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Novel switched Model Reference Adaptive Control for continuous Piecewise Affine systems

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Abstract— **This paper is concerned with the derivation of a novel model reference adaptive control (MRAC) scheme for piecewise-affine (PWA) continuous systems. A novel version of the minimal control synthesis algorithm, originally developed as a MRAC for smooth systems, is presented. The resulting adaptive algorithm is a switched feedback controller able to cope with uncertain continuous PWA systems. The proof of stability, based on the newly developed passivity theory for hybrid systems, is provided and the effectiveness of the new proposed control strategy is numerically tested.**

I. INTRODUCTION

Hybrid and switched dynamical systems are increasingly used to model a wide variety of physical devices. Examples include systems in automotive engineering [20] and [21], biological systems (see [7] and [8]) and computer science [11] to name just a few. Within the area of hybrid control, piecewise-affine systems have been the subject of much ongoing research, spanning from the investigation of well posedness ([9] and [14]), stability ([2], [5] etc.) and passivity ([22], [32], [33]) to the study of their structural stability [18], observability and controllability ([1] and [16]).

Much attention has also been focussed on the control of systems described by piecewise-affine models. For example, novel schemes have been proposed to control the dynamics of systems with friction in mechanics [10], [20], [21] switching power converters in Electronics [6], [30] and more generally complementarity and PWA systems [4], [23], [25], [26] and [31]. As is typical in control, an important issue is for the control action to provide a certain amount of robustness with respect to parameter uncertainties and unmodelled dynamics. Adaptive controllers have been long used to this aim to control the dynamics of uncertain smooth dynamical systems (see, for example [3], [15] and [17]).

Surprisingly, little attempts have been made in the existing literature to develop adaptive control strategies aimed at PWA systems. Some results in this direction were presented for instance in [13], [27] but mostly centered on specific classes of systems.

The aim of this paper is to propose a novel model reference adaptive strategy to control a wider class of PWA systems. The idea is to synthezise a switched model reference adaptive controller for continuous piecewise-affine systems based on the so-called Minimal Control Synthesis Algorithm (MCS) for smooth systems (see [28] and [29] for a description of the MCS and further details). In particular, we assume that the state space of both the plant and the reference model are divided in polyhedral cells; each being associated to the dynamics governed by a different affine system with the vector fields assumed to be continuous across the phase-space boundaries beween cells. Proof of asymptotic stability of the resulting closed-loop system is given by using an extension of passivity theory for hybrid systems and Popov inequalities. The efficiency of the adaptive switching strategy is validated numerically on a representative case study.

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II. STATEMENT OF THE PROBLEM AND ASSUMPTIONS

The aim of the control strategy presented in this paper is the tracking of a given piecewise affine reference model assuming that the plant dynamics are also described by means of a continuous piecewise affine model. The statement of the problem is:

Given the plant dynamics described by a n-dimensional continuous PWA system of the form:

$$\dot{x} = A_i x + Bu + B_i, \ \text{ if } \ x \in \Omega_i, \ \ i \in \{1, \dots N\}. \tag{1}$$

and a fixed n-dimensional PWA reference model given by

$$\dot{x}_m = A_j^m x_m + B^m r + B_j^m, \text{ if } x_m \in \Omega_j^m, \ j \in \{1, \dots M\}. \tag{2}$$

the problem is to find a control strategy which guarantees an asymptotically bounded tracking error between the state variables of the plant and the reference model, for all initial conditions.

For the sake of brevity we will restrict our attention to bimodal systems ($N = M = 2$). Note that the strategy presented in the paper and its proof can be applied to systems with any number of modes.

Each domain Ω_p and Ω_p^m ($p \in \{1, 2\}$) in (1)-(2) is supposed to be a polyhedral cell given by:

$$\begin{aligned}
\Omega_1 &\triangleq \left\{x \in I\!R^n : H^T x + h \leq 0\right\}, \\
\Omega_2 &\triangleq \left\{x \in I\!R^n : H^T x + h > 0\right\}, \\
\Omega_1^m &\triangleq \left\{x_m \in I\!R^n : H_m^T x_m + h^m \leq 0\right\}, \\
\Omega_2^m &\triangleq \left\{x_m \in I\!R^n : H_m^T x_m + h^m > 0\right\}.
\end{aligned} \tag{3}$$

The hyperplanes which define the cells in state space are assumed to be known and, in particular, the switching manifold for the plant is described by the hyperplane Σ as:

$$\Sigma : H^T x + h = 0, \tag{4}$$

whereas the switching manifold for the reference model, Σ_m, is given by:

$$\Sigma_m : H_m^T x + h^m = 0, \tag{5}$$

where $H^T = \{h_k\}$, h, $H_m^T = \{h_k^m\}$ and h^m are constant vectors of proper dimensions, assumed to be known.

