (12)$$

This equation can be expressed using composition property [2] of the Prandtl-Ishlinskii model as:

$$z = y_P[\gamma(v)] \quad (13)$$

where y_P is given in (2) and its analytical inverse model [5] which can be inserted into (13) is expressed as:

$$y_P^{-1}[z] = y_P^{-1} o(z) \quad (14)$$

Because $y_P o y_P^{-1}[v] = v$ and γ^{-1} exists, the analytical inversion of the generalized can be expressed as:

$$\gamma^{-1}(y_P^{-1}[z]) = v \quad (15)$$

□

The above analytical inverse can also be numerically implemented. Using discrete inputs $v \in C[0,T]$ with a step size of h, Generalized Prandtl-Ishlinskii model can be expressed with the discrete input $v(k)$ corresponding to an interval k $(k=0,1,2,3,\ldots,N=T/h)$ as:

$$y_{p\gamma}(k) = q\gamma(v(k)) + \sum_{j=1}^{n} p_j F_{r_j}[v](k)) \quad (16)$$

where n is the number of the generalized play operators that used in the identification. The inversion of this model is expressed numerically as:

$$y_{p\gamma}{}^{-1}(k) = \gamma^{-1}(q^{-1}v(k) + \sum_{j=1}^{n} \hat{p}_j F_{\hat{r}_j}[v](k)) \quad (17)$$

Based on the pervious description of the generalized Prandtl-Ishlinskii model, the parameters of the inverse can follow the classical Prandtl-Ishlinskii model and are expressed as [5]:

$$q^{-1} = \frac{1}{q} \quad (18)$$

$$\hat{r}_j = qr_j + \sum_{i=1}^{j-1} p_i(r_j - r_i) \quad (19)$$

$$\hat{p}_j = -\frac{p_j}{(q + \sum_{i=1}^{j} p_i)(q + \sum_{i=1}^{j-1} p_i)} \quad (20)$$

V. Simulation Results With Inverse Generalized PI Model

In this section, simulation results are carried out to compensate hysteresis of smart actuators of different input/output relationships. Hysteresis is obtainable using Generalized Prandtl-Ishlinskii model. The analytical inverse of the generalized Prandtl-Ishlinskii model is employed as a feedforward controller to compensate hysteresis nonlinearities.

An input signal of the form: $v(t)=1.6\sin(\pi t)+1.1\cos(4.3\pi t)$ is considered to evaluate minor as well as major hysteresis loops. The proposed envelope function (4) is used to construct the generalized Prandtl-Ishlinskii model. The chosen simulation parameters are: T=6, Δt=0.01, q=1.1. The proposed parameters for the generalized symmetric play operator are: $c_o=1$, $c_1=1$, $c_2=0$, and $c_3=0$. The threshold r is selected as:

$$r_j = 0.009j, \quad j=1,2,3,\ldots\ldots,n=100 \tag{21}$$

Figure 5 show the simulation results of the generalized Prandtl-Ishlinskii model and its inverse. The results show the capability of the model and its inverse to compensate saturated hysteresis loops. Moreover, the results show the capability of the inverse to compensate the hysteresis of minor loops which does not show saturated output.

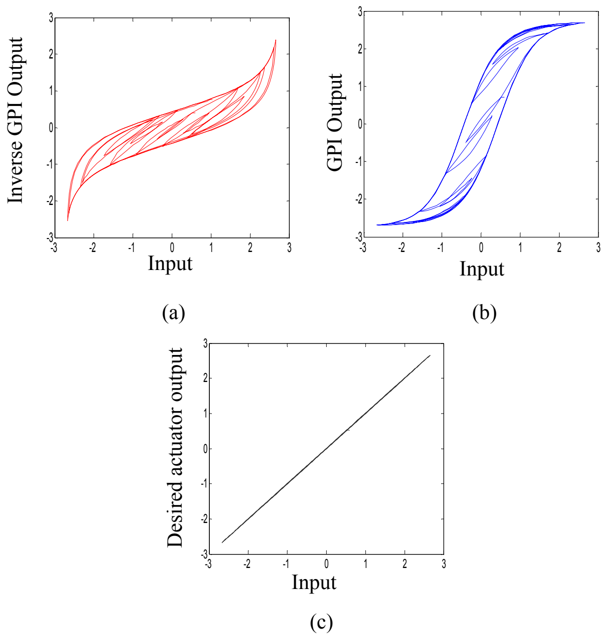


Fig.5. Compensation hysteresis using Generalized Prandtl-Ishlinskii model and its inverse (Inverse feedforward compensator). (a) Inverse generalized Prandtl-Ishlinskii model (b) Generalized Prandtl-Ishlinskii model. (c) Desired Output.

VI. Inverse Generalized Prandtl-Ishlinskii Model For Compensation in Piezoceramic Actuator

As shown in the previous section, simulation results show the capability of the generalized Prandtl-Ishlinskii model to compensate different hysteresis outputs. In this section, compensation hysteresis effects of piezoceramic actuator are carried out experimentally using generalized Prandtl-Ishlinskii model and its inverse. It should be mentioned that the piezoceramic actuator is used as an example to show the capability of the generalized model and its inverse to model and to reduce hysteresis of smart actuators.