Vector fields in (1) and (2) are supposed to be continuous across the boundaries, thus it is always possible to find two constant vectors, respectively G_H and G_{H_m}, so that the following equalities hold:

$$\left\{ \begin{array}{l} G_H H^T = A_1 - A_2, \\ h G_H = B_1 - B_2, \end{array} \right. \tag{6}$$

$$\left\{ \begin{array}{l} G_{H_m} H_m^T = A_1^m - A_2^m, \\ h^m G_{H_m} = B_1^m - B_2^m. \end{array} \right. \tag{7}$$

The plant and reference models are assumed to be in control canonical form given by:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & 1 \\ a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}, \tag{8}$$

$$B_e = \begin{bmatrix} 0 & \cdots & 1 \end{bmatrix}^T, \; B = bB_e, \tag{9}$$

$$B_i = b_i B_e, \quad i \in \{1, 2\}, \tag{10}$$

$$A_j^m = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & 1 \\ a_{j1}^m & a_{j2}^m & \cdots & a_{jn}^m \end{bmatrix}, \tag{11}$$

$$B_j^m = b_j^m B_e, \quad j \in \{1, 2\}, \tag{12}$$

$$B^m = b_m B_e. \tag{13}$$

where the matrices are in the companion form and belong to $I\!R^{n \times n}$ and all input vectors (element of $I\!R^n$) have at most the last entry different form zero. Note that all the entries on the last row of the matrices A_i, B_i and B are supposed to be unknown.

By using the canonical structure assumption, (6) and (7) can be rewritten as scalar equations, so that for some constants g_m and an unknown constant g it holds:

$$\left\{ \begin{array}{l} g h_k = a_{2k} - a_{1k}, \\ g h = b_1 - b_2, \quad k = 1 \ldots n, \end{array} \right. \tag{14}$$

$$\left\{ \begin{array}{l} g_m h_k^m = a_{2k}^m - a_{1k}^m, \\ g_m h^m = b_1^m - b_2^m, \quad k = 1 \ldots n. \end{array} \right. \tag{15}$$

III. MAIN RESULT

Our main result can be summarized as follows (the proof is sketched in section IV).

Theorem 1: Let α, β, ρ, ε_1, ε_2, θ_1^2, θ_2^1 and γ be some positive constants. Define (see also figure 1)

$$\sigma = \left\{ \begin{array}{ll} 1, & \text{if } x \in \Omega_1 \text{ and } x_m \in \Omega_1^m, \\ 2, & \text{if } x \in \Omega_2 \text{ and } x_m \in \Omega_1^m, \\ 3, & \text{if } x \in \Omega_2 \text{ and } x_m \in \Omega_2^m, \\ 4, & \text{if } x \in \Omega_1 \text{ and } x_m \in \Omega_2^m. \end{array} \right. \tag{16}$$

and let P_1 and P_2 be solutions of the following inequalities

$$P_1 A_1^m + A_1^{mT} P_1 + 2\varepsilon_1 P_1 B_e B_e^T P_1 \le 0, \tag{17}$$

$$P_2 A_2^m + A_2^{mT} P_2 + 2\varepsilon_2 P_2 B_e B_e^T P_2 \le 0, \tag{18}$$

$$P_2 A_1^m + A_1^{mT} P_2 + \frac{2}{\gamma^2 \theta_1^2} P_2 B_e B_e^T P_2 + \theta_1^2 P_1 B_e B_e^T P_1 \le 0, \tag{19}$$

$$P_1 A_2^m + A_2^{mT} P_1 + \frac{2}{\gamma^2 \theta_2^1} P_1 B_e B_e^T P_1 + \theta_2^1 P_2 B_e B_e^T P_2 \le 0. \tag{20}$$

The switched adaptive controller:

$$u(t) = K_\sigma(t) x(t) + K_R(t) r(t), \tag{21}$$

with

$$K_R(t) = \alpha \int_0^t y_e(\tau) r(\tau) d\tau + \beta y_e(t) r(t), \tag{22}$$

$$K_1(t) = \alpha \int_0^t y_e(\tau) x^T(\tau) d\tau + \beta y_e(t) x^T(t), \tag{23}$$

$$K_2(t) = K_1(t) + \tilde{K}(t), \tag{24}$$

$$K_3(t) = K_2(t) + \bar{K}(t), \tag{25}$$

$$K_4(t) = K_1(t) + \bar{K}(t). \tag{26}$$

where

$$\tilde{K}(t) = \left\{ \begin{array}{ll} \rho \int_{t_j}^t y_e(\tau) x^T(\tau) d\tau, & \text{if} \quad x \in \Omega_2, \\ 0 \quad \text{otherwise}, & \end{array} \right. \tag{27}$$

$$\bar{K}(t) = \left\{ \begin{array}{ll} \rho \int_{t_j^m}^t y_e(\tau) x^T(\tau) d\tau, & \text{if} \quad x_m \in \Omega_2^m, \\ 0 \quad \text{otherwise}. & \end{array} \right. \tag{28}$$

and

$$y_e \triangleq C_{e\sigma} x_e, \quad x_e \triangleq (x_m - x), \qquad C_{e\sigma} \triangleq B_e^T P_\sigma. \tag{29}$$

with

$$P_\sigma = \left\{ \begin{array}{ll} P_1, & \text{if} \quad x_m \in \Omega_1^m, \\ P_2, & \text{if} \quad x_m \in \Omega_2^m. \end{array} \right. \tag{30}$$

solves the problem described in section II under the assumption that no sliding is present in the closed-loop system.

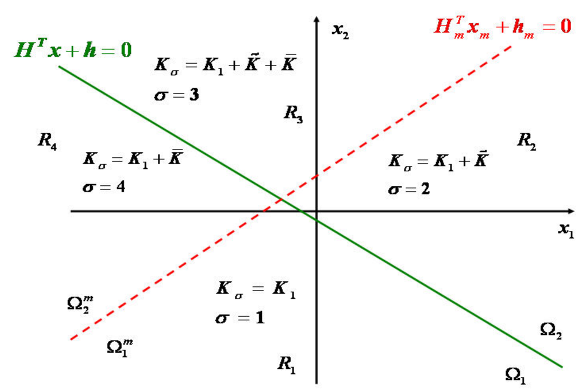

Fig. 1. Switching manifolds and adaptive gains.

Remarks

- The initial conditions of the adaptive gains are set as $K_R(0) = 0$, $K_1(0) = 0$, $\bar{K}(t_j^m) = \bar{K}(\bar{t}_{j-1}^m)$ and $\tilde{K}(t_j) = \tilde{K}(\tilde{t}_{j-1})$ where (as shown in figure 2) t_j^m is the time instant where the j-th transition from Ω_1^m to Ω_2^m occurs, $\bar{t}_{j-1}^m$ is the time instant where $(j-1)$-th transition from Ω_2^m to Ω_1^m occurs and similarly t_j and $\tilde{t}_{j-1}$ are the times of the transitions between Ω_1 to Ω_2 and viceversa. At the first commutation $\bar{K}(t_1^m) = 0$ and $\tilde{K}(t_1) = 0$.

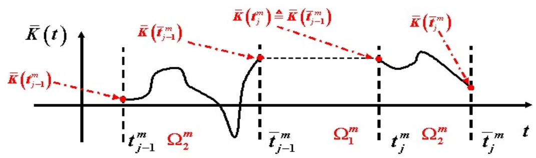

Fig. 2. A generic $\bar{K}$ evolution.