A. Experimental set up

A piezoceramic actuator (P-753.31C) from Physik Instrumente Company is used in the experiment. The actuator provided maximum displacement amplitude of 100 µm from its static equilibrium position, and it integrated a capacitive sensor (sensitivity=1µm/V) for measurement of actuator displacement response. The excitation module compromised a voltage amplifier (LVPZT, E-505) with affixed gain of 10, which provided excitation voltage to the actuator. The input voltage and output displacement signals were acquired using Q4 (Quanser) data acquisition card which is connected into 450 MHz PC.

B. Generalized Prandtl-Ishlinskii model for characterizing hysteresis of a piezoceramic actuator

In this section, the parameters of the generalized Prandtl-Ishlinskii model are defined using experimental results of piezoceramic actuator. The experimental data for the piezoceramic actuator was acquired under complex harmonic input $v(t)= 9\sin(2\pi t)+ 30\cos(4\pi t)$. The parameters of the generalized play operator and density function are selected as follows. The threshold r is chosen as:

$$r_j = \beta\, j,\ j = 1,\ldots\ldots,\ n = 5 \tag{22}$$

where β is a positive constant. A linear envelope function is proposed to model hysteresis in piezoceramics actuator:

$$\gamma = c_o v + c_1 \tag{23}$$

The proposed density function (9) is employed to construct the Generalized Prandtl-Ishlinskii model.

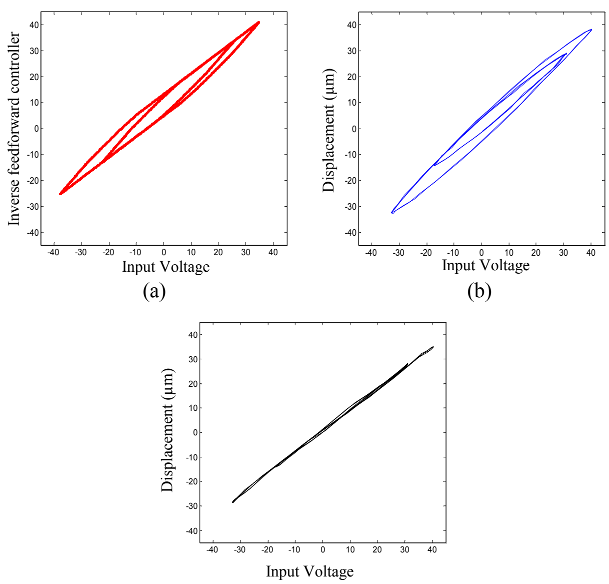
The model parameters $(q,\ \beta,\ \rho,\ \tau,\ c_o,$ *and* $c_1)$ were identified using MATLAB optimization toolbox. These Parameters are identified using (10). Solutions were attained for a number of starting values of the parameter vector, which converged to very similar parameter values. Identified parameters of the generalized Prandtl-Ishlinskii model are presented in Table 1.

Table 1: Identified parameters of the generalized Prandtl-Ishlinskii model using generalized play operator for the piezoceramic actuator.

Parameters	Generalized Prandtl-Ishlinskii model
β	4.9188
ρ	0.0211
τ	-0.0118
c_o	0.8157
c_1	-7.3757
q	1.0000

C. *Inverse Generalized Prandtl-Ishlinskii model for Compensation hysteresis in piezoceramic actuator*

Inverse generalized Prandtl-Ishlinskii model is employed to compensate the hysteresis effects in the piezoceramic actuator in real-time experiments. The parameters of the inverse are identified using equations (17), (18), (19), and (20). This inverse is used as feedforward compensator to reduce hysteresis nonlinearities in the output displacement of the actuator. Figures 6.a and 6.b show the generalized Prandtl-Ishlinskii model and its inverse, respectively. The measured output of the piezoceramic actuator is shown in Figure 6.c. These experimental results show that the inverse Prandtl-Ishlinskii model can successfully compensate hysteresis in the output displacement of the piezoceramic actuator.



(c)

Fig.6. Compensation hysteresis of piezoceramic actuator using generalized Prandtl-Ishlinskii model and its inverse (Inverse feedforward compensator). (a) Inverse generalized Prandtl-Ishlinskii model (b) Measured piezoceramics actuator displacement without control signal. (c) Output of the piezoceramics actuator with Inverse feedforward compensator.

VII. Conclusions

This paper presents a Generalized Prandtl-Ishlinskii model to characterize and to compensate hysteresis nonlinearities in different smart actuators. This model is formulated using generalized symmetric play operator and density function. As an illustration, comparison of the model results with the measured data for SMA, magnetostrictive, and piezoceramics actuators revealed reasonable good agreements among them. Analytical inversion for the generalized Prandtl-Ishlinskii model is derived. Simulation results show the capability of the inverse generalized Prandtl-Ishlinskii model (inverse feedforward compensator) to compensate hysteresis nonlinearities of input-output relationships. Furthermore, modeling and compensation of the hysteresis in the piezoceramic actuator using the formulated generalized model and its inverse are carried out experimentally.

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