- The problem stated in (17)–(20) is not as difficult to solve as it might appear at first. A solution can be obtained from the solution of a standard LMI problem which is well known in the wide literature on hybrid systems [5]; namely the problem of finding a symmetric positive definite matrix P so that:

$$PA_1^m + A_1^{mT} P < 0, \qquad PA_2^m + A_2^{mT} P < 0. \quad (31)$$

  From the solution of (31), inequalities (17)–(20) can be solved by imposing $P_1 = P_2 = P$ and finding some strictly, positive sufficiently small constants ε_1, ε_1, θ_1^2, θ_2^1 and a strictly positive, large enough constant γ, so that all the positive definite terms in (17), (18), (19) and (20) do not change the sign of the matrices in (31). Notice that (31) is not a restrictive condition since it is equivalent to require uniform global exponential stability of the reference model.
- The adaptation laws for K_R and K_1 in (22) and (23) are the same as those proposed for the standard MCS approach presented in [12]. Here the parameters α and β weigh the sensitivity of the gain dynamics to the proportional and integral terms and can be chosen heuristically.
- The assumption that no sliding is present in the closed loop system is not as strict as it might appear for PWA continuous systems. Ongoing work is addressing the possible presence of sliding and will be presented elsewhere.

IV. Proof of Stability

In what follows, theorem 1 is proven by using the novel theory of passivity for switched system [33]. The aim is to show that the closed-loop system is composed by the feedback of two passive switched systems, and thus it is passive and, therefore stable. Mainly, the proof is based on the following three steps:

A. Recast the error dynamics as a feedback system.

B. Use passivity theory for switched systems to prove the dissipativity of the feedforward dynamics.

C. Show that the feedback block satisfies the Popov inequality (feedback dynamics are passive).

In what follows, for the sake of brevity, each step of the proof is only sketched (for the complete proof and further application examples see [19])

A. Closed loop dynamics

From the definition of the control strategy in (21) and the tracking error given in (29), it is possible to describe the error evolution as:

$$\begin{aligned}\dot{x}_e = A_j^m x_e \quad &+ \quad \left(A_j^m - A_i - BK_\sigma\right) x \\ &+ \quad (B_m - BK_R)\, r + \left(B_j^m - B_i\right). \quad (32)\end{aligned}$$

Using the hypothesis of phase canonical structure, equation (32) can be recast in the compact form:

$$\dot{x}_e = A_j^m x_e + B_e \left[\phi_{ij\sigma} w + \left(b_j^m - b_i\right)\right], \qquad (33)$$

where

$$w \triangleq \begin{bmatrix} x \\ r \end{bmatrix}, \quad \phi_{ij\sigma} \triangleq \left[B_e^T \left(A_j^m - A_i\right) - bK_\sigma | b_m - bK_R\right]. \quad (34)$$

From (34), it is easy to represent the error dynamics as the feedback system shown in figure 3, where the feedforward block is described by the following equations:

$$\begin{aligned}\dot{x}_e &= \begin{cases} A_1^m x_e + B_e \xi = f_1(x_e) + g_1(x_e)\, u_1, & \text{if } x_m \in \Omega_1^m, \\ A_2^m x_e + B_e \xi = f_2(x_e) + g_2(x_e)\, u_2, & \text{if } x_m \in \Omega_2^m. \end{cases} \\ y_e &= \begin{cases} C_{e1} x_e = h_1(x_e), & \text{if } x_m \in \Omega_1^m, \\ C_{e2} x_e = h_2(x_e), & \text{if } x_m \in \Omega_2^m. \end{cases}\end{aligned} \qquad (35)$$

where ξ is the input signal to the feedforward system (see figure 3), whereas the functions $f_1, f_2, h_1, h_2, u_1, u_2$ allow to describe the linear bimodal system as a generic one.

Since the input to the error dynamics (see figure 3) is piecewise constant (therefore bounded), it follows that, if the feedback system in figure 3 is passive (namely, if both the feedforward and feedback path are passive) the tracking error is bounded.

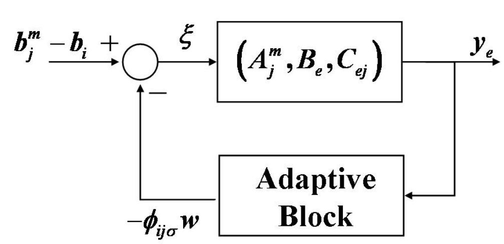

Fig. 3. Closed-loop error dynamics described as equivalent feedback system.

B. Feedforward dynamics

Using passivity theory for hybrid systems developed in [33], it is possible to choose, as supply rates for the two modes in (35), the two functions

$$\omega_i^i(u_i, h_i) = u_i^T h_i - \varepsilon_i h_i^T h_i, \quad i \in \{1, 2\}, \qquad (36)$$

and, as cross supply rates, the following two functions:

$$\begin{aligned}\omega_2^1(x_e, u_1, h_1, t) &= \frac{1}{2}\theta_2^1\left(\gamma^2 u_1^T u_1 - h_1^T h_1\right), && (37)\\ \omega_1^2(x_e, u_2, h_2, t) &= \frac{1}{2}\theta_1^2\left(\gamma^2 u_2^T u_2 - h_2^T h_2\right). && (38)\end{aligned}$$

The feedforward path is than a passive switched system as defined in [33], if the following inequalities are satisfied

$$\mathcal{L}_{f_1}(S_1) \leq -\varepsilon_1 h_1^T h_1 \leq 0 \quad (39)$$

$$\mathcal{L}_{f_2}(S_2) \leq -\varepsilon_2 h_2^T h_2 \leq 0 \quad (40)$$

$$\mathcal{L}_{f_1}(S_2) + \frac{1}{2\gamma^2\vartheta_2^1}\left(\mathcal{L}_{g_1}(S_2)\right)\left(\mathcal{L}_{g_1}(S_2)\right)) + \frac{1}{2}\vartheta_2^1 h_1^T h_1 \leq 0 \quad (41)$$

$$\mathcal{L}_{f_2}(S_1) + \frac{1}{2\gamma^2\vartheta_1^2}\left(\mathcal{L}_{g_2}(S_1)\right)\left(\mathcal{L}_{g_2}(S_1)\right) + \frac{1}{2}\vartheta_1^2 h_2^T h_2 \leq 0 \quad (42)$$

where $\mathcal{L}_v$ denotes the Lie derivative along the vector v and $S_j(x_e)$ are the two storage function chosen as

$$S_j(x_e) = \frac{1}{2}x_e^T P_j x_e, \quad j \in \{1, 2\}. \quad (43)$$

By choosing $C_{ej} \triangleq B_e^T P_j$, $j \in \{1, 2\}$, it is possible to prove that equations (39)–(42) are equivalent to the problem stated in (17)–(20), and thus the passivity of the feedforward dynamics can be guaranteed.

C. Feedback dynamics

In order to prove the passivity of the feedback dynamics in figure 3, it is sufficient to show that (see [17] and [24]):

$$\int_0^t -y_e \phi_{ij\sigma} w dt = \Gamma + \Delta \geq -c^2, \quad (44)$$

where

$$\begin{aligned}\Delta &= \int_0^t y_e\left(b_m - bK_R\right) r d\tau, && (45)\\ \Gamma &= \sum_{v=1}^{n} \Psi_v, && (46)\end{aligned}$$

with

$$\Psi_v = \int_0^t y_e\left[\left(a_{jv}^m - a_{iv}\right) - bK_{\sigma v}\right] x_v d\tau, \quad v \in \{1...n\}. \quad (47)$$

for some finite constant c. This holds if each term in (47) is greater than a finite negative constant.

Since both the input plant matrix and the input reference model matrix do not switch, by choosing the adaptive gain K_R as in (6), from the proof of the classical MCS control for smooth systems [12], it follows that the integral in (45) is greater than a finite negative constant.

Furthermore, since all addends in (46) have the same structure, it is sufficient to show that a generic addend is greater than a finite negative quantity.

Let z be the generic number of switchings from Ω_1^m to Ω_2^m in $[0, t]$ and q the number of switchings from Ω_1 to Ω_2. It is now possible to decompose the generic integral in (47) as:

$$\begin{aligned}\Psi_v &= \int_0^t -y_e\left[\left(a_{1v}^m - a_{1v}\right) - bK_{1v}\right] x_v d\tau\\ &+ \sum_{d=1}^{z}\int_{t_d^m}^{\bar{t}_d^m} -y_e\left[g_m h_v^m - b\bar{K}_v\right] x_v d\tau\\ &+ \sum_{l=1}^{q}\int_{t_l}^{\tilde{t}_l} -y_e\left[g h_v - b\tilde{K}_v\right] x_v d\tau.\end{aligned} \quad (48)$$

The adaptation law for K_1, according to the MCS theory [12], guarantees that the first integral is greater than a finite negative constant, whereas by using the adaptive laws for $\bar{K}$ and $\tilde{K}$ in (28) and (27), it follows that

$$\begin{aligned}&\sum_{d=1}^{z}\int_{t_d^m}^{\bar{t}_d^m} -y_e\left[g_m h_v^m - b\bar{K}_v\right] x_v d\tau\\ &+\sum_{l=1}^{q}\int_{t_l}^{\tilde{t}_l^m} -y_e\left[g h_v - b\tilde{K}_v\right] x_v d\tau \geq -\frac{\rho^{-1}}{2}\cdot\\ &\left[\left(g_m h_v^m - b\bar{K}_v\left(t_1^m\right)\right)^2 + \left(g h_v - b\tilde{K}_v\left(t_1\right)\right)^2\right] > -\infty.\end{aligned} \quad (49)$$

Hence the theorem remains proven.

V. A REPRESENTATIVE EXAMPLE

In order to validate the control strategy, we consider a bimodal piecewise affine system of the form (1) with:

$$\begin{aligned}A_1 &= \begin{bmatrix} 0 & 1\\ -2 & -3\end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1\\ -6 & 1\end{bmatrix}, && (50)\\ B_1 &= \begin{bmatrix} 0 & 30\end{bmatrix}^T, \quad B_2 = \begin{bmatrix} 0 & 10\end{bmatrix}^T, && (51)\\ B &= \begin{bmatrix} 0 & 4\end{bmatrix}^T, && (52)\\ H^T &= \begin{bmatrix} 1 & -1\end{bmatrix}, \quad h = 5, && (53)\\ x(0) &= \begin{bmatrix} 10 & 10\end{bmatrix}^T. && (54)\end{aligned}$$

The reference model is chosen as in (2) with:

$$\begin{aligned}A_1^m &= \begin{bmatrix} 0 & 1\\ -10 & -30\end{bmatrix}, \; A_2^m = \begin{bmatrix} 0 & 1\\ -12.5 & -32.5\end{bmatrix}, && (55)\\ B_1^m &= \begin{bmatrix} 0 & 15\end{bmatrix}^T, \quad B_2^m = \begin{bmatrix} 0 & 2.5\end{bmatrix}^T, && (56)\\ B^m &= \begin{bmatrix} 0 & 2\end{bmatrix}^T, && (57)\\ H_m^T &= \begin{bmatrix} 1 & 1\end{bmatrix}^T, \quad h_m = 5, && (58)\\ x_m(0) &= \begin{bmatrix} 15 & 0\end{bmatrix}^T. && (59)\end{aligned}$$

Furthermore, to emphasize the difference between the classical MCS and the one presented here, it is assumed that $\alpha = 10^{-6}$, $\beta = 10^{-7}$ and $\rho = 10^{-6}$. Notice that in applications, the adaptation constants are typically small so that the controlled system is not too reactive.

The performance of the novel switching adaptive controller are now compared with a classical MCS approach. Figures 4 and 5 show the tracking performance provided by

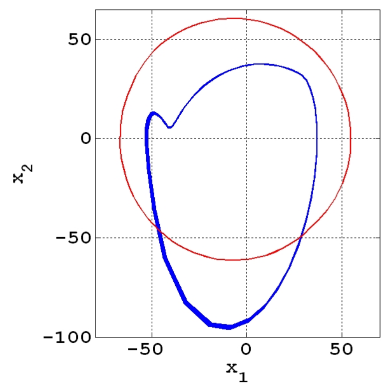

Fig. 4. Phase-space evolution of the closed-loop system under a classical MCS control when the transient is over: reference model (red) and controlled signal (blue)

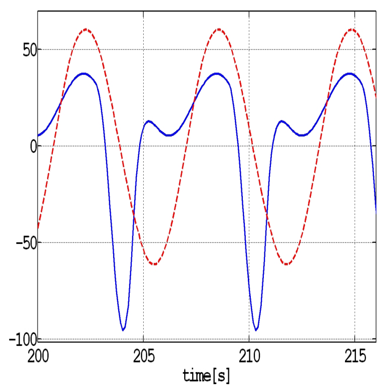

Fig. 5. Time history of x_{2m} and x_2 under a classical MCS control: reference model (red) and controlled signal (blue).

a classical MCS control. Obviously, the standard MCS is unable to guarantee an acceptable tracking error.

Defining $C_{e\sigma}$ as in (29) and (30), the MCS tracking performance can be improved as shown in figures 6 and 7. Still, the PWA nature of the plant makes the standard MCS unable to cope with the switchings from one cell to the other.

By using the complete novel control strategy introduced in section III, it is possible to obtain excellent tracking of the reference model as shown in figures 8 and 9. The corresponding control input is shown in figure 10.

VI. Conclusion

We presented a novel adaptive control strategy for piecewise affine systems based on the MCS algorithm. In our context both the plant and the reference model are supposed to be switching systems. The proof of the closed-loop stability is provided by using the novel concept of passivity for switched system [33] and the efficiency of the proposed method is validated by numerical simulations on representative case of study.

Ongoing work is aimed at extending this approach by evaluating the important case where sliding can occur in the close loop system and relaxing the continuity assumption on the plant and reference model at the boundary.

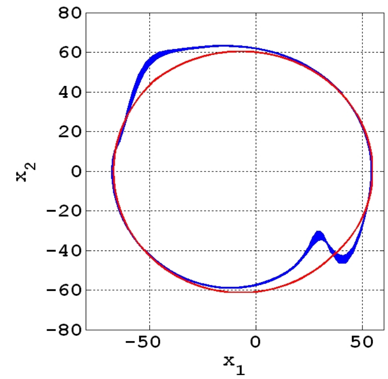

Fig. 6. Phase-space evolution of the closed-loop system under a classical MCS control and the new definition of $C_{e\sigma}$ when the transient is over: reference model (red) and controlled signal (blue).

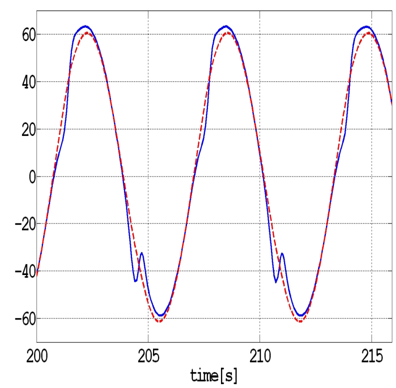

Fig. 7. Time history of x_{2m} and x_2 under a classical MCS control and the new definition of $C_{e\sigma}$: reference model (red) and controlled signal (blue).

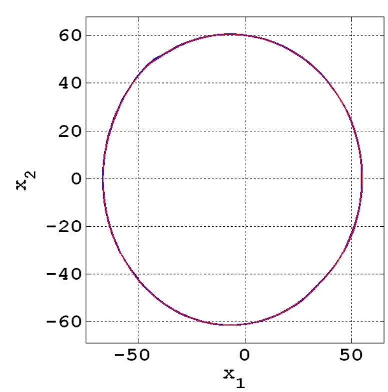

Fig. 8. Phase-space evolution of the closed-loop system under the new control strategy when the transient is over: reference model (red) and controlled signal (blue).

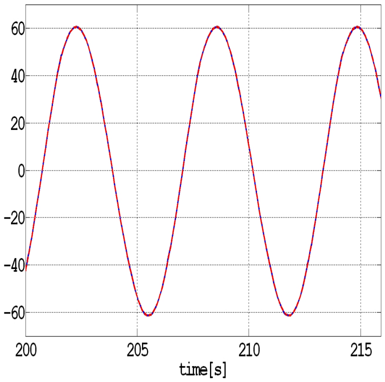

Fig. 9. Time history of x_{2m} and x_2 under the new control strategy: reference model (red) and controlled signal (blue).

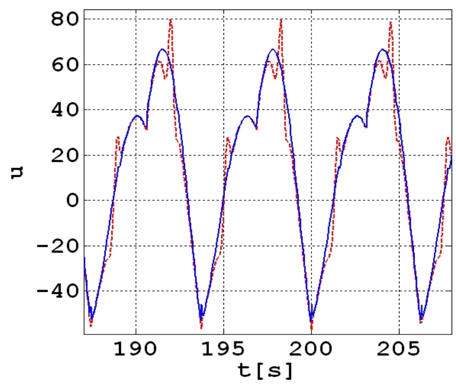

Fig. 10. Comparison between the complete new control action (blue solid line) and the control taking in account only the equation (29) and (30) (red dashed line).

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